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Alpha decay measured in single-particle units as a manifestation of nuclear collectivity

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ABSTRACT

A salient feature of quantum mechanics is the inherent property of collective quantum motion, when apparent independent quasiparticles move in highly correlated trajectories, resulting in strongly enhanced transition probabilities. To assess the extend of a collective quantity requires an appropriate definition of the uncorrelated average motion, often expressed by single particle units. A well known example in nuclear physics is the Weisskopf unit for electromagnetic transitions which reveals different aspects of collective motion. In this paper we define the corresponding single particle unit for alpha decay as induced by four uncorrelated/non-interacting protons and neutrons. Our definition facilitates an unified description of all alpha decay processes along the nuclear chart, revealing a simple mass dependence. The comparison of the uncorrelated decay rates with the experimentally observed ones, shows a significant enhancement of the decay rates pointing towards collective alpha like correlations in the nuclear ground state. As a limiting case, the formalism presented here is applied to proton decay revealing its single particle nature.

Alpha decay has been one of the most rewarding subjects in physics since Gamow was the first to apply the probabilistic interpretation of quantum mechanics to describe the penetrability of the Coulomb barrier by the α-particle [1]. The subsequent developments upon radioactive particle decay in nuclear physics have been outstanding [2,3]. At present, α-decay is crucial for the identification of unstable nuclei far from stability, particularly super heavy and proton rich nuclei [4]. Yet there are unsolved fundamental problems even today: One of these is whether the nuclear configuration interaction shell model is able to describe the clustering of the four nucleons which eventually constitute the α-particle from a microscopic point of view.

The understanding and quantification of collective motion in atomic nuclei have a long history. Enhanced decay probabilities in electromagnetic transitions are used to classify different excitation modes such as vibrations and rotations. These classifications of collectivity can be made through a reliable basic quantity, namely the single-particle Weisskopf unit (W.u.) [5]. Such a common reference enables one to differentiate between decays that are non collective and those that involve the coherent motion of many nucleons. Although called “unit”, the W.u. has not an universal value, since it depends upon the mass of the nucleus in question as well as upon the character of the transition ($E\lambda$ or $M\lambda$). Similarly, the collectivity of pairing correlations and its analogue to deformation in terms of symmetry breaking has been discussed to great extent in nuclear physics, see e.g. Refs. [6,7]. For alpha decay, one has found that the pairing interaction is important to describe the alpha clustering at the nuclear surface. Still, the pairing collectivity is far from sufficient to account for the alpha decay width in a microscopic fashion [3]. Indeed, several studies point towards the presence of alpha clustering in atomic nuclei [2,3].

To assess and describe the collectivity of the clustering of two neutron and protons into an alpha particle we define in this letter an unit which is equivalent to the W.u. We call it particle decay unit, p.d.u. The p.d.u. relates the measured probability of α decay to an averaged single configuration in the description of the mother nucleus. Our definition enables the appropriate comparison of all hitherto observed $l=0$ α decay on the same footing, avoiding the multitudes of effective quantities found at present in the literature [8–10]. In addition, the formalism presented in this paper will enable one to quantify the role played by α clustering in heavy nuclei.

Below we present in detail the formalism. We start with the Thomas expression for the α decay width [11],

$$\Gamma_\alpha(R) = \frac{\ln 2\hbar}{T_{1/2}} = \frac{\ln 2\hbar^2 k}{\mu} \frac{R^2|F_\alpha(R)|^2}{|H_l^+(\chi,\rho)|^2} \quad (1)$$

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which often is written as

$$\Gamma_\alpha(R) = \frac{\hbar^2 R}{\mu}|F_\alpha(R)|^2 P_\alpha(R) \tag{2}$$

where $P_\alpha(R) = kR/|H_l^+(\chi, \rho)|^2$ is the penetrability of the already formed α particle through the Coulomb and centrifugal fields starting at the point R, which is the distance between the mass centers of the daughter nucleus and α cluster. In these equations μ is the reduced mass, H_l^+ is the Coulomb-Hankel function describing the two-body system in the outgoing channel. Its arguments are $\rho = \mu\nu R/\hbar$ and $\chi = 2Z_\alpha Z_d e^2/\hbar\nu$ with ν the outgoing velocity of the α-particle. Z_α and Z_d are the charge numbers of the alpha and daughter nucleus, respectively. The function $F_\alpha(R)$ is the α formation amplitude, i.e., the mother wave function describing the motion of the α cluster in the field induced by the daughter nucleus at the point R. It is important to stress the difference between the exact treatment and the effective treatments mostly used in the literature. In Eq. (1) the evaluation of the formation amplitude is assumed to be performed within a microscopic framework [12–14]. At the point R in Eq. (1) the α-particle is already formed and only the Coulomb and centrifugal interactions are relevant.

In our formalism the formation amplitude is determined following the microscopic treatment [3], i.e.,

$$F_\alpha(R) = \int d\hat{R} d\xi_d d\xi_\alpha [\Psi_d(\xi_d)\phi_\alpha(\xi_\alpha) Y_l(\hat{R})]^* \Psi_m, \tag{3}$$

where ξ_d and ξ_α are the internal degrees of freedom determining the dynamics of the daughter nucleus and the α-particle. The wave functions $\Psi_d(\xi_d)$ and $\Psi_m(\xi_d, \xi_\alpha, \hat{R})$ correspond to the daughter and mother nuclei respectively. The intrinsic α-particle wave function has the form of a $n = l = 0$ $(0s)^4$ harmonic oscillator eigenstate in the neutron-neutron relative distances r_{nn}, as well as in the proton-proton distance r_{pp} and in the distance r_{np} between the mass centers of the nn and pp pairs [3],

$$\phi_\alpha(\xi_\alpha) = \sqrt{\frac{1}{8}}(\frac{\nu_\alpha}{\pi})^{9/4} exp[-\nu_\alpha(r_{nn}^2 + r_{pp}^2 + 2r_{pn}^2)/4] S_\alpha \tag{4}$$

where S_α is the α-spinor corresponding to the lowest harmonic oscillator wave function. The total angular momenta are $L = S = 0$. The quantity $\nu_\alpha = 0.574$ fm^{-2} is the α-particle harmonic oscillator parameter [15].

We consider decays involving uncorrelated states of even-even nuclei. We will focus our treatment on ground-state to ground-state transitions, implying that $l = 0$ and $Y_{l=0}(\hat{R}) = 1/\sqrt{4\pi}$. Uncorrelated decay means that the mother nucleus consists of the daughter nucleus times a pure configuration of a pair coupled to zero angular momentum times a similar proton pair, i.e.

$$\Psi_m(\xi_d, \xi_\alpha, \hat{R}) = (\varphi_\nu(\mathbf{r_1})\varphi_\nu(\mathbf{r_2}))_{00}(\varphi_\pi(\mathbf{r_3})\varphi_\pi(\mathbf{r_4}))_{00}\Psi_d(\xi_d) \tag{5}$$

Writing the single-particle wave functions $\varphi_i(\mathbf{r})$ in their radial, angular and spin components, these last two are canceled in the angular and spin integrals in Eq. (3). In order to perform the radial part of this integral it is convenient to write the mother wave function in terms of the relative coordinates $\mathbf{r_{nn}}$, $\mathbf{r_{pp}}$, $\mathbf{r_{pn}}$ and the center of mass coordinate $\mathbf{R}$. Since the Jakobian corresponding to the transformation from absolute to relative coordinates in the integral (3) is unity one can write

$$\Psi_m(\xi_d, \xi_\alpha, \hat{R}) = \phi(\mathbf{r_{nn}})\phi(\mathbf{r_{pp}})\phi(\mathbf{r_{pn}})\phi(\hat{R})\Psi_d(\xi_d) \tag{6}$$

where ϕ are the wave functions in relative coordinates. These functions may diverge at $r = 0$ and therefore we use the standard function $u(r) = r\phi(r)$. Following the method employed by Weisskopf, we assume that the radial single-particle wave function $u(r)$ in Eq. (5) is constant inside the mother nucleus, with radius R. As a result, the relative and center of mass radial wave functions inside the mother nucleus are constants. Notice that according to our prescription the nn, pp and pn wave functions vanish outside the nuclear surface, while $\phi(R)$, the wave function corresponding to the motion of the α particle center of mass, is constant inside the nucleus, but outside corresponds to an outgoing α particle, as seen below.

The normalization condition provides

$$\int_0^R (u(r)/r)^2 r^2 dr = RC^2 = 1 \tag{7}$$

where the constant C is the same for the pp, nn, pn and the center of mass wave functions inside the mother nucleus resulting in $C = 1/\sqrt{R}$.

The formation amplitude in Eq. (3) acquires the form,

$$\begin{aligned} F_\alpha(R) = &\int d\hat{R} \int r_{nn}^2 dr_{nn} r_{pp}^2 dr_{pp} r_{pn}^2 dr_{pn} \sqrt{\frac{1}{8}}(\frac{\nu_\alpha}{\pi})^{9/4} \\ &\times e^{-\nu_\alpha(r_{nn}^2 + r_{pp}^2 + 2r_{pn}^2)/4} \frac{1}{\sqrt{4\pi}} \frac{C^4}{r_{nn} r_{pp} r_{pn} R} \\ = &\int r_{nn} dr_{nn} r_{pp} dr_{pp} r_{pn} dr_{pn} \sqrt{\frac{1}{8}}(\frac{\nu_\alpha}{\pi})^{9/4} \\ &\times e^{-\nu_\alpha(r_{nn}^2 + r_{pp}^2 + 2r_{pn}^2)/4} \frac{\sqrt{4\pi}}{R^3} \end{aligned} \tag{8}$$

It is straightforward to perform the radial integrals. Thus for r_{nn} one obtains,

$$\int r_{nn} dr_{nn} exp[-\nu_\alpha r_{nn}^2/4] = \frac{2}{\nu_\alpha}. \tag{9}$$

The remaining integrals can be calculated in the same fashion. We are interested in the formation amplitude at the radius R and therefore integrate over the angle $\hat{R}$ (which provides a factor 4π). The formation amplitude at the nuclear surface becomes,

$$F_{\alpha;\mathrm{pdu}}(R) = \sqrt{\frac{1}{8}}(\frac{\nu_\alpha}{\pi})^{9/4}\sqrt{4\pi}\frac{C^4}{R}\frac{4}{\nu_\alpha^3} = \frac{\sqrt{8}\nu_\alpha^{-3/4}\pi^{-7/4}}{R^3} \tag{10}$$

which defines the particle decay unit (p.d.u.). It measures the α decay formation amplitude for decays from four uncorrelated single particle states. With $R = 1.2(A^{1/3} + 4^{1/3})$ fm one obtains

$$F_{\alpha;\mathrm{pdu}} = 0.335/(A^{1/3} + 4^{1/3})^3 \text{ fm}^{-3/2}. \tag{11}$$

In order to clarify the procedure that we are following here it is worthwhile to point out that the neutrons and protons form the α particle at the nuclear surface due to the clusterization induced by the pairing interaction. As Weisskopf did, we assume that inside the nuclear surface the α particle wave function has the constant value $u(\mathbf{r}) = C$. Outside the nuclear surface, i.e. at $r > R$ (where only the Coulomb and centrifugal interactions are relevant), the wave function of the outgoing α particle becomes

$$u(r) = r\phi(r) = N[H_l^+(\chi, \rho)] \tag{12}$$

where N is the matching constant. The independence of the Thomas expression upon the distance R (as pointed out above, R should be beyond the nuclear surface) has often been used in microscopic calculations of α decay to probe whether the results are reliable [16].

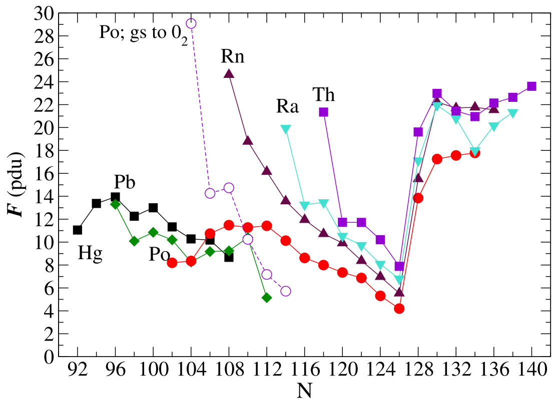

Fig. 1. α-particle formation probabilities in p.d.u. for the decays of the even-even isotopes as a function of the neutron numbers N of the mother nuclei.

Following Eq. (11), we extract the α decay formation amplitude measured in p.d.u. from the ratio between experiment and the corresponding p.d.u. value. For that we firstly extract the absolute value of the α decay formation amplitude from experiential decay half-life as

$$|F_\alpha^{Expt.}(R)| = \frac{(\ln 2)^{1/2}}{R\nu^{1/2}} \frac{|H_l^+(\chi,\rho)|}{(T_{1/2}^{Expt.})^{1/2}}. \tag{13}$$

Above "experimental" α decay formation amplitude is then expressed in p.d.u. as [17]

$$F_\alpha(pdu) = \frac{|F_c^{Expt.}(R)|}{F_{\alpha;\mathrm{pdu}}}. \tag{14}$$

Similar to the W.u. in electromagnetic decay, values that exceed the p.d.u. by an order of magnitude reflect an enhancement of α decay, pointing towards the presence of alpha clustering as a fundamental collective mode.

The value of the α formation amplitudes in p.d.u. of known α emitters in the mass $A = 180$ region and beyond are depicted in Fig. 1. The experimental half-lives are taken from Ref. [18] and references therein. The figure reveals distinctive features characterizing α decay. Thus and most conspicuous, the decay rates all exceed by far the value of a single particle unit. This feature indicates the presence of α clustering due to the correlated motion of neutrons and protons in the nuclear field generated by the daughter nucleus. This is attested by the need of adding cluster components in the shell model wave function in order to account for the experimental decay width [2,19]. Other important feature revealed by the figure is the shell closure at $N = 126$, reducing the probability for α clustering. For heavier isotopes, i.e. above the magic number 126, the p.d.u. approach a constant value, somewhat above 20 p.d.u. When $N > 126$, and $Z > 82$, neutrons and protons move above the magic shell gaps in similar orbits, contributing coherently to the pairing mode, thus enhancing the nucleon-nucleon clustering.

Below the magic number $N = 126$ the ground states of Po, Hg and Pb are determined by neutron hole excitations. Therefore continuum configurations, lying high in the spectrum, do not contribute appreciably to the clusterization process. As a result the collectivity of α clustering is reduced. This reduction of clustering, explains the reduced p.d.u. value, of about 10 units, seen in the Figure. Below $N = 126$ one can recognize two decay branches corresponding to the Po isotopes. The ground state to ground state decays of neutron deficient Po isotopes are strongly hindered due to the different deformations in the mother and daughter nuclei and the reduced overlap between their wave functions, see Ref. [20,21].

The mid-shell nuclei with $Z > 84$ show significant increase in p.d.u. Apparently the onset of deformation in those nuclei results

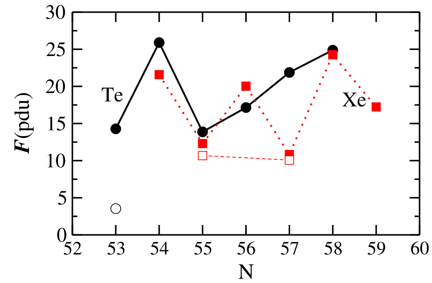

Fig. 2. α-decay formation amplitude in p.d.u. as a function of N for neutron-deficient Te (circle) and Xe (square) above ^{100}Sn. Open symbols correspond to the decays of α particles carrying orbital angular momentum $l = 2$. The experimental data are extracted from Ref. [25–28].

in an enhanced collectivity corresponding to α clustering. In particular, the α decay of the deformed ground state of ^{188}Po to the deformed 0_2^+ state in ^{184}Pb shows the largest value in p.d.u. and hence the largest collective α clustering.

We have evaluated the α decay formation amplitude for the magic nucleus ^{208}Pb, which is stable due to low Q value, following the microscopic treatment as described in Ref. [16]. What is striking is that the calculated α decay formation amplitude is nearly unitary in p.d.u. This result is quite reasonable since one expects minimal collectivity in the nucleus ^{208}Pb. It further validates the approximation we applied in deriving Eq. (10).

Fig. 1 reveals similar trends as the formation amplitude depicted in Ref. [3]. Indeed, the formation amplitude is calculated in a similar fashion as the p.d.u. is derived. The profound difference is that the p.d.u. enables an unified description, that is not dependent on model wave functions used to calculate the formation amplitude. In particular, the p.d.u. elucidates the physical process of alpha clustering, resulting in a global formula.

The collectivity manifested in alpha decay, goes beyond standard pairing collectivity. In addition to the correlated motion of protons and neutrons in time reversed orbits, one deals with the one induced by neutrons and protons moving coherently as constituents of the alpha particle itself. To account for those correlations, alpha-cluster components need to be present in the wave function. Alternatively, presentations where these correlations can be treated explicitly.

In Fig. 2 we show the formation amplitudes of nuclei above ^{100}Sn in p.d.u. The alpha decay properties of those nuclei have attracted much attention in recent studies because an expected superallowed α decay process here. This expectation is due to the enhanced neutron-proton interaction in nuclei close to the $N = Z$ line and hence an enhanced clustering [23,24,26,29]. The definition of p.d.u. enables now a direct comparison between these very different mass regions and to assess different aspects of collectivity on the same footing. Fig. 2 reveals that the formation amplitude of those nuclei follows the average general trend of the α available experimental data. Still, rather large fluctuations and uncertainties are attached to these values. Further experimental investigations are essential to clarify whether this mass region indeed experienced enhanced clustering effects.

The systematics of formation probabilities in available α decay data shows an increasing trend with decreasing mass number [3]. As our formula for p.d.u. shows, the formation of α particles scales with the nuclear volume, $1/A$. This important feature revealed by our results, needs to be taken into account in studies of α decays of trans-tin nuclei, in particular when comparing to heavy nuclei like e.g. ^{212}Po.

One can employ the α decay formation amplitude in Eq. (10) to go one step further and evaluate even the α decay width in p.d.u. This can be easily evaluated by using Eq. (2). Thus, the $l = 0$ decay width in p.d.u. is,

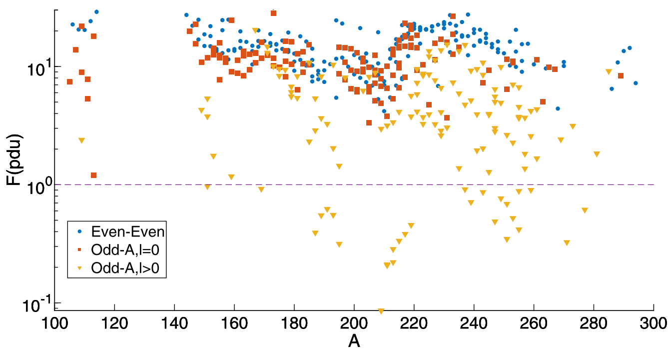

Fig. 3. Systematics of α-decay formation amplitudes in p.d.u. in odd-A nuclei as a function of A with $l=0$ (square) and $l>0$ (triangle) in comparison of those for $l=0$ even-even nuclei (circle). The experimental data are extracted from Ref. [27,28].

$$\Gamma_{\alpha;\text{pdu}}(R) = \frac{\hbar^2 k}{\mu} \frac{R^2 F^2_{\alpha;\text{pdu}}(R)}{|H_0^+(\chi,\rho)|^2} \approx \frac{\hbar^2 k}{\mu} \frac{R^2 F^2_{\alpha;\text{pdu}}(R)}{e^{2[\pi/2-2(\rho/\chi)^{1/2}+1/3(\rho/\chi)^{3/2}]\cdot\cot\beta}} \quad (15)$$

where $\cos^2\beta = \rho/\chi$. It has to be noticed that this width depends upon the decay Q-value, analogous to the energy dependence of the EM transitions. For details of the approximate form of the Coulomb function used in this equation, see Ref. [32].

The l-dependence of the alpha decay is not considered in our definition of p.d.u. That introduces an explicit l-dependence in the penetrability through the Coulomb and centrifugal barriers as well as in the formation amplitude (see, the Y_l term in Eq. (3)). The $l \neq 0$ alpha formation amplitude is difficult to evaluate without explicitly considering the angular momentum coupling of the valence particle wave functions entering Eq. (3) which, however, mostly reflect nuclear structure features. Therefore one can still expect that the p.d.u. can provide an useful measure for the collectivity in those $l \neq 0$ cases. We have found 8 cases of alpha decay from excited states of even-even nuclei with $l \neq 0$. In most of those cases the α formation measured in p.d.u. is significantly smaller than the $l=0$ cases. A typical example is the non-collective 18^+ state in ^{212}Po which has a p.d.u. value 12 orders of magnitude smaller than that of the ground state. A similar reduction is expected for the decays from non-collective high-spin isomeric states [22]. Many more $l \neq 0$ cases can be found in α decays from odd-A and odd-odd nuclei. In Fig. 3 we plotted α-decay formation amplitudes in p.d.u. in odd-A nuclei in comparison of those for even-even nuclei. In general, the p.d.u. values for $l=0$ decays from odd nuclei are slightly smaller but comparable with those of neighboring even-even nuclei, where the reduction can be attributed to the reduced pairing collectivity. On the other hand, the p.d.u. values for $l \neq 0$ decays can be significantly smaller. As can be seen from the figure, it falls below one in many cases. The case with the smallest p.d.u. value, and consequently the one with least collectivity, corresponds to the decay from $9/2^-$ state in ^{209}Bi which is a rather pure single-particle state, i.e. with no collectivity. The odd-odd nuclei show a similar trend.

It is still difficult to extend our alpha-decay derivation to heavier clusters due to the increasing complexity of the internal cluster constituents. On the other hand, one may test our approach for the limiting case of proton decay. Since the proton is already a constituent in the nucleus, the formation amplitude is just the proton wave function. For details, see Ref. [3]. With the same assumption as above the uncorrelated proton decay formation has the simple form,

$$F_{p;\text{pdu}}(R) = \frac{1}{R^{3/2}}. \quad (16)$$

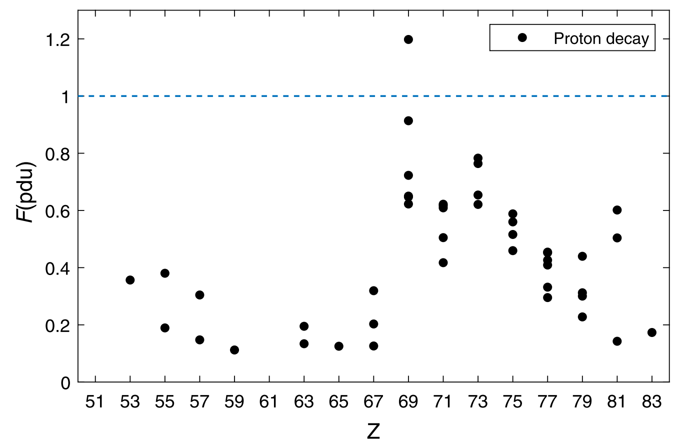

Fig. 4. Proton decay formation amplitude in p.d.u. extracted from known data [30,31] on decays from ground states and low-lying isomeric states.

Using this value, in Fig. 4 we show the proton formation amplitude in p.d.u. As expected, the p.d.u. values are smaller than unity. This is because a given p.d.u. value indicates a partial occupation of the state corresponding to a spectroscopic factor, upon which the emitted proton is moving in the daughter nucleus before decaying. Most decays in the figure show p.d.u. values between 0.1 and 0.8. The largest values correspond to the decays from the odd-odd nuclei ^{144,146}Tm, with values of 0.9 and 1.2, respectively. This enhancement is suggested to arise from the coupling of the decaying proton with the odd neutron [33].

In conclusion, we presented in this paper the single-particle limit of the α formation amplitude, which we call particle decay unit (p.d.u.). We also presented the value of the corresponding alpha-decay width in p.d.u. This unit enables an unified description of alpha-decay in nuclei. Thus a large value of the alpha formation amplitude in p.d.u. indicates that a collective mechanism is involved. The decay pattern reveals clearly that a truly microscopic description requires the explicit presence of α clustering elements in the nuclear wave function. An important feature revealed by our formalism is that the α formation amplitude in p.d.u. scales with the nuclear volume. Competing decay mechanisms within the same mother nucleus can be understood as changes of α clustering at the surface. One may expect a similar effect as induced by the competition between pairing and deformation in two-nucleon transfer reactions (see, e.g., Ref. [34]). As a limiting case and test of our approach, we apply the model to proton decay showing that this decay is uncorrelated. The definitions presented in this paper can be useful for quantifying the role played by α clustering in heavy nuclei, which may be expected to exhibit a strong correlation to the slope of the nuclear symmetry energy and the underlying nuclear equation of state [35]. The present definition aims to greatly enhance the understanding of α correlations in nuclei.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.physletb.2021.136373.

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