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Neutrinoless double beta decay theory

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Abstract. Within left-right symmetric model a generalization of the Majorana neutrino mass for the case of light and and heavy neutrinos is introduced and analyzed. Further, current status of calculation of the neutrinoless double beta decay matrix elements is shortly reviewed. An important connection between them and matrix element of double Gamow-Teller operator is established. A new way of fixing quenching of axial-vector coupling constant g_A is presented.

1. Introduction

The smallness of the neutrino masses suggests that they have a different origin with respect to other Standard Model particles and that neutrinos can be Majorana particles, which are their own antiparticles.

A distinctive signature of the Majorana nature of neutrino masses is the violation of lepton number by two units ($|\Delta L| = 2$), which manifests itself in neutrinoless double-beta decay ($0\nu\beta\beta$-decay) [1]

$$(A, Z) \to (A, Z + 2) + 2e^-, \tag{1}$$

and other processes like nuclear muon-to-positron conversion, or rare meson decays such as $K^+ \to \pi^- e^+ e^+$. However, the $0\nu\beta\beta$-decay is by far the most sensitive laboratory probe of total lepton number violation (LNV).

2. Particle physics aspects

2.1. Majorana neutrino mass

Usually, it is assumed that the conventional light neutrino exchange mechanism generated by left-handed V-A weak currents is the dominant mechanism of the $0\nu\beta\beta$-decay. It is the case when LNV scale is the GUT scale. The inverse half-life of the $0\nu\beta\beta$-decay takes the form [1]

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu}(g^{\rm eff}_A)^4 \left|M^{0\nu}_\nu\right|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2, \tag{2}$$

where $G^{0\nu}$, $g^{\rm eff}_A$ and $M^{0\nu}$ represent the known phase-space factor [3], the effective axial-vector coupling constant and the nuclear matrix element (NME) of the process, respectively. m_e is the mass of electron. The ultimate goal of the search for $0\nu\beta\beta$-decay is to determine the Majorana neutrino mass

$$m_{\beta\beta} = \left|\sum_{j=1}^{3} U^2_{ej} m_j\right|. \tag{3}$$

Here, U_{ej} and m_j ($j = 1, 2, 3$) are elements of Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix and masses of neutrinos, respectively. To deduce the value of $m_{\beta\beta}$ from a non-zero $0\nu\beta\beta$-decay rate measurement, $M^{0\nu}_{\nu}$ and $g^{\rm eff}_A$ have to be reliably calculated by the tools of nuclear structure theory.

The value of $m_{\beta\beta}$ can be evaluated with help of neutrino oscillation parameters by making an assumption about the mass of lightest neutrino and by choosing a type of spectrum (normal or inverted) and values of CP violating Majorana phases. In future experiments, a sensitivity a few tens of meV to $m_{\beta\beta}$ is planned to be reached. This is the region of the inverted hierarchy of neutrino masses. In the case of the normal mass hierarchy $m_{\beta\beta}$ is too small, a few meV, to be probed in the $0\nu\beta\beta$-decay experiments of the next generation [1].

2.2. Majorana neutrino mass for light and heavy neutrinos

Within left-right symmetric models, in which small neutrino masses naturally arise in the see-saw mechanism, the Majorana neutrino mass mechanisms of the $0\nu\beta\beta$-decay are considered [2].

The left-handed ν_{eL} and right-handed ν_{eR} weak eigenstate electron neutrinos are expressed as superpositions of the light and heavy mass eigenstate Majorana neutrinos ν_j and N_j as follows:

$$\nu_{eL} = \sum_{j=1}^{3} \left(U_{ej}\nu_j + S_{ej} N^C_j \right), \qquad \nu_{eR} = \sum_{j=1}^{3} \left(T^*_{ej}\nu^C_j + V^*_{ej} N_j \right). \tag{4}$$

The 3×3 block matrices in flavor space U, S, T, V form a 6×6 unitary neutrino mixing matrix, which diagonalizes the general 6×6 Dirac-Majorana neutrino mass matrix in the Lagrangian. As a result one ends up with 3 light m_i (i=1,2 and 3) and 3 heavy M_i (i=1,2 and 3) neutrino masses.

The inverse $0\nu\beta\beta$-decay half-life can be written as [4]

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu} (g^{\rm eff}_A)^4 \left| M^{0\nu}_{\nu} \right|^2 \left(\frac{m^{\nu N}_{\beta\beta}}{m_e} \right)^2 , \tag{5}$$

where the Majorana neutrino mass for light and heavy neutrinos $m^{\nu N}_{\beta\beta}$ takes the form

$$\left(m^{\nu N}_{\beta\beta}\right)^2 = \left| \sum_{j=1}^{3} \left(U^2_{ej}\ m_j + S^2_{ej}\ \frac{\langle p^2 \rangle}{\langle p^2 \rangle + M^2_j}\ M_j \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^{3} \left(T^2_{ej}\ m_j + V^2_{ej}\ \frac{\langle p^2 \rangle}{\langle p^2 \rangle + M^2_j}\ M_j \right) \right|^2 . \tag{6}$$

Here, $\lambda = M_{W_L}/M_{W_R}$ (M_{W_L} and M_{W_R} is the mass of light and heavy vector boson, respectively) and

$$\langle p^2 \rangle = m_{\rm p} m_{\rm e} \frac{M^{0\nu}_{\rm N}}{M^{0\nu}_{\nu}}, \tag{7}$$

which is interpreted as the mean square neutrino momentum in a nucleus ($\sqrt{\langle p^2 \rangle} \approx 200$ MeV). $M^{0\nu}_{\nu}$ and $M^{0\nu}_{\rm N}$ are nuclear matrix elements associated with exchange of light ($m_i < 1$ eV) and heavy ($M_i \gg 1$ GeV) neutrinos. Recall that for all nuclear structure methods of interest $\langle p^2 \rangle$ depends only weakly on the considered isotope [4]. A rather good proportionality of $M^{0\nu}_{\nu}$ and $M^{0\nu}_{\rm N}$ was concluded also by their statistical treatment in [5].

From Eqs. (5) and (6) it follows that the dominance of light or heavy neutrino mechanisms of the $0\nu\beta\beta$-decay can not be established by an observation of this process on different isotopes. This task requires an additional information or assumption concerning neutrino masses and mixing.

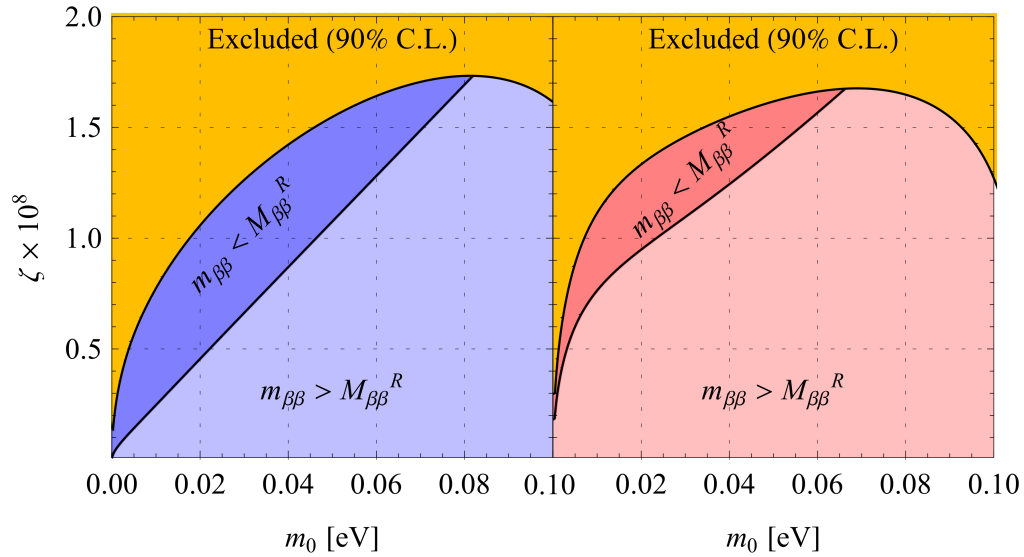

Figure 1. The region of the dominance of the $M^R_{\beta\beta}$ contribution over $m_{\beta\beta}$ contribution to $m^{\nu N}_{\beta\beta}$ for the see-saw type of neutrino mixing matrix given in Eq. (8) and by assuming $\zeta^2 \simeq m_i/M_i$ The cases of the normal and inverted hierarchy of neutrino masses are presented in the left and right panels, respectively. The current constraint on the $\eta_{\nu N}$ parameter is deduced from the lower limit on the $0\nu\beta\beta$-decay half-life of ^{136}Xe [6] by using nuclear matrix element calculated within the QRPA [7]. For square ratio of masses of left and right vector bosons $\lambda = 7.7 \cdot 10^{-4}$ is considered.

If the flavor universal mixing between the active and sterile neutrino sectors is assumed, the seesaw mixing matrix $\mathcal{U}$ takes the form [4]

$$\mathcal{U} \;=\; \begin{pmatrix} U & S \\ T & V \end{pmatrix} \simeq \frac{1}{\sqrt{1+\zeta^2}} \begin{pmatrix} U & \zeta\,\mathbf{1} \\ -\zeta\,\mathbf{1} & U^\dagger \end{pmatrix}. \tag{8}$$

Here, $\zeta \ll 1$ is the seesaw parameter ($1+\zeta^2 \simeq 1$). U and $U^\dagger$ are the PMNS neutrino mixing matrix and its hermitian conjugate, respectively. The seesaw relation $\zeta^2 = m_i/M_i$ for light and heavy neutrino masses is assumed. If the LNV scale is significantly larger than $\langle p^2\rangle_a$ we find

$$\left(m^{\nu N}_{\beta\beta}\right)^2 = m^2_{\beta\beta} \;+\; (M^{\mathrm{R}}_{\beta\beta})^2 \quad \text{with} \quad M^{\mathrm{R}}_{\beta\beta} = \lambda\,\zeta^2 \left|\sum_{j=1}^{3} (U^\dagger)^2_{ej}\,\frac{\langle p^2\rangle_a}{m_j}\right|. \tag{9}$$

Currently, the most stringent bound comes out from ^{136}Xe $0\nu\beta\beta$-decay experiment [6]. For this bound and $\lambda = 7.7 \cdot 10^{-4}$ (current upper bound from the accelerator experiments) the separate contributions of the light and heavy neutrinos to $0\nu\beta\beta$-decay are analyzed in Fig. 1 in the plane of the parameters ζ^2 and m_0 (the lightest neutrino mass) for the cases of the normal (left panel) and inverted hierarchy (right panel) of neutrino masses. We see again that for a chosen set of parameters the value of $M^{\mathrm{R}}_{\beta\beta}$ can be comparable with $m_{\beta\beta}$.

3. Nuclear physics aspects

3.1. Nuclear matrix elements

A significant progress has been achieved in the evaluation of the $0\nu\beta\beta$ decay nuclear matrix elements (NME) in the last decade. Nevertheless, there is still a spread by the factor 2-3

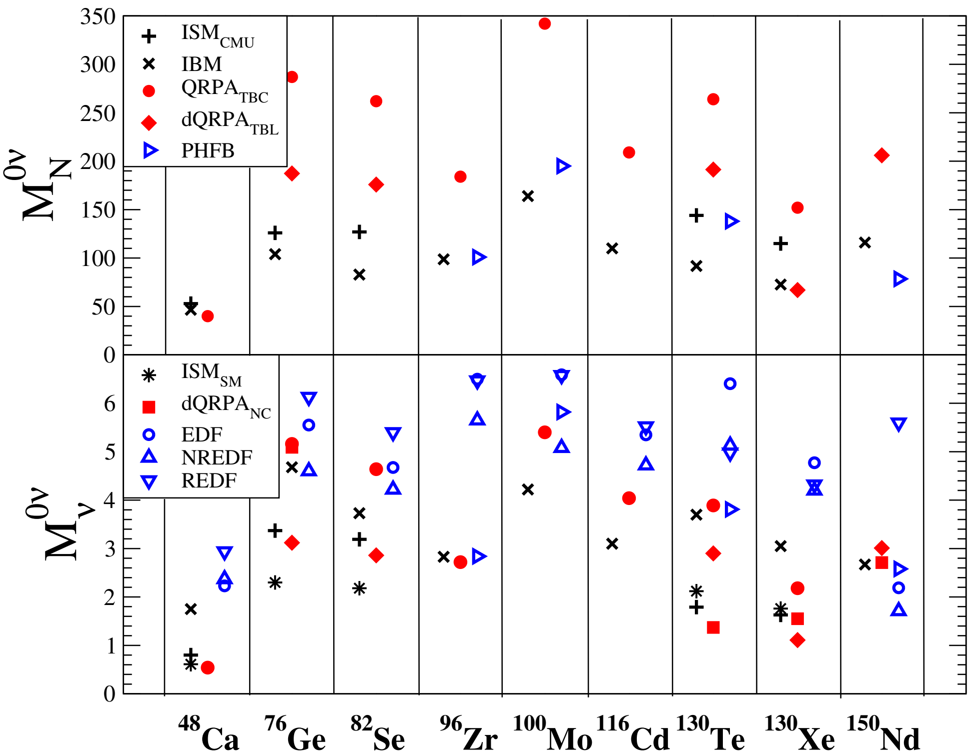

Figure 2. Nuclear matrix elements $M^{0\nu}_{\nu}$ and $M^{0\nu}_{\rm N}$ for $0\nu\beta\beta$-decay candidates calculated in the framework of different approaches: i) Interacting Shell Model (Strasbourg-Madrid (ISM$_{\rm SM}$, [10]) and Central Michigan University (ISM$_{\rm CMS}$, [11]) groups) and Interacting Boson Model (IBM, [12]) in black; ii) spherical (Tuebingen-Bratislava-Caltech (QRPA$_{TBC}$, [7]) group) and deformed (Tuebingen-Lanzhou-Bratislava (dQRPA$_{\rm TLB}$, [13]) and North Caroline (dQRPA$_{\rm NC}$, [14]) groups) Quasiparticle Random Phase Approximation in red; iii) Projected Hartree-Fock-Bogoliubov (PHFB, [15]), Non-relativistic Energy Density Functional (EDF, [16]) and (NREDF, [17]) and Relativistic EDF (REDF, [18]) methods in blue. In the case of ISM$_{\rm CMS}$, IBM, $QRPA_{TBC}$, dQRPA$_{\rm TLB}$ and PHFB [15]) calculations the Argonne two-nucleon short-range correlations (src) were considered. The $rmISM_{\rm SM}$ results are with Jastrow src. The effect of src was neglected in dQRPA$_{\rm NC}$, EDF, NREDF and REDF evaluation of NMEs. The non-quenched value of weak axial-vector coupling g_A and R = 1.2 fm $A^{1/3}$ are assumed.

between the $M^{0\nu}_{\nu}$ calculations using different nuclear models. While earlier evaluations of NMEs were performed mostly within the Quasiparticle Random Phase Approximation (QRPA) and interacting shell model (ISM), nowadays results of the interacting boson model (IBM), and of different versions of the energy density functional (EDF), are also available. It is generally accepted that all these models suffer from neglecting certain essential aspects of physics, different in each case. Currently, it is difficult, or impossible, to reliably assign the corresponding uncertainties in the resulting NMEs. For these nuclear model based methods the concrete issues that are widely discussed are the role of ground state correlations, deformation, the size of the model space, or the restoration of the SU(4) spin-isospin symmetry [8, 9]. The approaches with the "controlled errors", like no core shell model, coupled cluster methods, or Green's function Monte-Carlo are being developed. They are, however, so far applicable only to the light nuclear systems and not yet to the relatively heavy $0\nu\beta\beta$ decay candidate nuclei.

In Fig. 2 $M^{0\nu}_{\nu}$ and $M^{0\nu}_{\rm N}$ calculated within different nuclear structure methods are presented. We notice that the ISM results are significantly smaller when compared with results of different mean field approaches (PHFB, EDF, NREDF and REDF). The importance of relative deformation of initial and final nuclei in evaluation of NMEs is manifested by a difference between results of spherical and deformed QRPA.

For any mechanism responsible for the decay, the matrix element $M^{0\nu}$ consists of three parts, Fermi (F), Gamow-Teller (GT) and Tensor (T),

$$M^{0\nu} = M^{0\nu}_{GT} - \frac{M^{0\nu}_F}{(g^{\rm eff}_A)^2} + M^{0\nu}_T. \tag{10}$$

By considering closure approximation we have

$$\begin{aligned} M^{0\nu}_{GT} &= \langle f| \sum_{j,k}^{A} \tau^+_j \tau^+_k \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k \; P_{GT}(r_{kj})|i\rangle, \quad M^{0\nu}_F = \langle f| \sum_{j,k}^{A} \tau^+_j \tau^+_k \; P_F(r_{jk})|i\rangle, \\ M^{0\nu}_T &= \langle f| \sum_{j,k}^{A} \tau^+_i \tau^+_j [3(\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{jk})(\boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{jk}) - \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k] \; P_T(r_{jk})|i\rangle \; . \end{aligned} \tag{11}$$

Here $|i\rangle, |f\rangle$ are the ground state wave functions of the initial and final nuclei. $P_{GT}(r_{jk})$, $P_F(r_{jk})$ and $P_T(r_{jk})$ are the potentials that depend on the relative distance r_{jk} of the two nucleons. The sum is over all nucleons in the nucleus.

For the light and heavy neutrino mass mechanisms the dimensionless neutrino potential for the $K = GT, F$ and T parts are

$$\left(\begin{array}{c} P^{\nu}_K(r_{jk}) \\ P^{N}_K(r_{jk}) \end{array} \right) = f^2_{\rm src}(r_{jk}) \; \frac{2}{\pi g^2_A} R \int_0^\infty \; f_K(q r_{jk}) \; h_K(q^2) \left(\begin{array}{c} 1/\left(q(q+E_{\rm av})\right) \\ 1/\left(m_e m_p\right) \end{array} \right) q^2 dq \; . \tag{12}$$

Here, R is the nuclear radius added to make the potential dimensionless. The functions $f_{F,GT}(q r_{jk}) = j_0(q r_{jk})$ and $f_T(q r_{jk}) = -j_2(q r_{jk})$ are spherical Bessel functions. The functions $h_K(q^2)$ are defined in [8]. The light neutrino exchange potential depends rather weakly on average nuclear excitation energy $E_{\rm av}$. The function $f_{\rm src}(r_{jk})$ represents the effect of two-nucleon short range correlations [19].

Better insight into the structure of matrix elements can be gained by explicitly considering their dependence on the distance r between the two neutrons that are transformed into two protons in the decay. Thus we define the function $C^{0\nu}_{GT}(r)$ (and analogous ones for M_F and M_T) as

$$C^{0\nu-\nu,N}_{GT}(r) = \langle f| \sum_{j,k} \tau^+_j \tau^+_k \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k \; \delta(r - r_{jk}) P^{\nu,N}_{GT}(r_{jk})|i\rangle, \;\; \text{i.e.,} \;\; M^{0\nu-\nu,N}_{GT} = \int_0^\infty C^{0\nu-\nu,N}_{GT}(r) dr \; . \tag{13}$$

In other words, knowledge of $C^{0\nu-\nu,N}_{GT}(r)$ makes the evaluation of $M^{0\nu-\nu,N}_{GT}$ trivial. We note that the function $C(r)$ was first introduced in [8].

From the way the function $C^{0\nu-\nu,N}_{GT}(r)$ was constructed, it immediate follows that

$$C^{0\nu-\nu,N}_{GT}(r) = P^{\nu,N}_{GT}(r) \; C^{2\nu}_{GTcl}(r), \;\; \text{with} \;\; C^{2\nu}_{GTcl}(r) = \langle f| \sum_{j,k} \tau^+_j \tau^+_k \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k \; \delta(r - r_{jk})|i\rangle. \tag{14}$$

as already pointed out in [20]. $C^{2\nu}_{GTcl}(r)$ determines the two-neutrino double-beta decay closure matrix element $M^{2\nu}_{GTcl}$ as a result of integration over r. From (14) it follows that if $C^{2\nu}_{GTcl}(r)$ were known, the $C^{0\nu-\nu,N}_{GT}(r)$ can be easily constructed and hence also the $0\nu\beta\beta$-decay matrix element $M^{0\nu-\nu,N}_{GT}$. The analogous procedure can be followed, of course, also for $M^{0\nu-\nu,N}_F$ and $M^{0\nu-\nu,N}_T$. But Eq. (14) is much more general. Knowing $C^{2\nu}_{GTcl}(r)$ makes it possible to evaluate the corresponding matrix element for any neutrino potential $P^{0\nu}_{GT}(r)$. That represents, no doubt, a significant practical simplification. A better understanding of $C^{2\nu}_{GTcl}(r)$ function is a key to a reliable calculation of the double beta decay NMEs [9].

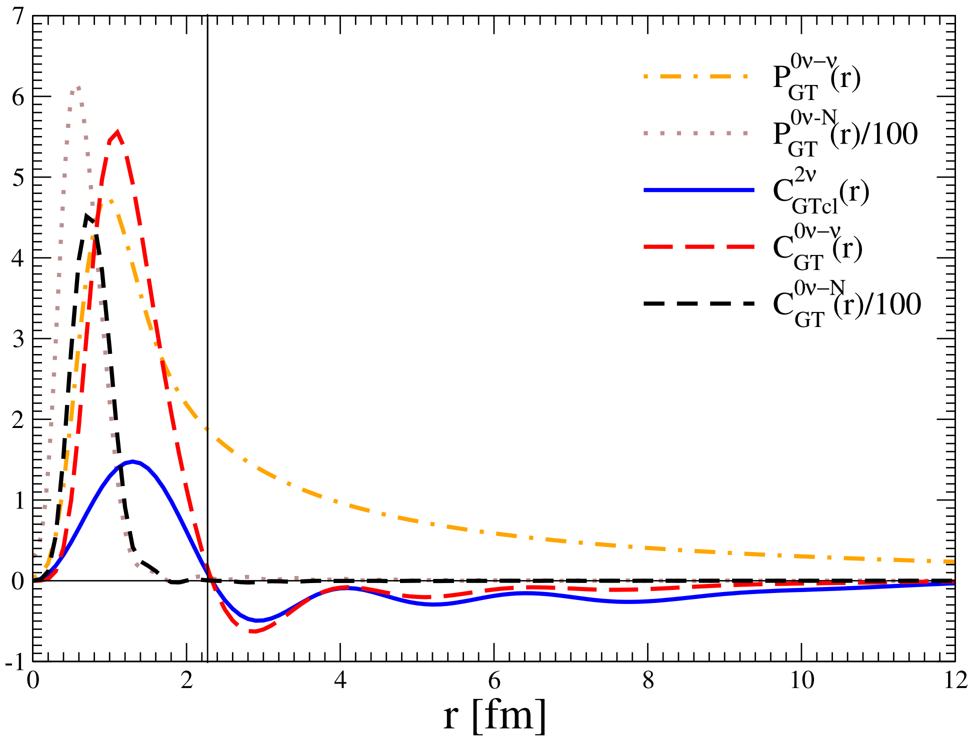

Figure 3. Neutrino exchange potentials $P_{\nu}^{0\nu}(r)$, $P_N^{0\nu}(r)$ and functions $C^{2\nu}_{GTcl}(r)$, $C^{0\nu-\nu,N}_{GT}(r)$ ($C^{0\nu-\nu,N}_{GT}(r) = P^{0\nu}_{\nu,N}(r)\ C^{2\nu}_{GTcl}(r)$) evaluated in the QRPA [7] for the $0\nu\beta\beta$-decay of ^{76}Ge.

3.2. Quenching of g_A

The problem of so-called quenching of the axial-vector weak current, often simplified by the concept of "g_A quenching", is of particular importance. There is not a consensus on its origin, but some studies indicate that a careful treatment of nuclear correlations and inclusion of the three-body interaction and of the corresponding two-body weak currents avoids the need for quenching [21].

Further progress in understanding of quenching could be achieved if related nuclear processes are going to be pursued both theoretically and experimentally. One example are matrix elements and differential characteristics of the two-neutrino double beta decay ($2\nu\beta\beta$), that require the evaluation of the states in the intermediate odd-odd nucleus, possible so far only in the QRPA and ISM. In this context the role of the states in the odd-odd nucleus relatively far from the ground state, i.e. near the region of the giant Gamow-Teller resonance, is crucial.

The $2\nu\beta\beta$ rate is expressed as the product of the fourth power of $\mathrm{g}_A^{\mathrm{eff}}$, NME $M^{2\nu}_{GT}$ and the phase-space factor $G^{2\nu}$, which is calculated with high accuracy [3], as follows [1]:

$$(T^{2\nu}_{1/2})^{-1} = (\mathrm{g}_A^{\mathrm{eff}})^4 |M^{2\nu}_{GT}|^2 G^{2\nu} \quad \text{with} \quad M^{2\nu}_{GT} = \sum_n \frac{\langle f|\sum_j \tau_j^+ \boldsymbol{\sigma}_j|1^+_n\rangle \cdot \langle 1^+_n|\sum_j \tau_k^+ \boldsymbol{\sigma}_k|i\rangle}{E_n - (E_i+E_f)/2} \qquad (15)$$

Here, E_n, E_i and E_f are energies of the n-th 1^+ state of the intermediate nucleus, 0^+ ground states of initial and final nuclei, respectively.

The usual procedure is that the value of $\mathrm{g}_A^{\mathrm{eff}}$ is deduced from the measured $T^{2\nu}_{1/2}$ once $M^{2\nu}_{GT}$ is theoretically evaluated, e.g., within the ISM or IBM. An incredible result $(\mathrm{g}_A^{\mathrm{eff}})^4 \simeq (1.269\ A^{-0.18})^4 = 0.063$ was obtained by the IBM calculation of $M^{2\nu}_{GT}$ within closure approximation in which the sum over virtual intermediate nuclear states was completed by

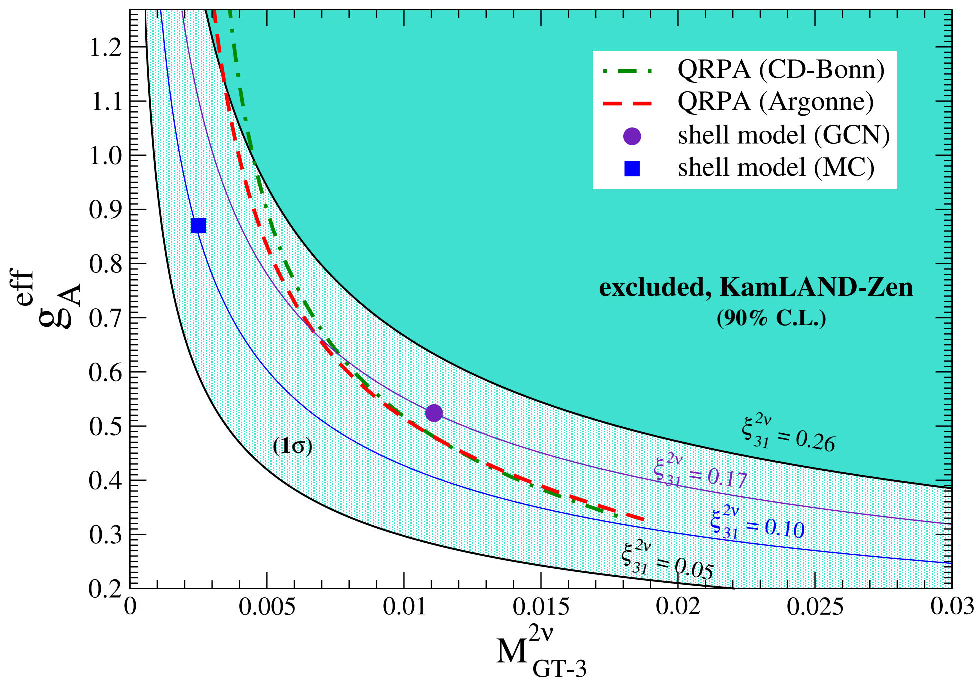

Figure 4. Effective axial-vector coupling constant $g_A^{\rm eff}$ versus the matrix element $M^{2\nu}_{GT-3}$ for the $2\nu\beta\beta$ decay of ^{136}Xe [24]. The solid blue circle and black square display the ISM results with the CGN5082 and MC interactions, respectively. The QRPA dependence is shown by the solid green curve.

closure after replacing $E_n - (Ei + Ef)/2$ by some average value $E_{\rm av}$ [22]. This result strongly disfavor searches for the signal of the $0\nu\beta\beta$-decay. Of course, the validity of the closure approximation is as good as the guess about the value of $E_{\rm av}$. In the QRPA calculation one fixes $g_A^{\rm eff}$ and adjusts the isoscalar particle-particle interaction to reproduce the $M^{2\nu}_{GT}$ that describes the experimental $T^{2\nu}_{1/2}$.

Recently, a more accurate expression for the $2\nu\beta\beta$ rate was derived [23]:

$$(T^{2\nu}_{1/2})^{-1} = (g_A^{\rm eff})^4 |M^{2\nu}_{GT-3}|^2 \frac{1}{|\xi^{2\nu}_{31}|^2} (G^{2\nu}_0 + \xi^{2\nu}_{31} G^{2\nu}_2), \qquad (16)$$

where $G^{2\nu}_0$ and $G^{2\nu}_2$ are the phase-space factors with different dependence on lepton energies. The new parameter $\xi^{2\nu}_{31} = M^{2\nu}_{GT-3}/M^{2\nu}_{GT}$ depends on M_{GT-3} and M_{GT}, NMEs with the first and third power of the energy denominators, respectively. While $M^{2\nu}_{GT}$ is sensitive to contributions from high-lying states in the intermediate odd-odd nucleus with $J^\pi = 1^+$, for M_{GT-3} typically only the lowest 1^+ state contributes.

By using of $2\nu\beta\beta$ decay data $\xi^{2\nu}_{31}$ can be determined experimentally with electron energy spectrum fits extracting the leading and second order contributions in Eq. (16). This new observable $\xi^{2\nu}_{31}$ allows to test and discriminate between QRPA and ISM calculations that all reproduce the experimental $T^{2\nu}_{1/2}$. In Fig. 4 the effective axial-vector coupling constant $g_A^{\rm geff}$ is displayed as a function of the matrix element M_{GT-3} for the $2\nu\beta\beta$-decay of ^{136}Xe. It is showed that $\xi^{2\nu}_{31} > 0.26$ (0.05) is excluded by the present KamLAND-Zen measurement at 90% (1σ) C.L. [24]. Further experimental $\xi^{2\nu}_{31}$ sensitivity improvements may distinguish between various nuclear structure scenarios.

The problem of the "g_A quenching" dilemma can be resolved also with help of muon capture in nuclei, a weak process with $\sim$ 100 MeV of momentum transfer, where many multipoles play an important role. Experimental work on this aspect is conducted at J-PARC [25]. Analysis of the results, as well as of the total muon capture rate, can shed light on the problem of origin of quenching. Yet another example is the study of heavy-ion double-charge exchange reaction (with $\Delta T = 2$), in particular of the ground state to ground state transitions, at LNC Catania (the NUMEN experiment [26]). This and analogous low energy pion double charge exchange reaction are potentially related to the NME of the $0\nu\beta\beta$ decay. That relation, however, requires further theoretical study.

4. Conclusions

In summary, the observation of $0\nu\beta\beta$-decay on two and more nuclear isotopes will allow one to deduce information about the size of $m^{\nu N}_{\beta\beta}$ but not about the relative contribution of the light or heavy neutrino-exchange mechanisms to the decay rate. An additional theoretical or experimental input about neutrino masses and mixing is needed to shed light on the particular role of each of these mechanisms.

The NMEs $M^{0\nu}_{\nu}$ and $M^{0\nu}_{\rm N}$ calculated within modern nuclear structure approaches still exhibit significant differences and further progress is highly requested. It is manifested that in solving this task a crucial role plays $C^{2\nu}_{GTcl}$ function associated with $2\nu\beta\beta$-decay closure matrix element $M^{2\nu}_{GTcl}$. Further, the origin of quenching of axial-vector coupling constant remains to be clarified. Both nuclear structure tasks might be solved with help of supporting measurements of the $2\nu\beta\beta$-decay, muon capture in nuclei and heavy ion double-charge exchange reactions.

Acknowledgments

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