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Constraining quintessence field dynamics with recent cosmological observations

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Abstract We present an investigation of a scalar field dark energy model in the context of the FLRW universe, focusing on the parameterization of the deceleration parameter $q(z)$ to study the evolution of cosmic acceleration. By employing extensive observational datasets—including 30 independent cosmic chronometer measurements, 17 additional baryon acoustic oscillation data points, and standard candle datasets from Pantheon Type Ia supernovae, Quasars, and Gamma-Ray Bursts—we provide constraints on cosmological parameters using advanced Markov chain Monte Carlo methods. Our analysis identifies a transition redshift of $z_t = 0.62$, marking the shift from decelerated to accelerated expansion, with a current deceleration parameter of $q_0 = -0.59$. The equation of state parameter confirms the dynamical behavior of quintessence, deviating slightly from a cosmological constant. Furthermore, the model demonstrates strong consistency with ΛCDM at lower redshifts while revealing distinct deviations at higher redshifts, which provides valuable insights into the late-time dynamics of the universe. By examining the evolution of cosmography parameters, energy density, pressure, and the scalar field equation of state, this study contributes the relevance of scalar field models as promising candidates for dark energy.

1 Introduction

In the early twentieth century, Albert Einstein was inspired to form a theory known as General Relativity in explaining how gravity behaves with regard to space and time. He indicated that matter and radiation were coupled to the geometry of space time. It was Einstein's approach when he equated the energy and momentum of matter and radiation with the curvature of space time in terms of the field equation: $R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij}$ [1]. The current state of our expanding cosmos is explained by cosmological models that are grounded in general relativity. Numerous experimental investigations provide evidence in favor of the current fast expansion phase [2–5]. This expansion is thought to be driven by dark energy, whereas dark matter is essential to the formation of the universe's large-scale structures [6–8]. A unexplained force answerable for the expansion of the universe is dark energy, which exerts a repulsive pressure. These findings have led to the development of a variety of models that describe the universe and into which the cosmological constant—a plausible dark energy candidate due to repulsion—will be incorporated [9]. Through the investigation of several theoretical models and cosmographic tests, Bamba et al. [10] examined the cosmology of dark energy in 2012 and eventually shown that degeneracies between parameters can be eliminated with accurate data analysis of big datasets. Moreover models of dark energy have been proposed to analyze the issues with the cosmological constant. These models include holographic dark energy (HDE) models [11–14] and variable equations of state parameters [15–18]. From the beginning of the universe to the present, Nojiri and Odintsov [19] examined its cosmic history in 2011. Nojiri et al. [20] recently presented a cosmological model that well captures the universe's late-time dynamics.

The universe's accelerated expansion has been clarified by a number of hypotheses that do not rely on the cosmological constant. In the Einstein-Gauss-Bonnet theory, $f(T)$, $f(R)$, $f(R, T)$, $f(R, G)$, and $f(Q)$ gravities are modified theories that yield various predictions about the nature of dark energy and the behavior of the universe [21–25]. A different picture for the universe without the idea of dark energy is presented by certain hypotheses. To explain observations, they need more parameters. Despite being backed by several experiments and fitting well with scientific results, the cosmological constant is ineffective to justify for the universe's

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inflationary epoch. Modified techniques that depict late-time cosmic expansion without the use of a cosmological term include Weyl, Lyra, Brans-Dicke, and others [26–29]. There has also been research on modified gravity theories, which appear to be stable and do not require dark energy [19, 20, 30]. 2011 saw the presentation of the cosmic history of the universe from its earliest beginnings in the modern era by Nojiri and Odintsov [19]. More recently, Nojiri et al. [20] looked into a cosmological model that may effectively explain the universe's late-time dynamics. The modified theory of gravity, which incorporates both positive and negative powers of curvature, is one of the important fields of study in this context.

Apart from modified theories, several cosmological models use scalar fields to explain the universe's current expansion phase as well as its early inflationary phase [31, 32]. These models assume that a scalar field (ϕ) represents dark energy. In these models, the scalar field and a diminishing potential, $V(\phi)$, provide the negative pressure. Scalar field cosmologies have been proposed in a number of research to explain the universe's dynamics [31–34]. References [33, 34] state that the quintessence model is a extra advantageous scalar field-based model that describes the current state of the universe while successfully avoiding the traditional fine-tuning and cosmic coincidence difficulties. Johri [35] introduced the concept of tracking by putting forward a clear path for the universe's progression based on the tracker potential. Observational evidence provided considerable support for this theory. In the literature, numerous quintessence models have been put out, such as those with non-minimal coupling between quintessence and dark matter [36, 37] and scalar field evolution driven by a non-canonical kinetic term [38]. In [39, 40], important uses of time-dependent equation of state (EoS) parameters in scalar-tensor theory are described. The existence of scalar fields in astrophysical situations is also acknowledged by several basic theories. Many cosmological models have recently been put up within various frameworks of scalar field theory [39–44].

Although a range of dynamical dark energy and modified gravity models have been suggested to account for late-time cosmic acceleration, it is vital to thoroughly examine and test these models against cosmological observations. The intricate nature of the physics behind late-time acceleration, combined with the ever-improving accuracy of observational data, highlights the importance of comprehensive analyses. Employing appropriate parameterizations that provide model-independent frameworks for describing late-time cosmic acceleration is key to achieving this [45–48]. In this context, a variety of parameterizations have been introduced for different physical and geometrical parameters such as the Hubble, the deceleration, and the equation of state parameters [48–54].

Inspired by the aforementioned discussions, in this study, we aim to investigate a scalar field dark energy model, namely quintessence model, focusing a parameterization of the deceleration parameter. For this purpose, we use measurements from several observational datasets, including the cosmic chronometers (CC), baryon acoustic oscillations (BAO), and standard candles (SC). The BAO dataset incorporates recent observations from multiple surveys, such as the sloan digital sky survey (SDSS), dark energy camera legacy survey (DECaLS), and 6dF Galaxy Survey BAO (6dFGS BAO), augmented by 17 additional BAO data points. The CC dataset consists of 30 independent Hubble parameter measurements derived from the differential age method of galaxies. For SC, we employ uncorrelated measurements from the Pantheon Type Ia supernova dataset, along with Quasars and Gamma-Ray Bursts. The numerical analysis is conducted using Markov chain Monte Carlo (MCMC) methods implemented with the Polychord sampler, and we also incorporate the R19 measurement of the Hubble constant to refine our constraints. This combined dataset enables us to compare our scalar field model with the ΛCDM model, providing insights into the universe's expansion history and late-time dynamics.

The structure of this paper is organized as follows: In Section II, we introduce the field equations for the quintessence model, providing the mathematical framework for understanding the dynamics of dark energy. Section III presents a parameterization of the deceleration parameter $q(z)$. In Section IV, we describe the methodology, highlighting the observational datasets and the statistical tools used for model evaluation. Section V explores the baryon acoustic oscillation (BAO) scale as a standard ruler in cosmology, while Section VI compares our proposed scalar field model with the ΛCDM model, discussing their alignment with observational data. In Section VII, we analyze cosmography parameters such as the deceleration (q), jerk (j), and snap (s) parameters. Section VIII investigates the physical characteristics of quintessence, including the evolution of energy density, pressure, and the equation of state parameter. Finally, Section IX summarizes the findings and concludes the study.

2 Field equations with quintessence

One of the biggest unanswered questions in cosmology is dark energy. There is considerable debate about its nature, particularly in relation to whether it functions as a origin tenure in Einstein's field equations. Consistent with a broad spectrum of cosmological evidence, quintessence is one of the most promising dark energy theories and a feasible alternate for the cosmological constant. The earliest physics answer to explain the universe's apparent accelerated expansion was this scalar field. These ideas were initially proposed in [55]. Various time-dependent dark energy concepts have now been proposed. A few of these have been categorized into "species of essence." In this case, the selection of quintessence is based only on the attractiveness or repulsiveness of the power it carries. Quintessence models' dynamical behavior can be described by a low rate of potential and a suitable kinetic energy ratio from the field.

The action, as in [56], is given by

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2}M_{\rm Pl}^2 R + \mathcal{L}_m + \mathcal{L}_\phi\right), \tag{1}$$

where $\mathcal{L}_m$ is the Lagrangian of the matter fields, and $\mathcal{L}_\phi$ is the Lagrangian of the scalar field, imparted

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi), \tag{2}$$

In these expressions, g is the determinant of the metric $g_{\mu\nu}$, $M_{\rm Pl} = (8\pi G)^{-1/2}$ is the abated Planck mass, R is the Ricci scalar, and $V(\phi)$ is a general self-coupling potential. The field ϕ should be positive for physically acceptable fields. We consider that there is no direct coupling between non-relativistic matter and the quintessence field (minimal interaction). Varying the action (1) with respect to $g_{\mu\nu}$ yields the gravitational field equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = M_{\rm Pl}^{-2}T_{\mu\nu}^{\rm Total}, \tag{3}$$

where

$$T_{\mu\nu}^{\phi} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 - g_{\mu\nu}V(\phi), \tag{4}$$

represents the energy-momentum tensor of the scalar field, with $(\partial\phi)^2 \equiv g^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi$, and, $T_{\mu\nu}^{m}$ being the energy-momentum tensor of the matter, $T_{\mu\nu}^{\rm Total} = T_{\mu\nu}^{m} + T_{\mu\nu}^{\phi}$. From equation (2), the usual Klein-Gordon equation is derived:

$$\nabla_\mu\nabla^\mu\phi - \frac{\partial V}{\partial\phi} = 0. \tag{5}$$

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric describes the most general homogeneous and isotropic space time. to begin with, consideration is given to the metric:

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2\right). \tag{6}$$

where $a(t)$ is the scale factor of the universe. The metric describes closed, flat, and open geometries for $k = +1,\ 0,\ -1$, respectively. We set the speed of light to $c = 1$. Observational data suggest a flat universe, leading to the reduced space time:

$$ds^2 = -dt^2 + a(t)^2\big(dr^2 + r^2\big(d\theta^2 + \sin^2\theta\,d\phi^2\big)\big), \tag{7}$$

In this setup, the Einstein field equations give us the following dynamical equations:

$$M_{\rm Pl}^{-2}\rho^{\rm Total} = 3\left(\frac{\dot a}{a}\right)^2 = 3H^2, \tag{8}$$

$$M_{\rm Pl}^{-2}p^{\rm Total} = -2\frac{\ddot a}{a} - \left(\frac{\dot a}{a}\right)^2 = (2q-1)H^2. \tag{9}$$

Here $H = \dot a/a$ is the Hubble parameter and $q = -a\ddot a/\dot a^2$ is the deceleration parameter.

The scale factor a governs the dynamics of the model. Total energy density and pressure are defined as $\rho_{\rm Total} = \rho_M + \rho_\phi$ and $p_{\rm Total} = p_M + p_\phi$, respectively. The energy density and pressure for the scalar field are defined as follows:

$$\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot\phi^2 - V(\phi).$$

for that reason the equation of state (EoS) parameter ω_ϕ for the scalar field describing dark energy is presented by

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)}. \tag{10}$$

It is important to note that the EoS of dark energy (quintessence) is a dynamic parameter, which can vary within the range $[-1, 1]$. When the potential dominates, ω_ϕ approaches -1, corresponding to a cosmological constant. Values of ω_ϕ less than -1 indicate a phantom regime, which we do not explore in this study. The conservation equation can be derived from equations (7) and (8) as

$$\dot\rho_{\rm Total} + 3(p_{\rm Total} + \rho_{\rm Total})\frac{\dot a}{a} = 0. \tag{11}$$

This fundamental equation outlines how matter interacts. In modern cosmology, two main categories of dark energy models are concluded: interacting dark energy models, in which there is coupling between cold dark matter and dark energy [57–59], and non-interacting dark energy models, in which matter components evolve independently [60–62]. Currently, no interaction beyond gravity has been observed between matter and dark energy. This work focuses exclusively on non-interacting dark energy models.

3 Parameterization of $q(z)$ and the related models

Investigating the cosmological expansion history using only the cosmological principle—that the cosmological principle holds that the universe is homogeneous and isotropic at vast scales—is known as cosmography. [63]. Using the Hubble parameter (H_0) and the deceleration parameter (q_0), several basic properties of the universe can be specified. It is possible to develop model-independent kinematics of cosmic expansion using these parameters. The scale factor $a(t)$ can be expanded as a Taylor series in the conventional approximation around the current time t_0, which is also the universe's current age [64]. One way to express this expansion is:

$$a(t) = 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2(t - t_0)^2 + \frac{1}{3!}j_0 H_0^3(t - t_0)^3 + \frac{1}{4!}s_0 H_0^4(t - t_0)^4 + \frac{1}{5!}l_0 H_0^5(t - t_0)^5 + \cdots, \tag{12}$$

The Hubble parameter, H_0, measures velocity; the deceleration parameter, q_0, measures acceleration; the jerk parameter, j_0, measures the change in acceleration; the snap parameter, s_0; and the lerk parameter, l_0. These kinematic variables are essential for differentiating between different dark energy models and for cosmographic investigation of the universe. Cosmological characteristics like the scale factor a, cosmic redshift z, and time t can be used to characterize the evolution of the deceleration parameter. However, the text adds that a Taylor expansion will be employed to describe the deceleration parameter because its precise mathematical form is unknown[65].

$$q(x) = q_0 + q_1\left(1 - \frac{x}{x_0}\right) + q_2\left(1 - \frac{x}{x_0}\right)^2 + \cdots, \tag{13}$$

It is well known that we can get a better fit for the observational data by expanding the expansion by adding more words. Only the first two words in the expansion will be taken into consideration, though, in order to keep the model simple. $q(z)$'s parameterization is provided by [66].

$$q(z) = q_0 + \frac{q_1 z(1 + z)}{1 + z^2}. \tag{14}$$

It is possible to obtain this more appropriate form of the deceleration parameter by replacing $\frac{x}{x_0} = 1 - \frac{z(1+z)}{1+z^2}$ into Eq. (14). The Hubble parameter and the accelerating redshift are represented by the following relations:

$$H(z) = H_0(1 + z)^{1+q_0}\left(1 + z^2\right)^{q_1/2}, \tag{14}$$

4 Methodology

We have used a large dataset, including the most recent (BAO) observations from several observational investigations, in our exploration of the universe's late-time cosmic acceleration. The sloan digital sky survey (SDSS) is one of the several sources from which these data points were collected. Other significant observations from prominent surveys, including (DES) [67], the dark energy camera legacy survey (DECaLS), and the 6dF Galaxy Survey BAO (6dFGS BAO) [68], are also included in our dataset. One of the main difficulties related to the analysis of the BAO dataset is correlations between measurements released in different epochs. Mock datasets are highly helpful in addressing this problem and estimating the organized unreliability. These mock datasets are generated using cosmological parameters via N- body simulations, allowing the covariance matrices required for the accurate analysis of BAO measurements to be calculated. Since our analysis is based on the amalgamation of data from several different experiments, it becomes difficult for the exact covariance matrix detailing the interrelations among the data points to be obtained. Therefore, we have performed a covariance analysis as suggested by [69]. This is the covariance matrix for uncorrelated data points: $C_{ii} = \sigma_i^2$. We ensure the symmetry of the covariance matrix by including non-diagonal entries to take into consideration possible correlations between data points. This strategy allows us to generate positive correlations between up to twelve pairs of data points that are selected at random, which makes up over 66.6% of the dataset. The magnitudes of the selected covariance elements, represented by C_{ij}, are set as $C_{ij} = 0.5\sigma_i\sigma_j$, where σ_i and σ_j are the 1σ errors associated with data points i and j, apart. The positions of these non-diagonal elements are chosen at random. As stated in [70], we add 17 more BAO data points to the dataset in order to restrict the parameters of our cosmological models. Additionally, we use the most recent Hubble parameter measurements made utilizing the cosmic chronometers (CC) approach that contains 30 independent data points covered in [71–74]. Using the GetDist package [75] and a nested sampler as implemented in the open-source program Polychord [76], we describe our findings. The uniform distribution prior that we select has the following parameters: $q_0 \in [-1, 0]$, $q_1 \in [0., 1]$, $H_0 \in [50, 100]$, and $r_d \in [100, 200]$, Mpc. As a further prior, R19, we have included the Hubble constant measurement, which yielded $H_0 = 74.03 \pm 1.42$, $(km/s)/Mpc$ at 68% CL [77]. Because we sought to avoid a very narrow prior for r_d influencing the specification of the Hubble parameter [78] in our study, it should be noted that the prior for r_d is somewhat large. In [0.9, 1.1] we employ a prior for the ratio $r_d/r_{d,fid}$ for the fiducial cosmology. The results for the BAO and BAO + R19 are shown in Fig. 1, as well as in Table 1. As we expected, including the correlations does not significantly change the results. The difference of no correlation to 30% correlated points is some 10%, like the results found in [79]. However, in our case, r_d and the fiducial cosmology are free parameters. Thus, although this method will likely underestimate the covariance matrices, it does give a rationale for treating the points as uncorrelated. We constrain the

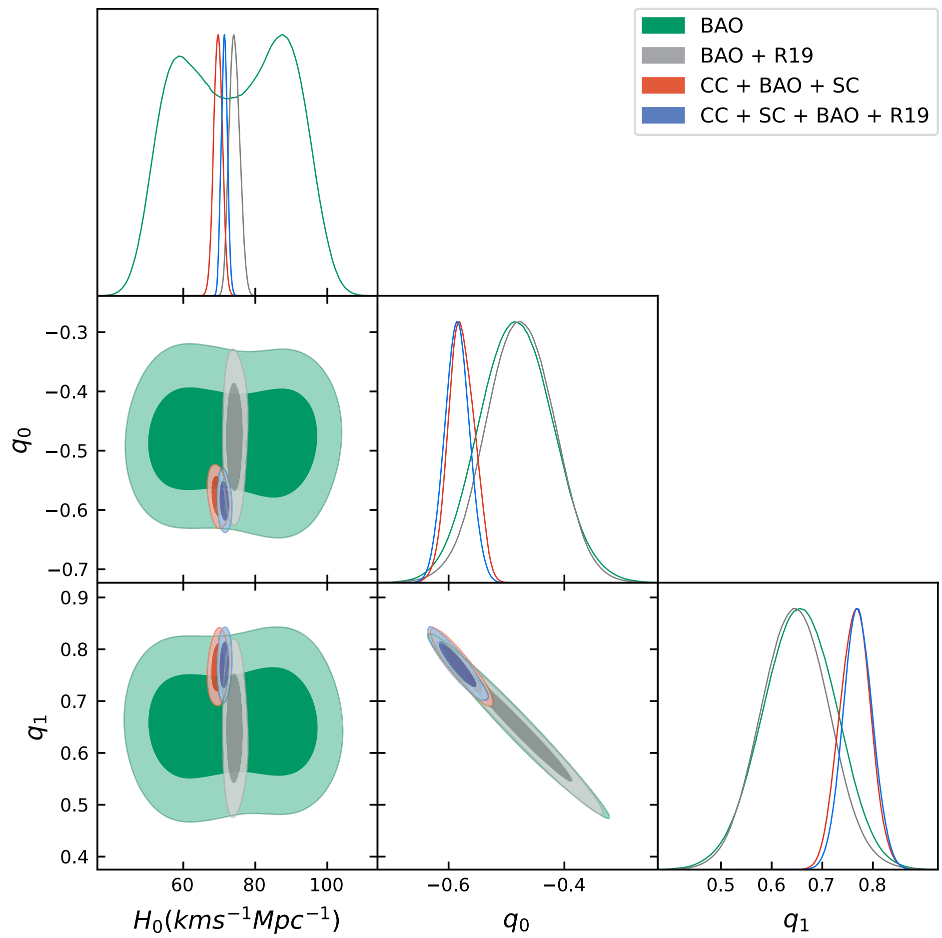

Fig. 1 Constraints on cosmological parameters derived from various observational datasets, showing the joint posterior distributions of H_0, q_0, and q_1 with 1σ and 2σ confidence levels for BAO, BAO + R19, CC + SC + BAO, and CC + SC + BAO + R19

Table 1 95% CL constraints on the cosmological parameters for the Λ CDM and our model are presented. The datasets used include: BAO alone, the BAO + CC + SC combination, and the Riess 2019 measurement as a Gaussian prior

MCMC Results

Model	Parameters	Priors	BAO	BAO+R19	CC+BAO+SC	CC+SC+BAO+R19
ΛCDM Model	H_0	[50,100]	$69.08^{+4.39}_{-6.25}$	$73.90^{+1.35}_{-2.78}$	$69.85^{+1.25}_{-2.38}$	$71.61^{+1.00}_{-1.93}$
	Ω_m	[0,1]	$0.25^{+0.02}_{-0.06}$	$0.25^{+0.02}_{-0.06}$	$0.26^{+0.01}_{-0.02}$	$0.26^{+0.01}_{-0.03}$
	Ω_Λ	[0,1]	$0.73^{+0.02}_{-0.03}$	$0.73^{+0.02}_{-0.06}$	$0.72^{+0.00}_{-0.01}$	$0.72^{+0.00}_{-0.01}$
	r_d	[100,200]	$149.50^{+10.03}_{-15.21}$	$139.44^{+2.91}_{-5.88}$	$146.54^{+2.59}_{-5.01}$	$143.29^{+2.21}_{-4.35}$
	$r_d/r_{d,fid}$	[0.9,1.1]	$1.00^{+0.06}_{-0.10}$	$0.94^{+0.02}_{-0.03}$	$0.99^{+0.01}_{-0.03}$	$0.96^{+0.01}_{-0.03}$
Model	H_0	[50,100]	$74.001^{+18.29}_{-22.57}$	$74.21^{+1.4}_{-2.3}$	$69.64^{+1.1}_{-2.3}$	$71.37^{+0.8}_{-1.4}$
	q_0	[-1,0]	$0.482^{+0.06}_{-0.11}$	$-0.475^{+0.05}_{-0.10}$	$-0.577^{+0.03}_{-0.02}$	$-0.585^{+0.02}_{-0.03}$
	q_1	[0,1]	$0.655^{+0.07}_{-0.13}$	$0.646^{+0.06}_{-0.12}$	$0.764^{+0.013}_{-0.05}$	$0.770^{+0.02}_{-0.05}$
	r_d	[100,200]	$140.88^{+30.5}_{-39.3}$	$134.62^{+3.06}_{-7.73}$	$146.46^{+2.34}_{-4.17}$	$143.17^{+1.61}_{-3.55}$
	$r_d/r_{d,fid}$	[0.9,1.1]	$0.99^{+0.06}_{-0.09}$	$0.998^{+0.06}_{-0.09}$	$1.004^{+0.06}_{-0.09}$	$0.99^{+0.07}_{-0.09}$

cosmological models not only with the BAO dataset but also with cosmic chronometers (CC) and standard candles (SC). The cosmic chronometers (CC) exploit the differential age evolutions of passive galaxies at different redshifts to directly constrain the Hubble parameter [80, 81]. This set consists of 30 uncorrelated CC measurements of $H(z)$, following [82]. For SC we employ uncorrelated measurements from the Pantheon Type Ia supernova dataset, compiled in [83] as implemented in [84], plus Quasars, as from Roberts et al[85] and Gamma-Ray Bursts [86]. We refer to the dataset containing BAO, CC, and SC as the "full" dataset.

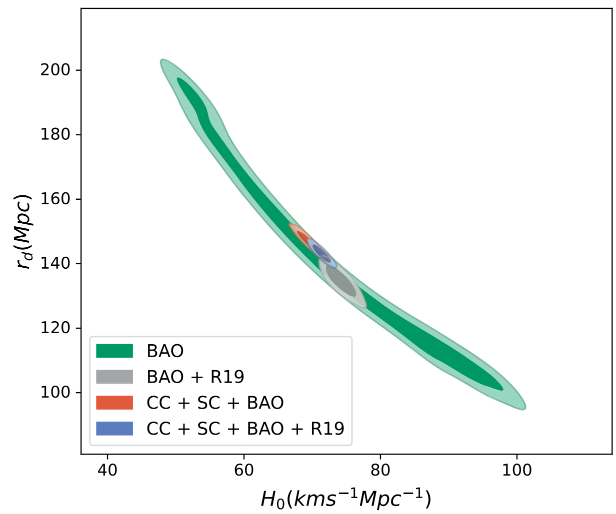

Fig. 2 Relationship between H_0 and r_d derived from various observational datasets, highlighting the constraints provided by BAO, BAO + R19, CC + SC + BAO, and CC + SC + BAO + R19

5 The baryon acoustic oscillation (BAO) scale

The BAO scale is a specific length scale observed within the distribution regarding galaxies, arising from early density fluctuations that caused acoustic waves to propagate through the primordial plasma of the universe. These fluctuations led to regions of higher and lower density. Once the universe cooled enough for atoms to form, free electrons and photons decoupled from baryons, leaving an imprint on the distribution of matter. The BAO scale, also known as the sound horizon, represents the distance sound waves traveled before the universe became transparent to radiation.

The BAO scale acts as a cosmic standard ruler, allowing astronomers to estimate distances to galaxies by measuring its angular size on the sky. This enables the study about the universe's geometry and expansion the past documentation. BAO measurements have been essential in precision cosmology, particularly in refining models of dark energy and the expansion of the universe. Surveys like the sloan digital sky survey (SDSS) have provided critical data on the BAO scale [87].

The BAO scale is discovered by the cosmic sound horizon, that depends on the speed of sound in the early universe and the expansion rate, described by the Hubble parameter $H(z)$. The sound horizon at the drag epoch is given by:

$$r_s(z_d) = \int_0^{z_d} \frac{c_s(z)}{H(z)} dz \tag{15}$$

where $c_s(z)$ is the speed of sound, and $H(z)$ is the Hubble parameter. The speed of sound is represented as:

$$c_s = \frac{1}{\sqrt{3(1+R)}} \tag{16}$$

where R is the baryon-to-photon density ratio, given by:

$$R = \frac{3}{4} \frac{\Omega_b h^2}{\Omega_\gamma h^2} \tag{17}$$

Here, Ω_b is the baryon density parameter, and Ω_γ is the photon density parameter. The BAO scale is typically measured at the drag epoch with a redshift z_d of approximately 1059.94 [88]. The BAO scale, r_d, is often quoted in megaparsecs (Mpc), with values ranging from 143.7 Mpc to 147.09 Mpc, depending on the study[89–91]. These measurements help refine cosmological models and improve our understanding of the universe's large-scale structure, dark energy, and fundamental physics.

Figure 2 illustrates the correlation between the Hubble constant (H_0) and the BAO scale (r_d) using different combinations of observational datasets. The results show how the inclusion of R19, CC, and SC data impacts the constraints on H_0 and r_d. Notably, the combined dataset (CC + SC + BAO + R19) provides tighter constraints, emphasizing the importance of utilizing multiple datasets to refine cosmological parameters. This analysis demonstrates the model's robustness and its consistency with observational data across different redshifts.

6 Comparing our model to CC dataset and ΛCDM with implications

To assess the alignment of our cosmological models with the observed data, we compared their predictions to the current cosmological datasets (denoted as CC datasets) in a comparative study. As a baseline, we also entailed the broadly recognized ΛCDM model,

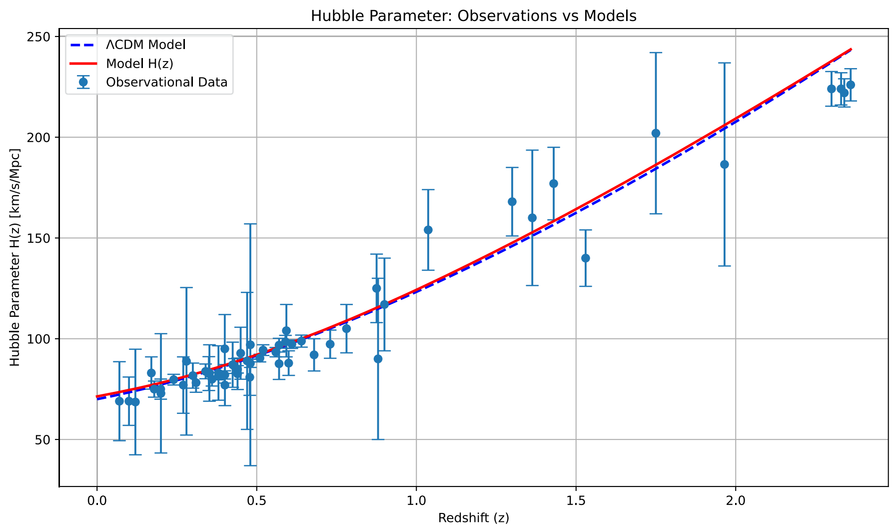

Fig. 3 Hubble parameter $H(z)$ as a function of redshift, comparing the predictions of our model with observational data and the standard ΛCDM model, along with 1σ and 2σ confidence levels

which uses the cosmological parameters $\Omega_{m0} = 0.3$ supposedly the matter density and $\Omega_{\Lambda 0} = 0.7$ for the dark energy density. The results of this comparison are illustrated in the figure. The figure shows that our model performs well in matching the CC dataset at lower redshifts ($z < 0.5$). The agreement suggests that our model can accurately describe the evolution of the universe at these redshifts. However, the figure also reveals noticeable discrepancies between our model's predictions and the CC dataset at higher redshifts. These discrepancies become more significant as the redshift increases, particularly beyond $z > 0.5$. At higher redshifts, the differences between our model and the CC dataset are important for cosmology, as they indicate a mismatch in the model's ability to explain the state of the universe at these early stages. This implies that although our model can describe the universe well at lower redshifts, it encounters test when extending to higher redshifts. Our study indicates that for redshifts $z < 0.5$, our model is in good agreement with the ΛCDM model. However, at redshifts higher than $z > 0.5$, our model significantly deviates from the ΛCDM model. The alignment at lower redshifts put forward that both models are in reasonable agreement when describing the universe's dynamics at present-day or near-present stages. Still, the deviation at higher redshifts is concerning, suggesting that the assumptions or parameters used in both models may not fully capture the universe's behavior at early times. We conclude that these discrepancies point to the need for further refinement or possible extensions of our models. It is crucial to explore ways to better account for the universe's behavior across a broader range of redshifts, particularly at higher redshifts where our current models show a breakdown in agreement with observed data. This will help in refining cosmological models and improving their ability to describe the full range of the universe's evolution.

Figure 3 depicts the evolution of the Hubble parameter $H(z)$ as a function of redshift, comparing the predictions of our scalar field model with the standard ΛCDM model and observational data. The plot highlights the alignment of the proposed model with observational data at lower redshifts, while revealing noticeable deviations at higher redshifts.

7 Cosmography parameters

Cosmography is an invaluable tool in modern cosmology, offering a detailed and comprehensive framework to study the expansion dynamics of the universe [92]. It provides a means of analyzing the observed data in conjunction with theoretical models, offering profound insights into the behavior of the universe across different epochs. In this work, we use cosmography to enhance our perception of cosmic dynamics relatively empirical data with theoretical models such as the BA model, JBP model, and the well-established ΛCDM model. This point of view is crucial for exploring the evolution of the universe across distinct redshifts, thereby supposing us to probe diverse cosmic periods. By doing so, we aim to intensify our knowledge of the past, present, and future of the universe, potentially uncovering new aspects of cosmic history. A central aspect of cosmography is the deceleration parameter (DP), which plays a significant role in understanding the expansion dynamics of the universe [92]. This parameter was first introduced by Edwin Hubble, and it is mathematically defined as: $q = -\frac{\ddot{a}}{aH^2}$, where $a(t)$ represents the scale factor of the universe as a function of time, $\dot{a}$ is the first derivative (the rate of change of the scale factor), and $\ddot{a}$ is the second derivative (the acceleration of the scale factor). The DP reveals critical information about the nature of the universe's expansion. A positive value of q suggests that the universe's expansion was slowing down in the past, which indicates that gravitational attraction from matter was dominant. A zero value of q corresponds to a "critical universe," where the expansion rate remained constant over time. A negative q indicates an accelerating expansion, a phenomenon attributed to the influence of dark energy. Beyond the deceleration parameter, the jerk parameter j provides an additional layer of understanding regarding cosmic dynamics [93]. The jerk parameter is defined as the

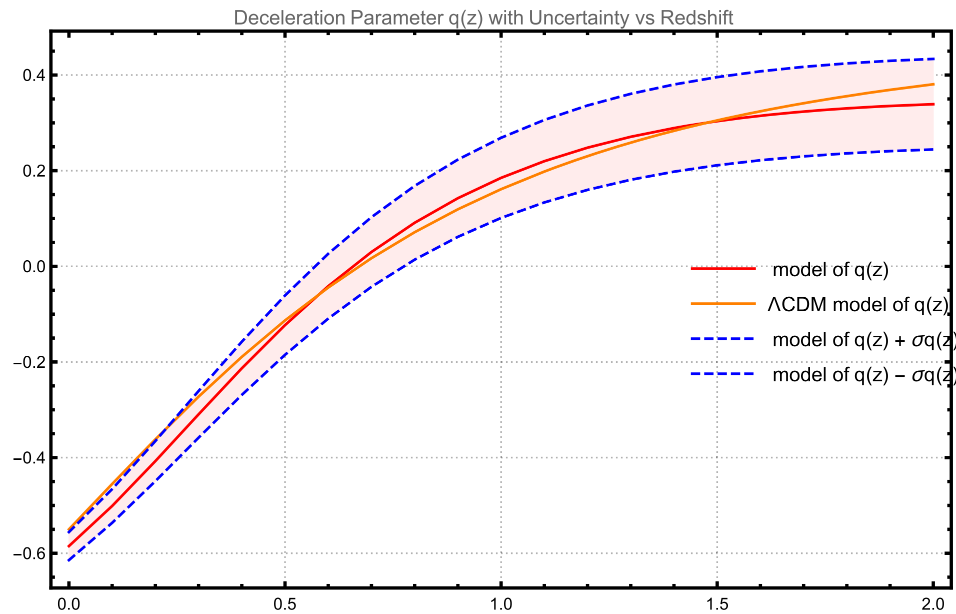

Fig. 4 Deceleration parameter $q(z)$ as a function of redshift, showing the evolution predicted by our model compared to the ΛCDM model, along with uncertainty bounds

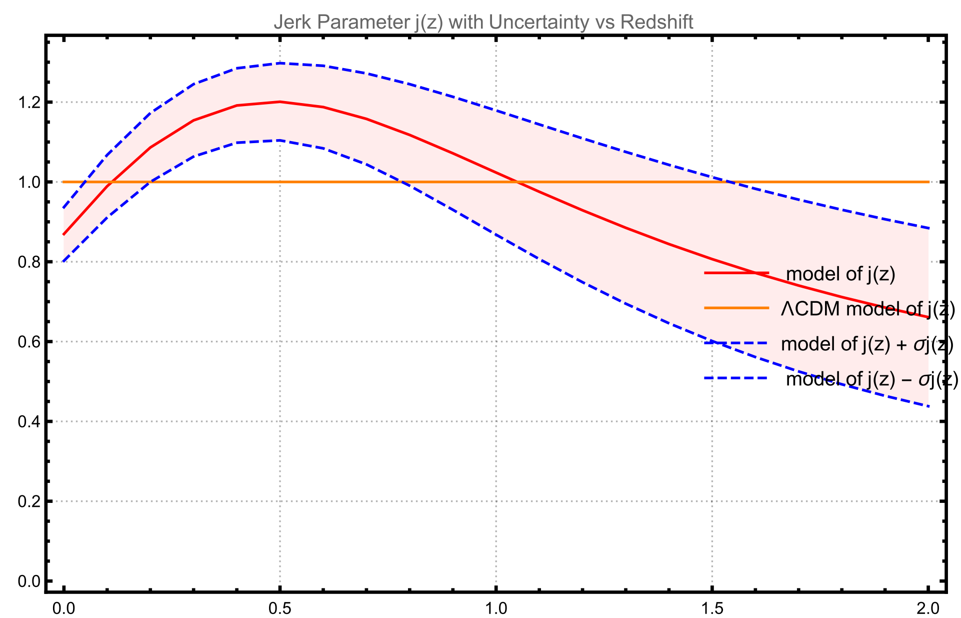

Fig. 5 Jerk parameter $j(z)$ as a function of redshift, showing the evolution predicted by our model compared to the ΛCDM model, along with uncertainty bounds

third derivative of the scale factor with respect to time: $j = \frac{1}{aH^3}\frac{d^3a}{dt^3}$. This parameter gives valuable insight into transitions in the expansion of the universe and helps us understand changes in the acceleration of cosmic expansion. It is essential for distinguishing between different models of dark energy, which may behave differently at various stages of the universe's evolution. In addition to the jerk parameter, the snap parameter, denoted by s, offers a deeper understanding of the universe's curvature and expansion. The snap parameter is the fifth time derivative of the scale factor and is given by: $s = \frac{1}{aH^4}\frac{d^4a}{dt^4}$. In the context of the standard ΛCDM model, where $j = 1$, the snap parameter can be expressed as: $s = -(2 + 3q)$ This relationship between q and s helps measure deviations from the expected behaviors predicted by the ΛCDM model and contributes to a more nuanced understanding of cosmic evolution dynamics. Together, these cosmographic parameters—deceleration, jerk, and snap—form a comprehensive set of tools in observational cosmology. They shed light on critical aspects of the universe, including the roles of dark matter and dark energy, and help determine the overall geometry of the universe. These parameters, by providing detailed information about the universe's expansion and its underlying components, are essential for refining cosmological models and enhancing our understanding of cosmic evolution.

Figure 4 demonstrates the evolution of the deceleration parameter $q(z)$ as a function of redshift, providing a direct insight into the transition from deceleration to acceleration in cosmic expansion. The plot compares the predictions of our scalar field model

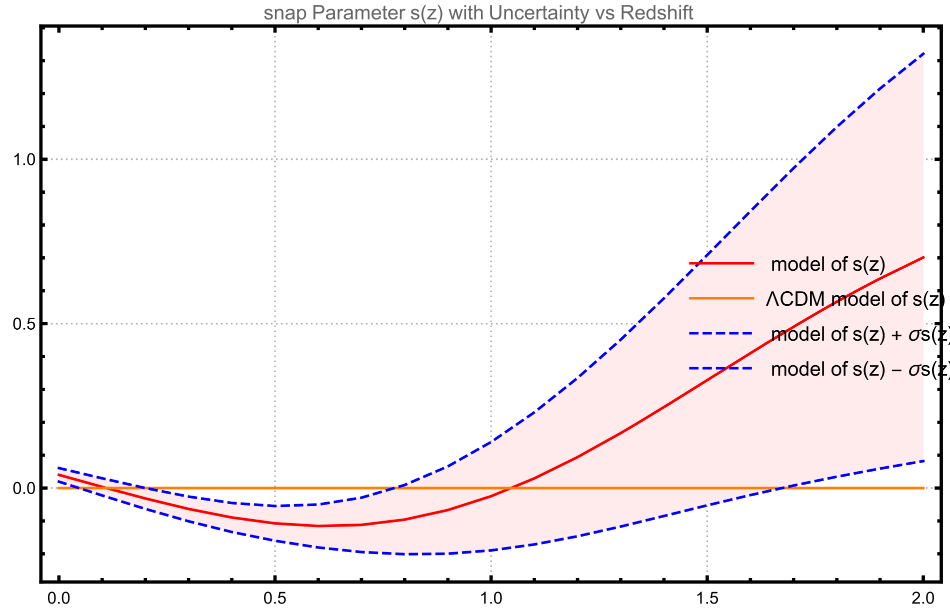

Fig. 6 Snap parameter ($s(z)$) as a function of redshift, showing the evolution predicted by our model compared to the ΛCDM model, along with uncertainty bounds

with the ΛCDM model, highlighting the alignment at lower redshifts and slight deviations at higher redshifts. The transition redshift $z_t = 0.62$ is a key feature, indicating the epoch at which the universe shifted from decelerating to accelerating expansion. The current value of the deceleration parameter is $q_0 = -0.59$. Figure 5 illustrates the evolution of the jerk parameter $j(z)$ as a function of redshift. The plot compares our scalar field model with the standard ΛCDM model, showing consistency at lower redshifts while revealing potential deviations at higher redshifts. The jerk parameter, which measures the rate of change of cosmic acceleration, remains positive throughout the examined redshift range, confirming that the universe's expansion is not only accelerating but also evolving dynamically. Figure 6 presents the evolution of the snap parameter $s(z)$ as a function of redshift. The plot reveals a clear distinction between the predictions of our scalar field model and the ΛCDM model, particularly at higher redshifts. This distinction highlights the ability of our model to capture subtle changes in the universe's expansion behavior.

8 Physical characteristics of quintessence as a dark energy source: a cosmic evolution

Equations (8) and (9) allow us to obtain the following formulas for the energy density and pressure of the quintessence field for quintessence as a dark energy candidate:

$$M_{\rm Pl}^{-2}\rho_\phi = 3H^2 - M_{\rm Pl}^{-2}\rho_M, \tag{18}$$

$$M_{\rm Pl}^{-2}p_\phi = (2q-1)H^2, \tag{19}$$

assuming pressure–matter ($p_M = 0$). For a universe consisting of two fluids (matter and scalar field), it is supposed that matter and dark energy do not interact. Hence, $\dot{\rho}_M + 3H\rho_M = 0$ and $\dot{\rho}_\phi + 3H\rho_\phi = 0$ are resulted from their independent conservation equations. The solution to the first equation is $\rho_M = ca^{-3} = c(1+z)^3$, where c is an integration constant. Given the density parameter Ω and the current values of $z = 0$, we find that $c = 3M_{\rm Pl}^2 H_0^2 \Omega_{M0}$, which implies that $\rho_M = 3M_{\rm Pl}^2 H_0^2 \Omega_{M0}(1+z)^3$.

8.1 Energy density and pressure of dark energy

By solving equations (18) and (19), we obtain expressions for the energy density and pressure of the quintessence field for the model:

$$M_{\rm Pl}^{-2}H_0^{-2}\rho_\phi = 3(1+z)^{2(1+q_0)}\left(1+z^2\right)^{q_1},$$

$$M_{\rm Pl}^{-2}H_0^{-2}p_\phi = \left[(2q_0-1) + \frac{2q_1 z(1+z)}{1+z^2}\right](1+z)^{2(1+q_0)}\left(1+z^2\right)^{q_1}.$$

Figures 7, 8, and 9 collectively explore the physical characteristics of the scalar field as a dark energy candidate. Figure 7 shows the evolution of the normalized energy density (ρ_ϕ), which decreases with redshift, indicating a dominant contribution to the universe's energy budget at late times. Figure 8 depicts the normalized pressure (p_ϕ), which remains negative across all redshifts, driving the accelerated expansion of the universe. Figure 9 highlights the evolution of the equation of state parameter (ω_ϕ), which lies within the quintessence region ($-1 < \omega_\phi < 0$) and approaches a value near -1 at present with the current value of $\omega_{\phi 0} = -1.01$,

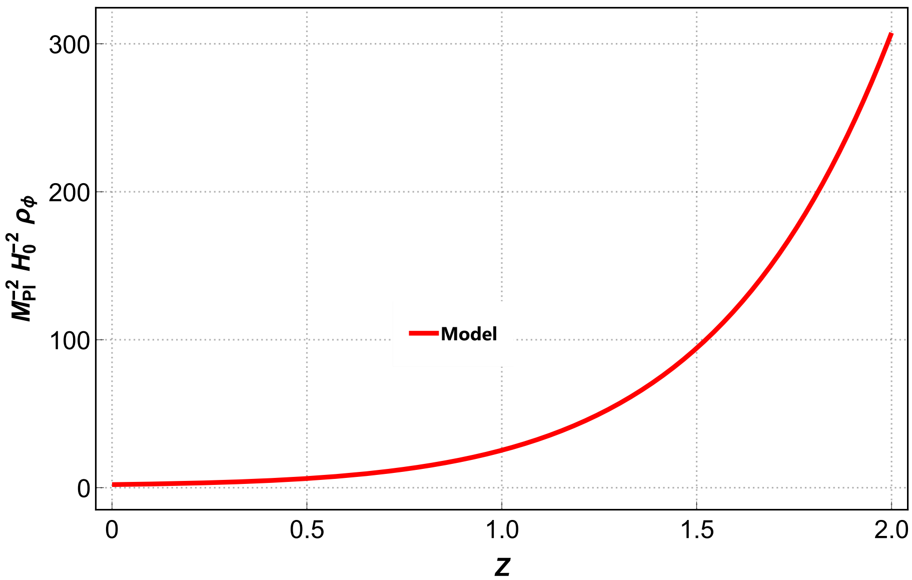

Fig. 7 Normalized energy density of the scalar field (ρ_ϕ) as a function of redshift, showing its evolution using the best-fit parameters derived from the observational data

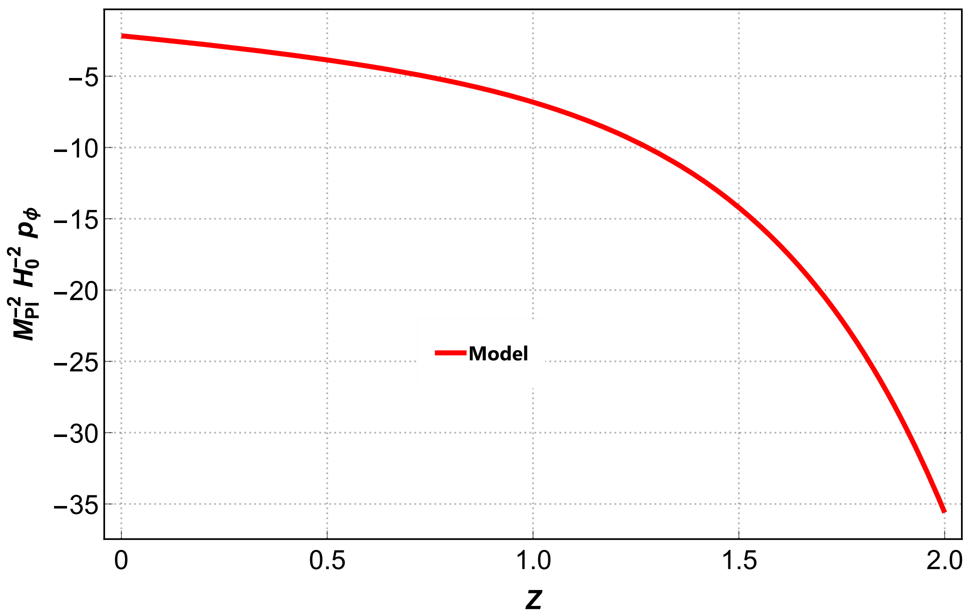

Fig. 8 Normalized pressure of the scalar field (p_ϕ) as a function of redshift, illustrating the negative pressure driving the accelerated expansion of the universe

suggesting behavior similar to a cosmological constant. Together, these figures emphasize the consistency of the scalar field model with observational data and its ability to explain the late-time dynamics of the universe.

9 Conclusion

In conclusion, this study highlights the significant role that the dark energy model in a scalar field plays to address the inconsistencies between early and late observation of the universe. With careful analysis of recent observational data, including $\mathcal{CC}$, $\mathcal{SC}$, $\mathcal{BAO}$, $\mathcal{R}19$ and combining all measurements, we have discussed the transition phases of key cosmological parameters relevant to understanding the evolution of the cosmic expansion. The deceleration parameter's transition from a decelerating phase to an accelerating phase, with the current value of $q_0 = -0.59$ and a transition redshift at $z_t = 0.62$, , points to a shift in the dominant force driving the expansion of the universe. This transition is accompanied by the positive jerk parameter $j_0 = 0.89$, which states that the universe is accelerating, and its acceleration is not constant but is accelerating itself. The value of the snap parameter being positive, $s_0 = 0.05$, means that this acceleration is not only happening but also that it is growing in acceleration. So, the expansion of the universe is not only dynamical but is deepening in dynamics. Another important factor brought to light in this study is the equation of state parameter, which is, $\omega_{\phi 0} = -1.01$ and thus within the quintessence region. It seems that dark energy does behave in a way

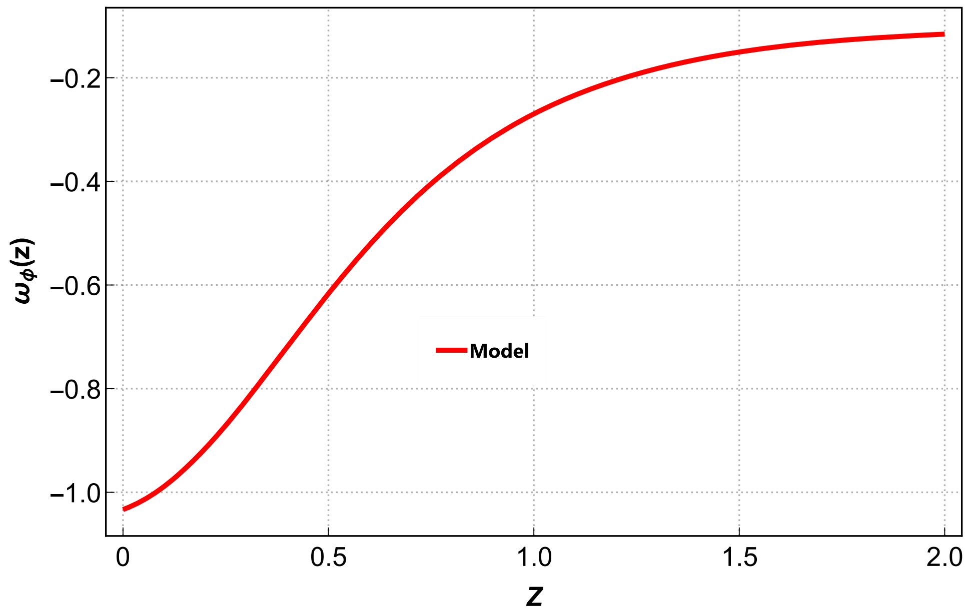

Fig. 9 Equation of state parameter (ω_ϕ) for the scalar field as a function of redshift, demonstrating its evolution and current value within the quintessence region

which is not a cosmological constant; there is some small evolution in the equation of state with time across the history of the cosmos—a critical requirement for resolving the inconsistencies found between measurements made of the early and late universe. The presence of positive energy density coupled with entirely negative pressure underpins the characteristics of scalar field dark energy, where the evolving dynamics of $\omega_{\phi 0}$ additionally, other parameters allow for a coherent model that addresses both the early universe observations (matter and radiation dominated) as well as late-time acceleration (dark energy dominated). This result confirms scalar field models, dark energy with quintessence-like behavior, as being consistent with the most updated data from the observational regions and suggesting very solid premises for further study. The ongoing tension between different observational timescales underscores the need for further refinements in our cosmological models. As future data and theoretical advances continue to unravel, this study serves as an important step in bridging the gap between the early and late universe, casting new light on the role of scalar field dark energy and its implications for our understanding of cosmic evolution.

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Data Availability No data were associated in the manuscript.

Declarations

Conflict of interests All authors declare that they have no conflict of interest.

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