A flux-scaling scenario for moduli stabilization and axion inflation in string theory

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We describe a type IIB string scenario in which tree-level moduli stabilization via geometric and non-geometric fluxes is achieved. We present stable non-supersymmetric vacua with all moduli fixed except for some massless axions. The moduli vacuum expectation values and their masses feature a specific scaling with the fluxes thereby allowing for parametric control. We discuss some phenomenological aspects of our scenario and explain how it provides an interesting framework for realizing inflation in string theory.

Keywords: Flux compactification; moduli stabilization; axion inflation.

1. Introduction

In string compactifications there are generically many moduli, namely massless scalars with a flat potential. Clearly, these fields must acquire a mass to avoid unobserved long-range forces. Moreover, phenomenological quantities such as gauge coupling constants depend on the moduli vevs. Thus, moduli stabilization is a crucial link in connecting string theory and low-energy phenomena. In the KKLT and large volume scenarios the axio-dilaton and complex structure moduli obtain masses at tree-level from a flux-induced potential while the Kähler moduli do it from non-perturbative contributions[1,2]. As a result, the masses of the latter are exponentially smaller. In our scenario the motivation is to have all moduli masses of the same order so that moduli stabilization and single-field inflation can be naturally combined. This requires all moduli masses to be greater than the Hubble scale and the mass of the inflaton Θ. Besides, the string and Kaluza-Klein (KK) scales must be above all these scales to justify working with an effective supergravity action. Therefore, we want to develop a string scenario that guarantees the hierarchy $M_{\rm Pl} > M_{\rm s} > M_{\rm KK} > M_{\rm mod} > H_{\rm inf} > M_{\Theta}$. To have all moduli masses of the same order it is logical to fix them at tree-level and this can be achieved taking into account non-geometric fluxes. The desired hierarchy of scales can be ensured thanks to flux scaling, which means that the moduli masses are determined by adjustable ratios of product of fluxes. Below we will briefly describe the flux-scaling scenario, discuss some phenomenological implications, and consider the aplication to inflation. More details and references can be consulted in Refs. 3, 4.

2. Flux-scaling scenario

We will work in the framework of $\mathcal{N} = 1$ type IIB Calabi-Yau (CY) orientifolds with O3- and O7-planes. The CY manifold $\mathcal{M}$ is characterized by the Kähler form J

and the holomorphic 3-form Ω. Under the orientifold projection the harmonic (1,1) and (1,2) forms can be even or odd and are counted by Hodge numbers $h^{1,1}_{\pm}$ and $h^{1,2}_{\pm}$ respectively. We will assume that $h^{1,2}_{+} = 0$. The low-energy effective theory has been obtained in Ref. 5. The moduli sector comprises the axio-dilaton S, even T_α and odd G^a Kähler moduli, $\alpha = 1, \ldots, h^{1,1}_{+}$, $a = 1, \ldots, h^{1,1}_{-}$, plus complex structure moduli U^i, $i = 1, \ldots, h^{1,2}_{-}$. More precisely, $S = e^{-\phi} - i\, C^{(0)}$, where ϕ is the dilaton and $C^{(0)}$ the R-R 0-form, $G^a = S\, b^a + i\, c^a$, and

$$T_\alpha = \frac{1}{2}\, \kappa_{\alpha\beta\gamma} t^\beta t^\gamma + i\, \rho_\alpha - \frac{i}{2}\, \kappa_{\alpha ab}\, c^a b^b - \frac{1}{4}\, e^\phi\, \kappa_{\alpha ab}\, G^a (G + \overline{G})^b\, . \tag{1}$$

Here t^α, ρ_α, c^a and b^a are the components of J, $C^{(4)}$, $C^{(2)}$ and B in the internal space. Besides, $\kappa_{\alpha\beta\gamma}$ and $\kappa_{\alpha ab}$ are triple intersection numbers of $\mathcal{M}$. Recall also that the 3-form Ω can be expanded in a symplectic basis as $\Omega = X^\lambda \alpha_\lambda - \mathcal{F}_\lambda \beta^\lambda$, where $\lambda = 0, \ldots, h^{1,2}_{-}$. The periods $\mathcal{F}_\lambda = \frac{\partial \mathcal{F}}{\partial X^\lambda}$ follow from a holomorphic prepotential that in the large complex structure limit has the form $\mathcal{F} = d_{ijk} \frac{X^i X^j X^k}{X^0}$. The complex-structure moduli are given by $U^i = -i\, \frac{X^i}{X^0}$.

The starting point for the moduli stabilization analysis is the F-term scalar potential of the 4-dimensional $\mathcal{N} = 1$ theory, namely

$$V_F = \frac{M_{\rm Pl}^4}{4\pi}\, e^K \left(K^{I\overline{J}} D_I W D_{\overline{J}} \overline{W} - 3 \left|W\right|^2 \right), \tag{2}$$

where $K_{I\overline{J}} = \partial_I \partial_{\overline{J}} K$, $D_I W = \partial_I W + (\partial_I K)\, W$, and the indices run over the moduli fields. At tree-level in the large-volume limit the Kähler potential is given by[5]

$$K = -\log\left(-i \int_{\mathcal{M}} \Omega \wedge \overline{\Omega} \right) - \log\bigl(S + \overline{S}\bigr) - 2 \log \mathcal{V}\, , \tag{3}$$

where $\mathcal{V} = \frac{1}{3!}\, \kappa_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma$ denotes the volume of the CY 3-fold in Einstein frame.

The superpotential is generated by fluxes. In addition to R-R and NS-NS 3-form fluxes $\mathfrak{F}$ and H we also consider the geometric and non-geometric fluxes F, Q and R required by T-duality[6]. The generalized superpotential can be written as[7–9]

$$W = \int_{\mathcal{M}} \left(\mathfrak{F} + \mathcal{D}\, \Phi_c^{\rm ev} \right) \wedge \Omega\, , \tag{4}$$

where $\Phi_c^{\rm ev} = i\, S - i\, G^a\, \omega_a - i\, T_\alpha\, \sigma^\alpha$, with ω_a and σ^α 2- and 4-forms in $\mathcal{M}$. The twisted differential $\mathcal{D}$ is defined as $\mathcal{D} = d - H\wedge\, - F \circ\, - Q \bullet\, - R \llcorner$. The operators $F\circ$, $Q\bullet$ and $R\llcorner$ acting on a p-form give respectively a $(p+1)$-, $(p-1)$- and $(p-3)$-form. For instance, acting on a 4-form $\mathcal{D}\sigma^\alpha = -\tilde{q}^{\lambda\alpha} \alpha_\lambda + q_\lambda{}^\alpha \beta^\lambda$, so that $q_\lambda{}^\alpha$ and $\tilde{q}^{\lambda\alpha}$ are the components of the non-geometric Q-flux. Here we are following the approach of Refs. 10, 11 to treat the generalized fluxes in the CY orientifold. There are similar parameters h_λ, $\tilde{h}^\lambda$, $f_{\lambda a}$ and $\tilde{f}^\lambda{}_a$, encoding H and F. The R-flux is eliminated by the orientifold projection and will not appear in W. The fluxes in $\mathcal{D}$ are constrained by Bianchi identities arising from the nilpotency condition $\mathcal{D}^2 = 0$. On the other hand, the R-R flux parameters $\mathfrak{f}_\lambda$ and $\tilde{\mathfrak{f}}^\lambda$ enter in tadpole cancellation conditions.

Expanding and integrating we obtain the superpotential[4]

$$W = -\left(\mathfrak{f}_\lambda X^\lambda - \tilde{\mathfrak{f}}^\lambda \mathcal{F}_\lambda\right) + i S\left(h_\lambda X^\lambda - \tilde{h}^\lambda \mathcal{F}_\lambda\right) \\ + i G^a\left(f_{\lambda a} X^\lambda - \tilde{f}^\lambda{}_a \mathcal{F}_\lambda\right) - i T_\alpha\left(q_\lambda{}^\alpha X^\lambda - \tilde{q}^{\lambda\alpha} \mathcal{F}_\lambda\right). \tag{5}$$

Notice that the generalized fluxes give terms depending on the Kähler moduli. This opens up the possibility to stabilize them at tree-level.

The task is to look for vacua of the flux-induced scalar potential with the desired properties. In our conventions the imaginary parts of the moduli are axions that do not enter in the Kähler potential. The real parts, dubbed saxions, determine the string coupling and the internal volume. We then want to fix them in a perturbative regime of weak coupling and large radius. For the axions we instead allow that some remain massless at a first stage to become massive after turning on additional fluxes. It is known that supersymmetric minima with unconstrained axions have tachyons[12]. We will then search for stable non-supersymmetric minima. Moreover, we demand that moduli vevs and masses are parametrically controlled by adjusting fluxes. The mass of the lightest massive axion, which is the inflaton candidate, is also parametrically or numerically controlled. Finally, the moduli masses have to be smaller than the string and KK scales. We found several models where these properties are realized.

To exemplify the flux-scaling vacua consider a simple model with $h_-^{2,1} = 1$ and $h_+^{1,1} = 1$, for which the Kähler potential in the large complex structure limit reads

$$K = -3\log(T + \overline{T}) - \log(S + \overline{S}) - 3\log\left(U + \overline{U}\right). \tag{6}$$

The underlying geometry can be seen as an isotropic six-torus. We will use the notation $S = s + ic$, $T = \tau + i\rho$ and $U = v + iu$. For the flux superpotential we take

$$W = -\mathfrak{f} - 3\tilde{\mathfrak{f}}U^2 - hUS - qUT. \tag{7}$$

In our conventions the fluxes are quantized and in this example they are unconstrained by Bianchi identities. Analyzing the resulting scalar potential we find a non-supersymmetric tachyon-free AdS minimum with axions fixed at $u = 0$ and $(hc + q\rho) = 0$, whereas the vevs of the saxions enjoy flux scaling of the form

$$\tau = -15\, v\, \frac{\tilde{\mathfrak{f}}}{q}, \quad s = -12\, v\, \frac{\tilde{\mathfrak{f}}}{h}, \quad v^2 = \frac{1}{3 \cdot 10^{\frac{1}{2}}}\, \frac{\mathfrak{f}}{\tilde{\mathfrak{f}}}. \tag{8}$$

As in other examples, the scaling can be inferred from the superpotential. Indeed note that all terms in W are proportional to $\mathfrak{f}$. For $h, q < 0 < \mathfrak{f}, \tilde{\mathfrak{f}}$, all vevs are positive. Besides, the flux $\mathfrak{f}$ does not contribute to any of the tadpoles so it can be large. Hence, by choosing $\mathfrak{f} \gg \tilde{\mathfrak{f}}, h, q \sim O(1)$, we can ensure that all moduli are fixed in the perturbative regime. The moduli masses in the canonically normalized basis turn out to be $M^2_{\text{mod},i} = \mu_i\, \frac{hq^3}{\mathfrak{f}^{\frac{3}{2}}\tilde{\mathfrak{f}}^{\frac{1}{2}}}\, \frac{M^2_{\text{Pl}}}{4\pi \cdot 2^7}$ with numerical values $\mu \approx \left(2.1, 0.37, 0.25\,; 1.3, 0.013, 0\right)$. The first (last) three eigenstates are saxions (axions). The massless mode is the axionic combination $(qc - h\rho)$. Observe that the lightest massive mode is axionic, and although not parametrically light, its mass is numerically light. We also find the ratios $\frac{M_{\rm mod}^2}{M_{\rm KK}^2} \sim hq\left(\frac{\tilde{\mathfrak{f}}}{\mathfrak{f}}\right)^{\frac{1}{2}}$ and $\frac{M_{\rm s}^2}{M_{\rm KK}^2} = 62.5\left(\frac{h}{q}\right)^{\frac{1}{2}}$. Therefore, for this model we can achieve a controlled hierarchy of mass scales $M_{\rm Pl} \underset{p}{\gtrsim} M_{\rm s} \underset{p}{\gtrsim} M_{\rm KK} \underset{p}{\gtrsim} M_{\rm mod}$.

We constructed and analyzed several models with non-supersymmetric flux-scaling AdS extrema. A characteristic feature is that for n moduli, $n+1$ fluxes in the superpotential must be turned on. In models with a larger number of Kähler moduli the extrema often have tachyons that can be stabilized by adding a D-term potential $V_D = \frac{M_{\rm Pl}^4}{2{\rm Re}(f)}\xi^2$ due to a stack of N D7-branes wrapping some internal 4-cycle Σ and equipped with some $U(1)$ gauge flux. The D-term potential depends on the Kähler moduli through the gauge kinetic function f and the Fayet-Iliopoulos (FI) term of the $U(1)$, given by $\xi = \frac{1}{\mathcal{V}}\int_\Sigma J \wedge c_1(L)$, where $c_1(L)$ is the first Chern class of the $U(1)$ line bundle. By virtue of the Freed-Witten anomaly cancellation conditions the FI vanishes at the AdS supersymmetric minimum but also at the non-supersymmetric extremum with same ratio of moduli vevs. Thus, the extremum is not shifted but the mass of the Kähler tachyon is uplifted. We also studied soft supersymmetry breaking masses in a MSSM realized on D7-branes. For instance, since generically the F-terms are not zero, gaugino masses $M_a = \frac{1}{2}\big({\rm Re}\, f_a\big)^{-1} F^i\partial_i f_a$ are typically of the same order of $M_{\rm mod} \sim 10^{14}\,{\rm GeV}$. However, in a particular toy model we arranged to have $F^T = 0$ and then M_a, being generated by α' corrections, can be lowered. Concerning uplift of the cosmological constant, in principle it can be achieved by including a potential due to anti D3-branes but we could only find examples with unrealistic values of the parameters involved. More recently, by adding an extra anti D3-brane contribution to the scalar potential, we were indeed able to obtain Minkowski and dS vacua, although not continuously connected to the initial AdS ones[13]. One persistent problem with the models is that it is not always possible to attain moduli masses lower than the KK scale.

3. Inflation

The current limit on the ratio of tensor-to-scalar perturbations is $r < 0.113$[14]. We also know that for $r > 0.01$ the Lyth bound implies that the inflaton Θ rolls over trans-Planckian field values[15]. Thus, it is important to study large-field inflation in the context of string theory which is purported to be UV complete. In particular, string theory should explain how perturbative corrections to the inflaton potential can be suppressed. A compelling mechanism is the shift symmetry of axions that generically appear in string compactifications. The symmetry has to be broken and it is interesting to do it by fluxes as in the proposal of F-term axion monodromy inflation[16–18]. Turning on fluxes can also stabilize other moduli and break supersymmetry. A novel feature in our models is that the axions from Kähler moduli can get a potential induced by non-geometric fluxes. We will consider two possibilities for the inflaton. It can be a massless axion that becomes massive, as proposed in Refs. 19, 20, or it can be a lightest axion from the beginning.

In the first approach the idea is to initially fix most moduli by turning on a particular set of fluxes that leaves a massless axion which is then the inflaton candidate. At a second step we rescale the original fluxes and introduce secondary ones to generate a potential for the inflaton. Schematically the superpotential has the form $W = \lambda W_0 + f_{\rm ax}\Delta W$. By choosing $\lambda \gg f_{\rm ax}$ it is possible to make $M_\Theta < M_{\rm mod}$. However it is then difficult to keep $M_{\rm KK} > M_{\rm mod}$.

In the second approach the inflaton is the lightest axion already present and we take into account the backreaction of the massive moduli during the slow roll of the inflaton[21]. We worked out a toy example with only S and T moduli and particular flux potential given by

$$V = \lambda^2 \left(\frac{(hs+\tilde{\mathfrak{f}})^2}{16 s \tau^3} - \frac{6hqs - 2q\tilde{\mathfrak{f}}}{16 s \tau^2} - \frac{5q^2}{48 s \tau} \right) + \frac{\theta^2}{16 s \tau^3}\,, \tag{9}$$

where $\theta = hc + q\rho$. When $\lambda = 1$ this potential is due to $W = -i\tilde{\mathfrak{f}} + ihS + iqT$, but the axion θ is not the lightest mode. Taking λ large makes the axion lighter than the saxions. Integrating out the heavy saxions, and adding an uplift to Minkowski, we find the backreacted potential shown in Fig. 1 for $\lambda = 10$. For small values of θ the potential is quadratic, for large values it reaches a plateau, and in the intermediate region it is linear. For $\theta/\lambda \gg \tilde{\mathfrak{f}}$, in terms of the inflaton with canonical kinetic energy, the potential is actually of Starobinsky type, namely

$$V_{\rm back}(\Theta) = \frac{25}{216}\,\frac{h\,q^3\,\lambda^2}{\tilde{\mathfrak{f}}^2}\left(1 - e^{-\gamma\,\Theta}\right), \tag{10}$$

where $\gamma^2 = 28/(14+5\lambda^2)$. To determine how the value of the scalar-to-tensor ratio varies with λ requires a numerical analysis[3]. For instance, it is found that $r \sim 0.08$ for $\lambda = 20$.

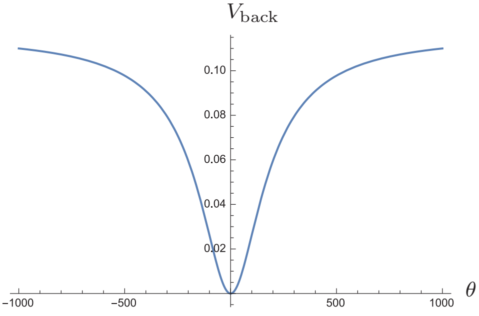

Fig. 1. Backreacted potential for $h = 1\,, q = 1\,, \tilde{\mathfrak{f}} = 10$ and $\lambda = 10$.

4. Final comments

In this work we have elaborated on a large scale scenario of moduli stabilization which includes non-geometric fluxes in order to fix all closed string moduli at tree-level. We were able to construct non-supersymmetric non-tachyonic anti de Sitter vacua characterized by a definite scaling of vevs and masses with the fluxes. In this way we gained parametric control over these quantities, thereby allowing to stabilize the moduli in their perturbative regime while keeping their masses below the string and KK scales.

We discussed some phenomenological questions such as soft supersymmetry breaking masses, but the main interest resides in aplications to inflation. We showed that the flux-scaling vacua provide a natural set-up for F-term axion monodromy inflation. An uplift to Minkowski or de Sitter was assumed but later we found proper mechanisms by adding an anti D3-brane or a D-term containing geometric and non-geometric fluxes[13].

The uplift of the 4-dimensional effective theory with non-geometric fluxes to a full solution of string theory in 10 dimensions is an open question. Some progress in this direction is the recent derivation of the scalar potential with non-geometric fluxes from dimensional reduction of double field theory on a Calabi-Yau 3-fold[22].

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