# Spin dynamics of triaxial nuclei with a quasiparticle alignment

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**Abstract.** The dynamics of nuclei with a triaxial core and a non-axial rigid quasiparticle alignment is described with a Schrödinger equation constructed from a semiclassical formalism. The use of the total angular momentum projection as a continuous variable and the separation of the potential energy allows the interpretation of the spectra in terms of anharmonic wobbling oscillations and tilted axis rotations. The experimental realization of the model is presented for the $h_{11/2}$ quasiparticle bands of the $^{133}$La nucleus, which is found to exhibit a novel tilted-axis wobbling mode of excitation.

## 1 Introduction

The precession of the total angular momentum around a body fixed intrinsic axis of the nucleus defines in simple terms the so called nuclear wobbling motion [1]. It is a direct consequence of the triaxial deformation, and is therefore of central importance for the experimental identification of triaxially deformed nuclei. In this study, we present a theoretical formalism for the description of diverse wobbling regimes involving quasiparticle alignments. It is based on a Schrödinger equation for a projection variable deduced from the semiclassical treatment of the Particle Rotor Model (PRM). In this way, it is possible to extend the notion of wobbling motion to lower spin states, as well as beyond the harmonic oscillation and axial alignments. As a demonstrative example of the experimental realization of the model, the case of wobbling bands in the $^{133}$La nucleus is presented.

## 2 Collective Hamiltonian

Through the semiclassical description [2, 3] of a system composed of a triaxial core coupled to a set of rigidly aligned quasiparticles, one can construct a collective Hamiltonian for the wobbling excitation mode [4]. The observed transverse wobbling bands are usually built on quasiparticles above the Fermi surface, which favor rotations around the short axis of the mean field defined by the nuclear density distribution of the triaxial core [5]. Considering a quasiparticle spin $j$ aligned between the short and medium body fixed axes, the prescribed Schrödinger equation can be written using the following Hamiltonian operator

$$\hat{H}_c = -\frac{1}{2}\frac{1}{\sqrt{B(x)}}\frac{d}{dx}\frac{1}{\sqrt{B(x)}}\frac{d}{dx} + V(x), \tag{1}$$

where

$$[B(x)]^{-1} = \frac{(2I-1)(I^2-x^2)(\mathcal{J}_l-\mathcal{J}_s)}{2\mathcal{J}_s\mathcal{J}_l} + \frac{j\cos\alpha\sqrt{I^2-x^2}}{\mathcal{J}_s}, \tag{2}$$

$$V(x) = \frac{I(\mathcal{J}_l+\mathcal{J}_s)}{4\mathcal{J}_l\mathcal{J}_s} + \frac{I^2}{2\mathcal{J}_m} + \frac{(2I-1)(I^2-x^2)(\mathcal{J}_m-\mathcal{J}_s)}{4I\mathcal{J}_m\mathcal{J}_s} - \frac{j\cos\alpha\sqrt{I^2-x^2}}{\mathcal{J}_s} - \frac{xj\sin\alpha}{\mathcal{J}_m} + \frac{B''(x)}{8[B(x)]^2} - \frac{9[B'(x)]^2}{32[B(x)]^3}, \tag{3}$$

are the coordinate-dependent effective mass and respectively the wobbling potential. Its variable $x$ is associated with the total angular momentum projection on the medium axis, while $\alpha$ is the tilting angle of the single particle alignment from the short axis. The collective Hamiltonian is state dependent because it changes with the total angular momentum $I$. The moments of inertia (MOI) $\mathcal{J}_k(k=s,m,l)$ are indexed by the short, medium and long identification of the body-rigid semi-axes. When the hydrodynamic model is used [1],

$$\mathcal{J}_k = \frac{4}{3}\mathcal{J}_0\sin^2\left(\gamma - \frac{2k\pi}{3}\right), \tag{4}$$

the correspondence between indexes $\{1,2,3\} \equiv \{l,s,m\}$ is realized for $\gamma \in (60°, 120°)$. Note that the quasiparticle spin can be associated with a single nucleon [4] or with a pair of nucleons [6, 7].

The solutions $F_{Ip}(x)$ of the collective Hamiltonian (1) in respect to the $dx$ integration metric, are determined by diagonalization in a basis of particle in the box $|x| \leq I$ wave functions. The order of the solution $p$ is related to

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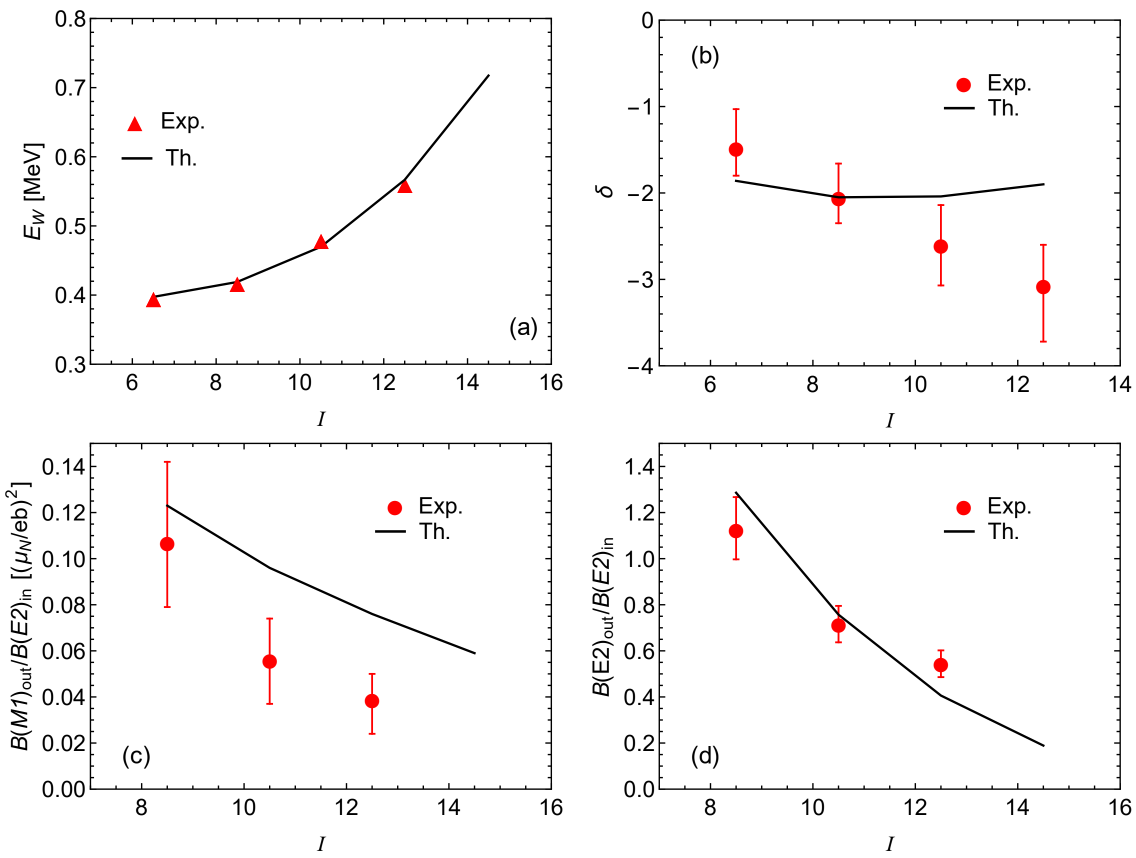


**Figure 1.** Calculated and experimental wobbling energies (a), mixing ratios $\delta_{I\to I-1}$ (b), transition probability ratios $B(M1)_{out}/B(E2)_{in}$ (c), and the dimensionless $B(E2)_{out}/B(E2)_{in}$ ratios (d), are plotted as a function of the total angular momentum $I$. The mixing ratios and "*out*" transitions are from $(I, n_W = 1)$ states to $(I-1, n_W = 0)$ states, whereas the "*in*" $E2$ transitions are from the same initial state to a $(I-2, n_W = 1)$ state within the same wobbling band.

the wobbling quantum number as $n_W = p - 1$, while the density probability distribution $\rho_{Ip}(x) = |F_{Ip}(x)|^2$ is used to define the coefficients of the total wave function expansion in rotation matrices:

$$|\Psi_{IMp}\rangle = \sum_{K=-I}^{I} F_{Ip}(K)|IKM\rangle. \quad (5)$$

## 3 Experimental realization

The theoretical energy formula used to describe the experimental data is expressed as

$$E(I, n_W) = E_{diag}(\mathcal{J}_0, \gamma, \alpha; In_W) + CI(I+1) + E_0, \quad (6)$$

where the energy obtained from the diagonalization of Eq.(1) is amended with a reference energy $E_0$ which encompass the constant single-particle contribution, and a rotational correction which does not break the rotational symmetry of the system. Besides $C$ and $E_0$, the model also has three parameters employed in the diagonalization procedure: triaxiality $\gamma$, tilting angle $\alpha$, and the scaling MOI $\mathcal{J}_0$.

This model successfully described the wobbling bands in $^{105}$Pd, $^{133}$La, and $^{135}$Pr odd mass nuclei [4] based on a $j = 11/2$ quasiparticle, while its axial alignment variant was employed for the study of wobbling bands in even-even nuclei [6, 7]. The transverse wobbling motion in odd mass nuclei has been also described initially through a harmonically approximated potential (3) [3, 8]. The case of $^{133}$La nucleus is especially interesting, as it has a rather distinct dynamical behaviour from the established reference pictures of transverse and longitudinal wobblers. For this nucleus, the parameter values $\gamma = 101°$, $\alpha = 32°$, $\mathcal{J}_0 = 41$ MeV$^{-1}$, $E_0 = 325.89$ keV, and $C = 8.049$ keV [4] are determined by minimizing the sum of the rms values for the wobbling energy

$$E_W(I) = E(I,1) - \frac{1}{2}\left[E(I+1,0) + E(I-1,0)\right] \quad (7)$$

and the associated energy levels (6) of the yrast band ($n_W = 0$) and the excited wobbling band ($n_W = 1$). For the obtained triaxiality $\gamma = 101°$, the distribution of MOI about the long, short and medium axes are 5.8, 23.5 and 52.7 MeV$^{-1}$, respectively. It can be easily shown that, up to a scale and a reference energy, the evolution with spin of the wobbling energy depends only on the triaxial deformation and the tilting angle. Therefore, the alignment and deformation characteristics of the system uniquely define its general spectral properties. As one can see in Fig. 1 (a), the calculated wobbling excitation energy is

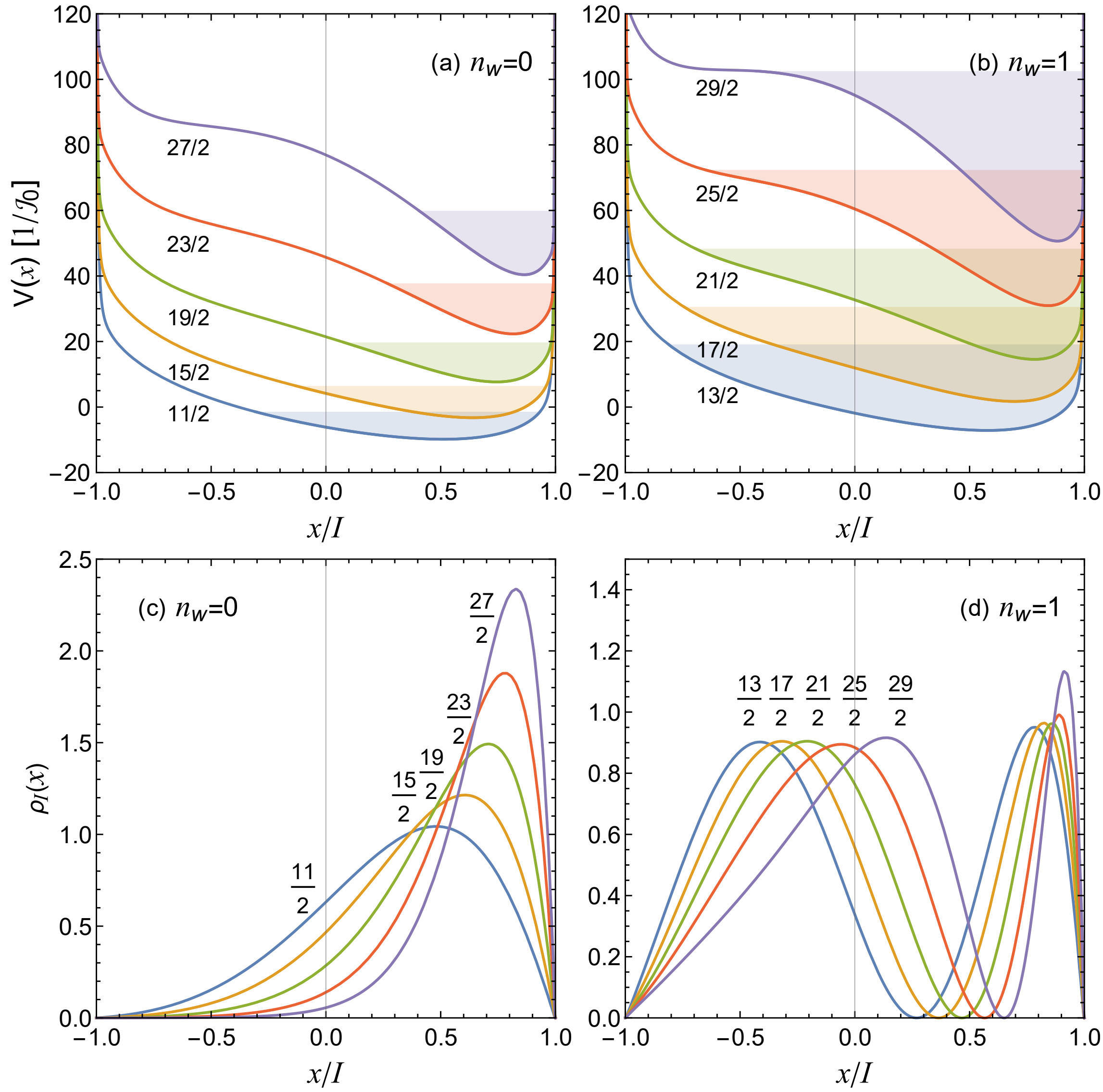


**Figure 2.** The evolution with total angular momentum of the quantum wobbling potential dependence on the projection variable $x$, for the $n_W = 0$ (a) and $n_W = 1$ (b) states of the $^{133}$La nucleus with available experimental counterparts. The corresponding energy levels are identified with the same color filling of the potential well. The probability distribution for each considered state is visualised in panels (c) and (d).

in good agreement with experiment, which increases with spin. This dynamical evolution is usually associated with a longitudinal wobbling mode [9], where the quasiparticle spin aligns to the medium axis of the core rotation. Although the obtained $\alpha = 32°$ tilting, is substantial, the quasiparticle spin is still closer to the short axis. This implies that the quasiparticle retains a strong particle nature [10], which is in agreement with the transverse alignment and wobbling suggested for the neighboring $^{135}$Pr nucleus [11]. The non-negligible tilting has however an important effect on the system's dynamics, and was also obtained in PRM calculations [12].

The wave function (5) depends only on the independent $\gamma$ and $\alpha$ parameters, and therefore the spin dependence of the electromagnetic properties is also given in large by these two parameters. Figs.1(b),(c), and (d) show that the parameters obtained from fits on energy data, describe in a satisfactory manner the evolution with spin of some electromagnetic observables. The electromagnetic transitions are calculated in the standard manner:

$$B(EM\lambda, I_i p_i \to I_f p_f) = \frac{\left|\langle\Psi_{I_f p_f}||T_\lambda(EM\lambda)||\Psi_{I_i p_i}\rangle\right|^2}{2I_i + 1}. \quad (8)$$

The $M1$ operator

$$T_{1\mu}(M1) = \sqrt{\frac{3}{4\pi}}\mu_N g_{eff} \sum_{\nu=0,\pm1} j_\nu D^1_{\mu\nu}, \quad (9)$$

is defined in terms of the single-particle spin components and is factorized by the nuclear magneton $\mu_N$ and an effective gyromagnetic factor $g_{eff} = g_j - g_R$ gathering the corresponding values of the core ($R$) and of the involved single-particle orbital $j$.

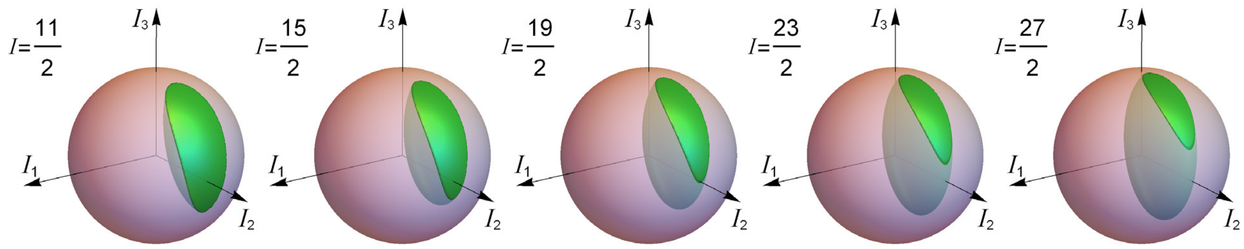


**Figure 3.** Classical precession trajectory of the total angular momentum vector along the yrast band of the $^{133}$La nucleus, defined by the intersection of constant energy (ellipsoid) and total angular momentum squared (sphere) in the space of classical angular momentum projections in units of $I$. The energies are taken as solutions of the Scrödinger equation.

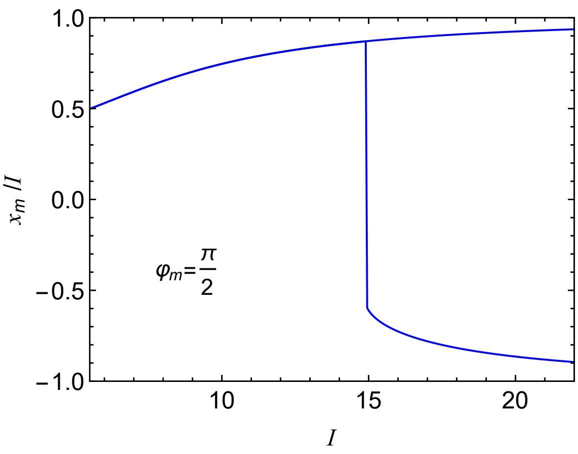


**Figure 4.** The projection coordinate $x_m$ of the classical energy minima for the $^{133}$La nucleus. The corresponding azimuth angle $\varphi_m$ is invariant due to the alignment geometry.

The $E2$ transition rates employ a second order contribution

$$T_{2\mu}(E2) = Q_0\left(q_{2\mu} + \chi\,[q \times q]_{2\mu}\right), \tag{10}$$

in quadrupole moments

$$q_{2\mu} = \beta\left[\cos\gamma D^2_{\mu 0} + \frac{\sin\gamma}{\sqrt{2}}\left(D^2_{\mu 2} + D^2_{\mu -2}\right)\right], \tag{11}$$

where $Q_0$ is an empirical quadrupole moment and $\beta$ is the quadrupole deformation whose value is taken from Ref.[14]. When considering ratios of $E2$ transition probabilities, the $Q_0$ scale is eliminated. It is found that quite a small factor $\chi = -0.04$ is needed to produce the best agreement with experiment. In the case of ratios between magnetic and electric transition, the resulting scaling factor $Q_0/g_{eff}$ is fixed such that to reproduce the best experimental $\delta_{21/2\to 19/2}$ data point for the mixing ratio [15]:

$$\delta_{I_i\to I_f} = 8.78\cdot 10^{-4} E_{if}\frac{\langle\Psi_{I_ip_i}||T_2(E2)||\Psi_{I_fp_f}\rangle}{\langle\Psi_{I_ip_i}||T_1(M1)||\Psi_{I_fp_f}\rangle}, \tag{12}$$

where $E_{if}$ is the transition energy given in MeV units.

The satisfactory description of experimental data validates the hypotheses on which the model is based. The major advantage of the model however consists in its phenomenological interpretation of the spectral properties by the motion of a single continuous variable associated with the total angular momentum projection. This can be inferred from Fig.2, where the separated potential and its corresponding wave function are plotted as a function of the projection variable, for each considered state of the $^{133}$La nucleus. The results suggest that the angular momentum vector oscillates asymmetrically and anharmonically around an equilibrium direction with a substantial tilting from the short axis. This is reflected in the single and double bell shape of the ground and excited state probability distributions, which persist as the total angular momentum increases, however with a gradual movement of the equilibrium direction towards the medium axis. This evolution is in correspondence with the wobbling potential, whose second high energy potential minimum forms at $I \geq 31/2$ and is quite unstable. In what concerns the coordinate-dependent mass function (2), it is symmetrical in the projection variable regardless of triaxiality or tilting and is flatter for lower spin.

The semiclassical origin of the model can be used to translate the quantum dynamics discussed above into a classical context. This is realized by visualizing in Fig.3 the classical trajectories of the total angular momentum for the yrast states. In the space of classical components of the spin defined by the projection variable and an azimuth angle, these trajectories are determined by the intersection of a spherical surface and a displaced ellipsoid corresponding to the two constants of motion, total angular momentum squared and respectively the energy. Fig.3 shows that the spin vector revolves in a precessional-like motion around the stationary points (potential minima), whose evolution with spin is depicted in Fig.4. It must be stressed that the motion of the angular momentum vector is not uniform, having different velocities at certain points of its trajectory [13], which can be obtained by integrating the classical equations of motion.

The fact that this picture cannot transpire from the projected probability distribution depicted in Fig.2, demonstrates the usefulness of the semiclassical description which can provide simultaneous quantum and classical views on the system's dynamics. The correspondence between the two alternative descriptions is obvious from the tilting of the precession toward the medium axis as angular momentum increases. Moreover, the trajectories become tighter.

## 4 Conclusions

A Schrödinger equation which has the total angular momentum projection as a continuous variable, is obtained from the classical energy function associated with a semiclassical treatment of a system composed of a triaxial core and a rigid quasiparticle spin alignment. A numerical application of the model is presented for the description of experimental bands based on a $h_{11/2}$ proton observed in the $^{133}$La nucleus. The semiclassical upbringing of the Schrödinger equation provided a correlation between the well reproduced experimental spectral characteristics of the considered bands and the classical interpretation of their associated dynamics. As a result, the model predicts that the $^{133}$La nucleus undergoes a transition from the transverse wobbling mode towards longitudinal wobbling. This transitional regime is an example of a new revolving tilted axis wobbling motion. It is worth mentioning that a similar theoretical formalism has been also used for the description of chiral bands [16–19].

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