Classical Realisations of w_∞

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ABSTRACT

We describe the classical geometry of linear and non-linear realisations of the w_∞ algebra in two-dimensional field theories. We also discuss the relation of gauged w_∞-invariant models with $N-1$ independent scalar fields to W_N symmetry.

There has been much interest recently in the various possible extensions of the Virasoro algebra that could occur as world-sheet symmetry algebras of $d = 2$ field theories. The realisation that there exist not only spin-1 extensions of the Virasoro algebra such as the Kac-Moody algebras, but also the w-algebras [1], which contain conformal spins higher than 2, has opened up new possibilities for world-sheet symmetries.

The left-handed Virasoro algebra is generated by the T_{++} component of the stress tensor, which may be Fourier analysed into the L_n generators:

$$T_{++} = \sum_{n=-\infty}^{\infty} e^{inx^+} L_n, \tag{1}$$

where $x^+ = x + t$ is considered for this purpose to be a circular coordinate. At the classical level with which we shall mainly be concerned in this article, there is no central extension of the Virasoro algebra, which thus has the commutation relations

$$[L_m, L_n] = (m-n)L_{m+n}. \tag{2}$$

Under the action of the Virasoro algebra, a *primary field* $W_{(s)}(x^+, x^-)$ of conformal spin s is one whose Fourier components $W_{(s)\,n}$ satisfy the commutation relations

$$[L_m, W_{(s)\,n}] = ((s-1)m - n)\, W_{(s)\,m+n}. \tag{3}$$

The W_N algebras [1] extend the spin-2 Virasoro algebra by the inclusion of generators with conformal spins $3 \oplus 4 \oplus \cdots \oplus N$. As can be seen from the need for covariance of the commutators of the new higher-spin generators under the Virasoro subalgebra, there must be *non-linear* combinations of generators on the right-hand side of at least the highest spin commutator. For example, the W_3 algebra has a term on the right-hand side of the $[W_m, W_n]$ commutator that is quadratic in the Virasoro generators L_m, for this is the only way to construct the term of conformal spin 4 that is necessary by Virasoro covariance of the commutation relations, without having an independent spin-4 generator itself.

In the limit of $N \to \infty$, however, the need for such non-linearity can be pushed off indefinitely. Thus, the w_∞ algebra [2] recovers the structure of an ordinary Lie algebra:

$$[\hat{v}^i_m, \hat{v}^j_n] = ((j+1)m - (i+1)n)\, \hat{v}^{i+j}_{m+n}, \tag{4}$$

where the superscript indices i, j correspond to conformal spins $s-2$, and run over integral values $i, j \geq 0$. Thus, the Virasoro generators are the spin-2 generators $v^0_m = L_m$ and the spin-3 generators are the v^1_m, etc. The algebra (4) contains only the leading conformal spin $i + j + 2$ required by the Virasoro algebra. One can also consider a deformation of this algebra in which sub-leading conformal spins also make an appearance — the algebra W_∞ [3,4], from which w_∞ as given above may be recovered by a contraction. Here, we shall be concerned only with the w_∞ algebra (4).

A natural realisation of the w_∞ algebra (4) is in terms of area-preserving, or symplectic, diffeomorphisms of a cylinder [3] $S^1 \times R$, to which we may give coodinates (w, y), and on which we may expand a general function $f(w, y)$ in a complete Fourier $\times$ polynomial basis of functions

$$v_m^\ell = -\mathrm{i}e^{\mathrm{i}mw}y^{\ell+1}, \qquad (5)$$

in terms of which the algebra (4) is realised as a Poisson bracket algebra $\{v_m^i, v_n^j\}$, where the Poisson brackets are defined on the cylinder (w, y) in the standard fashion:

$$\{f, g\} = \partial_w f \partial_y g - \partial_y f \partial_w g. \qquad (6)$$

Alternatively, one may introduce the hamiltonian vector field operators

$$\hat{v}_m^\ell = \{v_m^\ell, \bullet\}, \qquad (7)$$

where a function to be operated on by $\hat{v}_m^\ell$ would be placed at the location of the bullet $\bullet$. For the basis (5), these vector fields are given explicitly by

$$\hat{v}_m^\ell = e^{\mathrm{i}mw}\left(my^{\ell+1}\partial_y + \mathrm{i}(\ell+1)y^\ell\partial_w\right); \qquad (8)$$

one may then verify that the algebra (4) is realised by ordinary commutators of the differential operators $\hat{v}_m^\ell$.

There is a seemingly trivial extension of the w_∞ algebra (4) that will nonetheless play an important part in the rest of our discussion. One can readily see that the algebra (4) will also close if one allows the upper indices i, j to range over integral values in the range $i, j \geq -1$. In that case, a set of conformal spin-1 generators $\hat{v}_m^{-1} = me^{\mathrm{i}mw}\partial_y$ is included into the algebra, corresponding to the y-independent basis functions $v_m^{-1} = -\mathrm{i}e^{\mathrm{i}mw}$. This extended algebra is called $w_{1+\infty}$.

The above *linear* realisations of w_∞ and $w_{1+\infty}$ are not themselves of the type that we expect to have for fields on the worldsheet (x^+, x^-). The type of realisation that we are looking for in this context is a generalisation of the Virasoro realisation on a scalar field φ,

$$\delta\varphi = k(x^+)\partial_+\varphi. \qquad (9)$$

Note that since we are concerned at present with the realisation of a *single chiral copy* of the Virasoro algebra, the parameter $k(x^+)$ (equivalent to k_m, $-\infty < m < \infty$) depends only on the x^+ coordinate and the transformation of φ involves only a ∂_+ derivative. This "semi-local" structure of the Virasoro transformation laws, involving only the x^+ coordinate and with x^- having essentially the rôle of an inert "time", requires a generalisation to w_∞ and $w_{1+\infty}$ in terms of fields $\varphi(x^+, x^-)$ where only the x^+ coordinate is involved in the transformations. Thus, we are essentially looking for realisations in terms of functions of only one variable — the "time" variable x^- will be important only when we consider the construction of a Lagrangian. Scalar functions of only one variable represent an infinitely smaller field content than that of the linear realisation described above. Thus, we are necessarily looking for *nonlinear* realisations of w_∞, $w_{1+\infty}$ to generalise the standard Virasoro realisation.

A nonlinear realisation built according to the classic structure [5,6] with a $\mathcal{G}/\mathcal{H}$ coset will require both $\mathcal{G}$ and $\mathcal{H}$ to be "large", if the coset is to be parametrised in terms of the "small" field content of scalar fields over one dimension. In fact, $\mathcal{H}$ should differ from $\mathcal{G}$ only by the omission of generators whose linear realisation involves one-dimensional fields. This is exactly the relation between w_∞ and $w_{1+\infty}$ that we have discussed above, so it is natural for our purposes to try the coset $w_{1+\infty}/w_\infty$[7].

In order to construct a coset-space nonlinear realisation $w_{1+\infty}/w_\infty$, it is necessary to know how to make a *finite* $w_{1+\infty}$ transformation on fields defined on the cylinder (w, y). From the Poisson bracket form of the algebra, one sees that infinitesimal transformations of a function $f(w, y)$ are given by $f \to \tilde{f}$ where

$$\tilde{f} = f + \{\Lambda, f\}, \qquad (10)$$

in which Λ is the infinitesimal parameter of the transformation. Exponentiating, we obtain the transformation with finite Λ,

$$\tilde{f} = f + \{\Lambda, f\} + \frac{1}{2!}\{\Lambda, \{\Lambda, f\}\} + \cdots. \qquad (11)$$

Note that, like our Virasoro transformation of a scalar field (9), these transformations are *active*, i.e., they transform fields only and not the coordinates at the point of evaluation, $f(w,y) \to \tilde{f}(w,y)$. If we rewrite the finite transformation (11) as an Einstein-style tranformation for a scalar field $f(w,y) \to \tilde{f}(\tilde{w},\tilde{y}) = f(w,y)$, we derive the corresponding coordinate transformation $(w,y) \to (\tilde{w},\tilde{y})$. Then one can verify that (11) does indeed yield an area-preserving, or symplectic, diffeomorphism:

$$\det\left(\frac{\partial(\tilde{w},\tilde{y})}{\partial(w,y)}\right) = 1. \qquad (12)$$

Since the basis functions v_m^{-1} corresponding to the $w_{1+\infty}/w_\infty$ coset are y-independent, projection of an expression linear in generators of $w_{1+\infty}$ into the coset is easily effected by setting $y \to 0$. Correspondingly, the coset transformations are parametrised by functions of w alone, $\Lambda(w)$, and similarly the Goldstone fields φ_m for the $w_{1+\infty}/w_\infty$ nonlinear realisation can be assembled into a field $\varphi(w,x^-)$, where, as before, x^- is the inert "time". At this stage, it is appropriate to make the identification $w \leftrightarrow x^+$. The vector field built from the Goldstone field φ is

$$\hat{\varphi} = \{\varphi, \bullet\} = \varphi' \partial_y, \qquad (13)$$

where $\varphi' = (\partial/\partial x^+)\varphi = \partial_w \varphi$. Thus, the finite $w_{1+\infty}/w_\infty$ element parametrised by φ is $\exp(\varphi'\partial_y)$.

We may now derive the non-linear transformations in the usual way, multiplying on the left by a general element g of $w_{1+\infty}$:

$$g e^{\varphi'\partial_y} = e^{\tilde{\varphi}'\partial_y} \tilde{h}, \qquad (14)$$

where $\tilde{h}$ is an element of $\mathcal{H} = w_\infty$. For a transformation in the coset, $g = e^{\Lambda'\partial_y} \in w_{1+\infty}/w_\infty$, the transformation of φ is just an inhomogeneous shift:

$$\varphi \to \tilde{\varphi} = \varphi + \Lambda(x^+). \qquad (15)$$

This transformation is indeed not linear, in the sense that it is not homogeneous in φ, and thus fulfils our expectations for the transformation of a Goldstone field under a transformation from its own coset. More unusual, however, is what happens to φ under a transformation from the *denominator* group w_∞.

For a transformation by an element of the w_∞ denominator, we take an element $g \in w_\infty$ in the infinitesimal neighborhood of the identity, $g = 1 + \hat{\lambda}$, and make the standard rearrangement:

$$e^{-\varphi'\partial_y} \hat{\lambda} e^{\varphi'} \partial_y = \delta\varphi' + \tilde{\hat{\lambda}}, \qquad (16)$$

where $\delta\varphi' = \tilde{\varphi}' - \varphi'$. For the infinitesimal w_∞ transformation generated by $\lambda = k_\ell(x^+)y^{\ell+1}$, one then finds $\delta\varphi' = (k_\ell(\varphi')^{\ell+1})'$, so

$$\delta\varphi = k_\ell(x^+)(\partial_+\varphi)^{\ell+1}, \qquad \ell = 0, 1, \ldots, \infty. \qquad (17)$$

The whole set of numerator and denominator transformations (15) and (17) can be summarised by

$$\delta\varphi = k_\ell(x^+)(\partial_+\varphi)^{\ell+1}, \qquad \ell \geq -1. \qquad (18)$$

The unorthodox feature of the above construction, namely the nonlinearity under transformations by the denominator group, arises because the $w_{1+\infty}/w_\infty$ coset is too small to form a linear realisation of the w_∞ denominator. More precisely, as one can see from (4), the commutator of a $\mathcal{G}/\mathcal{H}$ generator with an $\mathcal{H}$ generator gives back a generator in $\mathcal{H}$, except for commutators with the $i = 0$ level of the $\mathcal{H}$ algebra, i.e., with the Virasoro subalgebra, which is actually linearly realised as one can see from (9).

In the nonlinear realisation of $w_{1+\infty}$, we have a situation somewhat similar to the case of $d = 4$ conformal symmetry $SO(4,2)$, where the structure of the nonlinear realisation is made more clear by recourse to the "inverse Higgs effect" [9]. In this, one constructs a larger coset $\mathcal{G}/\mathcal{H}'$ which satisfies $[\mathcal{G}/\mathcal{H}', \mathcal{H}'] \subset \mathcal{G}/\mathcal{H}'$, so that $\mathcal{G}/\mathcal{H}'$ does actually form a linear realisation of $\mathcal{H}'$. Then one looks for a set of *covariant constraints* to bring the number of independent Goldstone fields down to the smaller size actually sought. The possible covariant constraints are easily constructed using Cartan differential forms, according to the standard theory of nonlinear realisations [5,6]. The analog of $\mathcal{H}'$ in the $w_{1+\infty}$ case [10] is the Virasoro algebra, according to which the $w_{1+\infty}$ generators $\hat{v}_m^i$ break up into Virasoro representations of conformal spin $s = i+2$. In order to realise $w_{1+\infty}$ according to the nonlinear realisation theory of refs [6], we should associate the x^+ worldsheet coordinate to some coset generator of the algebra, just as one does for $SO(4,2)$ with $x^\mu \leftrightarrow P^\mu$. Here, we associate [10] x^+ to the $L_{-1} = \hat{v}_{-1}^0$ generator. For the Virasoro algebra, this gives the transformations $L_n : x^+ \to x^+ + \delta x^+$, where $\delta x^+ = k_n(x^+)^{n+1}$. In this realisation, the Virasoro algebra splits up into *singular* ($n < -1$) and *non-singular* ($n \geq -1$) sectors. Moreover, the non-singular sector closes onto itself, as can be seen from the algebra (2). Thus, we may consistently restrict the discussion to the non-singular sector of the Virasoro algebra and the corresponding non-singular sector of $w_{1+\infty}$, which consists of the generators $\hat{v}^i_{m\geq-(i+1)}$. The denominator subalgebra $\mathcal{H}'$ for this realisation is the stability subalgebra of the point $x^+ = 0$, with generators $\hat{v}^0_{m\geq 0}$. The boundary generators $\hat{v}^i_{-(i+1)}$ will correspond to the irreducible coset elements that remain after the inverse Higgs effect, analogously to x^μ and $\sigma(x)$ in the $d = 4$ conformal case. The result of the inverse Higgs analysis is given by the following diagram of the $w_{1+\infty}$ generators:

Map of the non-singular $w_{1+\infty}$ generators

	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
	o	x	x	x	x	x	x	x	⋯
⋮		o	x	x	x	x	x	x	⋯
$i=2$			o	x	x	x	x	x	⋯
$i=1$				o	x	x	x	x	⋯
$i=0$	—	—	—	—	•	◇	◇	◇	⋯
$i=-1$						o	x	x	⋯
						|			
					m =	0			

In this diagram, the generators corresponding to reducible Goldstone fields that can be eliminated by covariant constraints in the inverse Higgs effect are indicated by x, the irreducible Goldstone fields that survive the inverse Higgs effect are indicated by o, the $\hat{v}^0_{-1}$ generator corresponding to the world-sheet coordinate x^+ is indicated by •, and the generators of the denominator subalgebra $\mathcal{H}'$ are indicated by ◇.

Introducing the notation $\varphi^i_m(x^+)$ for the Goldstone field corresponding to the $w_{1+\infty}$ generator $\hat{v}^i_m$, we see that the boundary generator in the bottom row of the diagram is the one corresponding to the x^+-independent transformations in the $w_{1+\infty}/w_\infty$ coset, and so the Goldstone field φ_0^{-1} is the one that corresponds to the $w_{1+\infty}/w_\infty$ Goldstone field $\varphi(x^+)$ of our earlier discussion. Indeed, after eliminating all of the reducible Goldstone fields in the diagram by the inverse Higgs covariant derivative conditions, the transformation of $\varphi_0^{-1}(x^+) = \varphi(x^+)$ is exactly given by eq. (18) as required, after re-expressing the transformations in the same active form that we used before. Thus, similarly to the $d = 4$ conformal case, we have recovered the nonlinear transformations of the minimal nonlinear realisation $\mathcal{G}/\mathcal{H}$ from an extended discussion starting from a coset $\mathcal{G}/\mathcal{H}'$ that transforms linearly under $\mathcal{H}'$, then eliminating as many as possible reducible Goldstone fields by the covariant derivative conditions of the inverse Higgs effect.

In both the present case and in the $d = 4$ conformal example, one actually gets more than was asked for. In the $d = 4$ conformal case, one wants to understand the nonlinear proper conformal transformations of x^μ, which are initially realised in the fashion of ref. [8] using the coset $SO(4,2)/\mathcal{H}$, where $\mathcal{H}$ consists of the unorthodox Poincaré subalgebra $(SO(3,1)\otimes\{K^\mu\})$ times the dilations D. After the inverse Higgs effect, one reobtains the desired x^μ transformations, but finds that there is also a Goldstone field, $\sigma(x^\mu)$, which is necessary in general to make a dilation-covariant coupling to general matter fields. Of course, certain *specific* Lagrangians may not involve this dilation Goldstone field — for example, classical $d = 4$ Yang-Mills theory is already conformally invariant by itself and does not need any help from the dilation Goldstone field. This is not the case, on the other hand, for the massless spin-two Pauli-Fierz theory (general relativity linearised in $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$), whose conformally invariant realisation does require coupling to the dilaton. Moreover, as is well-known, even if a theory is conformally-invariant at the classical level without coupling to the dilation Goldstone field $\sigma(x)$, conformal anomalies may require such coupling at the quantum level. Indeed, a standard way to calculate the conformal anomaly for a given theory is to provide a dilation Goldstone field that is classically decoupled, and then to calculate the quantum-induced coupling to it.

In the $w_{1+\infty}$ case, we get lots more than the expected Virasoro transformations of x^+ and the nonlinear transformations (18) of $\varphi(x^+, x^-)$. Aside from the world-sheet coordinate x^+, which is also a survivor of the inverse Higgs effect, the surviving irreducible Goldstone fields are:

$$\varphi = \varphi^0_{-1},\ \varphi^1_{-2},\ \varphi^2_{-3}, \cdots, \varphi^i_{-(i+1)}, \cdots \qquad (19)$$

Thus, general $w_{1+\infty}$-invariant theories cannot be made with just the Goldstone field $\varphi(x^+, x^-)$, although *specific* theories might be. An example of such a specific $w_{1+\infty}$-invariant theory is just the $d = 2$ free field theory for $\varphi(x^+, x^-)$ itself:

$$I_\varphi = \tfrac{1}{2}\int d^2x \partial_+\varphi\partial_-\varphi, \tag{20}$$

which one can verify is invariant under the nonlinear transformations (18) without any need for coupling to the higher Goldstone fields in (19). As in the case of the $d = 4$ dilaton field, however, this classical decoupling may be violated by anomalies. Thus, one should be on guard for the recoupling of the second and higher members of the list in (19) at the quantum level. Moreover, coupling of the multiplet (19) to other worldsheet fields should in general be expected to involve the whole set.

The above discussion has concerned only the "rigid" realisation of w_∞, which might better be considered "semi-local" since the transformation parameters in (18) are functions of x^+ only. We next consider the case of *gauged* w_∞, in which the parameters are allowed to become arbitrary functions of x^+ and x^-, and to achieve this one has to couple to gauge fields A_i, $i \geq 0$. Thus, we are dealing with a theory of "w_∞-gravity" coupled to φ. As was shown in [12] for the case of W_3, this gauging is particularly simple for the case of the *chiral* W-realisations generalising just the left-handed copy of the Virasoro algebra (2). This chiral gauging of W_3 trivially generalises to the chiral w_∞ case. The non-chiral cases are more involved, with two sets of gauge fields A_i, $\bar{A}_i$ and infinite series of "seagull" couplings nonlinear in these gauge fields [12]. These infinite series are equivalent to the use of cross-coupled auxiliary fields F_i, $\bar{F}_i$ that give rise to "nested covariant derivatives" as a result of their algebraic but non-polynomial field equations [13]. For our present purposes, however, we shall be concerned with the simplest case of chiral gauged w_∞ [7], for we wish to examine the features of the gauged model analogous to those found above in the semi-local case.

When we let the parameters in the transformations (18) depend on both world-sheet coordinates, $k_\ell(x^+) \to k_l(x^+, x^-)$, the action (20) will be invariant only if we include Noether coupling terms for the gauge fields A_ℓ, $\ell \geq 0$. The gauged action then becomes

$$\begin{aligned} I_{\text{local}} = \int d^2x \big(&\tfrac{1}{2}\partial_+\varphi\partial_-\varphi \\ &- \sum_{\ell=0}^{\infty} \frac{1}{\ell+2} A_\ell (\partial_+\varphi)^{\ell+2}\big), \end{aligned} \tag{21}$$

and the gauge field transformations required when the φ transformations (18) become fully local are

$$\begin{aligned} \delta A_\ell = \partial_- k_\ell - \sum_{j=0}^{\ell} \big[&(j+1)A_j\partial_+ k_{\ell-j} \\ &- (\ell-j+1)k_{\ell-j}\partial_+ A_j\big]. \end{aligned} \tag{22}$$

The Noether currents $(\partial_+\varphi)^{\ell+2}$ coupled to the A_ℓ in (21) might perhaps better be considered semi-local "charges", since the free φ equation of motion $\partial_-\partial_+\varphi = 0$ implies that they are all independent of the "time" x^-, even without integration over x^+. As a consequence, they are not really all independent for the single φ-component case that we have been studying for simplicity up to now, since $\partial_-\partial_+\varphi = 0 \Rightarrow \partial_-(\partial_+)^{\ell+2} = 0$.

This lack of independence is at the root of another infinite set of local symmetries of the action (22), under which the $A_{\ell \geq 1}$ gauge fields undergo arbitrary shifts [7]:

$$\begin{aligned} \delta A_0 &= -\sum_{\ell \geq 1} \frac{2}{\ell+2}\alpha_\ell(\partial_+\varphi)^\ell \\ \delta A_\ell &= \alpha_\ell \qquad \ell \geq 0, \end{aligned} \tag{23}$$

where $\alpha_\ell(x^+, x^-)$, $\ell \geq 1$ is a set of local parameters. These local "shift" symmetries are akin to the Steuckelberg symmetry that arises when massive Maxwell theory is made into a gauge theory by the addition of a "spurion" scalar field. The shift symmetries can be fixed by the gauge conditions

$$A_\ell = 0 \qquad \ell \geq 1. \tag{24}$$

Thus, the infinite set of w_∞ gauge fields A_ℓ can be "telescoped" down to just A_0, which couples to the one truly independent current in the action (25). Moreover, the eliminable gauge fields correspond exactly to the w_∞ generators $\hat{v}_m^{\ell \geq 1}$ whose Goldstone fields decouple in the actions (20,21).

The telescoping phenomenon goes differently if one takes a multiplet of scalars φ^{AB} valued in the adjoint representation of some Lie group $\mathcal{T}$ [7]. For example, if φ is valued in the adjoint representation of $SU(N)$, then the action is similar to (21) except that we must now trace over the $\mathcal{T}$ indices:

$$I_{\mathcal{T}} = \int d^2x \left[\tfrac{1}{2}\mathrm{tr}(\partial_+\varphi\partial_-\varphi) - \sum_{\ell=0}^{\infty} \frac{1}{\ell+2} A_\ell \mathrm{tr}(\partial_+\varphi)^{\ell+2}\right]. \tag{25}$$

The difference in the telescoping behaviour now arises because some of the currents $\mathrm{tr}(\partial_+\varphi)^{\ell+2}$ are trully independent; the number of such independent currents is given by the rank of the group $\mathcal{T}$. Thus, for $\mathcal{T} = SU(N)$, there are $N-1$ independent currents $\mathrm{tr}(\partial_+\varphi)^2, \cdots, \mathrm{tr}(\partial_+\varphi)^N$, which couple to the irreducible gauge fields $A_0, A_1, \cdots, A_{N-2}$. The shift symmetries in this case permit one to gauge to zero all the higher gauge fields $A_{\ell \geq N-1}$. Moreover, in order to maintain the gauge $A_{\ell \geq N-1} = 0$ on performing w-transformations, compensating shift transformations are needed. The remaining fields and transformation laws then agree with those for a gauged chiral W_N model, the nonlinearity in the W_N algebra arising from the combination of w_∞ with the compensating shift transformations. The same agreement occurs (after field redefinitions) between the non-chiral gauged W_3 theory of ref. [13] and the gauge-fixed $\mathcal{T} = SU(3)$ version of the w_∞ gauging of ref. [7]. Thus, from w_∞ we come back to W_N in a finite component model. The restriction on the telescoping of the higher gauge fields down to those of a W_N gauging actually can be done somewhat more economically than we have presented above. It is not necessary to take a full set of fields valued in the adjoint representation of $\mathcal{T}$ — all that is necessary is to let the fields φ take their values in *diagonal* $\mathcal{T}$-matrices [14], in order to preserve the independence of the currents $\mathrm{tr}(\partial_+\varphi)^2, \cdots, \mathrm{tr}(\partial_+\varphi)^N$. How to describe this $\mathcal{T}$-valued "telescoping" structure in the languange of nonlinear realisations that we started with remains a problem for further research.

References

1. A.B. Zamolodchikov, *Theor. Mat. Fiz.* **65** (1985) 347;
   V.A. Fateev and S. Lukyanov, *Int. J. Mod. Phys.* **A3** (1988) 507.
2. A. Bilal, *Phys. Lett.* **227B** (1989) 406;
   I. Bakas, *Phys. Lett.* **228B** (1989) 57.
3. C.N. Pope, L.J. Romans and X. Shen, *Phys. Lett.* **236B** (1990) 173.
4. C.N. Pope, L.J. Romans and X. Shen, *Nucl. Phys.* **B339** (1990) 191.
5. S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2239;
   C.L. Callan, Jr., S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2247;
   C.J. Isham, *Nuovo Cimento* **59A** (1969) 356.
6. D.V. Volkov, *Fiz. Elem. Chastits At. Yadra* **4** (1973) 3;
   V.I. Ogievetsky, in *Proc. 10th Winter School of Theoretical Physics in Karpacz*, Vol. 1 (Wrocław, 1974).
7. E. Bergshoeff, C.N. Pope, L.J. Romans, E. Sezgin, X. Shen and K.S. Stelle, *Phys. Lett.* **243B** (1990) 350.
8. G. Mack and Abdus Salam, *Ann. Phys. (N.Y.)* **53** (1969) 174.
9. A.B. Borisov and V.I. Ogievetsky, *Theor. Mat. Fiz.* **21** (1974) 329;
   E.A. Ivanov and V.I. Ogievetsky, *Theor. Mat. Fiz.* **25** (1975) 164.
10. E. Sezgin and K.S. Stelle, to appear.
11. E.A. Ivanov and S.O. Krivonos, *Lett. Math. Phys.* **7** (1983) 523;
    K. Schoutens, *Nucl. Phys.* **B292** (1987) 974.
12. C.M. Hull, *Phys. Lett.* **240B** (1989) 110.
13. K. Schoutens, A. Sevrin and P. van Nieuwenhuizen, *Phys. Lett.* **243B** (1990) 245.
14. K. Li and C.N. Pope, in *Proc. Trieste Summer High-Energy Workshop, July 1990.*