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Two- and three-meson decay modes of the τ -lepton in the Monte Carlo generator TAUOLA

Olga Shekhovtsova

Radzikowskiego str. 152, Cracow 31-342, Poland
Akademicheskaya str. 1, Kharkov 61202, Ukraine

E-mail: shekhovtsova@kipt.kharkov.ua

Abstract. The current status of the two- and three-meson decay study of τ -lepton using the Monte Carlo generator TAUOLA is overviewed.

1. Introduction

The precise experimental data for tau lepton decays collected at B-factories (both Belle [1] and BaBar [2]) provide an opportunity to measure the Standard Model (SM) parameters, such as the strong coupling constant, the quark-mixing matrix, the strange quark mass etc, and for searching new physics, beyond the SM. Leptonic decay modes of the τ -lepton allow to test the universality of the lepton couplings to the gauge bosons [3].

Hadronic decays, in addition to the low energy e^+e^- data, give an information about the hadronization mechanism and resonance dynamics in the energy region of 1-2 GeV where the methods of the perturbative QCD cannot be applied. Unlike the e^+e^- annihilation into hadrons, the τ -lepton decays weakly and thus provides access to both vector and axial-vector currents. In addition, systematical comparison between τ -lepton and e^+e^- data for corresponding hadronic channels allows to test the conservation of the vector current hypothesis [4]. The τ -lepton data are also applied to estimate the hadronic vacuum polarization contribution to the anomalous magnetic momentum of the muon [5]. In addition hadronic decays of the τ -lepton are a tool in high-energy physics to study Higgs boson features [6]. Also hadronic flavour-violating and CP violating decays of τ lepton allow to search for new physics scenario [7].

Being the main hadronic decay modes, the two- and three-meson decays of the τ -lepton play a key role in the studies mentioned above. The current status of the part of the Monte Carlo generator TAUOLA related to the two- and three-meson modes is discussed in this note. The article is organized as follows: Section 2 is devoted to a general presentation of the two- and three-meson currents while Sections 3 and 4 describe the numerical test and fit of the dominant modes to experimental data. Section 5 summarizes this note.

2. Current status of the MC TAUOLA for two- and three-meson states

Since the 90's TAUOLA [8] has been the main MC generator to simulate τ decays. It has been used to simulate decays of the τ lepton by the experimental collaborations CLEO, ALEPH, and at both B-factories BaBar and Belle. The simulation of hadronic decay modes in the original



version of TAUOLA was done on the basis of the theoretical results within the Vector Meson Dominance (VMD) approximation. In this approach a hadronic current is modeled by a weighted product of Breit-Wigner functions. This approach was contested in [9]: the investigation of the low energy behaviour of the $\tau \rightarrow \pi\pi\pi\nu_\tau$ amplitude demonstrates that the VMD amplitude reproduces only the leading order χ PT result but contradicts the next-to-leading-order χ PT prediction [10]. Moreover, the model was not sufficient to describe the CLEO $KK\pi$ data [11]. To reproduce the data the CLEO collaboration reshaped the model by introducing two ad-hoc parameters that spoil the QCD normalization for Wess-Zumino contribution. The parameters are obtained by fitting to data and demonstrated more than 60% deviation from the predicted Wess-Zumino behaviour. However, before making conclusion that the Wess-Zumino anomaly normalization is spoiled it should be checked whether an oversimplified theoretical approximation, like VMD, was applied.

The alternative approach based on the Resonance Chiral Lagrangian ($R\chi L$) [12] was proposed in [13, 14]. The computations done within $R\chi L$ are able to reproduce the low-energy limit of χ PT at least up to next-leading-order and demonstrate the right falloff in the high energy region [13, 14]. The hadronic currents for the main two-meson ($\pi\pi$, $K\pi$, KK) [15, 16, 17] and three-meson ($\pi\pi\pi$ and $KK\pi$) [13, 14] decay modes were calculated in the framework of $R\chi L$ and were implemented in TAUOLA [18]. The set covers more than 88% of the total hadronic τ width.

2.1. Two-meson decay modes in TAUOLA

In the general case, an hadronic current of a two-meson tau-lepton decay mode is described by the two Lorentz structures and corresponding them two hadronic form factors. For a decay $\tau \rightarrow h_1(p_1) + h_2(p_2) + \nu_\tau$, the hadronic current reads

$$J_{P_1 P_2}^\mu = N(P_1 P_2)[c_1(p_1 - p_2)^\mu F^+(s) + (p_1 + p_2)^\mu F^-(s)], \quad (1)$$

where $s = (p_1 + p_2)^2$, $\Delta_{12} = m_1^2 - m_2^2$. The normalization factor $N(P_1 P_2)$ depends on the decay channel.

The function F^+ is the vector form factor ($F^V \equiv F^+$) and F^- is related to the scalar form factor F^S : $F^-(s) = (F^S(s) - F^+(s)) \cdot \Delta_{12}/s$. Therefore the two-meson hadronic current is re-written as

$$J_{P_1 P_2}^\mu = N(P_1 P_2)[(p_1 - p_2 - \frac{\Delta_{12}}{s}(p_1 + p_2))^\mu F^V(s) + \frac{\Delta_{12}}{s}(p_1 + p_2)^\mu F^S(s)]. \quad (2)$$

The form factors F^V and F^S are associated to the $L = 1$ and $L = 0$ waves of the final hadronic system. Fixing the $N(\pi^-\pi^0) = 1$, the other modes normalization form factors are

$$N(K^-K^0) = \frac{1}{\sqrt{2}}, \quad N(\pi^-K^0) = \frac{1}{\sqrt{2}}, \quad N(\pi^0K^-) = \frac{1}{2}. \quad (3)$$

The expression (2) together with normalizations factors (3) are applied in TAUOLA [18].

The exact appearance of both form factors are specified by the theoretical model describing hadronization. In the isospin symmetry limit, $m_{\pi^-} = m_{\pi^0}$ and $m_{K^-} = m_{K^0}$, the scalar form factor F^S vanishes for the two-pion and two-kaon decay modes and the current is described by the vector form factor F^V only. In the original version of TAUOLA [8] the scalar form factor was not included, as the generator was mainly oriented on the two-pion mode simulation. However, in the case of the $K\pi$ channels the scalar form factor [19] gives an essential contribution and has to be taken into account. For this reason it was added in the simulation in the updated version of the code [18].

Let us move to the theoretical parametrizations for the two-meson form factors applied to TAUOLA and we start with the dominant two-pion mode ($Br(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) \simeq 25.52\%$). The scalar two-pion form factor was put to zero and the four theoretical results described below were implemented for the vector two-pion form factor in TAUOLA.

The original version of TAUOLA [8] for the two-pion decay contained only

- Kühn-Santamaría (KS) parametrization [20], *i.e.*

$$F_\pi^V(s) = \frac{1}{1 + \beta + \gamma} (BW_\rho(s) + \beta BW_{\rho'}(s) + \gamma BW_{\rho''}(s)) \quad (4)$$

$$BW(s) = \frac{M^2}{M^2 - s - i\sqrt{s}\Gamma_{\pi\pi}(s)},$$

thus the pion form factor was written as a weighted sum of the P -wave Breit -Wigner (BW) functions of the $\rho(770)$, $\rho(1450)$, $\rho(1700)$ vector resonances with their relative strengths 1, α and β . The denominator of BW functions includes only an imaginary part of the two-pion loop contribution through the energy dependent width ($\Gamma_{\pi\pi}(s) = \Gamma_\rho \cdot s/m_\rho^2 \cdot \sigma_\pi^3(s)/\sigma_\pi^3(m_\rho^2)$ with $\sigma_\pi(s) = \sqrt{1 - 4m_\pi^2/s}$) of the resonances.

To follow state of the art of theoretical calculations for the pion form factor and experimental data analysis on the τ decay into two pions, three other parametrizations of the pion vector form factor were added.

- Gounaris-Sakurai parametrization [21]. It was applied to data analysis by the Belle, ALEPH and CLEO Collaborations.

$$F_\pi^V(s) = \frac{1}{1 + \beta + \gamma} (BW_\rho^{GS}(s) + \beta BW_{\rho'}^{GS}(s) + \gamma BW_{\rho''}^{GS}(s)), \quad (5)$$

$$BW^{GS}(s) = \frac{M^2 + dM\Gamma_{\pi\pi}(s)}{M^2 - s + f(s) - i\sqrt{s}\Gamma_{\pi\pi}(s)},$$

For the precise formalism for the function $f(s)$ and parameter d I refer the reader to [22]. Also this parametrization is a weighted sum of BW contributions related to the ρ mesons. As in the case of KS only the two-pion loop contribution is taken into account in the energy dependent width of the ρ -meson states, however, both real (through $f(s)$) and imaginary ($\Gamma_{\pi\pi}$) parts are included. The GS parameters, as the resonance masses, widths and relative strengths, were fixed to the Belle fit values, Table VII in [22] (the case 'Fit results (fixed $|F(0)|^2$)' was chosen).

- Parametrization based on the R χ L approach [15].

$$F_\pi^V(s) = \frac{1 + \sum_{i=\rho,\rho'} \frac{F_{V_i} G_{V_i}}{F^2} \frac{s}{M_i^2 - s}}{1 + \left(1 + \sum_{i=\rho,\rho'} \frac{2G_{V_i}^2}{F^2} \frac{s}{M_i^2 - s} \right) \frac{2s}{F^2} \left[B_{22}^\pi(s) + \frac{1}{2} B_{22}^K(s) \right]}, \quad (6)$$

This parametrization is obtained directly from the Resonance Lagrangian (see for details [15]). Only two states of the vector ρ resonance contribute to the form factor, however their contribution is included as a weighted sum separately in the nominator and the denominator. Both pion and kaon loops are taken into account in this parametrization through the functions $B_{22}^{\pi;K}$. The relation between $Im(B_{22}^\pi(s))$ and $\Gamma_{\pi\pi}(s)$ can be found in Section 2 of [23]. The model parameters F_{V_i} and G_{V_i} as well as the resonance masses were fitted to the ALEPH (only the $\sqrt{s} < 1.2$ GeV points were taken into account) data [24], the one available at that moment. Only the $\sqrt{s} < 1.2$ GeV points were taken into account, and presented in Table I of [15].

- Combined R χ L parametrization [16].

It combines two approaches: dispersion approximation in the low energy region ($s < s_0$) and modified R χ L result at the high tail of the spectrum ($s > s_0$).

The low energy part of the spectrum ($s < s_0$) is presented by the Omnès solution of the 3-subtracted dispersion relation [25] and was calculated in [16]:

$$F_\pi^V(s) = \exp \left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3 (s' - s - i\epsilon)} \right], \quad (7)$$

where the parameters α_1 , α_2 are the subtraction constants and δ_1^1 is the $I = 1$, $J = 1$ two-pseudoscalar scattering phase shift. The expression for δ_1^1 via the simplified version of the form factor is presented in Eq. (16) of [16].

The high energy tail of the spectrum ($s > s_0$) is based on the modified R χ L result and includes contributions from the $\rho(770)$, $\rho(1450)$, $\rho(1700)$:

$$F_\pi^V(s) = \frac{M_\rho^2 + (\beta + \gamma)s}{M_\rho^2 - s + \frac{2s}{F^2} M_\rho^2 \left[B_{22}^\pi(s) + \frac{1}{2} B_{22}^K(s) \right]} - \frac{\beta s}{M_{\rho'}^2 - s + \frac{192\pi s \Gamma_{\rho'}}{M_{\rho'} \sigma_\pi^3(s)} B_{22}^\pi(s)} - \frac{\gamma s}{M_{\rho''}^2 - s + \frac{192\pi s \Gamma_{\rho''}}{M_{\rho''} \sigma_\pi^3(s)} B_{22}^\pi(s)}.$$

Both pion and kaon loops are taken into account. The s_0 parameter is supposed to satisfy $1.0\text{GeV}^2 < s_0 < 1.5\text{GeV}^2$ [16]. In order to get the numerical values of the model parameters in Eqs. (7) and (8) the fit to the Belle data [22] was done, its results are summarized in Table 1 and Eqs. (29), (30) in [16].

The $K^- K^0$ decay channel. Up to the normalization factor the vector form factor for the $\tau^- \rightarrow K^- K^0 \nu_\tau$ coincides with the vector form factor of the two-pion mode. The two parametrizations are currently included in TAUOLA: the KS approach (4) and the R χ L one, Eq. (8). More details and discussion can be found in [23].

For the $(K\pi)^-$ modes both scalar and vector form factors are essential. The vector form factor was included within the two parametrizations [17] and Eqs. (17), (18) of [19]. Both parametrizations are based on the R χ L approach, however, the final state interaction effects are treated in different ways. More details about these parametrizations can be found in Sections 2.4 and 5.4 of Ref. [18] and in the theoretical papers [17, 19]. The scalar $K\pi$ form factor was calculated on the basis of the private code of M. Jamin that corresponds to the numerical solution of a set of coupled dispersion relations specified in [26].

2.2. Three-meson decay modes in TAUOLA

When the τ lepton decays into three hadrons and a neutrino: $\tau \rightarrow h_1(p_1) + h_2(p_2) + h_3(p_3) + \nu_\tau$, Lorentz invariance determines the decomposition of the hadronic current in terms of five Lorentz invariant structures multiplied by hadronic form factors (F_i)

$$J^\mu = N \{ T_\nu^\mu [c_1(p_2 - p_3)^\nu F_1(q^2, s_1, s_2) + c_2(p_3 - p_1)^\nu F_2(q^2, s_1, s_2) + c_3(p_1 - p_2)^\nu F_3(q^2, s_1, s_2)] + c_4 q^\mu F_4(q^2, s_1, s_2) - \frac{i}{4\pi^2 F^2} c_5 \epsilon^\mu{}_{\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F_5(q^2, s_1, s_2) \}, \quad (8)$$

where as usual $T_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$ denotes the transverse projector, $q^\mu = (p_1 + p_2 + p_3)^\mu$ is the total momentum of the hadronic system and the two-pion invariant mass squared is

$s_i = (p_j + p_k)^2$. The normalization factor N is $N = \cos \theta_{Cabibbo}/F$ for modes with an even number of kaons and $N = \sin \theta_{Cabibbo}/F$ for an odd kaon number modes. The TAUOLA value for the c_i coefficients is presented in Table 1 in Ref. [18].

I would like to stress that the decomposition (9) is model independent while the hadronic functions $F_i(q^2, s_1, s_2)$, $i = 1...5$ are model dependent. The vector form factor (F_5) vanishes for the three-pion modes due to the G-parity conservation. Of the three hadronic form factors F_i , $i = 1, 2, 3$ corresponding to the axial-vector part of the hadronic tensor, only two are independent. In strangeness-conserving decays the pseudoscalar form factor F_4 is proportional to m_π^2/q^2 [18], so that is suppressed with respect to F_i , $i = 1, 2, 3$.

As in the case of the two-meson decay modes I start with the parametrizations implemented in TAUOLA for the predominant three-meson modes $\tau \rightarrow \pi\pi\pi\nu_\tau$ ($Br(\tau \rightarrow \pi\pi\pi\nu_\tau) \simeq 18.67\%$).

Currently the following three-pion form factors are available in TAUOLA.

- CPC version [8].

It corresponds to the original version of TAUOLA and includes only the dominant $a_1 \rightarrow \rho\pi$ production mechanism. The form factor is a product of the Breit-Wigner for the a_1 and ρ meson.

- CLEO parametrization [27].

It is based on the Dalitz plot analysis carried out by the CLEO collaboration for $\tau^- \rightarrow \pi^0\pi^0\pi^-\nu_\tau$. The analysis included the following intermediate states: $a_1 \rightarrow (\rho; \rho')\pi$ (where $(\rho; \rho')\pi$ can be either in S- or in D-wave), $a_1 \rightarrow \sigma\pi$, $a_1 \rightarrow f_2(1270)\pi$, $a_1 \rightarrow f_0(1370)\pi$. As expected the $\rho\pi$ S-wave amplitude with a branching fraction 70% was dominant, the next one was the σ -meson part (about 16%) and the Dalitz plot analysis demonstrated that it could not be neglected. It is interesting to mention also that the $\rho' \equiv \rho(1450)$ state showed up more strongly in the D-wave amplitude than in the S-wave amplitude. All resonances were modelled by Breit-Wigner functions and the hadronic current was written as a weighted sum of their product.

- Modified R χ L parametrization [30].

Initially the R χ L three-pion current based on [14] was implemented in the TAUOLA code, see Section II.A of [18]. The R χ L three-pion current was written as a sum of the chiral contribution corresponding to the direct vertex ($W^- \rightarrow (\pi\pi\pi)^-$), single-resonance contributions (*e.g.* $W^- \rightarrow (\rho\pi)^-$) and double-resonance contributions (*e.g.* $W^- \rightarrow a_1^- \rightarrow (\rho\pi)^-$). Only contributions from the intermediate vector and axial-vector were included in the original R χ L hadronic currents. More details about the parametrization can be found in Section II.A of [18].

However, the comparison of the R χ L predictions with the BaBar spectra (see Section 4), demonstrated a disagreement in the low energy tail of the two-pion spectra, both $\pi^+\pi^-$ and $\pi^-\pi^-$, Fig.1 in [29], and required a modification of the current. We ascribed this disagreement to the scalar resonance absence in the theoretical studies in [14] and to solve the problem an $f_0(500) = \sigma$ scalar resonance contribution was added. I would like to stress that the σ -meson contribution was included phenomenologically, *i.e.* it was not based on any Lagrangian. Our unique restriction was in requiring the same structure for the current as it was for the vector resonance contribution. In other words the hadronic currents had to contain a single-resonance part ($W^- \rightarrow \sigma\pi^-$) and a double-resonance part ($W^- \rightarrow a_1^- \rightarrow \sigma\pi^-$). In addition, the σ -resonance was modelled by a BW function. Hereinafter this approach will be refereed to as 'modified R χ L'.

Also effects related with the Coulomb interaction of the final-state pions was included in TAUOLA. The far-field approximation was applied to estimate the Coulomb interaction effects, *i.e.* the final-state pions were treated as stable point-like objects and the three pion interaction was treated as a superposition of the two pion ones.

Channel	Width, [GeV]		
	PDG	Isospin-averaged masses	Phase space with physical masses
$\pi^- \pi^0$	$(5.778 \pm 0.35\%) \cdot 10^{-13}$	$(5.2283 \pm 0.005\%) \cdot 10^{-13}$	$(5.2441 \pm 0.005\%) \cdot 10^{-13}$
$\pi^0 K^-$	$(9.72 \pm 3.5\%) \cdot 10^{-15}$	$(8.3981 \pm 0.005\%) \cdot 10^{-15}$	$(8.5810 \pm 0.005\%) \cdot 10^{-15}$
$\pi^- \bar{K}^0$	$(1.9 \pm 5\%) \cdot 10^{-14}$	$(1.6798 \pm 0.006\%) \cdot 10^{-14}$	$(1.6512 \pm 0.006\%) \cdot 10^{-14}$
$K^- K^0$	$(3.60 \pm 10\%) \cdot 10^{-15}$	$(2.0864 \pm 0.007\%) \cdot 10^{-15}$	$(2.0864 \pm 0.007\%) \cdot 10^{-15}$
$\pi^- \pi^- \pi^+$	$(2.11 \pm 0.8\%) \cdot 10^{-13}$	$(2.1013 \pm 0.016\%) \cdot 10^{-13}$	$(2.0800 \pm 0.017\%) \cdot 10^{-13}$
$\pi^0 \pi^0 \pi^-$	$(2.10 \pm 1.2\%) \cdot 10^{-13}$	$(2.1013 \pm 0.016\%) \cdot 10^{-13}$	$(2.1256 \pm 0.017\%) \cdot 10^{-13}$
$K^- \pi^- K^+$	$(3.17 \pm 4\%) \cdot 10^{-15}$	$(3.7379 \pm 0.024\%) \cdot 10^{-15}$	$(3.8460 \pm 0.024\%) \cdot 10^{-15}$
$K^0 \pi^- \bar{K}^0$	$(3.9 \pm 24\%) \cdot 10^{-15}$	$(3.7385 \pm 0.024\%) \cdot 10^{-15}$	$(3.5917 \pm 0.024\%) \cdot 10^{-15}$
$K^- \pi^0 K^0$	$(3.60 \pm 12.6\%) \cdot 10^{-15}$	$(2.7367 \pm 0.025\%) \cdot 10^{-15}$	$(2.7711 \pm 0.024\%) \cdot 10^{-15}$

Table 1. The τ decay partial widths. For each channel, the PDG value [31] is compared with the MC prediction. The third column includes results with isospin averaged masses, whereas for the last column physical masses were used.

The $K\pi K$ decay modes, *i.e.* $K^- \pi^- K^+$, $K^0 \pi^- \bar{K}^0$ and $K^- \pi^0 K^0$ final states. As for the $\pi\pi\pi$ decay mode the MC TAUOLA contains three parametrizations: CPC, based on the CLEO analysis and R χ L one. To see the details of these parametrizations I refer the reader to Refs. [8], [11] and Sections II.B and II.C of [18], respectively.

The Table 1 summarizes the TAUOLA update within the R χ L framework and contains the R χ L predictions for the decay width for every updated channel.

As one can see a difference between Table 1 and PDG values is 5-30% depending on the channel. To improve it a fit to available experimental data was done, see Section 4.

For the rest of three-meson decay modes, $\eta\pi^-\pi^0$ was computed in [28] ($K\pi\pi$ has not been published yet) in the framework of R χ L, only the CPC parametrization [8] is included in TAUOLA.

3. Numerical tests and technical points

Since Ref. [8] has been published, numerical tests of TAUOLA MC functioning have not been repeated in a systematic way, despite the technical precision requirements are much higher now and reach sub-per mil level. Therefore the TAUOLA update required to revisit the numerical stability of the generator and of the multiple integration. The following steps were done:

- check of the phase space integration;
- numerical tests for every R χ L current including a comparison with semi-analytical results.

The details on this study can be found in Section IV of [18] on the sample of the $\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ decay mode and for the other channels they are collected in Ref. [33].

From the technical point of view the R χ L version of the hadronic form-factors are chosen invoking CALL INIRCHL(1) prior to TAUOLA initialization whereas the old version (either CLEO or VMD) corresponds to CALL INIRCHL(0). The numerical values of the model parameters as well as the flags to choose different types of R χ L parametrizations for 3π (simple or modified R χ L, to run with or without $\rho(1450)$, to switch on/off the Coulomb interaction), $K\pi$ (to run with or without the scalar form factor) are fixed in the file: *value_parameters.f*.

4. Fit to experimental data for $\tau \rightarrow \pi^- \pi^0 \nu_\tau$ and $\tau \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ modes.

The predicted values of the partial widths in Table 1 from the previous section corresponded to the 'default' numerical values of the $R\chi L$ parameters specified in Table IV of [18]. However, the model uncertainty varies from 10% to 30% depending on a $R\chi L$ parameter, thus a fit to experimental distributions had to be carried out in order to adjust numerical values of the model parameters. In this Section the fit result to the Belle and BaBar data for the predominant two-meson ($\tau \rightarrow \pi^- \pi^0 \nu_\tau$) and three-meson ($\tau \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$) channels are collected.

In all parametrizations (4)-(7) for the two-pion mode, except for the $R\chi L$ one, the pion form factor is given by interfering amplitudes from the known isovector meson resonances $\rho(770)$, $\rho'(1450)$ and $\rho''(1700)$ with relative strengths 1, β and γ . Although one could expect from the quark model that β and γ are real, the parameters were allowed to be complex (following the Belle, CLEO and ALEPH analysis) and their phases were left free in the fits.

The fit results to the Belle pion form factor [22] are presented in Figs. 1. The best fit occurs with the GS pion form factor ($\chi^2 = 95.65$), as could be expected, since this parametrization corresponds to the one applied for the Belle analysis, and the worst one is with the $R\chi L$ one ($\chi^2 = 156.93$), which is not able to reproduce the high energy tail.

The two main differences of the $R\chi L$ parametrization compared to the others are: 1) the ρ'' -meson absence, 2) a real value of the $\rho'(1450)$ -meson strength. To check the influence of the $\rho''(1700)$ on the $R\chi L$ result, the $\rho''(1700)$ was added in the same way as it was done for $\rho'(1450)$. However, this inclusion did not improved the result (see Fig. 2 of [23]). That brings to a conclusion: the fault in reproducing the high energy tail is related to the real value of the $F_{V_i} G_{V_i} / F^2$ parameters which is associated with the β and γ parameters in the other parametrizations. In an effective field theory, like $R\chi L$, complex values come only from loops, *e.g.* missing loop contributions in the ρ' and ρ'' propagators, most probably related for the four-pion loops, could be responsible for the disagreement and the complex value of the β and γ parameters might mimic missing multi-particle loop contributions. The same conclusion was reached in Ref. [15] where it was stressed that the two-pseudoscalar loops could not incorporate all the inelasticity needed to describe the data and other multiparticle intermediate states could play a role.

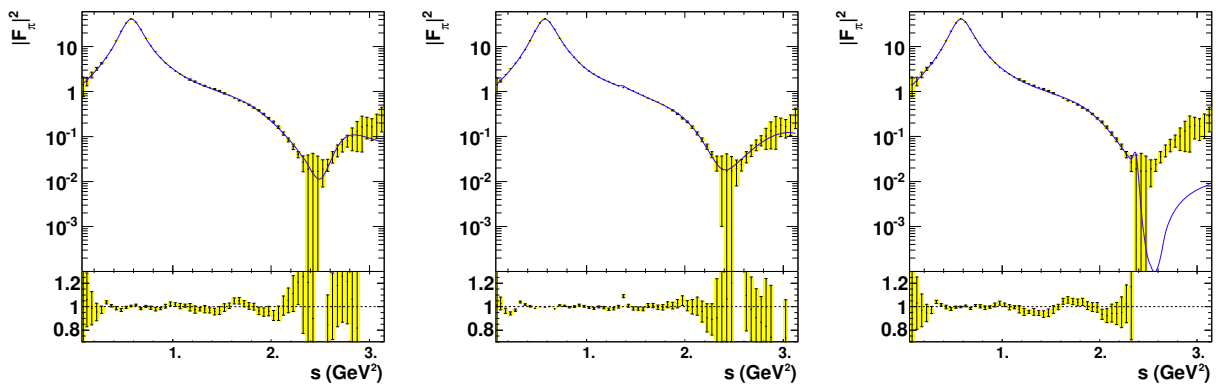


Figure 1. The pion form factor fit to Belle data [22]: the GS parametrization (left panel), the combined $R\chi L$ parametrization (central panel), the $R\chi L$ parametrization with $\rho''(1700)$ resonance (right panel). At the bottom: the ratio of a theoretical prediction to the data is given.

For the $\tau \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ decay the one-dimensional distributions of the two- and three-pion invariant mass spectra calculated on the base of the modified $R\chi L$ parametrization have been fitted in [30] to the BaBar preliminary data [32]. The first comparison of the $R\chi L$

	M_ρ	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	M_σ	Γ_σ	F	F_V
Min	0.767	1.35	0.30	0.99	0.400	0.400	0.088	0.11
Max	0.780	1.50	0.50	1.25	0.550	0.700	0.094	0.25
Fit	0.771849	1.350000	0.448379	1.091865	0.487512	0.700000	0.091337	0.168652
	F_A	$\beta_{\rho'}$	α_σ	β_σ	γ_σ	δ_σ	R_σ	
Min	0.1	-0.37	-10.	-10.	-10.	-10.	-10.	
Max	0.2	-0.17	10.	10.	10.	10.	10.	
Fit	0.131425	-0.318551	-8.795938	9.763701	1.264263	0.656762	1.866913	

Table 2. Numerical ranges of the $R\chi L$ parameters used to fit the BaBar data for three pion mode [32]. The approximate uncertainty estimates are collected in Table III of Ref. [30]. The $M_{\rho'}$ and Γ_σ best fit values are observed to be at the boundary of the physically motivated range of variation that we allowed for them.

prediction, specified in [18], showed a disagreement in the low energy tail of both $\pi^-\pi^-$ and $\pi^+\pi^-$ spectra [30], whereas the modified $R\chi L$ approach was in line with the BaBar data in the whole energy spectra, Fig. 2.

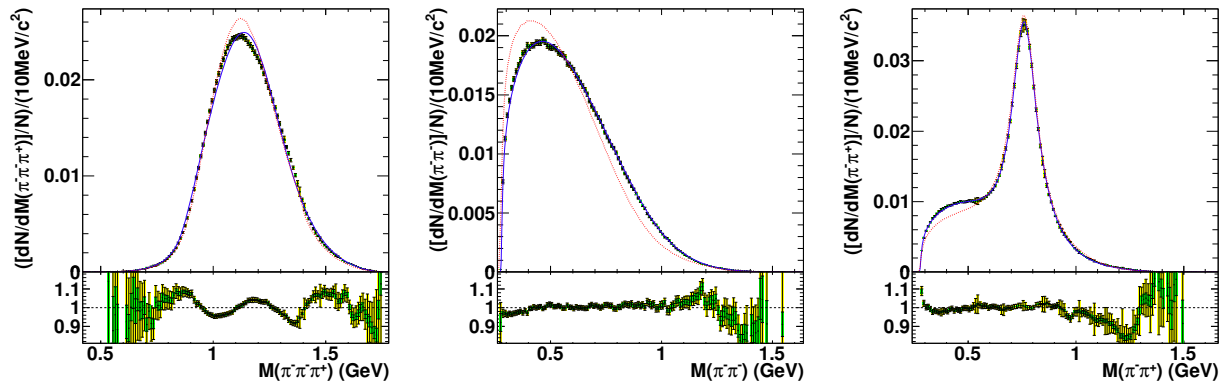


Figure 2. The $\tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$ decay invariant mass distribution of the three-pion system (left panel) and the two-pion system (central and right panels). The BaBar data [32] are represented by the data points, with the results from the modified $R\chi L$ current as described in the text (blue line) and the old fit from Cleo [27] (red-dashed line) overlaid.

The study of the Coulomb interaction of the final pions, which might be important near the production threshold and might mimic the missing effects responsible for the disagreement between the $R\chi L$ prediction [14] and the BaBar data, demonstrated that the Coulomb interaction alone without the σ meson could not describe the low energy tail of the experimental spectra.

Based on the fitting of the modified $R\chi L$ spectra to the preliminary BaBar data the numerical values of the $R\chi L$ parameters were adjusted, Table 2.

For the discussion of the systematic and statistical uncertainties in the fit, technical details and the computing strategy of the fitting procedure I refer to [30].

Finally, using the obtained numerical values of the model parameters, the $\pi^0\pi^0\pi^-$ partial width was estimated: $\Gamma(\tau^- \rightarrow \pi^0\pi^0\pi^-\nu_\tau) = (2.1211 \pm 0.0016\%)$ that is only 1% higher than the central PDG value [31] and is within the errors cited by PDG.

5. Conclusion

A set of the theoretical parametrizations for the two- and three-meson decay modes of the τ -lepton installed in the MC event generator was reviewed in this note.

Also it was discussed the results of fits for the two- and three-pion decay modes to the Belle and preliminary BaBar data. In the case of the two-pion decay modes the three parametrizations were chosen for the pion form factor: a) the Gounaris-Sakurai model, b) the $R\chi L$ approach, c) a framework combining the resonance approach together with the dispersion approximation. All three models reproduced well the energy spectrum up to 1.05 – 1.15 GeV while the theoretical result on the basis of the $R\chi T$ was not able reproduce the experimental spectrum for the higher energy region. For the $\pi^-\pi^-\pi^+$ decay mode the fit to the preliminary BaBar data was done using the modified $R\chi L$ hadronic current. The result can be summarized as following: the theoretical distributions were in line with the BaBar data for both two- and three-pion invariant mass distributions in the whole energy spectra. A small discrepancy between theoretical spectra and experimental data can be explained by missing resonances in the model, such as the axial-vector resonance $a_1'(1600)$, the scalar resonance $f_0(980)$ and the tensor resonance $f_2(1270)$. The first study on the $f_2(1270)$ resonance inclusion within the $R\chi L$ approach in the three-pion decay mode of the τ -lepton was recently presented in [34].

The last point to be mentioned in this note is the second-class current study in the τ -lepton decays. Presently the second-class current decays are not observable yet and their measurement is a key point of the τ physics study at Belle-II. However, according to theoretical predictions (including the $R\chi L$ prediction [35]), the main second-class current decay mode $\tau^- \rightarrow \pi^- \eta \nu_\tau$ should be finally discovered at the forthcoming Belle-II. Therefore, the future upgrade of TAUOLA should include also the second-class current simulation.

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- [1] S. Ryu *et al.* [Belle Collaboration], 2014 *Phys. Rev. D* **89** 072009.
- [2] B. Aubert *et al.* [BaBar Collaboration], 2008 *Phys. Rev. Lett.* **100** 011801.
- [3] B. Aubert *et al.* [BaBar Collaboration], 2010 *Phys. Rev. Lett.* **105** 051602.
- [4] F. Jegerlehner and A. Nyffeler, 2009 *Phys. Rept.* **477** 1.
- [5] M. Davier, A. Höcker, B. Malaescu and Z. Zhang, 2011 *Eur. Phys. J. C* **71** 1515. Erratum: [*Eur. Phys. J. C* **72** (2012) 1874].
- [6] G. Aad *et al.* [ATLAS and CMS Collaborations], 2016 *J. High Energy Phys.* JHEP08(2016)045.
- [7] A. Celis, V. Cirigliano and E. Passemar, 2014 *Phys. Rev. D* **89** 013008.
- [8] S. Jadach, J. H. Kühn and Z. Was, *Comput. Phys. Commun.* **64** (1990) 275. S. Jadach, Z. Was, R. Decker and J. H. Kühn, 1993 *Comput. Phys. Commun.* **76** 361.
- [9] D. Gómez Dumm, A. Pich and J. Portolés, 2004 *Phys. Rev. D* **69** 073002.
- [10] G. Colangelo, M. Finkemeier and R. Urech, *Phys. Rev. D* **54** (1996) 4403.
- [11] T. E. Coan *et al.* [CLEO Collaboration], 2004 *Phys. Rev. Lett.* **92** (2004) 232001.
- [12] G. Ecker, J. Gasser, A. Pich and E. de Rafael, 1989 *Nucl. Phys. B* **321** 311.
- [13] D. G. Dumm, P. Roig, A. Pich and J. Portolés, 2010 *Phys. Rev. D* **81** 034031.
- [14] D. G. Dumm, P. Roig, A. Pich and J. Portolés, 2010 *Phys. Lett. B* **685** 158.
- [15] J. J. Sanz-Cillero and A. Pich, 2003 *Eur. Phys. J. C* **27** 587.
- [16] D. Gómez Dumm and P. Roig, 2013 *Eur. Phys. J. C* **73** 2528.
- [17] M. Jamin, A. Pich and J. Portolés, 2008 *Phys. Lett. B* **664** 78.
- [18] O. Shekhovtsova, T. Przedzinski, P. Roig and Z. Was, 2012 *Phys. Rev. D* **86** 113008.
- [19] D. R. Boito, R. Escribano and M. Jamin, 2009 *Eur. Phys. J. C* **59** 821.
- [20] J. H. Kühn and A. Santamaría, *Z. Phys. C* **48** (1990) 445.
- [21] G. J. Gounaris and J. J. Sakurai, *Phys. Rev. Lett.* **21** (1968) 244.
- [22] M. Fujikawa *et al.* [BELLE Collaboration], 2008 *Phys. Rev. D* **78** 072006.

- [23] O. Shekhovtsova, 2015 *JETP Lett.* **102** 329 [*Pisma Zh. Eksp. Teor. Fiz.* **102** (2015) no.6, 366].
- [24] R. Barate et al. [ALEPH Collaboration], 1997 *Z. Phys. C* **76** 15.
- [25] F. Guerrero and A. Pich, 1997 *Phys. Lett. B* **412** 382.
- [26] M. Jamin, J. A. Oller and A. Pich, 2002 *Nucl. Phys. B* **622** 279.
- [27] D. M. Asner et al. [CLEO Collaboration], 2000 *Phys. Rev. D* **61** 012002.
- [28] D. Gómez Dumm and P. Roig, *Phys. Rev. D* **86** (2012) 076009.
- [29] O. Shekhovtsova, I. M. Nugent, T. Przedzinski, P. Roig and Z. Was, 2014 *Nucl. Phys. Proc. Suppl.* **253 - 255** 73.
- [30] O. Shekhovtsova, T. Przedzinski, P. Roig and Z. Was, *Phys. Rev. D* **86** (2012) 113008. I. M. Nugent, T. Przedzinski, P. Roig, O. Shekhovtsova and Z. Was, 2013 *Phys. Rev. D* **88** 093012.
- [31] K. Nakamura et al. [Particle Data Group], 2010 *J. Phys. G* **37** 075021.
- [32] I. M. Nugent [BaBar Collaboration], 2014 *Nucl. Phys. Proc. Suppl.* **253-255** 38.
- [33] <http://annapurna.ifj.edu.pl/~wasm/RχL/RχL.html>.
- [34] J. J. Sanz-Cillero and O. Shekhovtsova 2017 Refining the scalar and tensor contributions in $\tau \rightarrow \pi\pi\pi\nu_\tau$ decays. Preprint arXiv:1707.01137 [hep-ph].
- [35] R. Escribano, S. González-Solis and P. Roig, *Phys. Rev. D* **94** (2016) no.3, 034008. E. A. Garcés, M. H. Villanueva, G. L. Castro and P. Roig, arXiv:1708.07802 [hep-ph].