Type II Flux Compactifications and Orientifold Intersections

by

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Abstract

It is well known that the universe is expanding, seemingly driven by a near-constant scalar potential. Much work has been done attempting to construct such potentials using flux compactifications in string theory setups. This thesis will discuss a nongeometric model and its associated scalar potential. It will then discuss and classify local minima of the scalar potential. Certain ingredients in the model, namely D-branes and O-planes, are discussed in further detail. In addition to their application in flux compactifications, intersections of these objects are used when building models of particle physics. These intersections will be discussed. It will be shown that supergravity solutions for two perpendicularly intersecting localized sources in flat space do not exist for a generic diagonal metric Ansatz. Sections 2-6 are based on work from[1]. Sections 7-9 are based on work from[2].

1 Introduction

Shortly after the big bang, the universe was expanding exponentially in a process called inflation. Today, the universe is also expanding exponentially, albeit at a much smaller rate. Today's expansion is caused by dark energy. A cartoon of this is shown below.

Based on observation, it can be stated that the universe is isotropic (it

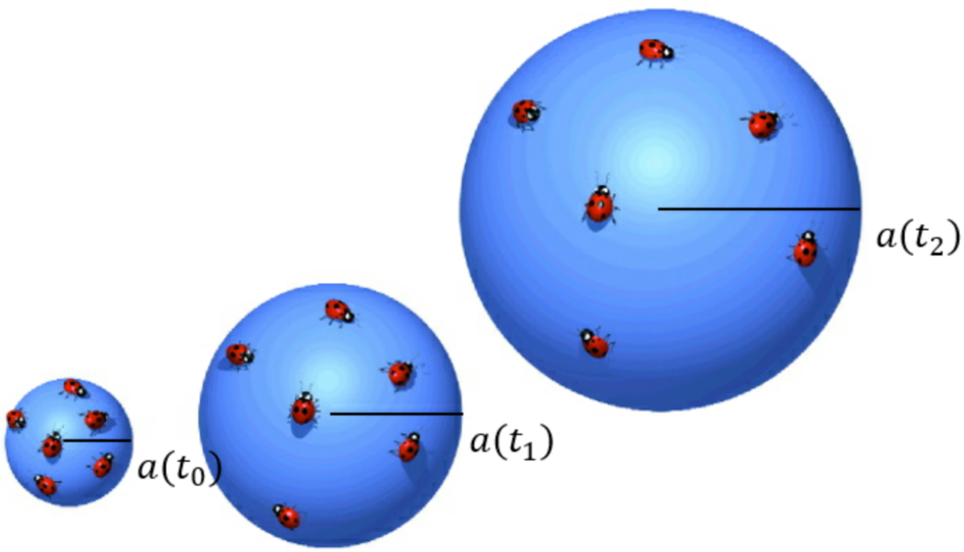

Figure 1.1: Inflation, Dark energy as ladybugs (galaxies) on an expanding balloon (spacetime)

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appears the same in every direction) and homogeneous (it appears the same from every location) at large scales. The only three-dimensional spaces that have these properties are flat space, a 3-sphere, and a hypersphere. The line element in any of these three spaces is given by the Friedmann-Robertson-

Walker metric, written below in spherical coordinates:

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right). \tag{1.1}$$

Here, $K = -1$ corresponds to the hypersphere, $K = 0$ to flat space, and $K = 1$ to the sphere. Both K and r are dimensionless. $a(t)$ is what contains the dimensionality of the spatial component of the metric, and thus determines the size scale of the universe.

The primary difference between the two periods of exponential growth mentioned above is the value of the time constant. $a_{inf}(t) \sim e^{H_{inf}t}, a_0(t) \sim e^{H_0 t}$ and $H_{inf} \gg H_0$. The question then becomes "How did H go from a large value during inflation to its small value now?" Both values of H are proportional to $\sqrt{\Lambda}$, where Λ is Einstein's cosmological constant. Thus, any change in H would have to correspond to a change in Λ. This means that the cosmological constant is not actually constant, it is merely approximately constant in the relevant time windows and changed dramatically during a transitional period[3].

1.1 A Changing Cosmological Constant

With Λ no longer being a constant, further investigation into how it behaves is necessary. To do so, we must first introduce the concept of a scalar field. Simply put, a scalar field is a function that assigns a scalar value to every point in spacetime. An example of a scalar field is temperature, whose spatial dependence is depicted below:

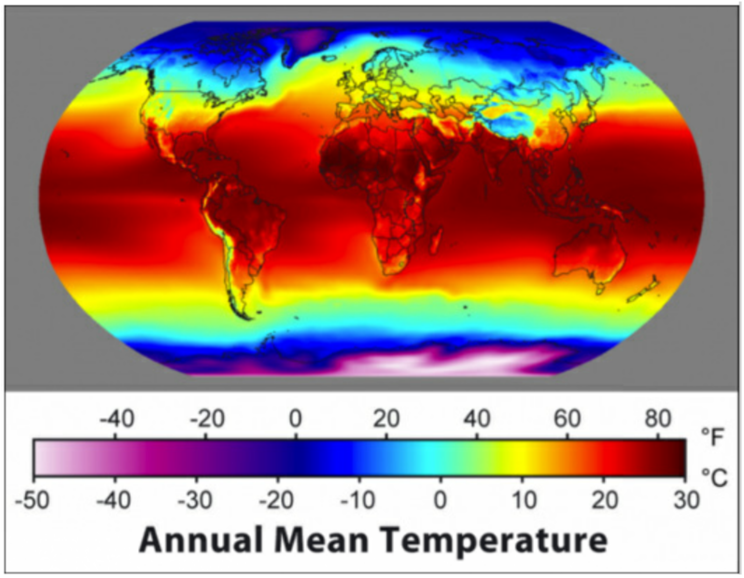


Figure 1.2: Temperature variance over spatial coordinates

The temperature scalar field assigns a higher value to equatorial regions and lower value to polar regions, as expected. Of course, temperature also varies in time, be it day-to-day, season-to-season, or year-to-year.

Now, consider a scalar field $\phi(t, \vec{x})$ with the following action:

$$S = \int d^4x \sqrt{g_{FRW}} \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \right). \tag{1.2}$$

It can be shown that the energy density and pressure of such a scalar field are:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\frac{(\nabla\phi)^2}{a^2} + V(\phi), \qquad P_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}\frac{(\nabla\phi)^2}{a^2} - V(\phi). \tag{1.3}$$

For small spatial and temporal derivatives of ϕ, these equations reduce to $\rho_\phi \approx -P_\phi \approx V(\phi)$. The energy density and pressure of a cosmological constant are given by a similar relationship: $\rho_\Lambda = -P_\Lambda = \Lambda$. So, a potential $V(\phi)$ of a slowly-changing scalar field ϕ could play the role of a cosmological constant.

1.2 Lorentz and Poincaré Symmetries

The isotropy of the universe mentioned in section 1 implies that the universe is symmetric under rotations. The nature of the universe's expansion namely that the rate at which the distance between two objects increases is proportional to the distance between the two objects, is indicative of a scale invariant system. This means that the universe is symmetric under changes in scale, or boosts. By combining these two symmetries, one achieves Lorentz symmetry. The generators of the Lorentz group are three rotations, one about each of the three spatial directions and three boosts, one along each of the three spatial directions. The rotations can be written as $J_i = -i\epsilon_{ij}^{k} x^j \partial_k$ where $i, j, k \in \{1, 2, 3\}$ and ϵ is the Levi-Civita tensor. The boosts can be written as $K_i = -i(x^i \partial_t + t\partial_i)$. One can calculate the commutator of two rotations, two boosts, and one of each.

$$[J^i, J^j] = i\epsilon_k^{ij} J^k \qquad [K^i, K^j] = -i\epsilon_k^{ij} J^k \qquad [J^i, K^j] = i\epsilon_k^{ij} K^k \quad (1.4)$$

Note that the commutator of two boosts is a rotation, so the boosts do not form a closed subgroup of the Lorentz group. The homogeneity of the universe mentioned in section 1 implies that the universe is symmetric under spatial translations. However, recalling Equation 1.1, the universe is not invariant under time translations. If we limit to either small length scales (making the impact of expansion negligible) or small time scales (reducing the actual expansion), we can say the universe is approximately symmetric under time translations. Combining Lorentz symmetry with translational symmetry results in Poincaré symmetry. The generators of the translational symmetery group are the the four-momenta P_μ. These generators obey the commutation relations

$$[P_\mu, P_\nu] = 0 \qquad [P_i, J_j] = -i\epsilon_{ij}^{k} P_k \qquad [P_i, K_j] = -i\delta_{ij} P_0. \tag{1.5}$$

for $\mu, \nu \in \{0, 1, 2, 3\}$ and $i, j, k \in \{1, 2, 3\}$. The commutators involving P_0 are

$$[P_0, J_i] = 0 \qquad [P_0, K_i] = iP_i. \tag{1.6}$$

Lorentz and Poincaré symmetries are described above for three spatial dimensions and one time dimension. However, one can have these symmetries in different numbers of dimensions in certain models. We will take advantage of this in section 9.

1.3 Spinors and Spinor Notation

Before discussing spinors, one must first understand the Clifford algebra. The Clifford algebra is defined as a set of four objects γ^μ for $\mu \in \{0, 1, 2, 3\}$ obeying the anticommutation relation

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \tag{1.7}$$

where $\eta^{\mu\nu}$ is the flat space metric. From the above anticommutation relation, one can write three conditions on the four γ^μ:

$$\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu, \qquad (\gamma^0)^2 = -1, \qquad (\gamma^i)^2 = 1 \text{ for } i \in \{1, 2, 3\}. \tag{1.8}$$

The simplest expression for objects that obey these conditions are the following 4×4 block matrices:

$$\gamma^0 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}. \tag{1.9}$$

where $\mathbf{0}$ and $\mathbf{1}$ denote two by two zero and identity matrices respectively, and σ^i are the Pauli matrices. The commutator of two γ matrices can be calculated:

$$S^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu] = \frac{1}{2}(\gamma^\mu\gamma^\nu - \eta^{\mu\nu}\mathbf{1}_{4\times4}). \tag{1.10}$$

$S^{\mu\nu}$ are 16 4×4 matrices where the μ and ν indices indicate which of the 16 matrices is being used. To explicitly refer to the columns and rows of $S^{\mu\nu}$, one must add two more indices α, and $\beta \in \{0, 1, 2, 3\}$. So, the explicit matrix form is $(S^{\mu\nu})^{\alpha}_{\beta}$. One can now introduce a Dirac spinor field $\psi^{\alpha}(x)$, which transforms under $S^{\mu\nu}$. The transformation operator can be written in terms of the Lorentz transformation operator Λ

$$\Lambda = \exp(\frac{1}{2}\Omega_{\rho\sigma}M^{\rho\sigma}), \qquad S(\Lambda) = \exp(\frac{1}{2}\Omega_{\rho\sigma}S^{\rho\sigma}). \tag{1.11}$$

where M is a collection of the Poincaré transformations, and $M^{\rho\sigma}$ is an individual transformation. Now, one can write the transformation of ψ under $S(\Lambda)$:

$$\psi^{\alpha}(x) \to S(\Lambda)^{\alpha}_{\beta}\psi^{\beta}. \tag{1.12}$$

So, the transformation is a rotation when

$$S(\Lambda) = \begin{pmatrix} \exp(i\vec{\varphi} \cdot \vec{\sigma}/2) & \mathbf{0} \\ \mathbf{0} & \exp(i\vec{\varphi} \cdot \vec{\sigma}/2) \end{pmatrix}. \tag{1.13}$$

where the components of $\vec{\varphi}$ are the angles of rotation. This can be accomplished by choosing

$$S^{ij} = \frac{1}{2} \begin{pmatrix} \mathbf{0} & \sigma^{\mathbf{i}} \\ -\sigma^{\mathbf{i}} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{0} & \sigma^{\mathbf{j}} \\ -\sigma^{\mathbf{j}} & \mathbf{0} \end{pmatrix}, \tag{1.14}$$

which can be rewritten using the Levi-Civita tensor as follows

$$S^{ij} = -\frac{i}{2}\epsilon_{ijk}\begin{pmatrix}\sigma^k & \mathbf{0}\\ \mathbf{0} & \sigma^k\end{pmatrix}. \tag{1.15}$$

Additionally, one must choose $\Omega_{\mu\nu} = -\epsilon_{\mu\nu\tau}\varphi^\tau$. Choosing an angle $\varphi = \{2\pi, 0, 0\}$, the transfomation becomes

$$S(\Lambda) = \begin{pmatrix}\exp(i\pi\sigma^1) & \mathbf{0}\\ \mathbf{0} & \exp(i\pi\sigma^1)\end{pmatrix} = -\mathbf{1}_{4\times4}. \tag{1.16}$$

So, the spinor ψ picks up a minus sign under rotations by 2π about the x^1 axis. The same can be shown for rotations by 2π about the other two axes. One can similarly construct matrices describing the transformation of spinors under boosts. To distinguish between a rotation and a boost, we will call Λ_r the Lorentz transformation of a rotation and Λ_b that of a boost. We can now write

$$S(\Lambda_b) = \begin{pmatrix}\exp(\vec{\chi}\cdot\vec{\sigma}/2) & \mathbf{0}\\ \mathbf{0} & \exp(-\vec{\chi}\cdot\vec{\sigma}/2)\end{pmatrix}. \tag{1.17}$$

where the components of $\vec{\chi}$ are the boost magnitudes in each direction. The key difference between the rotations and boosts is the sign difference between the two nonzero components. Given the block diagonal nature of both transformation matrices, it is convenient split the 4-component spinor into two

2-component spinors

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}. \tag{1.18}$$

By doing this, the transformation matrices can be split up, so the upper left component only acts on ψ_+ and the lower right component only acts on ψ_-. Doing this, one can see that the 2-component spinors transform in the same way under rotations, but oppositely under boosts:

$$\psi_\pm \xrightarrow{\Lambda_r} exp(i\vec{\varphi}\cdot\vec{\sigma}/2)\psi_\pm, \qquad \psi_\pm \xrightarrow{\Lambda_b} exp(\pm\vec{\chi}\cdot\vec{\sigma}/2)\psi_\pm. \tag{1.19}$$

Spinors that transform in this way are called Weyl spinors, with ψ_+ being a left-handed Weyl spinor and ψ_- a right-handed Weyl spinor. With this in mind, we will use the following relabeling:

$$\psi_+ \to \psi_L, \qquad \psi_- \to \psi_R. \tag{1.20}$$

1.4 Supersymmetry

Supersymmetry (SUSY) is an extension of Poincaré symmetry that maps bosons to fermions and vice versa. Consider a SUSY charge Q along with bosonic and fermionic states $|b\rangle$ and $|f\rangle$ respectively. The following expressions show the mapping:

$$Q|b\rangle = |f\rangle, \qquad Q|f\rangle = |b\rangle. \tag{1.21}$$

In order to map an object with integer spin (a boson) to an object with half integer spin (a fermion), the charge must have half integer spin. Thus, the SUSY charge is a fermionic operator and accordingly obeys anticommutation relations. Equation 1.21 conceptually shows how a SUSY charge acts on particle states. A more detailed form is needed to write an expression for the anticommutation relations.

First, recall from subsection 1.3 that Dirac spinors can be broken into left and right-handed Weyl spinors that transform identically under rotations and differently under boosts. Let us now consider SUSY charges of the form Q_α, which acts on the left-handed Weyl spinor and $\bar{Q}_{\dot{\alpha}}$, which acts on the right-handed Weyl spinor. The spinor can be written as $\psi = \begin{pmatrix} \psi_L^\alpha & \psi_R^{\dot{\alpha}} \end{pmatrix}$ where $\alpha, \dot{\alpha} \in \{1, 2\}$. The anticommutators are as follows

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu, \qquad \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \tag{1.22}$$

where P_μ are the four-momenta from subsection 1.2. One can now look at how the SUSY charges interact with the momenta:

$$[P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_{\dot{\alpha}}] = 0. \tag{1.23}$$

Recall from subsection 1.2 that the four-momenta are the generators of the translational component of Poincaré symmetry. One can also show that the SUSY charges commute with the rotation and boost components of Poincaré symmetry. The supersymmetry described above has one species of supercharge Q_α along with its conjugate. Generally, supersymmetry can have more than one species of charge Q^i_α where $i \in \{1, ..., \mathcal{N}\}$. A supersymmetry with three species of charge is called $\mathcal{N} = 3$ supersymmetry.

1.5 String Theories

There are five consistent supersymmetric string theories: Type I, Type IIA, Type IIB, Heterotic $SO(32)$, and Heterotic $E(8) \times E(8)$, each involving different types of strings[4].

1.5.1 Types of Strings

Before we can discuss the types of string theories, we must discuss the types of strings. There are two broad categories of strings: open and closed. One can describe both types of strings using one time-like coordinate τ and one space-like coordinate σ. Together, they are referred to as the worldsheet coordinates. For higher dimensional objects, there will be more than two such coordinates. In this case, they are collectively referred to as worldvolume coordinates. The worldsheet coordinates can be embedded into a D-dimensional spacetime by defining fields $X^\mu(\tau, \sigma)$. The dependence of X^μ on τ describes how the string moves through spacetime, both via propogation and oscillation. The dependence of X^μ on σ describes how the embedding changes based on the position along the string. Open strings have two distinct endpoints obeying either Neumann or Dirichlet boundary conditions.

Neumann boundary conditions are written as $\partial_\sigma(X^\mu)|_{\sigma\in\{0,l_s\}} = 0$ for at least one value of μ. Endpoints are free to move along each direction with Neumann boundary conditions, but are fixed in each direction with Dirichlet boundary conditions.

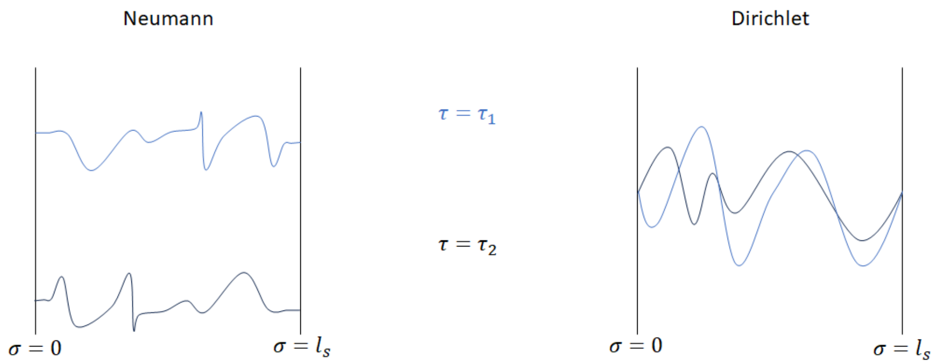


Figure 1.3: For both Neumann and Dirichlet, the horizontal axis is a spacetime direction with Dirichlet boundary conditions (the endpoints of the string cannot leave the vertical lines). For the Neumann plot, the vertical axis is a spacetime dimension with Neumann boundary conditions (the endpoints can move along the vertical lines, but must be flat coming in). For the Dirichlet plot, the vertical axis is a spacetime dimension with Dirichlet boundary conditions (the endpoints are fixed to a single point, but can approach that point from any direction).

If a string endpoint has p Neumann boundary conditions along spatial directions, the p directions form a p-dimensional hyperplane called a Dp-brane. Dp-branes will be further discussed in subsection 1.6. For a closed string, σ is constrained to $[0, l_s]$ where l_s is the length of the string. It is possible for the two endpoints of an open string to meet. When this happens, they can stick together and become a closed string. On a closed string, σ is periodic with a period of l_s. Thus, one can write the boundary conditions for a closed string as $X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + nl_s)$ where $n \in \mathbb{Z}$.

In addition to being open or closed, strings can also be oriented or unoriented. When discussing the string coordinate σ, we gave the strings orientation: one end had to have $\sigma = 0$ and the other end $\sigma = l_s$. In Figure 1.4, the arrows point in the direction of increasing σ. So, the open string on the top has the left endpoint defined as $\sigma = 0$ and the right as $\sigma = l_s$. The open string on the bottom is reversed. Both closed strings have $0 = l_s$ on the right-most point of the circle, with σ increasing as you go counterclockwise for the top string and clockwise for the bottom string.

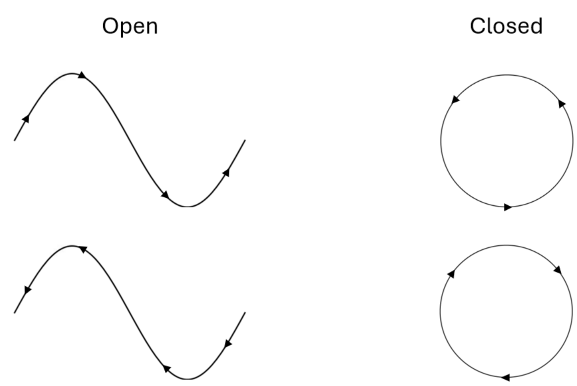


Figure 1.4: Open and closed strings with opposite orientations.

With a well defined orientation of a string, one can distinguish between vibrations moving in opposite directions. These vibrations are commonly referred to as "left-movers" and "right-movers."

Mathematically, changing from one orientation to another can be done by the following mapping:

$$\Omega : \sigma \rightarrow l_s - \sigma. \tag{1.24}$$

One can construct states invariant under this mapping by superimposing strings with opposite orientations. For example, a superposition of oppositely oriented closed strings would look something like $|S\rangle = \frac{1}{\sqrt{2}}(|CW\rangle + |CCW\rangle)$ where $|CW\rangle$ denotes a string with clockwise orientation and $|CCW\rangle$ denotes one with counterclockwise orientation. Ω acts on them as follows:

$$\Omega|CW\rangle = |CCW\rangle, \qquad \Omega|CCW\rangle = |CW\rangle. \tag{1.25}$$

Because each of the two states appear on equal footing in the definition of $|S\rangle$, Ω leaves $|S\rangle$ invariant. Such a state is called unoriented, as there is no distinction between left and right for open strings or clockwise and counterclockwise for closed strings.

1.5.2 Properties of Five Theories

Type I string theory contains both open and closed unoriented strings. The absence of oriented strings in type I means there is no distinction between left and right movers. Thus, there is no difference in how the two behave. The other four string theories contain only closed oriented strings. The difference between them is in how they handle the differences between left and right movers.

Types IIA and IIB each pair up two fermions, with type IIA pairs fermions with opposite handedness and IIB pairing fermions with like handedness. In the two heterotic string theories, left-moving bosonic vibrations exist in 26 dimensions and right-moving bosonic and fermionic vibrations exist in 10 dimensions. This presents a problem: the theory lives in two different numbers of dimensions. This can be resolved by introducing a gauge group to reduce the 26 dimensional bosonic theory to 10 dimensions. This can be thought of as compactifying the left movers on a 16-dimensional torus. There are two lattices that lead to consistent theories: $SO(32)$ and $E(8) \times E(8)$. This choice is the distinction between the two heterotic theories.

The five string theories can be transformed into one another using dualities. The process of moving between theories consequently converts observables into one another. One example of a duality is T-duality, described by the mapping $l_s \rightarrow \frac{1}{l_s}$. Applying this mapping to a theory exchanges momentum and winding number. So, the momentum in one theory becomes the winding number in another, and vice versa. When such a transformation can be done from one theory to another, the two theories are called dual, and are considered equivalent. The two string theories used from this point forward, Type IIA and Type IIB are related via T-duality.

1.6 Fields and Charged Objects

There are five massless 10D fields in type IIA string theory that we will focus on: $g_{\mu\nu}$, ϕ, $B^{(2)}$, $C^{(1)}$, and $C^{(3)}$. The graviton, $g_{\mu\nu}$, encodes information about the geometry of 10D spacetime. The dilaton, ϕ, encodes the string coupling. The Kalb-Ramond field, $B^{(2)}$ can be thought of as a generalization of electromagnetism to the worldsheet of a string. These three fields all arise from massless excitations of bosonic strings. The Ramond-Ramond fields $C^{(1)}$ and $C^{(3)}$ can be thought of as generalizations of the photon A_μ. $C^{(1)}$ provides a more direct comparison as they are both rank-1 tensors, but $C^{(3)}$ is an analogous rank-three tensor.

Just as the photon couples to charged particles, the Ramond-Ramond fields couple to charged objects. A general Ramond-Ramond field $C^{(p+1)}$ couples to a Dp-brane. The equivalent in standard electromagnetism is the case $p = 0$, where A_μ is $C^{(1)}$ and couples to point particles moving in time (0+1 dimensions), such as the up quark. The charge of a Dp-brane is fixed by the value of p. For example, all D0-branes have the same charge, and all D2-branes have the same charge, but D0-branes and D2-branes do not have the same charge.

There is one more class of charged object called O-planes. The "O" stands for orientifold. An orientifold plane spans the set of fixed points under a symmetry, and are therefore entirely dependent on the symmetries in the system. Similarly to Dp-branes, the charge of an O-plane is fixed by its dimensionality. Moreover, the charge of an O-plane can be written in terms of its Dp-brane counterpart with the following forumla:

$$q_{Op} = -2^{p-4} q_{Dp} \tag{1.26}$$

Note that for us O-planes have a negative charge and tension and Dp-branes have a positive charge and tension [5].

2 Toroidal Compactification

Supersymmetric string theory requires ten dimensions; four that we see (one time, three spatial), and six that we do not. The six dimensions that we do not see must be sufficiently small to remain undetected. These are called compactified dimensions, and there are countless ways to do these compactifications. The gold standard of string theory compactifications (not just for 10D theories) is a class of topological spaces called Calabi-Yau manifolds, as they guarantee Ricci-flatness.

One example of such a compactification is three two-tori: $\mathcal{T}^2 \times \mathcal{T}^2 \times \mathcal{T}^2$ (Figure 2.1). The ten-dimensional metric for such a compactfication would be block-diagonal and would look something like:

$$\begin{pmatrix} g_{ij,4\times 4} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}^{(1)}_{2\times 2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}^{(2)}_{2\times 2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{T}^{(3)}_{2\times 2} \end{pmatrix}.$$

The six real coordinates can be rewritten into three complex coordinates and their conjugates: $z^a = y^{2a-1} + iy^{2a}$ and $\bar{z}^a = y^{2a-1} - iy^{2a}$ for $a \in \{1, 2, 3\}$. The three complex coordinates are referred to as holomorphic coordinates,

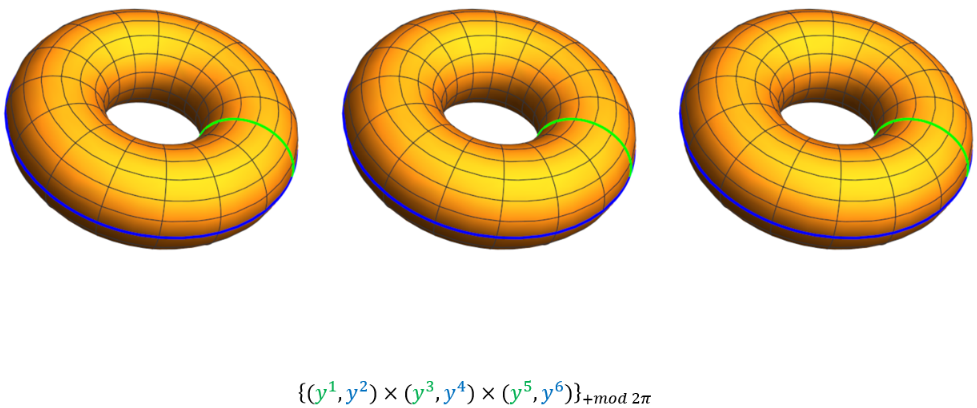


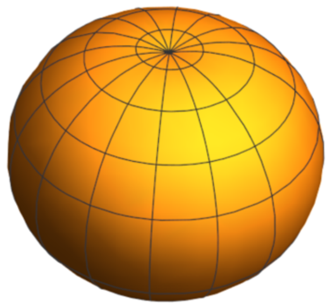
Figure 2.1: Three-Torus Compactification

and their conjugates are referred to as antiholomorphic coordinates.

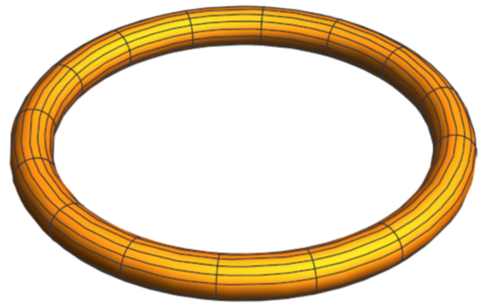
Each of the three tori can be defined in terms of the radii of the two circles that form it. From there, one can define the following fields[5]:

$$\mathrm{Im}(T^{(a)}) \sim r^{(2a)} r^{(2a-1)},\ \mathrm{Im}(U^{(a)}) \sim \frac{r^{(2a)}}{r^{(2a-1)}}. \tag{2.1}$$

Here, $\mathrm{Im}(T^{(a)})$ describes the overall volume of torus a and $\mathrm{Im}(U^{(a)})$ describes the shape of torus a. They are called the volume modulus and complex structure modulus respectively. The figure below shows the physical significance of the complex structure modulus:



Small Im(U)



Large Im(U)

2.1 Symmetries

Next, one can introduce symmetries to the system. First, let $\alpha = e^{\pi i/3}$ [6]

$$\frac{\mathcal{T}^6}{\mathbb{Z}_3} : (z^1, z^2, z^3) \to (\alpha^2 z^1, \alpha^2 z^2, \alpha^2 z^3). \tag{2.2}$$

Due to the periodicity of the each of the coordinates, families of fixed points can be found for all three z^a by solving the following equation

$$z^a + n_{2a-1}(2\pi r_{2a-1}) + in_{2a}(2\pi r_{2a}) = \alpha^2 z^a, \text{ with } a \in \{1, 2, 3\}. \tag{2.3}$$

Solving this for z^a results in

$$z^a = \frac{\pi n_2 r_2}{\sqrt{3}} - \pi n_1 r_1 + i(-\frac{\pi n_1 r_1}{\sqrt{3}} - \pi n_2 r_2). \tag{2.4}$$

One can orbifold by another $\mathbb{Z}_3$ symmetry that acts as

$$\frac{\mathcal{T}^6}{\mathbb{Z}_3 \times \mathbb{Z}_3} : (z^1, z^2, z^3) \to (\alpha^2 z^1 + \frac{1+\alpha}{3}, \alpha^4 z^2 + \frac{1+\alpha}{3}, z^3 + \frac{1+\alpha}{3}). \quad (2.5)$$

One can set up similar equations to find fixed points, but only z^1 and z^2 have solutions:

$$\begin{aligned} z^1 + n_1(2\pi r_1) + i n_2(2\pi r_2) &= \alpha^2 z^1 + \frac{1+\alpha}{3} \\ z^2 + n_3(2\pi r_3) + i n_4(2\pi r_4) &= \alpha^4 z^2 + \frac{1+\alpha}{3}. \end{aligned} \quad (2.6)$$

An example fixed point for the first torus (ignoring the other two torii) is $z^1 = \frac{\sqrt{3}-1}{3} - i$ where $n_1 = n_2 = 1$ and $r_1 = 1, r_2 = 2$. However, there are no true fixed points of this symmetry because there are no values for which z_3 is fixed.

Next consider the following symmetry:

$$\frac{\mathcal{T}^6}{I} : (z^1, z^2, z^3) \to (\bar{z}^1, \bar{z}^2, \bar{z}^3) \quad (2.7)$$

or, back in the original y^i coordinates,

$$\frac{\mathcal{T}^6}{I} : (y^1, y^2, y^3, y^4, y^5, y^6) \to (y^1, -y^2, y^3, -y^4, y^5, -y^6). \quad (2.8)$$

From here, it is clear that the 6+1-dimensional space defined by $\mathbb{R}^{3,1} \times (y^1, 0, y^3, 0, y^5, 0)$ is left invariant under the transformation. Combining I with the worldsheet parity operator Ω_p and the left moving fermion number

$(-1)^{F_L}$ creates an orientifold. So, this symmetry results in an O6-plane in the external space and y^1, y^3, y^5 directions.

2.2 Compactified Fields

With an understanding of how space is compactified, one can now investigate how compactifications impact the fields discussed in subsection 1.6. In this section, we will discuss how the 10D fields manifest in the form of seven fields and eleven fluxes. Six of these fields, $T^{(a)}$ and $U^{(a)}$ are partially described in section 2. However, nothing has been said about $Re(T^{(a)})$, or our final field S. $Re(T^{(a)})$ can be defined in terms of the Kalb-Ramond field as follows:

$$Re(T^{(a)}) \sim \int_{\mathcal{T}^{(a)}} B^{(2)}. \tag{2.9}$$

Similarly, $Re(S)$ can be defined in terms of $C^{(3)}$ along the directions that are unaffected by the orientifold described in subsection 2.1 and $Im(S)$ can be defined in terms of the dilaton:

$$Re(S) \sim \int_{y^1,y^3,y^5} C^{(3)} \qquad\qquad Im(S) \sim e^{-\phi}. \tag{2.10}$$

In addition to their contributions to the seven fields, the Kalb-Ramond and Ramond-Ramond fields manifest as fluxes threading the compactified dimensions or filling the external dimensions. This is done by calculating the field strength for each of them.

$$
\begin{aligned}
dC^{(1)} &= F_{(2)} = f_2^{(1)} dy^1 \wedge dy^2 + f_2^{(2)} dy^3 \wedge dy^4 + f_2^{(3)} dy^5 \wedge dy^6 \\
dB &= H = h_s dy^2 \wedge dy^4 \wedge dy^6 + h_u^{(1)} dy^2 \wedge dy^3 \wedge dy^5 \\
&\quad + h_u^{(2)} dy^1 \wedge dy^4 \wedge dy^5 + h_u^{(3)} dy^1 \wedge dy^3 \wedge dy^6 \\
dC^{(3)} &= F_{(4)} = f_6^{(1)} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 + f_4^{(1)} dy^3 \wedge dy^4 \wedge dy^5 \wedge dy^6 \\
&\quad + f_4^{(2)} dy^1 \wedge dy^2 \wedge dy^5 \wedge dy^6 + f_4^{(3)} dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4
\end{aligned}
\tag{2.11}
$$

2.3 Landau-Ginzburg Models

Landau-Ginzburg orbifold models provide a way of analytically continuing Calabi-Yau compactifications to small volume and can even be used to describe the mirror dual of compactification on a rigid Calabi-Yau manifold[7]. A Landau-Ginzburg theory is determined by the superpotential $\mathcal{W}_{ws}(\Phi_i)$, which is a quasi-homogeneous analytic function of the worldsheet chiral superfields Φ_i. In this thesis, following[8], we will consider two models. Firstly, we consider the 1^9 model with a superpotential given by

$$\mathcal{W}_{ws} = \sum_{i=1}^{9} \Phi_i^3 \,, \tag{2.12}$$

and secondly we will consider the 2^6 model with a superpotential given by

$$\mathcal{W}_{ws} = \sum_{i=1}^{6} \Phi_i^4 . \tag{2.13}$$

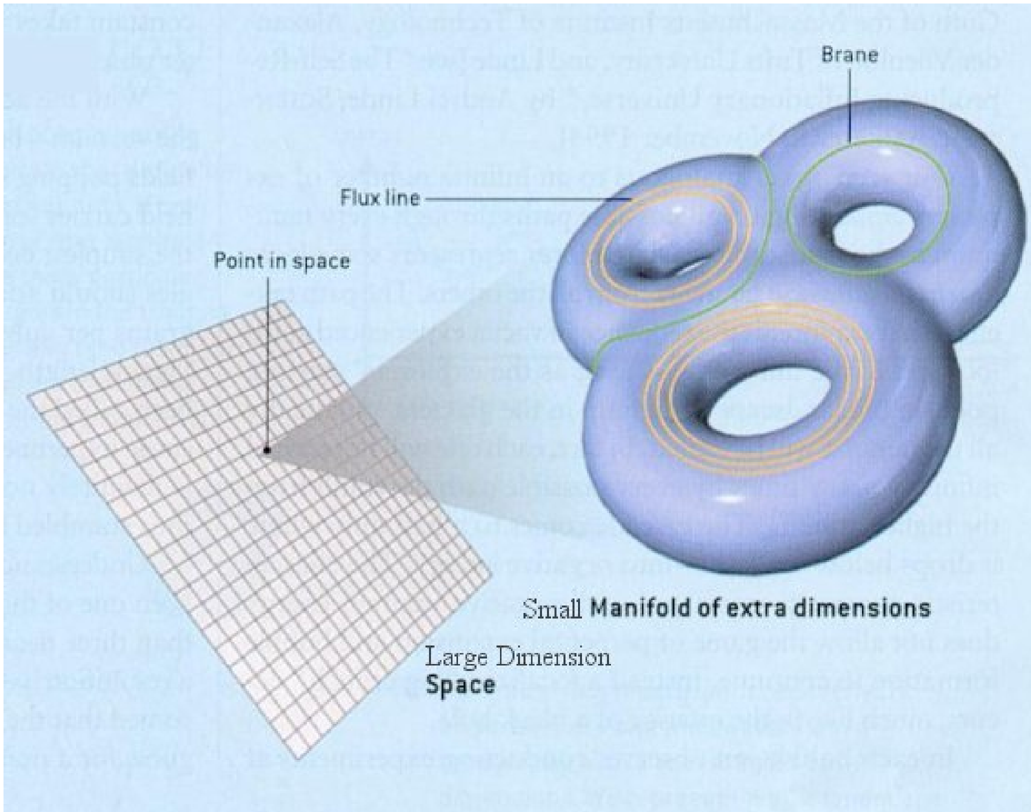


Figure 2.2: Schematic sketch of a string compactification. At each point of the 3+1 dimensional spacetime we have an internal tiny space that is threaded by electric and magnetic fields lines and wrapped by Dp-branes and Op-planes.

In the 1^9 model one can orbifold by the $\mathbb{Z}_3$ symmetry $\Phi_i \to \omega\Phi_i$ where $\omega = e^{\frac{2\pi i}{3}}$, while in the 2^6 model we use the $\mathbb{Z}_4$ symmetry with $\omega = e^{\frac{\pi i}{2}}$. For the 1^9 orientifold, σ_1 in [8], one combines worldsheet parity with $(\Phi_1, \Phi_2, \Phi_3, ..., \Phi_9) \to -(\Phi_2, \Phi_1, \Phi_3, ..., \Phi_9)$. The orientifold for the 2^6 model is the σ_0 orientifold in [8] that acts on the fields as $(\Phi_1, \Phi_2, \Phi_3, ..., \Phi_6) \to e^{2\pi i/8}(\Phi_1, \Phi_2, \Phi_3, ..., \Phi_6)$. In both of the cases one ends up with O3-planes whose charge can be cancelled by turning on F_3 and H_3 fluxes and/or by adding D3 branes.

Before turning on the fluxes, it is easy to check which are the corresponding Calabi-Yau (CY) manifolds. We need to compute the dimensions of the ring of superprimary chiral operators $R = \frac{C[\Phi]}{\partial_j \mathcal{W}_{ws}(\Phi)]}$. The (c, c) ring correspond to $(2, 1)$ harmonic forms while the chiral-antichiral ring (c, a) corresponds to $(1, 1)$ forms. One can then count the number of monomials with i holomorphic coordinates and j antiholomorphic coordinates and label that number $h_{i,j}$. For the 1^9 model it is easy to check that there are $h_{2,1} = 63$ monomials $\Phi_i \Phi_j \Phi_k$ which are invariant under the $\mathbb{Z}_3$ and the orientifold action. One also obtains $h_{1,1} = 0$ [8], that is, there are no corresponding Kähler moduli in the would be CY manifold. It corresponds to the mirror of the $\frac{T^6}{\mathbb{Z}_3 \times \mathbb{Z}_3}$ discusssed above. Thus we see that in the absence of fluxes the model is dual to a DGKT construction, i.e. to a compactification of type IIA on a rigid CY manifold.[1] Similarly, for the 2^6 orientifold one obtains $h^{1,1} = 0$ and $h_{2,1} = 90$ and it corresponds to the mirror of $\frac{T^6}{\mathbb{Z}_4 \times \mathbb{Z}_4}$.

3 Nongeometric Model

The $\mathbb{Z}_3 \times \mathbb{Z}_3$ symmetry discussed in subsection 2.1 fixes $U^{(a)}$ such that $U^{(a)} \sim S$. This will not be used immediately, but keep it in mind later in this section.

For a Calabi-Yau manifold with metric $g_{I\bar{J}}$, one can define a Kähler po-

[1]The actual model that was explicitly worked out by DGKT is a slightly different $\frac{T^6}{\mathbb{Z}_3 \times \mathbb{Z}_3}$ that differs from this model in the twisted sector [9].

tential using[10]:

$$g_{I\bar{J}} = \frac{\partial^2 K}{\partial \phi^I \partial \bar{\phi}^{\bar{J}}}. \tag{3.1}$$

From there, one can calculate the Kähler potential for this setup up to a constant[8],[9]:

$$K = -\ln(-i(S-\bar{S})) - \sum_{a=1}^{3} \ln(-i(T^{(a)} - \bar{T}^{(a)})) + \ln(-i(U^{(a)} - \bar{U}^{(a)})). \tag{3.2}$$

This potential, however, only takes into account the geometry and string coupling. To account for the internal fluxes and Dp-branes and Op-planes, one can introduce a superpotential[8],[9]:

$$\begin{aligned} W &= f_6 + f_4^{(1)} T^{(1)} + f_4^{(2)} T^{(2)} + f_4^{(3)} T^{(3)} \\ &\quad + f_2^{(1)} T^{(2)} T^{(3)} + f_2^{(2)} T^{(1)} T^{(3)} + f_2^{(3)} T^{(1)} T^{(2)} \\ &\quad + f_0 T^{(1)} T^{(2)} T^{(3)} + h_s S + h_u^{(1)} U^{(1)} + h_u^{(2)} U^{(2)} + h_u^{(3)} U^{(3)}. \end{aligned} \tag{3.3}$$

Now, one can introduce the covariant derivative:

$$D_I W = \partial_I W + W \partial_I K. \tag{3.4}$$

Using these ingredients, the overall potential can be written as:

$$V(\phi^I, \bar{\phi}^{\bar{J}}) = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3W\bar{W}) \tag{3.5}$$

where $K^{I\bar{J}} = ((\partial_I \partial_{\bar{J}} K)^{-1})^T$ and $\phi^I \in \{S, T^{(1)}, T^{(2)}, T^{(3)}, U^{(1)}, U^{(2)}, U^{(3)}\}$. It is straightforward to calculate the derivative of this potential:

$$\begin{aligned}\partial_I V &= e^K(\partial_I K(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3W\bar{W}) + \partial_I(K^{I\bar{J}}) D_I W D_{\bar{J}} \bar{W} \\ &\quad + K^{I\bar{J}} \partial_I(D_I W) D_{\bar{J}} \bar{W} \; + K^{I\bar{J}} D_I W \partial_I(D_{\bar{J}} \bar{W}) \\ &\quad - 3\partial_I(W)\bar{W} - 3W\partial_I(\bar{W})). \end{aligned} \tag{3.6}$$

Now consider $D_I W = D_{\bar{J}} \bar{W} = 0$. Setting Equation 3.4 to 0 shows that $\partial_I W = -W\partial_I K$. Using this, plugging in 0 for any instances of untouched covariant derivatives and taking advantage of the fact that $\bar{W}$ is only a function of the antiholomorphic coordinates, one can simplify the derivative to:

$$\partial_I V = e^K(-3\partial_I(K)W\bar{W} + 3W\partial_I(K)\bar{W}) = 0. \tag{3.7}$$

Thus, $D_I W = D_{\bar{J}} \bar{W} = 0 \Rightarrow \partial_I V = 0$. Extrema of the potential found by calculating the covariant derivative of the superpotential are called supersymmetric (SUSY) extrema.

3.1 T-Duality, Equating Fields

By making the three tori identical, one can equate each of the three T fields, each of the three U fields, and any fluxes associated with them. This significantly simplifies the Kähler potential and superpotential:

$$K = -\ln(-i(S - \bar{S})) - 3\ln(-i(T - \bar{T})) - 3\ln(-i(U - \bar{U})). \tag{3.8}$$

$$W = f_6 + f_4 T + f_2 T^2 + f_0 T^3 + h_s S + h_u U. \tag{3.9}$$

One can use T-duality to move from Type IIA string theory to Type IIB string theory. For the purposes of this thesis, it is sufficient to say that T-duality swaps the roles of the complex strucure moduli and the volume moduli ($T \leftrightarrow U$) and repackages the flux variables. This means that T is now fixed to S and U is free. This fixed T is why the model is called nongeometric. Using this, one can rewrite the Kähler potential and superpotential:

$$K = -4\ln(-i(S - \bar{S})) - 3\ln(-i(U - \bar{U})). \tag{3.10}$$

$$W = (f^0 - Sh^0)U^3 - 3(f^1 - Sh^1)U^2 + 3(f_1 - Sh_1)U + (f_0 - Sh_0). \tag{3.11}$$

Note that this does not look like the form of the superpotential in subsection 2.3. This is the spacetime superpotential, whereas that was the worldsheet superpotential. For example, the 1^9 model has nine fields, each appearing as cubic terms in the superpotential. Mixed terms, such as $\Phi_1\Phi_2\Phi_3$ are neglected. The U field above is related to the coefficients of such terms. This is the form of the Kähler potential and superpotential that will be used for section 4, section 5, and section 6.

3.2 Tadpole Cancellation

Just as in electromagnetism, the field lines from any source must either end at another source or go off to infinity. However, in a compactified space, it is impossible to be infinitely far away from a source. This means that *every* field line must begin and end at a source. That is only possible if the net charge from all the sources is exactly zero. In this model, there are three sources discussed earlier in the paper: D3-branes, O3-planes, and the fluxes. From Equation 1.26, the charge of an O3-plane is $-\frac{1}{2}$ of the charge of a D3-brane. Thus,

$$N_{D3} + N_{\text{flux}} - \frac{N_{O3}}{2} = 0. \tag{3.12}$$

There are two values of interest for N_{O3} that arise from these models: 24 and 80. Thus, $N_{D3} + N_{\text{flux}} \in \{12, 40\}$ and

$$N_{\text{flux}} = \int_M F_3 \wedge H_3 = -h^0 f_0 - 3h^1 f_1 + h_0 f^0 + 3h_1 f^1. \tag{3.13}$$

3.3 Non-renormalization theorem

In this subsection we first recall the absence of perturbative and non-perturbative corrections to the superpotential [8,9]. First of all, α' corrections are already taken into account in the LG theory. Thus, one only has to focus on g_s perturbative and non-perturbative corrections. However, it was argued in [8,9] that the superpotential does not receive any perturbative or non-perturbative corrections at all, which follows for example from the non-renormalization of the BPS tension of a D5-brane domain wall but also passes other non-trivial checks [8]. This means that the superpotential is exact even at strong coupling. Note however, that the Kähler potential can and will receive perturbative and non-perturbative corrections, which is something we will return to in the next paragraph. The cautious reader might worry about the familiar brane instanton corrections to the IIB superpotential. Let us therefore recall that our models have $h^{1,1} = 0$ and thus no Euclidean D3-brane instantons. The absence of D(-1) instantons was argued for in footnote 6 in [8] as follows: Since the D(-1) instantons do not depend on the volume and they are not there in the decompactification limit due to higher supersymmetry, they should also not appear here. This is also consistent with the recent analysis in [11], which trivially covers our setup since we have $h^{1,1} = 0$ and therefore no 4-cycles and no D7 branes or O7 planes. Alternative it was argued for the absence of any brane instanton corrections in [9] using the duality to the type IIA setting of DGKT: There the only 3-cycle in models with $h^{2,1} = 0$ has H_3 flux and therefore there are no brane instantons [12].

When studying Minkowski vacua we will assume that the non-renormalization theorem holds and the superpotential receives no corrections even in those vacua where g_s is of order 1 or larger. The conditions for supersymmetric Minkowski vacua are $\partial_i W = W = 0$ and do not depend on the Kähler potential. Thus, the very existence of Minkowski vacua does not change even if one includes arbitrary corrections because those can only appear in the Kähler potential. Previously, such explicit supersymmetric, fully stabilized Minkowski vacua were constructed in [8,9,13]. However, it was stated in [9] that these are necessarily at strong coupling[2] and thus receive large cor-

[2]We find that they cannot be at parametrically weak coupling but there are certainly

rections to the Kähler potential. This then leads to the following important question: Are these truly fully stabilized 4d $\mathcal{N}=1$ Minkowski vacua or can the corrections to K lead to flat directions?

We will prove here that even arbitrary, unknown corrections to K cannot lead to flat directions in these models: We assume that one has been able to a find a fully stabilized SUSY Minkowski vacuum as was the case in [8,9,13] (see also section 4 below). Then the Hessian matrix of second derivatives of the scalar potential has only positive eigenvalues and is given by[3]

$$H_{i\bar{j}} = \partial_i \partial_{\bar{j}} V = (\partial_i \partial_k W) K^{k\bar{\ell}} (\partial_{\bar{\ell}} \partial_{\bar{j}} \overline{W}), \tag{3.14}$$

or in matrix form

$$H = \mathcal{W} \mathcal{K} \overline{\mathcal{W}}\,. \tag{3.15}$$

Now compute the determinant

$$\det\, H = \det\, \mathcal{W}\, \det\, \mathcal{K}\, \det\, \overline{\mathcal{W}} = |\det\, \mathcal{W}\,|^2 \det\, \mathcal{K}\,. \tag{3.16}$$

Given that *all* eigenvalues of H were positive to begin with we can conclude that $|\det\, \mathcal{W}\,|^2 > 0$.

Now let us take into account arbitrary and unknown corrections to the Kähler potential and denote the inverse Kähler metric after including all

examples with $g_s < 1$.

[3]For simplicity we work here with the Hessian. The actual masses squared are the eigenvalues of $H_{i\bar{j}} K^{\bar{j}k}$. However, given that the Kähler metric is positive definite, this does not change our conclusion.

these corrections $\mathcal{K}_c$. The new Hessian for this corrected Minkowksi vacuum is now given by

$$H_c = \mathcal{W}\mathcal{K}_c\overline{\mathcal{W}}\,. \tag{3.17}$$

We again compute the determinant

$$\det\, H_c = \det\, \mathcal{W}\, \det\, \mathcal{K}_c\, \det\, \overline{\mathcal{W}} = |\det\, \mathcal{W}\,|^2 \det\, \mathcal{K}_c\,. \tag{3.18}$$

Since the superpotential did not receive any corrections we have from above that $|\det\, \mathcal{W}\,|^2 > 0$. Since the Kähler metric controls the kinetic terms of the scalar fields, its eigenvalues have to be positive. This remains true even after including arbitrary corrections and therefore $\det\, \mathcal{K}_c \neq 0$. This, combined with the preservation of $|\det \mathcal{W}|^2$ implies that $\det\, H_c \neq 0$. Thus, *all* the eigenvalues of H_c must be nonzero.

In supersymmetric Minkowski vacua, eigenvalues of the Hessian matrix have to be positive for stabilized moduli or zero for flat directions. It was just shown that the eigenvalues of H_c are nonzero, so we can conclude that these Minkowski vacua cannot have flat directions even when including unknown and arbitrary corrections to the Kähler potential.

One can actually prove also the existence of AdS vacua at strong coupling using the non-renormalization of W [8]. While this is not so important since there are infinite families of AdS vacua with parametrically weak coupling, let us nevertheless briefly recall the argument. For supersymmetric AdS vacua, satisfying $D_iW = \partial_i W + (\partial_i K)W = 0$, the $\partial_i K$ term can receive corrections. The authors of[8] expanded the corrected Kähler potential around the minimum which one can choose to be at $\phi^i = 0$, so that $K_c = K + f(\phi^i) + \bar{f}(\bar{\phi}^{\bar{\imath}}) + \phi^i \bar{\phi}^{\bar{\jmath}} g_{i\bar{\jmath}}(\phi^i, \bar{\phi}^{\bar{\imath}})$. At the minimum $\phi^i = 0$ the only correction to $\partial_i K$ arises from $f(\phi^i)$. However, this can be interpreted as a Kähler transformation: $K \to K + f + \bar{f}$, $W \to We^{-f}$, which changes $D_i W \to e^{-f} D_i W$. Therefore, supersymmetric AdS vacua satisfying $D_i W = 0$ cannot disappear even when including arbitrary unknown Kähler corrections. However, for example the mass spectrum is expected to be corrected (within the limits allowed by $\mathcal{N} = 1$ supergravity).

Finally, there is no argument for preventing corrections to non-supersymmetric vacua. So, if one finds them at strong coupling, they could disappear or become unstable when including string loop corrections.

4 Fully stabilized $\mathcal{N} = 1$ Minkowski vacua

As mentioned previously, the first fully stabilized 4d $\mathcal{N} = 1$ Minkowski vacua were found in[8]. In the dual type IIA case, such vacua do not exist in geometric compactifications[14,15], which means that in the type IIB models at least two components of the H_3 flux have to be turned on. It was also shown in[9] that these IIB Minkowski vacua are never arising at large complex structure, i.e. on the dual type IIA side they cannot arise at large volume. However, as we reviewed above the Landau-Ginzburg models take all α' corrections into account and therefore do not require us to be at large complex structure. It was furthermore stated [9] that these Minkowski vacua are confined to strong coupling. Given the non-renormalization theorem from the previous section, we can trust Minkowski vacua even at strong coupling. However, we also find that only parametrically weak coupled solutions are forbidden in this setup and $g_s < 1$ is possible with a model dependent lower bound on g_s. In the next subsection we present a new infinite family of fully stabilized supersymmetric Minkowski vacua and in the following subsection we discuss how this family of solutions fits into the swampland program.

4.1 Minkowski solutions

In order to find Minkowski vacua we have to solve $W = \partial_S W = \partial_U W = 0$ for the W given in equation (3.11) above. A particular family of solutions with properly integer quantized fluxes arises for

$$f^0 = -4, \quad f^1 = 0, \quad f_1 = 0, \quad f_0 = 4, \quad h^0 = -3 - h_0, \quad h^1 = 1, \quad h_1 = -1\,. \tag{4.1}$$

Here $h_0 \in \mathbb{Z}$ is a free parameter that actually does not appear in the tadpole condition since N_{flux} in equation (3.13) reduces to $N_{\text{flux}} = 12$ independent of h_0. Thus, this is a solution to the 1^9 model which does not require D3 branes since the fluxes cancel the negative O3 plane charge.

The moduli are stabilized at the following values

$$\text{Re}(U) = -\tfrac{1}{2}\,, \qquad \text{Im}(U) = \frac{\sqrt{3}}{2}\,,$$

$$\mathrm{Re}(S) = \tfrac{6+4h_0}{3+h_0(3+h_0)}\,, \qquad\qquad \mathrm{Im}(S) = \frac{2\sqrt{3}}{3+h_0(3+h_0)}\,. \tag{4.2}$$

While the complex structure modulus is stabilized at a fixed value, the inverse string coupling $\mathrm{Im}(S)$ changes when we vary the free parameter $h_0 \in \mathbb{Z}$. It takes on its maximal value of $\mathrm{Im}(S) = 2\sqrt{3}$ for $h_0 = -1$ and for $h_0 = -2$. For $h_0 \to \pm\infty$ we enter parametrically strong coupled regions with $\mathrm{Im}(S) \propto 1/h_0^2$. We stress again that even in this parametrically strong coupled regime there are no corrections to W due to the above non-renormalization theorem.

The positive masses squared for the two complex scalar fields in the Minkowski vacuum are given by

$$m_\pm^2 = \frac{(11 \pm 4\sqrt{7})(3+h_0(3+h_0))^3}{192\sqrt{3}}. \tag{4.3}$$

We see that in the limit $h_0 \to \pm\infty$ the masses grow like h_0^6. For the largest inverse string coupling value $\mathrm{Im}(S) = 2\sqrt{3} \approx 3.46$ which is obtained for $h_0 = -1$ and for $h_0 = -2$, the masses squared reduce in both cases to $m_-^2 = \frac{11-4\sqrt{7}}{192\sqrt{3}} \approx 0.00125$ and $m_+^2 = \frac{11+4\sqrt{7}}{192\sqrt{3}} \approx 0.0649$.

4.2 Minkowski vacua and the swampland

It is easy to find string compactifications that give rise to 4d Minkowski vacua with $\mathcal{N} \geq 2$, for example, by compactifying type II string theory on a Calabi-Yau manifold or a torus. However, to the best of our knowledge all these Minkowksi vacua with $\mathcal{N} \geq 2$ have flat directions, i.e. massless scalar fields. These flat directions can be protected by the high amount of supersymmetry. However, in 4d $\mathcal{N} = 1$ theories there is no such protection and it is expected that all flat direction would be lifted by corrections which likely leads to runaway directions. To the best of our knowledge, the Minkowski vacua first discovered in[8,9] are the only fully stabilized $\mathcal{N} = 1$ Minkowski vacua that arise in full-fledged string theory constructions. Given that corrections to the scalar potential are not forbidden by $\mathcal{N} = 1$ supersymmetry, one would have thought that it would not be possible to really argue for the existence of these vacua when including all perturbative and non-perturbative corrections. However, the non-renormalization of the superpotential[8] and our argument above about the mass matrix are implying that these vacua do indeed exist in a strongly coupled corner of string theory.

Given the more recent objections to the existence of dS vacua in string theory[16,17], the very existence of fully stabilized 4d $\mathcal{N} = 1$ Minkowksi vacua was questioned as well. The reason is that any small, SUSY breaking, positive energy contribution to the scalar potential turns these Minkowksi vacua into metastable dS solutions. Following this logic, the authors of[18] conjectured that strongly stabilized AdS vacua should be forbidden. Here by strongly stabilized one means that the mass of the lightest field satisfies $m_{\text{light}} L_{AdS} \gg 1$, where L_{AdS} is the length scale of the AdS space. This AdS moduli conjecture seems to imply that if we take the limit $L_{AdS} \to \infty$ to go to Minkowksi space, then $m_{\text{light}} \to 0$ in contradiction with the Minkowski vacua discussed here and previously in[8,9,13]. Note however, that these Minkowski vacua cannot arise as the limit of any of the infinite families of AdS solutions that we find in these models and that will be discussed in the next subsection. Likewise, there is no obvious small SUSY breaking correction or change to the model that leads to dS vacua. All string loop corrections do not change W and only modify the values of the positive masses squared of the scalar fields in the Minkowski vacuum. Changing some flux quanta to break supersymmetry is a large effect and the same probably applies to any other change given that the complex structure modulus is stabilized at order 1 and we are at strong coupling. However, it would definitely be interesting to study this further.

The existence of these vacua and the absence of corrections is surprising, maybe even more so given the recent paper[19] that finds that generically in quantum gravity any allowed correction should appear. The exception to this rule is stated in the same paper and is formalized in the supersymmetric genericity conjecture[19]. This conjecture says that quantities that are protected in higher supersymmetric theories should only vanish in lower supersymmetric theories, if the lower supersymmetric theory is related to a higher supersymmetric theory. In particular, the authors discuss 4d $\mathcal{N} = 1$ Minkowski vacua with everywhere vanishing superpotential, $W = 0$. They find that the equation $W = 0$ can only survive all corrections if the theory is related to a higher dimensional theory via for example a simple orbifold projection. While our setup with fluxes and a non-zero W generate by those fluxes is not covered by the analysis in[19], our findings seem nevertheless compatible with the supersymmetric genericity conjecture since our setups are simple orbifolds of toroidal models that preserve higher amounts of supersymmetry.

Summarizing, these non-geometric type IIB setups give rise to fully stabilized 4d $\mathcal{N} = 1$ Minkowksi vacua that seem to survive all stringy corrections, which makes them to our knowledge the only full-fledged string theory constructions of this type. These vacua arise only at relatively strong coupling in a barely studied part of the string landscape.

5 Infinite families of AdS vacua

In this section we study exemplary families of AdS solutions that arise in these non-geometric type IIB flux compactifications. As discussed above, due to the non-renorma-lization of W even supersymmetric AdS solutions at strong coupling will persist when including all potential corrections. However, for example the masses and the cosmological constant in these solutions might get significantly modified when we are not at weak string coupling. All the different families of AdS solutions that we present below, allow us to go to parametrically weak coupling and thus we have parametric control over them. This enables us to perform trustworthy and detailed studies even when these solutions are not supersymmetric. Given that the exact number of O3 planes in these infinite families plays essentially no role, we will restrict ourselves to the 1^9 model with $N_{O3} = 24$. We will introduce representative examples to illustrate the different behaviors that these infinite families display. Firstly, we present families that are dual to the AdS vacua found in DGKT [6] but we also find other infinite families of AdS vacua that arise in our more general setup. Secondly, we study interesting and very different sets of solutions, where by increasing the number of D3 branes the contribution of the fluxes to the tadpole can become negative and very large. In the $N_{\text{flux}} \to -\infty$ limit the number of D3 branes needs to become infinite $N_{D3} \to \infty$ as well, in order to satisfy the tadpole condition. We discuss how all these solutions fit into the web of swampland conjectures at the end of this section.

5.1 Infinite families of AdS vacua without D3 branes

5.1.1 The DGKT dual

In [9] two infinite families of SUSY AdS solutions were presented. The first solution is related to the infinite family of SUSY AdS vacua that were found in DGKT [6].[4] To find it one has to necessarily set three H_3 flux quanta to zero, $h^0 = h^1 = h_1 = 0$. The tadpole condition (3.12) then implies

$$h_0 = \frac{12}{f^0}\,, \tag{5.1}$$

which means that due to flux quantization $f^0 \in \{1, 2, 3, 4, 6, 12\}$. We will not plug in any specific flux values but keep in mind that f^0 and h_0 are bounded due to tadpole cancellation condition but the other fluxes are not.

[4] In the second SUSY AdS solution in subsection 4.3.2 in [9], there seems to be a typo. We find that either $\text{Im}(U)$ or $\text{Im}(S)$ are necessarily negative, so this does not seem to be a physically meaningful solution.

One can easily solve $D_S W = D_U W = 0$ and find that the axio-dilaton is stabilized at

$$\mathrm{Re}(S) = \frac{f_0 (f^0)^2 + 3 f_1 f^0 f^1 - 2(f^1)^3}{12 f^0}\,, \qquad \mathrm{Im}(S) = 2\sqrt{\frac{5}{3}}\frac{(f_1 f^0 - (f^1)^2)^{\frac{3}{2}}}{9 f^0}\,, \tag{5.2}$$

while the complex structure modulus is stabilized at

$$\mathrm{Re}(U) = \frac{f^1}{f^0}\,, \qquad \mathrm{Im}(U) = \sqrt{\frac{5}{3}}\frac{(f_1 f^0 - (f^1)^2)^{\frac{1}{2}}}{f^0}\,. \tag{5.3}$$

Given that f_1 is unconstrained by the tadpole, we can make it large and even send it to infinity. In that limit the string coupling $1/\mathrm{Im}(S)$ becomes parametrically small and the complex structure modulus becomes parametrically large. This is the mirror dual of the large volume, weak coupling families of AdS vacua that arise in type IIA flux compactifications if one makes the F_4 flux large[6].

The scalar potential at the minimum is

$$V_{AdS} = \frac{-19683\sqrt{\frac{3}{5}}(f^0)^3}{3200(f_1 f^0 - (f^1)^2)^{\frac{9}{2}}}\,. \tag{5.4}$$

The four masses squared in this family can be conveniently expressed in terms of the above value of the scalar potential as

$$m^2 = \left\{\frac{10}{3}, 6, \frac{70}{3}, \frac{88}{3}\right\}|V_{AdS}|\,. \tag{5.5}$$

Since the AdS radius in 4d is given by $R_{AdS} = \sqrt{3/|V_{AdS}|}$ one finds surprisingly that all the masses squared in AdS units, i.e. all $m^2 R_{AdS}^2$, are integers. This was recently discovered in [20] (see also [21]). Furthermore, the integers are such that the operator scaling dimensions in the dual CFT_3, i.e.

$$\Delta = \frac{1}{2}\left(3 + \sqrt{9 + 4m^2 R_{AdS}^2}\right) = \{5, 6, 10, 11\}\,, \tag{5.6}$$

are integers as well [20,22,23]. This fascinating feature of this family of AdS vacua currently awaits an explanation and we check below in our other families whether the same is true or not.

Given that we want to compare our infinite families with the AdS distance conjecture, it is important to determine the mass scale of a tower of states that becomes light in the large flux limit. In the dual DGKT construction [6] the large flux limit corresponds to a large volume limit and the KK scale sets the scale of a tower with a mass scale that goes to zero as the flux quanta go to infinity. Using mirror symmetry, as further discussed in appendix A, we can determine the dual mass scale of a tower that becomes light in this large flux limit[5]

$$m_{\text{tower}}^2 \sim \frac{1}{\text{Im}(U)\text{Im}(S)^2} \sim \frac{(f^0)^3}{(f_1 f^0 - (f^1)^2)^{\frac{7}{2}}} \sim \frac{1}{f_1^{\frac{7}{2}}}\,. \tag{5.7}$$

As we discuss below, the AdS distance conjecture [24], constrains the param-

[5] By mirror symmetry the large volume limit becomes a large complex structure limit in which winding modes should become light and lead to this tower of states.

eter α that relates the mass scale of the tower to the cosmological constant via $m_{\text{tower}} \sim |\Lambda|^{\alpha}$. In this solution we have $\alpha = 7/18$ since

$$m_{\text{tower}} \sim \frac{1}{f_1^{\frac{7}{4}}} \sim |V_{AdS}|^{\frac{7}{18}} \,. \tag{5.8}$$

5.1.2 SUSY families with $\alpha = 1/2$

Next we discuss another infinite family of AdS vacua that is also parametrically controlled but not dual to the DGKT model since we have two H_3 flux quanta turned on. In particular, we fix the following fluxes

$$f^0 = 0\,, \qquad f_1 = 0\,, \qquad h^0 = -3\,, \qquad h^1 = 0\,, \qquad h_0 = 0\,. \tag{5.9}$$

The tadpole condition in equation (3.12) is satisfied if we set $f_0 = 4 - h_1 f^1$ and we are left with two free flux parameters $h_1, f^1 \in \mathbb{Z}$. In this solution the real parts of S and U are equal to zero and the imaginary parts are given by

$$\begin{aligned} \text{Im}(S) &= \left[\frac{\left(-16 f^1 h_1 + 3\left(9 + \sqrt{81 + 24 f^1 h_1(-4 + f^1 h_1)}\right)\right)}{2h_1^2} \right] \text{Im}(U)\,, \\ \text{Im}(U) &= \frac{\sqrt{\frac{9 f^1 h_1 + 2(-9 + \sqrt{81 + 24 f^1 h_1(-4 + f^1 h_1)})}{f^1}}}{\sqrt{15}}\,. \end{aligned} \tag{5.10}$$

In the limit $f^1 \to \infty$ (and for negative $h_1 < 0$) we find the following scaling of the moduli

$$\begin{aligned}
\mathrm{Im}(U) &\approx \frac{\sqrt{(9-4\sqrt{6})h_1}}{\sqrt{15}}\,, \\
\mathrm{Im}(S) &\approx \frac{\sqrt{6+8\sqrt{\frac{2}{3}}f^1}}{\sqrt{-h_1}}\,.
\end{aligned} \tag{5.11}$$

So, we have parametric control since we can go to parametrically small string coupling. We can in principle also make the complex structure modulus large by an appropriate choice of h_1, however, this is not necessary since the Landau-Ginzburg model already takes all α' corrections into account [8].

In the above limit of very large f^1 the value of the potential at the minimum is given by

$$V_{AdS} \approx -\frac{27(-h_1)^{\frac{5}{2}}}{32\sqrt{1329+544\sqrt{6}}}\frac{1}{(f^1)^2}\,. \tag{5.12}$$

Comparing the mass of the light tower from equation (A.6) with the value of the scalar potential in this limit, we find that $m_{\text{tower}} \sim |V_{AdS}|^{\frac{1}{2}}$, i.e. $\alpha = 1/2$.

In the limit where $f^1 \to \infty$ the masses squared are,

$$\begin{aligned}
{m_{1\pm}}^2 &= \frac{2}{9}\left(17+\sqrt{6}\pm\sqrt{127+46\sqrt{6}}\right)|V_{AdS}|\,, \\
{m_{2\pm}}^2 &= \frac{1}{9}\left(25-2\sqrt{6}\pm\sqrt{337+68\sqrt{6}}\right)|V_{AdS}|\,.
\end{aligned} \tag{5.13}$$

The smallest of these masses squared,

$$m_{2-}^2 = \frac{1}{9}(25 - 2\sqrt{6} - \sqrt{337 + 68\sqrt{6}})|V_{AdS}| \approx -0.260|V_{AdS}|,$$

is above the Breitenlohner-Freedman bound $m_{BF}^2 = -\frac{3}{4}|V_{AdS}|$ [25], as required by supersymmetry. Obviously, none of these masses are integers in AdS units and the same is true for the dual conformal scaling dimensions. Since we kept h_1 finite in this example, the complex structure remains finite and therefore the mirror dual type IIA families should have likewise a fixed finite volume, which might (or might not) be related to the absence of integer conformal scaling dimensions.

5.1.3 Non-supersymmetric AdS vacua

Lastly, we discuss here a single non-supersymmetric family of AdS vacua. We have not performed an all encompassing search for such solutions but given that they exist in the type IIA models of DGKT and are related to the supersymmetric solutions by simple sign flips of F_4 flux quanta, they have to exist here as well. We found one such family that is related to the supersymmetric AdS solution discussed above in subsection 5.1.1, by setting $f^1 = f_0 = 0$ and flipping the sign of f_1.

Concretely, for $h^0 = h^1 = h_1 = f^1 = f_0 = 0$, and f^0 essentially fixed by the tadpole as in equation (5.1) above, we find a one parameter family of non-SUSY AdS vacua parameterized by f_1. The real parts of the two moduli vanish in this family, Re(S) =Re(U) = 0 and the imaginary parts are given by

$$\mathrm{Im}(U) = \sqrt{\frac{5}{3}}\sqrt{-\frac{f_1}{f^0}}\,, \qquad\qquad \mathrm{Im}(S) = \frac{2}{9}\sqrt{\frac{5}{3}}(-f_1)^{\frac{3}{2}}\sqrt{f^0}\,. \tag{5.14}$$

So, we see that both grow in the limit $f_1 \to -\infty$ and we have parametric control over these non-supersymmetric solutions. The scalar potential is given by

$$V_{AdS} = -\sqrt{\frac{3}{5}}\frac{19683}{3200(f^0)^{\frac{3}{2}}(-f_1)^{\frac{9}{2}}}\,. \tag{5.15}$$

Since the moduli and the cosmological constant scale as for the supersymmetric counter part in subsection 5.1.1 above, one again finds $\alpha = 7/18$.

The four masses squared for these solutions are given by

$$m^2 = \left\{\frac{70}{3}, \frac{40}{3}, 6, -\frac{2}{3}\right\}|V_{AdS}|\,. \tag{5.16}$$

The smallest of these masses squared, $m^2 = -\frac{2}{3}|V_{AdS}|$, is above the Breitenlohner-Freedman bound $m^2_{BF} = -\frac{3}{4}|V_{AdS}|$[25] and this solutions is stable, although in this case this is not guaranteed by supersymmetry.

We note that the masses squared above again give rise to dual conformal dimensions $\Delta = \{10, 8, 6, 2 \text{ or } 1\}$ that are all integers. This was previously noticed for non-supersymmetric DGKT solutions in[20,22] and it would be interesting to extend the general analysis of[23] to non-supersymmetric AdS vacua.

5.2 AdS vacua with a large number of D3 branes

Given the fact that supersymmetric fluxes in this setup can contribute to the tadpole condition in the same way as O3 planes, we do not necessarily need the latter, however, we will keep them in the models below. We can furthermore ask whether we can find infinite families of supersymmetric vacua where a flux contribution in the tadpole can cancel an arbitrarily large number of D3 branes. This is indeed the case and we will present below two exemplary families where $N_{\text{flux}} \to -\infty$, $N_{D3} \to \infty$ while the tadpole $N_{\text{flux}} + N_{D3} = N_{O3}/2 = 12$ is satisfied. To the best of our knowledge such types of solution have never been discussed in the flux compactification literature before. We will present them below and then discuss potential problems and detailed features of these solutions in more detail below in subsection 5.3.

5.2.1 An infinite family with $\alpha = 1/2$ and $\mathbf{N_{D3}} \to \infty$

We will set the following four fluxes to zero $f^1 = f_0 = h^0 = h_1 = 0$. Then we solve the SUSY equations $D_S W = D_U W = 0$. We find supersymmetric AdS solutions with Re(S) =Re(U)=0 and the imaginary parts are stabilized at

$$\text{Im}(U) = \sqrt{\frac{-3f^0h_0 - 9f_1h^1 + \sqrt{9(f^0h_0)^2 + 74f_1f^0h_0h^1 + 81(f_1h^1)^2}}{2f^0h^1}}\,, \quad (5.17)$$

$$\text{Im}(S) = \left(\frac{-3f^0h_0 + 9f_1h^1 + \sqrt{9(f^0h_0)^2 + 74f_1f^0h_0h^1 + 81(f_1h^1)^2}}{8h_0h^1}\right)\text{Im}(U)\,.$$

The tadpole equation (3.12) in this case reduces to

$$-3h^1 f_1 + h_0 f^0 + N_{D3} = 12\,. \tag{5.18}$$

Keeping h_0 and f^0 fixed and choosing a positive h^1, we can send $f_1 \to \infty$. This gives rise to an infinite family of solution that requires an ever growing number of D3 branes to be present, with $N_{D3} \propto f_1$. For simplicity we study the particular example $h^1 = f^0 = 1$. In the $f_1 \to \infty$ limit the moduli are approximately given by

$$\mathrm{Im}(U) \approx \frac{\sqrt{5h_0}}{3}\,, \qquad \mathrm{Im}(S) \approx \frac{3\sqrt{5}f_1}{4\sqrt{h_0}}\,.$$

Thus we are at parametrically weak coupling and we can even make $\mathrm{Im}(U)$ very large by choosing an appropriate fixed but arbitrarily large value for h_0.

In the limit where f_1 goes to infinity we have:

$$V_{AdS} \approx -\frac{2(h_0)^{3/2}}{25\sqrt{5}f_1^2}\,. \tag{5.19}$$

In the large f_1 limit the mass of the light tower (in Planck units) is

$$m^2_{\text{tower}} \sim \frac{1}{\mathrm{Im}(U)\mathrm{Im}(S)^2} \approx \frac{16\sqrt{h_0}}{15\sqrt{5}f_1^2}\,, \tag{5.20}$$

which corresponds to $\alpha = 1/2$. The masses squared in this limit are

$$m_{1\pm}{}^2 \approx \frac{1}{27}(41 \pm 4\sqrt{181})|V_{AdS}|\,, \qquad m_{2\pm}{}^2 \approx \frac{1}{27}(26 \pm \sqrt{181})|V_{AdS}|\,. \quad (5.21)$$

The smallest mass squared, $m_{1-}^2 \approx \frac{1}{27}(41-4\sqrt{181})|V_{AdS}| \approx -0.475|V_{AdS}|$, is above the Breitenlohner-Freedman bound $m_{BF}^2 = -\frac{3}{4}|V_{AdS}|$ [25], as required by supersymmetry.

5.2.2 An infinite family with $\alpha = 3/2$ and $\mathbf{N_{D3}} \to \infty$

Lastly, we present an infinite family that gives rise to a different value of α, while still requiring an ever growing number of D3 branes. We choose the following fixed flux values

$$f^1 = 1, \quad f_0 = 1, \quad h^0 = 0, \quad h^1 = -1, \quad h_1 = 0, \quad h_0 = -1, \quad f^0 = 1\,, \quad (5.22)$$

leaving us with f_1 as the free parameter. There exist then supersymmetric AdS vacua in which the moduli take on the following values

$$\mathrm{Re}(U) = 0\,, \qquad \mathrm{Im}(U) = \frac{\sqrt{-3 - 9f_1 + \sqrt{9 + f_1(74 + 81f_1)}}}{\sqrt{2}}\,, \quad (5.23)$$

$$\mathrm{Re}(S) = -1\,, \qquad \mathrm{Im}(S) = \left(\frac{3 - 9f_1 - \sqrt{9 + f_1(74 + 81f_1)}}{8}\right)\mathrm{Im(U)}\,.$$

The tadpole equation (3.12) in this case reduces to

$$3f_1 + N_{D3} = 13\,. \tag{5.24}$$

In the limit $f_1 \to -\infty$ the above tadpole requires $N_{D3} \sim -3f_1 \to \infty$. The value of the scalar potential in this limit is

$$V_{AdS} \approx -\frac{729}{32768(-f_1)^{\frac{1}{2}}}\,. \tag{5.25}$$

The moduli scale for $f_1 \to -\infty$ like

$$\mathrm{Im}(S) \approx \frac{8\sqrt{-f_1}}{3}\,, \qquad \mathrm{Im}(U) \approx 3\sqrt{-f_1}\,, \tag{5.26}$$

and therefore

$$m^2_{\text{tower}} \sim \frac{1}{\mathrm{Im}(U)\mathrm{Im}(S)^2} \sim \frac{1}{(-f_1)^{\frac{3}{2}}}\,. \tag{5.27}$$

This actually means that $m_{\text{tower}} \sim |V_{AdS}|^{\frac{3}{2}}$, i.e. $\alpha = 3/2$ in this case.

In the limit where $f_1 \to -\infty$ the masses become

$$m^2 \approx \left\{6, \frac{10}{3}, \frac{22}{7}, -\frac{8}{27}\right\} |V_{AdS}|\,. \tag{5.28}$$

The masses squared are above the Breitenlohner-Freedman bound $m^2_{BF} = -\frac{3}{4}|V_{AdS}|$ [25], as required by supersymmetry. Interestingly the first two masses squared give again rise to dual conformal scaling dimensions that are integers, while the later two give rise to fractional scaling dimensions: $\Delta = \{6, 5, 11/3, 8/3\}$.

5.3 AdS vacua and the swampland

Many explicit and widely studied constructions of AdS vacua in string theory exhibit the following two features: First, there are usually some light fields whose masses are comparable (or smaller) than the AdS scale $M_{AdS} = 1/R_{AdS} = \sqrt{|V_{AdS}|/3}$ and this was conjectured to be true in all string compactifications in[18]. Second, the most widely studied AdS vacua in string theory are of Freund-Rubin type[26,27] or exhibit similar features, by which we mean that the size of the internal space R_{KK} is not parametrically smaller than R_{AdS}. This property was recently studied for example in[22,28–34] and has led to the AdS distance conjecture[24] that states that for infinite families of AdS vacua with $V_{AdS} \to 0$, there exist a tower of massive states with masses that satisfy $m_{\text{tower}} \sim |V_{AdS}|^{\alpha}$ for some positive α of order one. The strong version of this conjecture says that for supersymmetric AdS vacua $\alpha = 1/2$. This conjecture has been refined in[35,36]. Lastly, it was conjectured that no stable AdS vacua exist at all[37] and all these conjectures have been used to derive important implications for the standard model of particle physics[38–43].

Against the backdrop of the above results, let us start by examining our infinite families of AdS vacua. First, let us note that in all the above families of solution the masses of the light fields S and U are always of the same order as $\sqrt{|V_{AdS}|}$. This means that they are all consistent with the AdS/moduli conjecture proposed in[18].

Let us now look at the $N_{\text{flux}} = 12$ solutions, which do not require the presence of D3 branes and that were discussed above in subsection 5.1. The supersymmetric AdS solutions with $\alpha = 7/18$ violate the strong version of the AdS distance conjecture. A refined version of the conjecture was proposed in[36] where a 4d discrete $\mathbb{Z}_k$ 3-form gauge symmetry was identified in the DGKT model and the following refined conjecture was proposed: $m_{\text{tower}} \sim \sqrt{k|V_{AdS}|}$. Given that our family of solutions is mirror dual to the DGKT AdS vacua we have a discrete $\mathbb{Z}_{f_1}$ symmetry and our solutions indeed satisfy $m_{\text{tower}} \sim \sqrt{f_1|V_{AdS}|}$.[6]

The next family of supersymmetric AdS vacua that we discuss above satisfies the strong version of the AdS distance conjecture since it has $\alpha = 1/2$. This absence of scale separation was also discovered in related IIA models in[44].

This leaves us with a non-supersymmetric family of AdS solutions that is also dual to DGKT and that has $\alpha = 7/18$. This is again consistent with the refined AdS distance conjecture due to the presence of a discrete symmetry that is unaffected by a simple sign flip of a flux quanta. Since these solutions are non-supersymmetric they are predicted to decay perturbatively or non-perturbatively[37]. Given that we find that the masses squared of S and U are above the Breitenlohner-Freedman bound[25], it is not clear whether there

[6]One could in principle work this out explicitly following[36]: A 3-form gauge field with $U(1)$ gauge group arises from $F_7 = dC_6$ wrapping an internal 3-cycle. This 3-form gauge field couples to the F_3 flux component f_1 and the complex structure axion $\text{Re}(U)$, which leads to the breaking of the symmetry to $\mathbb{Z}_{f_1}$. However, given the non-geometric nature of our compactifications things are more involved and it is easiest to simply rely on mirror symmetry.

is a perturbative instability. Studying all possible non-perturbative decay channels or trying to identify one explicit non-perturbative decay channel is a daunting task, so we restrict ourselves here to referring to a related study of non-supersymmetric AdS vacua in the dual DGKT model[45].

Finally, let us discuss the most interesting families of supersymmetric 4d $\mathcal{N} = 1$ AdS vacua, namely the new families that allow for the inclusion of an arbitrarily large number of D3 branes and that are discussed in subsection 5.2. While the first one has $\alpha = 1/2$ and is therefore consistent with the strong version of the AdS distance conjecture, the second one has $\alpha = 3/2$, which means that the light tower is becoming light much more quickly. These solution can be made consistent with the strong version of the AdS distance conjecture by demanding $\alpha \geq 1/2$, as is already discussed in the original paper[24]. Nevertheless, given that these vacua with $\alpha = 3/2$ are different from all the other solutions which had $\alpha = 1/2$ or smaller, they are interesting and deserve further study.

Since the later two families of supersymmetric AdS vacua have an ever increasing number of D3 branes one should worry about what that means exactly. In geometric compactifications we would expect an ever growing number of light open string modes associated with these N_{D3} branes. Concretely, for N_{D3} branes at separate locations the number of light open string degrees of freedom should grow like N_{D3}. If there is a an actual potential being generated for the D3 brane position moduli, then it seems likely that they all settle into the minimum.[7] We can of course also always choose to place all the N_{D3} on top of each other and since they are mutually BPS there should be no force between them. This would then lead to a number of light degrees of freedom that grows even faster like N_{D3}^2. Due to the species bound[46–49], this leads to a UV cutoff that goes like $\Lambda_{UV} \sim M_{pl}/\sqrt{N_{D3}^2} = M_{pl}/N_{D3}$. In our first family of AdS vacua one finds that $\Lambda_{UV} \sim 1/f_1 \sim m_{\text{tower}}$. So, the UV cutoff from the species bound scales in the same way as the infinite tower of light modes. In the second example with $\alpha = 3/2$ one finds that $\Lambda_{UV} \sim 1/f_1 \sim m_{\text{tower}}^{\frac{4}{3}}$. This means that the species bound is even lower than the tower of light states that comes down rather quickly in this case anyways. Note that the previous discussion is based on the geometric intuition that might well carry over to these non-geometric setups. However, the actual open string spectrum for D3 branes in these model was not worked out in the previous literature. We leave it as an interesting task for the future to check the light open string degrees of freedoms in these models.

Slightly disconnected from the different AdS conjectures discussed above, we lastly would like to point out the most interesting and most surprising feature of these AdS solutions with $N_{D3} \to \infty$: The fluctuations along the AdS_4 directions of the open string modes on these branes should give rise to gauge groups with arbitrarily large rank. For example, if we place all

[7] At least in a geometric compactification the moduli space is compact so there are no runaway directions and for a non-trivial potential there has to exist a global minimum. Any potential that is generated for the D3 brane position moduli should be small in our limit of parametrically weak string coupling, so these position moduli should be light.

N_{D3} branes on top of each other one would naively expect an $SU(N_{D3}/2)$ gauge group.[8] String universality in higher dimensions with higher amount of supersymmetry leads to fairly low ranks for the gauge group, which seems in stark contrast with the solutions above. This is a by now very active area of research following the initial work of[50–54]. However, there is no argument in the literature that forbids 4d $\mathcal{N} = 1$ (not scale separated) AdS solutions with an $SU(N)$ gauge group for arbitrarily large N. Furthermore, there exist families of AdS_7 solutions with arbitrarily large gauge group rank (see for example[55–59] for early work on this). So, it seems reasonable that related AdS_4 solutions do exist as well in the barely explored part of the string landscape that we study here. As discussed in section IV of[24], since the AdS_7 solutions are not scale separated one should think of the gauge group as living on a defect in the higher dimensional $AdS_7 \times S^4/\mathbb{Z}_k$ theory. For this AdS_7 case one can increase the gauge group rank by making k large and this does not lead to a decompactification. However, in our setup when we increase the rank of the gauge group we send the cosmological constant to zero $V_{AdS} \to 0$. The internal space is also not geometric. So, although our solutions are not scale separated and there is a tower of light string modes, it is not necessarily natural to think of the D3 branes as defects in a higher dimensional geometric space. We again add as a word of caution that the open string spectrum for these D3 branes has not been worked out and

[8]The tadpole condition in equation (3.12) counts D3 branes in the covering space, hence there can be at most $\lfloor N_{D3}/2 \rfloor$ freely moving D3 branes in the quotient space. If N_{D3} is odd then one D3 brane would necessarily be stuck on top of an O3 plane.

therefore it could hypothetically not contain any massless open strings or no gauge fields at all. It would be very interesting to check this explicitly and we hope to do this in the future.

6 de Sitter vacua

Lastly, we would like to comment on the existence of dS vacua in this setup. Given that the Kähler potential can receive string loop corrections, one finds that non-supersymmetric solutions can cease to exist, if corrections are large. Thus, unless they are at weak coupling one should not trust non-supersymmetric solutions. All dS solutions in the models discussed here will have a string coupling that is not that much smaller than 1 and it is therefore not clear whether they can be trusted. Nevertheless, we discuss them for the following two reasons: Firstly, they were recently studied in [13] and in the dual type IIA picture in [60] and we would like to comment on and extend these previous results. Secondly, dS vacua are notoriously difficult to construct in purely classical scalar potentials [61] and only very few explicit solutions without tachyons exist in the literature [62–65]. Therefore, it is interesting to check whether they also exist in our simple models or not.

Unstable dS solutions, i.e. solutions with a tachyonic direction and the correct tadpole for the 1^9 model, $N_{\text{flux}} = 12$, were found in [13]. Interestingly, the authors of that paper performed a scan over flux values that do not satisfy the tadpole condition and they found that stable dS vacua exist for a large $N_{\text{flux}} \sim \mathcal{O}(100)$. They also noticed that the ratio of stable dS vacua to all randomly generated vacua grows with N_{flux} (see figure 9 in [13]). The smallest N_{flux} value that was giving rise to a stable dS solution in figure 9 in [13] is larger than 66 and the smallest, explicitly listed, stable dS solution in table 5 of that paper has $N_{\text{flux}} = 74$. While this is substantially larger than the allowed $N_{\text{flux}} = 12$ in the 1^9 model, it is not that much larger than the allowed $N_{\text{flux}} = 40$ in the 2^6 model.

6.1 Explicit dS solutions

An explicit tachyonic dS extremum with $N_{\text{flux}} = 12$ was previously found in [13]. The corresponding fluxes are

$$\begin{aligned} &f^0 = 4, \quad &&f^1 = 8, \quad &&f_1 = 7, \quad &&f_0 = -17, \\ &h^0 = 1, \quad &&h^1 = 1, \quad &&h_1 = 1\,, \quad &&h_0 = -2\,. \end{aligned} \tag{6.1}$$

Given that $N_{\text{flux}} = 12$ this is a solution to the 1^9 model which does not require D3 branes since the fluxes cancel the negative O3 plane charge. The moduli are stabilized at the following values

$$\begin{aligned} \text{Re}(U) &\approx 0.544\,, \qquad & \text{Im}(U) &\approx 1.11\,, \\ \text{Re}(S) &\approx 7.72\,, \qquad & \text{Im}(S) &\approx 5.19\,. \end{aligned} \tag{6.2}$$

The value of the scalar potential is given by $V_{dS} \approx 1.72 \times 10^{-4}$. The masses squared for the four real scalar fields in the unstable dS extremum are given by

$$m_1^2 \approx 0.0226\,, \qquad m_2^2 \approx 0.0157\,, \qquad m_3^2 \approx 0.00143\,, \qquad m_4^2 \approx -0.00119\,. \tag{6.3}$$

So, there are unstable dS solutions like the one above and, as mentioned previously, there are also metastable dS vacua, if one ignores the tadpole and lets $N_{\rm flux}$ become fairly large. Therefore, one should ask what the lowest possible value for $N_{\rm flux}$ is that still gives rise to metastable dS solutions. We have not been able to answer this question in full generality. However, we noticed that unstable and metastable dS solutions still exist when we set four fluxes to zero: $f^1 = f_0 = h^0 = h_1 = 0$. We then studied the full parameter space spanned by the remaining four fluxes, while ignoring the tadpole. This led us to discover infinite families of solutions that transition from AdS to unstable dS and then to metastable dS, if we vary the fluxes. Within these family we identified the smallest possible $N_{\rm flux}$ that has integer quantized fluxes and gives rise to metastable dS solutions. We find that the only possible value below $N_{O3}/2 = 40$ for the 2^6 model is $N_{\rm flux} = 30$.[9] For this value there are four different metastable dS solutions. Three have $\text{Im}(S) < 1$ and are therefore expected to receive substantial string loop corrections. The

[9]The next larger values of $N_{\rm flux}$ that give rise to metastable dS solutions in our restricted model with only four non-zero fluxes are $N_{\rm flux} = \{59, 60, 61\}$. This is too large to be compatible with the tadpole cancellation condition.

fourth one with the fluxes

$$\begin{aligned} &f^0 = 33, \quad &&f^1 = 0, \quad &&f_1 = -1, \quad &&f_0 = 0, \\ &h^0 = 0, \quad &&h^1 = -1, \quad &&h_1 = 0\,, \quad &&h_0 = 1\,, \end{aligned} \tag{6.4}$$

has a metastable dS vacuum at

$$\begin{aligned} \mathrm{Re}(U) &= 0\,, \qquad & \mathrm{Im}(U) &\approx 0.299\,, \\ \mathrm{Re}(S) &= 0\,, \qquad & \mathrm{Im}(S) &\approx 1.32\,. \end{aligned} \tag{6.5}$$

The value of the scalar potential is given by $V_{dS} \approx 0.00524$. The masses squared for the four real scalar fields in the dS minimum are given by

$$m_1^2 \approx 3.31\,, \qquad m_2^2 \approx 1.29\,, \qquad m_3^2 \approx .302\,, \qquad m_4^2 \approx 0.0999\,. \tag{6.6}$$

Given that $N_{\rm flux} = 30$ this is a solution to the 2^6 model which does require $N_{D3} = 10$ D3 branes. Thus, there should be additional light open string moduli associated with those D3 branes.

It would be interesting to extend our full analysis beyond the restriction $f^1 = f_0 = h^0 = h_1 = 0$ and check whether there exist metastable dS solutions in these models that are at smaller string coupling and/or that do not require D3 branes in order to satisfy the tadpole. Due to the mirror symmetry that relates our above models to models with H_3 flux and non-geometric Q flux there should be also a connection to the metastable dS solution found in

2009 in[66]. Note however, that the latter also required geometric and/or non-geometric fluxes in the type IIB duality frame since $h^{1,1} \neq 0$ and thus they are less controlled then the models we discussed in this paper due to the risk of large α' corrections.

6.2 dS extrema and the swampland

The very existence of dS vacua in string theory was first questioned in[16,17] and a variety of refined dS swampland conjectures were proposed in 2018 in for example[67–71]. All of these conjectures forbid metastable dS solutions. However, given that our metastable dS solution above is expected to receive substantial string loop corrections, it does not invalidate these conjectures. The previously discovered unstable dS extremum of[13] has $e^{\phi} \approx .5$ and does not require D3 branes. It is thus in much better shape, however, given that it is unstable with large $|\eta| \approx 7$ it is not really incompatible with any of the refined dS swampland conjectures.

It would be interesting to study this simplified model or related more complicated setups to see whether one can find metastable dS vacua at weak coupling and without D3 branes. While there is no obstruction to this, it was recently shown in the context of type IIA flux compactifications that dS solutions cannot exist in a parametrically controlled region[72,73]. While these papers mostly focused on geometric type IIA flux compactifications they also discuss more exotic ingredients like non-geometric fluxes which makes them applicable to all the type IIA flux compactifications that are the mirror dual of our type IIB setup. Thus, they actually apply also to our non-geometric type IIB models. This means there should be no parametrically controlled dS solutions, i.e. no solutions with a free flux parameter that we can send to infinity to get $\mathrm{Im}(S) \to \infty$. However, there is no obvious reason why well-controlled dS solutions with $\mathrm{Im}(S) \gg 1$ cannot exist in the setup discussed in this thesis.

7 Introduction to Intersecting Sources

The following is based on [2].

For more than two decades string theory compactifications with intersecting D-branes and O-planes have played an important role in string phenomenology. On the one hand, intersecting D-brane models are used to obtain particle physics models that can resemble the supersymmetric standard model and extension thereof, see for example the review article [74]. On the other hand, orientifold planes are needed in flux compactifications to partially break supersymmmetry and to provide a source of negative energy in the scalar potential, see for example [75,76] for early review articles. For flux compactifications on toroidal orbifolds the orientifold planes generically intersect in the internal space. So, both settings lead to supergravity equations of motion that have localized sources that intersect in a non-trivial way.

For such intersecting sources one then has to solve the equations of motion for the electromagnetic field strengths that are being sourced. This is rather simple since the equations are linear and the field strengths for each individual source can simply be added up. However, Einstein's equations are non-linear and extremely hard to solve. This has led to the often-employed simplification of a so-called smearing of the sources over their transverse directions. Mathematically speaking one replaces the delta function sources with constants, which dramatically simplifies the equations of motion. If one does that, one would then have to try to understand how close such a smeared solution is to the actual localized solution one started with, which is not an easy question to answer[77–79].

One can of course try to solve the equations of motions for intersecting objects without smearing or by only partially smearing the sources. For example, one could smear only over the mutual transverse directions of all sources, or one smears the sources only over directions that are transverse to one and parallel to another, etc. This leads to a plethora of possibilities that are discussed for example in the review article[80] (see also[81] for an earlier review article). The upshot of this endeavor is that fully localized solutions are known essentially only for parallel sources and in all other cases one has to do at least some partial smearing in order to solve the equations of motion. One exception is the case of two intersecting NS5-branes extending along $x^0, x^1, x^2, x^3, x^4, x^5$ and $x^0, x^1, x^6, x^7, x^8, x^9$ respectively (without any mutually transverse directions), see[82] for a discussion of this solution.

Within the swampland program[83] in string theory many flux compactifications have recently been revisited and scrutinized. In particular, flux compactifications of massive type IIA give rise to infinite families of weakly coupled 4d $\mathcal{N} = 1$ AdS vacua [6,84]. The viability of these solution was questioned for example by the AdS swampland conjecture [24]. One criticism pertaining to these type IIA flux compactifications is that they are using smeared orientifolds planes, i.e., the full 10d supergravity equations of motion have not been explicitly solved [85]. Two papers recently revisited this problem [86,87] and found approximate solution with localized sources (see also [23,30,31,33,88–98] for closely related recent work). These approximate solutions in [86,87] arose from an expansion in the large F_4-flux quanta and they capture the leading order backreaction of the localized orientifold planes. However, at this order the actual effects of the intersection of the O-planes is not taken into account. It would therefore be extremely important to extend these approximate solutions to higher order. However, given the importance of intersecting sources in many parts of string theory, a broader approach is also certainly warranted.

In this thesis we study the equations of motion for two localized Dp-branes or Op-planes in flat space. We take them to intersect perpendicular with four Neumann/Dirichlet directions and $(p-2)$ common directions (often denoted Dp $\perp$ Dp $(p-2)$). This means the setup preserves 8 supercharges, which allows us study the SUSY transformations of the fermions. Demanding that these vanish, as required for a supersymmetric solution, we find that a fully localized solution cannot exist for a generic diagonal metric Ansatz, even when allowing for fully generic fluxes. While this might come as a surprise, similar results were previously obtained. For example, it was shown in [99] that no solution can exist for localized, intersecting D3/D5-branes.

8 Review of a single source

In this section we will solve the equations of motion of type II supergravity coupled to a stack of Dp-branes or an Op-plane in 10d flat space. Such a solution is textbook material [100] but we review it here to set up our notation and to remind the reader of some features that will be important in the next section.

8.1 Type II supergravity

We are using the notation and conventions of [77] but we will change to string frame. The trace reversed Einstein equations are given by

$$\begin{aligned} R_{ab} \;=\;& -2\nabla_a\partial_b\phi + \frac{1}{4}g_{ab}\left(2g^{cd}\partial_c\phi\partial_d\phi - \nabla^2\phi\right) + \frac{1}{2}|H|^2_{ab} - \frac{1}{8}g_{ab}|H|^2 \\ &+ \sum_{n\leq 5} \mathrm{e}^{2\phi}\Big(\tfrac{1}{2(1+\delta_{n5})}|F_n|^2_{ab} - \tfrac{n-1}{16(1+\delta_{n5})}g_{ab}|F_n|^2\Big) + \frac{1}{2}\mathrm{e}^{\phi}\left(T^{loc}_{ab} - \frac{1}{8}g_{ab}T^{loc}\right) . \end{aligned} \tag{8.1}$$

The sum over n includes all even/odd numbers from 0 to 5 for IIA/IIB. The δ_{n5} is the Kronecker delta, and squares of q-forms are defined via $|A|^2_{\alpha\beta} = \frac{1}{(q-1)!}A_{\alpha a_2\ldots a_q}A_\beta{}^{a_2\ldots a_q}$, $|A|^2 = \frac{1}{q!}A_{a_1\ldots a_q}A^{a_1\ldots a_q}$. We restrict to parallel (stacks) of Dp-branes or Op-planes so that the local stress tensor is given by

$$T^{loc}_{\mu\nu} = \mu_p \, g_{\mu\nu} \, \delta(p) \, . \tag{8.2}$$

Here μ_p is negative for Dp-branes and positive for an Op-plane.[10] $\delta(p)$ denotes a delta function that localizes us on the $p+1$ dimensional world volume of the source. For multiple parallel Dp-branes or Op-planes $\delta(p)$ should be understood as a sum of δ-functions. μ, ν are denoting the directions along the worldvolume of the source and $g_{\mu\nu}$ is the pullback of the spacetime metric g_{ab} to the worldvolume of the source.

The equation of motion for the dilaton is given by

$$\nabla^2 \phi = 2 g^{ab} \partial_a \phi \partial_b \phi - \frac{1}{2}|H|^2 + \sum_{n<5} \tfrac{5-n}{4} \mathrm{e}^{2\phi} |F_n|^2 - \tfrac{p-3}{4} \mathrm{e}^{\phi} \mu_p \delta(p) \, . \tag{8.3}$$

Plugging the above into equation (8.1), we find that it simplifies to

$$\begin{aligned} R_{ab} \;\; = \;\; & -2\nabla_a \partial_b \phi + \frac{1}{2}|H|^2_{ab} + \sum_{n\leq 5} \mathrm{e}^{2\phi} \left(\tfrac{1}{2(1+\delta_{n5})} |F_n|^2_{ab} - \tfrac{1}{4(1+\delta_{n5})} g_{ab} |F_n|^2 \right) \\ & + \frac{1}{2}\mathrm{e}^{\phi} \left(T^{loc}_{ab} - \frac{1}{2} g_{ab} \mu_p \delta(p) \right) \, . \end{aligned} \tag{8.4}$$

In the absence of NS5-branes, the Bianchi identities for the field strengths

[10] While we do not need the exact values, the charge and tension of a stack of N_p Dp-branes is $-N_p \tilde{\mu}_p = -N_p (2\pi\sqrt{\alpha'})^{-p}/\sqrt{\alpha'}$. The charge and tension of an Op-plane is $-2^{p-5}\tilde{\mu}_p$ in the quotient space. The quantity appearing in our equations is $\mu_p = -N_p 2\kappa_{10}^2 \tilde{\mu}_p = -N_p (2\pi\sqrt{\alpha'})^{7-p}$ for a stack of Dp-branes and $\mu_p = 2^{p-5}(2\pi\sqrt{\alpha'})^{7-p}$ for an Op-plane.

are

$$
\begin{aligned}
dH &= 0\,, \\
dF_n &= H \wedge F_{n-2} - \mu_{8-n}\delta_{n+1}(8-n)\,,
\end{aligned}
\tag{8.5}
$$

where $\delta_{n+1}(8-n)$ is a shorthand notation for the delta function $\delta(8-n)$ multiplied by a normalized $(n+1)$ volume form transverse to the source.

The equations of motion for the gauge fields in the absence of NSNS sources are given by

$$
\begin{aligned}
\mathrm{d}\left(\mathrm{e}^{-2\phi} \star H\right) &= -\frac{1}{2}\sum_{n\leq 10} \star F_n \wedge F_{n-2}\,, \\
d\left(\star F_n\right) &= -H \wedge \star F_{n+2} - (-1)^{\frac{n(n-1)}{2}} \mu_{n-2}\delta_{11-n}(n-2)\,.
\end{aligned}
\tag{8.6}
$$

The equations of motion for the RR fields can be obtained from the Bianchi identities in equation (8.5) by using that $F_n = (-1)^{\frac{(n-1)(n-2)}{2}} \star F_{10-n}$.

For supersymmetric solutions one has to require that the SUSY transformations of the fermions vanish. This provides a simpler set of first order equations that often completely fixes the system and thereby automatically solves the Einstein and dilaton equations. We use the conventions of[75,101] so that the transformations of the gravitino and gaugino are given by

$$
\begin{aligned}
\delta_\epsilon \psi_a &= \left(\partial_a + \frac{1}{4}\underline{\omega_a} + \frac{1}{4}\underline{H_a}\mathcal{P}\right)\epsilon + \frac{1}{8}e^{\phi}\sum_n \frac{1}{1+\delta_{n5}}\,\underline{F_n}\Gamma_a\mathcal{P}_n\epsilon\,, \\
\delta_\epsilon \lambda &= \left(\underline{\partial\phi} + \frac{1}{4}\underline{H}\mathcal{P}\right)\epsilon + \frac{1}{4}e^{\phi}\sum_n (-1)^n(5-n)\underline{F_n}\mathcal{P}_n\epsilon\,.
\end{aligned}
\tag{8.7}
$$

The sum over n includes all even/odd numbers from 0 to 5 for IIA/IIB. As above $a = 0, 1, \ldots, 9$ is a curved space index and we denote the corresponding tangent space indices as $A, B = 0, 1, \ldots, 9$. The underlined quantities are given by

$$\begin{aligned} &\underline{\omega_a} = \omega_a{}^{AB}\Gamma_{AB}\,, &&\underline{H_a} = \tfrac{1}{2}H_{abc}\Gamma^{bc}\,, \qquad \underline{H} = \tfrac{1}{3!}H_{abc}\Gamma^{abc}\,,\\ &\underline{F_n} = \tfrac{1}{n!}F_{a_1\ldots a_n}\Gamma^{a_1\ldots a_n}\,, &&\underline{\partial\phi} = \partial_a\phi\Gamma^a\,, \end{aligned} \tag{8.8}$$

where $\Gamma^{a_1a_2\ldots a_n} = \Gamma^{[a_1}\Gamma^{a_2}\ldots\Gamma^{a_n]}$, and we also define $\Gamma_{10} = \Gamma_{012\ldots 9}$. Furthermore, we have that

$$\begin{aligned} &\mathcal{P} = \Gamma_{10} \quad \text{in IIA}, && \mathcal{P} = -\sigma_3 \quad \text{in IIB},\\ &\mathcal{P}_n = (\Gamma_{10})^{\frac{n}{2}} \quad \text{in IIA}, && \mathcal{P}_n = \begin{cases} \sigma_1 \text{ for } \frac{n+1}{2} \text{ even},\\ \mathrm{i}\sigma_2 \text{ for } \frac{n+1}{2} \text{ odd}, \end{cases} \quad \text{in IIB}\,. \end{aligned} \tag{8.9}$$

The spinor ϵ in type IIA has 32 real components, which could be split into two 16 component Majorana-Weyl spinors with opposite chiralities: $\Gamma_{10}\epsilon_1 = \epsilon_1$, $\Gamma_{10}\epsilon_2 = -\epsilon_2$. For IIB $\epsilon = (\epsilon_1, \epsilon_2)^T$ is a doublet of two 16 component Majorana-Weyl spinors with positive chirality so that $\Gamma_{10}\epsilon_i = \epsilon_i$. The Pauli matrices σ_i above act on this doublet.

In the presence of Dp-branes along the first $p+1$ directions or when doing the corresponding orientifold projection we break half of the supersymmetry via the following projection (involving the flat space Γ-matrices)

$$\epsilon_2 = \Gamma_{01\ldots p}\epsilon_1 \, . \tag{8.10}$$

8.2 A single p-dimensional source

We consider first a single Op-plane or a stack of Dp-branes. These localized objects are magnetic sources for F_{8-p} due to their Chern-Simons coupling to C_{p+1}. So, the only sourced RR-field is $F_{8-p} = \star F_{p+2}$. We can set all other RR-fields and the NSNS-flux H equal to zero.

We can choose our coordinates in such a way that the Op-plane or the stack of Dp-branes extend along x^μ with $\mu = 0, 1, \ldots, p$ and are located at the origin in the transverse directions $x^i = 0$, for $i = p+1, p+2, \ldots, 9$. This then preserves an $SO(p,1) \times SO(9-p)$ symmetry group, where the first $SO(p,1)$ factor is enhanced to the full Poincaré group. The most general metric Ansatz that is compatible with these symmetries is

$$g = e^{2A_1(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2A_2(r)}\delta_{ij}dx^i dx^j \, . \tag{8.11}$$

Here $e^{2A_1(r)}$ and $e^{2A_2(r)}$ can only depend on $r = \sqrt{(x^{p+1})^2 + \ldots + (x^9)^2}$, the overall transverse distance from the localized source.

The solution to the equations above in subsection 8.1 can be found in the textbook[100] (10.38) and we write it as

$$e^{-4A_1(r)} = e^{4A_2(r)} = 1 - \frac{\tilde{\mu}_p}{r^{7-p}}\,, \tag{8.12}$$

$$e^{\phi(r)} = e^{\phi_0 + (p-3)A_1(r)}\,, \tag{8.13}$$

$$C_{p+1}(r) = \left(1 - e^{4A_1(r)}\right) e^{-\phi_0}\; dx^0 \wedge dx^1 \wedge \ldots \wedge dx^p\,. \tag{8.14}$$

Here e^{ϕ_0} is the asymptotic value of the dilaton infinitely far away from the source. We fixed the metric to be asymptotically Minkowski and we chose $C_{p+1}(r)$ to asymptotically vanish. We also defined $\tilde{\mu}_p = (-1)^{p+1} e^{\phi_0} \frac{\Gamma(\frac{9-p}{2})}{(7-p)\pi^{\frac{9-p}{2}}} \frac{\mu_p}{2}$.

Note that due to the minus sign in equation (8.12) and the fact that $\tilde{\mu}_p$ is positive for an Op-plane, there is actually a singularity at a finite distance $r = \tilde{\mu}_p^{\frac{1}{7-p}}$ from the Op-plane. This singularity is at a distance that is of the order of the string length, $l_s = 2\pi\sqrt{\alpha'}$. At this point stringy corrections modify the equations of motion and remove this singularity.

Since we will need this later, we derive here explicitly the solution to the non-trivial Bianchi identity (cf. equation (8.5)). We rewrite it using the transverse metric determinant $g_{9-p} = e^{2(9-p)A_2}$ as follows

$$\begin{aligned} dF_{8-p} &= -\mu_p\, \delta_{9-p}(p) \\ &= -\mu_p\, \delta_{9-p}(p) \star_{9-p} 1 \\ &= -\mu_p \frac{1}{\sqrt{g_{9-p}}}\, \delta(x^{p+1})\delta(x^{p+2})...\delta(x^9)\, \sqrt{g_{9-p}}\; dx^{p+1} \wedge dx^{p+2} \wedge ... \wedge dx^9 \\ &= -\mu_p\, \delta(x^{p+1})\delta(x^{p+2})...\delta(x^9)\, dx^{p+1} \wedge dx^{p+2} \wedge ... \wedge dx^9 \end{aligned}$$

$$= -\mu_p\, \tilde{\delta}(\vec{r})\, \tilde{\star}_{9-p} 1\,. \tag{8.15}$$

The tilde indicates that we are working with the flat space metric so there is no warp factor dependence anymore. The solution is given by

$$F_{8-p} = \tilde{\star}_{9-p}\, d \left(\frac{\tilde{\mu}_p}{r^{7-p}} \right)\,, \tag{8.16}$$

since

$$\begin{aligned} dF_{8-p} &= \mathrm{d}\tilde{\star}_{9-p} d \left(\frac{\tilde{\mu}_p}{r^{7-p}} \right) \\ &= (-1)^p (\tilde{\star}_{9-p} 1) \tilde{\nabla}^2 \left(\frac{\tilde{\mu}_p}{r^{7-p}} \right) \\ &= (-1)^{p+1} (\tilde{\star}_{9-p} 1) \tilde{\mu}_p \frac{2(7-p)\pi^{\frac{9-p}{2}}}{\Gamma(\frac{9-p}{2})} \tilde{\delta}(\vec{r}) \\ &= (\tilde{\star}_{9-p} 1)\, \mu_p\, \tilde{\delta}(\vec{r})\,. \end{aligned} \tag{8.17}$$

Summarizing, we see that it is possible to solve the supergravity equations exactly for a single source. Similarly, one can solve the equations of motion for parallel sources that are located not necessarily at $\vec{r} = 0$ but at different positions $\vec{r}_0^{(\alpha)}$, $\alpha = 1, 2, \ldots$. In this case we can simply add up the individual solutions for each source and the solution is given by

$$\begin{aligned} e^{-4A_1(\vec{r})} = e^{4A_2(\vec{r})} &= 1 - \sum_\alpha \frac{\tilde{\mu}_p^{(\alpha)}}{\left| \vec{r} - \vec{r}_0^{(\alpha)} \right|^{7-p}}\,, \\ e^{\phi(\vec{r})} &= e^{\phi_0 + (p-3)A_1(\vec{r})}\,, \end{aligned}$$

$$C_{p+1}(\vec{r}) = \left(1 - e^{4A_1(\vec{r})}\right) e^{-\phi_0}\; dx^0 \wedge dx^1 \wedge \ldots \wedge dx^p \,. \tag{8.18}$$

Note that the Bianchi identities in equation (8.5) are linear and we can always simply add up the field strengths for any arbitrarily complicated configuration of sources. However, it is highly unusual, and special to this case of parallel sources, that the non-linear general relativity equation in (8.1) is also solved if we simply add up solutions.

9 Two perpendicularly intersecting sources

In this section we want to solve the equations of motion for two perpendicularly intersecting p-dimensional sources in flat space. These could be either two Op-planes or two stacks of Dp-branes or one of each. We restrict to $1 \leq p \leq 6$ so that we can have four directions that are along one of the objects and transverse to the other and there is at least one common transverse direction. The configuration that preserves eight supercharges in flat space is shown below.

Directions	0	...	$p-2$	$p-1$	p	$p+1$	$p+2$	$p+3$	...	9
First source	×	×	×	×	×	-	-	-	-	-
Second source	×	×	×	-	-	×	×	-	-	-

The above intersecting sources respect an $SO(p-2,1) \times SO(2) \times SO(2) \times SO(7-p)$ symmetry.[11] The first $SO(p-2,1)$ group is actually enhanced

[11]For the special case of $p = 1$ there are no directions common to both sources and therefore no $SO(p-2,1)$ factor. However, this does not affect our reasoning.

to the full Poincaré group. This symmetry group together with the specific source configuration shown above allows the metric (warp factors) to only depend on $\rho_1 = \sqrt{(x^{p+1})^2 + (x^{p+2})^2}$, $\rho_2 = \sqrt{(x^{p-1})^2 + (x^p)^2}$ and $\rho_T = \sqrt{(x^{p+3})^2 + ... + (x^9)^2}$. We make the following diagonal metric Ansatz

$$\begin{aligned} ds^2 = & e^{2A_1(\rho_1,\rho_2,\rho_T)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2A_2(\rho_1,\rho_2,\rho_T)}\left((dx^{p-1})^2 + (dx^p)^2\right) \\ & + e^{2A_3(\rho_1,\rho_2,\rho_T)}\left((dx^{p+1})^2 + (dx^{p+2})^2\right) + e^{2A_4(\rho_1,\rho_2,\rho_T)}\left((dx^{p+3})^2 + ... + (dx^9)^2\right), \end{aligned} \tag{9.1}$$

with $\mu, \nu = 0, 1, \ldots, p-2$. Poincaré invariance ensures that the first part of the metric is generic and there cannot be any off-diagonal terms like for example $g_{\mu\rho_1}dx^\mu d\rho_1$ since there are no invariant constant vectors of $SO(p-2,1)$. Non-constant vectors like $\eta_{\mu\nu}x^\mu dx^\nu$ are forbidden by translational invariance. However, in general there could be terms involving $d\rho_1 d\rho_T$, etc. and also terms involving the corresponding angles $d\theta_1$ and $d\theta_2$ when going to polar coordinates, $(x^{p+1}, x^{p+2}) \to (\rho_1, \theta_1)$ and $(x^{p-1}, x^p) \to (\rho_2, \theta_2)$. Here we are restricting to a diagonal metric to make the problem tractable. Since the source setup is invariant under the exchanges $x^{p-1} \leftrightarrow x^p$ and $x^{p+1} \leftrightarrow x^{p+2}$ we can impose the same symmetry on the metric Ansatz, making equation (9.1) the most general diagonal metric Ansatz compatible with the source configuration.

We choose to work with Cartesian coordinates that have the following property that will be important below

$$
\begin{aligned}
\partial_{x^{p-1}} e^{2A_n(\rho_1,\rho_2,\rho_T)} &= \frac{x^{p-1}}{\rho_2}\partial_{\rho_2} e^{2A_n(\rho_1,\rho_2,\rho_T)}\,, \\
\partial_{x^{p}} e^{2A_n(\rho_1,\rho_2,\rho_T)} &= \frac{x^{p}}{\rho_2}\partial_{\rho_2} e^{2A_n(\rho_1,\rho_2,\rho_T)}\,, \\
\partial_{x^{p+1}} e^{2A_n(\rho_1,\rho_2,\rho_T)} &= \frac{x^{p+1}}{\rho_1}\partial_{\rho_1} e^{2A_n(\rho_1,\rho_2,\rho_T)}\,, \\
\partial_{x^{p+2}} e^{2A_n(\rho_1,\rho_2,\rho_T)} &= \frac{x^{p+2}}{\rho_1}\partial_{\rho_1} e^{2A_n(\rho_1,\rho_2,\rho_T)}\,. \qquad (9.2)
\end{aligned}
$$

For the dilaton the most general Ansatz is $\phi = \phi(\rho_1, \rho_2, \rho_T)$. We also define the transverse coordinates for the two O-planes

$$
\begin{aligned}
r_1 &= \sqrt{\rho_1^2 + \rho_T^2} = \sqrt{(x^{p+1})^2 + (x^{p+2})^2 + (x^{p+3})^2 + ... + (x^9)^2}\,, \\
r_2 &= \sqrt{\rho_2^2 + \rho_T^2} = \sqrt{(x^{p-1})^2 + (x^{p})^2 + (x^{p+3})^2 + ... + (x^9)^2}\,. \qquad (9.3)
\end{aligned}
$$

Using the metric Ansatz as given in equation (9.1), we seek the solution for the above source configuration. We first solve the linear Bianchi identity (cf. equation (8.5))

$$
dF_{8-p} = -\mu_p^{(1)}\delta_{9-p}^{(1)}(p_1) - \mu_p^{(2)}\delta_{9-p}^{(2)}(p_2)\,. \qquad (9.4)
$$

We solve the above equation by writing $F_{8-p} = F_{8-p}^{(1)} + F_{8-p}^{(2)} + F_{8-p}^{(c)}$, where

$dF^{(c)}_{8-p} = 0$ is closed[12] and

$$dF^{(1)}_{8-p} = -\mu^{(1)}_p \delta^{(1)}_{9-p}(p_1) \qquad \text{and} \qquad dF^{(2)}_{8-p} = -\mu^{(2)}_p \delta^{(2)}_{9-p}(p_2)\,. \tag{9.5}$$

So, this manifests the linearity of the electromagnetic equations and allows us to simply add up the two fields strengths for the two sources, i.e., we can add up the results for two single sources in flat space. The solution for the first source is (cf. equation (8.16))

$$F^{(1)}_{8-p} = \tilde{\star}^{(1)}_{9-p} d \left(\frac{\tilde{\mu}^{(1)}_p}{r_1^{7-p}} \right) . \tag{9.6}$$

From equation (9.6) we can read off the non-zero components of $F^{(1)}_{8-p}$

$$\begin{aligned}
F^{(1)}_{8-p} =& \tilde{\star}^{(1)}_{9-p} d \left(\frac{\tilde{\mu}^{(1)}_p}{r_1^{7-p}} \right) \\
=& - \frac{\tilde{\mu}^{(1)}_p (7-p)}{r_1^{8-p}} \tilde{\star}^{(1)}_{9-p} dr_1 \\
=& - \frac{\tilde{\mu}^{(1)}_p (7-p)}{r_1^{9-p}} \tilde{\star}^{(1)}_{9-p} \left(x^{p+1} dx^{p+1} + x^{p+2} dx^{p+2} + ... + x^9 dx^9 \right) \\
=& \frac{\tilde{\mu}^{(1)}_p (7-p)}{r_1^{9-p}} \Big(x^{p+1} dx^{p+2} \wedge dx^{p+3} \wedge ... \wedge dx^9 \\
& \qquad - x^{p+2} dx^{p+1} \wedge dx^{p+3} \wedge ... \wedge dx^9 \\
& \qquad + ...
\end{aligned}$$

[12] We are indebted to Daniel Junghans for pointing out this additional closed piece in F_{8-p}.

$$+ (-1)^p \left(x^9 dx^{p+1} \wedge dx^{p+2} \wedge ... \wedge dx^8\right)\Big) \,. \tag{9.7}$$

Explicitly we find the following component that we will use below

$$\left(F^{(1)}_{8-p}\right)_{(p+1)(p+3)(p+4)...9} = -\frac{\tilde{\mu}^{(1)}_p (7-p)}{r_1^{9-p}}\, x^{p+2} \,. \tag{9.8}$$

$F_2^{(2)}$ can be obtained by exchanging x^{p+1}, x^{p+2} with x^{p-1}, x^p in equation (9.7). In particular, it has the component

$$\left(F^{(2)}_{8-p}\right)_{(p-1)(p+3)(p+4)...9} = -\frac{\tilde{\mu}^{(2)}_p (7-p)}{r_2^{9-p}}\, x^{p} \,. \tag{9.9}$$

Note that the above $F_{8-p} = F^{(1)}_{8-p} + F^{(2)}_{8-p} + F^{(c)}_{8-p}$ is the most generic and exact solution to the Bianchi identity in equation (8.5). It is independent of our particular metric Ansatz since the warp factors do not appear.

9.1 The Einstein and dilaton equations

Now we can look at Einstein's equations from equation (8.1) that reduce to

$$\begin{aligned} R_{ab} =& -2\nabla_a \partial_b \phi + \frac{1}{4} g_{ab}(2 g^{cd}\partial_c\phi\partial_d\phi - \nabla^2\phi) \\ & e^{2\phi}\left(\frac{1}{2(1+\delta_{(8-p))5})}|F_{8-p}|^2_{ab} - \frac{7-p}{16(1+\delta_{(8-p)5})} g_{ab}|F_{8-p}|^2\right) \\ & + \frac{1}{2} e^{\phi}(T^{loc}_{ab} - \frac{1}{8} g_{ab} T^{loc}) \,. \end{aligned} \tag{9.10}$$

Calculating the Ricci scalar for the above metric Ansatz in equation (9.1) we find for $a = p-1$, $b = p+1$ (essentially from equation (9.2) but also via an explicit computation) that

$$R_{(p-1)(p+1)} = x^{p-1}x^{p+1}f_R(\rho_1, \rho_2, \rho_T)\,, \tag{9.11}$$

where $f_R(\rho_1, \rho_2, \rho_3)$ is a specific function that one can calculate from the above metric Ansatz in equation (9.1). The important point is that the entire $R_{(p-1)(p+1)}$ component of the Ricci tensor is proportional to derivatives with respect to x^{p-1} and x^{p+1}. This then leads (cf. equation (9.2)) to the above prefactor $x^{p-1}x^{p+1}$ in front of $f_R(\rho_1, \rho_2, \rho_3)$.

Likewise we find that the dilaton Ansatz $\phi = \phi(\rho_1, \rho_2, \rho_T)$ leads to

$$-2\nabla_{p-1}\partial_{p+1}\phi = x^{p-1}x^{p+1}f_\phi(\rho_1, \rho_2, \rho_T)\,. \tag{9.12}$$

Let us assume first that the F_{8-p}-flux is simply the superposition of the fluxes from the two single sources as might be expected due to the linearity of the corresponding equation (9.4). That means we are setting the closed piece $F^{(c)}_{8-p}$ to zero in the F_{8-p}-flux. This also means that neither the other RR-fluxes nor the H-flux are sourced.

All non-diagonal entries of the metric in equation (9.1) vanish and the source terms vanish away from the sources as well. Therefore, the off-diagonal entry of the Einstein equation (9.10) for $(ab) = (p-1\,p+1)$ is given by

$$
\begin{aligned}
R_{p-1\,p+1} &= -2\nabla_{p-1}\partial_{p+1}\phi + \frac{1}{2}e^{2\phi}|F_{8-p}|^2_{p-1\,p+1} \\
x^{p-1}x^{p+1}f_R &= x^{p-1}x^{p+1}f_\phi(\rho_1,\rho_2,\rho_T) \\
&\quad + e^{2\phi}\frac{1}{2(p-1)!}F_{8-p,p-1\,a_1\ldots a_{7-p}}g^{a_1b_1}\ldots g^{a_{7-p}b_{7-p}}F_{8-p,p+1\,b_1\ldots b_{7-p}} \\
&= x^{p-1}x^{p+1}f_\phi(\rho_1,\rho_2,\rho_T) \\
&\quad + e^{2\phi}\frac{1}{2}F_{8-p,p-1\,p+3\ldots 9}g^{p+3p+3}\ldots g^{99}F_{8-p,p+1\,p+3\ldots 9} \\
&= x^{p-1}x^{p+1}f_\phi(\rho_1,\rho_2,\rho_T) \\
&\quad + e^{2\phi}\frac{1}{2}\tilde{\mu}_p^{(2)}(7-p)\frac{x^p}{{r_2}^{9-p}}e^{-2(7-p)A_4}\tilde{\mu}_p^{(1)}(7-p)\frac{x^{p+2}}{{r_1}^{9-p}}\,.
\end{aligned}
\tag{9.13}
$$

We rewrite this as

$$
x^{p-1}x^{p+1}(f_R - f_\phi) = x^p x^{p+2}\left(\frac{e^{2\phi-2(7-p)A_4}}{2}\frac{\tilde{\mu}_p^{(1)}(7-p)}{{r_1}^{9-p}}\frac{\tilde{\mu}_p^{(2)}(7-p)}{{r_2}^{9-p}}\right)\,. \tag{9.14}
$$

The above equation has to be true for all $x^{p-1}, x^p, x^{p+1}, x^{p+2}$. In particular, the left-hand-side is odd under the sign flips $x^{p-1} \to -x^{p-1}$ or $x^{p+1} \to -x^{p+1}$ and even under the sign flips $x^p \to -x^p$ or $x^{p+2} \to -x^{p+2}$. Since ϕ, A_4, r_1 and r_2 are all functions of (ρ_1,ρ_2,ρ_T), we find that the symmetry properties of the right-hand-side are exactly opposite. This means the left- and right-hand-side have to vanish independently. Since the dilaton and the component e^{2A_4} of the diagonal metric cannot vanish everywhere we conclude that the vanishing of the right-hand-side implies that

$$\tilde{\mu}_p^{(1)}\tilde{\mu}_p^{(2)} = 0\,. \tag{9.15}$$

The above equation implies that one of the two sources is absent. Or, if we insist that both of the intersecting sources are present, we have shown that there is no solution to the supergravity equations of motion for our two intersecting localized sources with our generic diagonal metric Ansatz. To make this proof fully general, we have to allow for the closed piece $F_{8-p}^{(c)}$ as solution to the Bianchi identity (9.4). Then this closed form piece can source the H-flux and other RR-fluxes via the equations of motion for the fluxes given above in (8.6). Thus, in order to give a full proof we have to actually allow for all possible RR-fluxes and the most generic H-flux compatible with our $SO(p-2,1) \times SO(2) \times SO(2) \times SO(7-p)$ symmetry group. This makes the Einstein and dilaton equations too complicated to analyze directly. Therefore, in the next subsection we study the spinor equations and show that there is indeed no supersymmetric localized solution to the supergravity equations of motion.

9.2 Spinor equations for the most generic fluxes

Let us discuss the most generic forms that are invariant under the assumed symmetry group $SO(p-2,1) \times SO(2) \times SO(2) \times SO(7-p)$ for the backreacted solution. Since the first factor $SO(p-2,1)$ is enhanced to the full Poincaré group, the only invariant forms are the always present 0-form and its Hodge dual which is the volume form that is proportional to $dx^0 \wedge dx^1 \wedge \ldots \wedge dx^{p-2}$. The other three spaces all have an $SO(n)$ symmetry so we can discuss them together: In addition to the 0-form and the dual volume form, there are two more forms. There is one 1-form which is d acting on the radial coordinates, $d\rho_1$, $d\rho_2$, $d\rho_T$ in our case, and then there is the dual $(n-1)$-form. For an $SO(2)$ symmetry this would be another 1-form, which we denote $d\theta_1$ and $d\theta_2$, where (ρ_i, θ_i) are simply polar coordinates. For the $SO(7-p)$-symmetry we would go to spherical coordinates $(\rho_T, \theta_T^{(1)}, \theta_T^{(2)}, \ldots, \theta_T^{(6-p)})$ and an invariant $(6-p)$-form is given by $\sin(\theta_T^{(1)})^{5-p} \sin(\theta_T^{(2)})^{4-p} \ldots \sin(\theta_T^{(4-p)})^2 \sin(\theta_T^{(5-p)})\, d\theta_T^{(1)} \wedge \theta_T^{(2)} \wedge \ldots \wedge \theta_T^{(6-p)}$. Lastly, we note that all functions like the metric, the warp factors, the dilaton or the prefactors that appear in front of the forms when spelling out the fluxes, can only depend on ρ_1, ρ_2, ρ_T due to the preserved symmetry.

Let us give a concrete example to clarify the above discussion. We choose $p = 6$ and want to find a localized solution that describes two O6-planes (or D6-branes) that extend along the directions $(x^0, x^1, x^2, x^3, x^4, x^5, x^6)$ and $(x^0, x^1, x^2, x^3, x^4, x^7, x^8)$, respectively. We take them to be localized at the origin in their transverse spaces. We assume that the metric is given by equation (9.1) above for $p = 6$. We take all warp factors and the dilaton to be functions of the three variables

$$\rho_1 = \sqrt{(x^5)^2 + (x^6)^2}\,, \qquad \rho_2 = \sqrt{(x^7)^2 + (x^8)^2}\,, \qquad \rho_T = x^9\,. \tag{9.16}$$

We then make the most generic flux Ansatz that is compatible with an $SO(4,1) \times SO(2) \times SO(2)$ symmetry group

$$
\begin{aligned}
F_2 &= F_2^{(1)} + F_2^{(2)} + F_2^{(c)}\,, \\
F_2^{(1)} &= \tilde{\star}_3^{(1)} d\left(\frac{\tilde{\mu}_6^{(1)}}{\sqrt{\rho_1^2+\rho_T^2}}\right)\,, \\
F_2^{(2)} &= \tilde{\star}_3^{(2)} d\left(\frac{\tilde{\mu}_6^{(2)}}{\sqrt{\rho_2^2+\rho_T^2}}\right)\,, \\
F_2^{(c)} &= \sum_{i=1}^{10} f_2^{(i)}(\rho_1,\rho_2,\rho_T)\, Y_i^2\,, \\
F_4 &= \sum_{i=1}^{5} f_4^{(i)}(\rho_1,\rho_2,\rho_T)\, Y_i^4\,, \\
H &= \sum_{i=1}^{10} h^{(i)}(\rho_1,\rho_2,\rho_T)\, Y_i^3\,.
\end{aligned}
\tag{9.17}
$$

Here the $f_2^{(i)}$, $f_4^{(i)}$, $h^{(i)}$ are unknown functions and the Y_i^2, Y_i^3, Y_i^4 denote the invariant and closed forms that form a basis of invariant forms. Since the $f_2^{(i)}(\rho_1,\rho_2,\rho_T)$ are generic functions and the Y_i^2 include for example $d\theta_1 \wedge d\theta_2$, this Ansatz does not yet satisfy $dF_2^{(c)} = 0$. We furthermore allow for a constant and non-zero F_0. The two 16 component spinors that are present in 10d flat space are constrained due to the presence of the O6-planes (or D6-branes) and have to satisfy

$$
\begin{aligned}
\epsilon_2 &= \Gamma_{0123456}\epsilon_1\,, \\
\epsilon_2 &= \Gamma_{0123478}\epsilon_1\,.
\end{aligned}
\tag{9.18}
$$

This breaks one quarter of the supersymmetry and leaves us with 8 real independent spinor components. The fully backreacted solution should preserve these eight supercharges. We therefore assume that these eight spinors are independent (and also functions of (ρ_1, ρ_2, ρ_T)).

We now demand that there is a supersymmetric solution and therefore demand that the spinor transformations in equation (8.7) satisfy $\delta_\epsilon \psi_a = \delta_\epsilon \lambda = 0$. This leads directly to $F_0 = F_4 = H = 0$, while F_2 has to be of the following form

$$\begin{aligned} F_2 &= \tilde{\star}_3^{(1)} d\left(\frac{\tilde{\mu}_6^{(1)}}{\sqrt{\rho_1^2+\rho_T^2}}\right) + \tilde{\star}_3^{(2)} d\left(\frac{\tilde{\mu}_6^{(2)}}{\sqrt{\rho_2^2+\rho_T^2}}\right) \\ &\quad + f_2^{(3)}(\rho_1,\rho_2,\rho_T) d\theta_1 \wedge d\rho_1 + f_2^{(8)}(\rho_1,\rho_2,\rho_T) d\theta_1 \wedge d\rho_T \\ &\quad + f_2^{(7)}(\rho_1,\rho_2,\rho_T) d\theta_2 \wedge d\rho_2 + f_2^{(9)}(\rho_1,\rho_2,\rho_T) d\theta_2 \wedge d\rho_T \\ &= \left(f_2^{(3)}(\rho_1,\rho_2,\rho_T) + \frac{\mu_{6,1}\rho_1\rho_T}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_1 \wedge d\rho_1 \\ &\quad + \left(f_2^{(8)}(\rho_1,\rho_2,\rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_1 \wedge d\rho_T \\ &\quad + \left(f_2^{(7)}(\rho_1,\rho_2,\rho_T) + \frac{\mu_{6,2}\rho_2\rho_T}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_2 \wedge d\rho_2 \\ &\quad + \left(f_2^{(9)}(\rho_1,\rho_2,\rho_T) - \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_2 \wedge d\rho_T \,. \end{aligned} \tag{9.19}$$

Recall that we have made a fully generic Ansatz for the closed piece in F_2 and we have not yet imposed that it is actually closed.

Let us briefly discuss the above solution in equation (9.19). We see that without imposing the Bianchi identities and equations of motions for the fluxes we can only have a very limited number of flux components $f_2^{(i)}(\rho_1, \rho_2, \rho_T)$ in addition to the source terms. These extra flux components actually combine with the source terms which makes perfect sense. For example, we know that there are solutions for a single source and we can for example use the $f_2^{(i)}(\rho_1, \rho_2, \rho_T)$ to remove one of the sources and then we actually reproduce the result for a single source discussed above in subsection 8.2. Here however, we are interested in solutions that describe two intersecting sources and we therefore do not want to cancel any source terms. We therefore proceed to study the remaining equations of motion.

We want that the source terms containing $\mu_{6,1}$ and $\mu_{6,2}$ give rise to the delta function sources and that the rest is closed (see the discussion around equation (9.4) above). Thus, we have to demand that $dF_2 = 0$ away from the source and therefore we find that

$$\begin{aligned}
\partial_{\rho_2} f_2^{(3)}(\rho_1, \rho_2, \rho_T) &= 0\,, \\
\partial_{\rho_1} f_2^{(7)}(\rho_1, \rho_2, \rho_T) &= 0\,, \\
\partial_{\rho_2} f_2^{(8)}(\rho_1, \rho_2, \rho_T) &= 0\,, \\
\partial_{\rho_1} f_2^{(9)}(\rho_1, \rho_2, \rho_T) &= 0\,. \qquad (9.20)
\end{aligned}$$

Additionally, the spinor equations in (8.7) did not only set most of the flux components to zero but they also fixed the first derivatives of the warp factors via the spin connection term in $\delta_\epsilon \psi_a = 0$ and the first derivatives of the dilaton via $\delta_\epsilon \lambda = 0$. Concretely, they fix $\partial_{\rho_1} e^{A_2(\rho_1,\rho_2,\rho_T)}$ and $\partial_{\rho_2} e^{A_2(\rho_1,\rho_2,\rho_T)}$ to be two different functions of the warp factors, the dilaton, the $f_2^{(i)}(\rho_1,\rho_2,\rho_T)$ and the source terms

$$\begin{aligned}
\partial_{\rho_1} e^{A_2(\rho_1,\rho_2,\rho_T)} &= F_1(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T)\,, \\
\partial_{\rho_2} e^{A_2(\rho_1,\rho_2,\rho_T)} &= F_2(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T)\,. \qquad (9.21)
\end{aligned}$$

Now we can impose the conditions above in equation (9.20) and the following consistency condition

$$\begin{aligned}
0 &= \partial_{\rho_2}\partial_{\rho_1} e^{A_2(\rho_1,\rho_2,\rho_T)} - \partial_{\rho_1}\partial_{\rho_2} e^{A_2(\rho_1,\rho_2,\rho_T)} \qquad (9.22) \\
&= \partial_{\rho_2} F_1(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T) - \partial_{\rho_1} F_2(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T) \\
&= \frac{e^{A_2-2A_4+2\phi}}{2\rho_1\rho_2}\left(f_2^{(8)}(\rho_1,\rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}}\right)\left(f_2^{(9)}(\rho_2,\rho_T) - \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}}\right).
\end{aligned}$$

Since the prefactor in the above equation cannot vanish everywhere, we see that

$$\left(f_2^{(8)}(\rho_1,\rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}}\right)\left(f_2^{(9)}(\rho_2,\rho_T) - \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}}\right) = 0\,. \qquad (9.23)$$

This shows that there is no fully localized solution with our generic diagonal metric Ansatz. The above equation requires us to at least partially remove (or smear) one of the sources.

Let us pursue the above further by setting without loss of generality

$$f_2^{(9)}(\rho_2, \rho_T) = \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2 + \rho_T^2)^{\frac{3}{2}}} \,. \tag{9.24}$$

This cancels the last term in F_2 above in equation (9.19) and the closure $dF_2 = 0$ then imposes the additional constraint that

$$f_2^{(7)}(\rho_1, \rho_2, \rho_T) = \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2 + \rho_T^2)^{\frac{3}{2}}} + f(\rho_2)\,, \tag{9.25}$$

where $f(\rho_2)$ is an undetermined function. With that F_2 becomes

$$\begin{aligned} F_2 \;\; = \;\; & \left(f_2^{(3)}(\rho_1, \rho_T) + \frac{\mu_{6,1}\rho_1\rho_T}{(\rho_1^2 + \rho_T^2)^{\frac{3}{2}}} \right) d\theta_1 \wedge d\rho_1 \\ & + \left(f_2^{(8)}(\rho_1, \rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_2^2 + \rho_T^2)^{\frac{3}{2}}} \right) d\theta_1 \wedge d\rho_T \\ & + f(\rho_2) d\theta_2 \wedge d\rho_T \,. \end{aligned} \tag{9.26}$$

So, we have effectively removed the second source completely. Actually the equations of motion for F_2 fix $f(\rho_2) = c\rho_2$ and using that in the solution to the spinor equations, we find that all derivatives of the warp factors and the dilaton with respect to ρ_2 vanish: $\partial_{\rho_2} e^{A_i} = \partial_{\rho_2} e^{\phi} = 0$. This is indicative of a smeared source and we indeed see from

$$dF_2 \supset d(f(\rho_2) d\theta_2 \wedge d\rho_T) = c\, d\rho_2 \wedge d\theta_2 \wedge d\rho_T \,, \tag{9.27}$$

that we can have at best a smeared second source in which the delta function source (see equation (9.4)) is replaced with the constant c. Thus, in addition to proving the absence of a solution with two fully localized sources, our equations pass consistency checks and do not forbid solutions with partially smeared sources.

We have repeated the above analysis for two intersecting sources with $p = 1, 2, 3, 4, 5$ and explicitly reproduced the same absence of localized solutions. This might have been expected from T-duality invariance of type II string theory, however, there is an important subtlety: If we have an Op-plane (or a Dp-brane) in flat space, then we can T-dualize along any of its worldvolume directions. The reason is that the dilaton, metric and everything else does not depend on these coordinates. This leads to an O$(p-1)$-plane (or a D$(p-1)$-brane) that is actually smeared over the direction we T-dualized. Similarly, we cannot T-dualize along a transverse direction since these are not isometries. We would have to first smear the source along this transverse direction and then we can T-dualize to get a $(p+1)$-dimensional source. So, strictly speaking we cannot use T-duality invariance in the strict sense and therefore we checked the equations for each $p = 1, 2, 3, 4, 5, 6$ explicitly.

10 Conclusion

This dissertation discussed a model for a scalar potential, from cosmological motivation, to ingredients arising from string theory and compactified dimensions. These ingredients were then brought together to form a Kähler potential and a superpotential. Those two potentials were combined to form an overall potential. This overall potential was explored to find Minkowski, AdS, and dS minima. These minima were compatible with a tadpole condition for either 24 or 80 O3-planes and were checked to see if they could give rise to weakly coupled string interactions.

A single-parameter family of Minkowski minima was discussed. Every member of this family as positive masses squared, so they are all stable. The entire family also has $N_{\text{flux}} = 12$, so the tadpole condition was satisfied for 24 O3-planes and no D3-branes. The maximum value of $\text{Im}(S)$ for this family was $2\sqrt{3}$.

Three families of AdS solutions were discussed. Two of them were SUSY and one was not. All three satisfied the tadpole condition for 24 O3-planes and no D3-branes. One of the SUSY solutions and the non-SUSY solution had integer square masses when expressed in AdS units. One of the SUSY solutions and the non-SUSY solution had a negative mass direction. The stability of the SUSY solution is protected by the m_{BF} bound. $\text{Im}(S)$ could be made arbitrarily large with the correct choice of fluxes for all three AdS solutions.

One dS solution was discussed. This solution has $N_{\text{flux}} = 30$, so the tadpole condition could be satisfied with 80 O3-planes and 10 D3-branes. $\text{Im}(S)$ has a value of 1.32. All of the masses squared are positive, so it is stable in all directions.

The surprising result from section 9, that shows the absence of localized supergravity solutions for intersecting objects in flat space, raises many important questions: Does the result also hold for a generic non-diagonal metric? Can such setups be described explicitly in the full string theory? Does our result carry over to compactifications? Here, we will briefly discuss these questions. However, we will not be able to answer them and leave many avenues for further research.

First it seems clear that intersecting sources can arise in string theory and corresponding solutions will exist. Our two intersecting Op-planes can arise from a single orientifold projection combined with a $\mathbb{Z}_2$ orbifold of flat space. For example, we can do an orientifold involution consisting of the worldsheet parity operator Ω_p and a spatial involution that flips the signs of x^7, x^8, x^9. This leads to a single O6-plane localized at $x^7 = x^8 = x^9 = 0$. Doing a $\mathbb{Z}_2$ orbifold that flips the signs of x^5, x^6, x^7, x^8 then introduces a second O6-plane localized at $x^5 = x^6 = x^9 = 0$. In principle one should be able to study the full string theory on such an orientifolded orbifold of flat space. Supergravity as a low energy approximation of the full string theory might simply not allow for a solution because we neglect higher derivative corrections, string loop corrections and/or did not include the full spectrum of the string states.

For the case of intersecting stacks of D-branes in particular we neglected all the open strings on the D-branes. These open strings give rise to gauge theories and one can study the dynamics of these gauge theories. It is possible that the gauge dynamics leads to a (partial) smearing of the D-branes and partially smeared solutions do certainly exist, see for example[102–105]. Some of these papers also discuss the near core (near horizon) limit of these brane setups and manage to find localized solutions in this limit. For the particular case of two intersecting D6-branes or O6-planes one can also try to lift things to M-theory and try to find a solution in 11d supergravity. Such a lift of two intersecting D6-branes was discussed in[106]

Let us mention that it is known that multiplying together the two harmonic functions (warp-factors) for the two sources cannot solve the localized equations of motion but rather requires smearing (see for example[102] eqns. (1)-(2) and references therein). We reproduce the same result with a generic diagonal metric Ansatz. A loophole to our findings is exploited by the only (to us) known fully localized supergravity solution of two intersecting branes[82]. The two intersecting NS5 branes in this setup have no mutually transverse direction since they extend along 012345 and 016789. In our equations we crucially use the fact that there are $7-p>0$ transverse directions.[13] It would be interesting to study further brane setups without mutually transverse directions.

We crucially assumed here that there is an unbroken $SO(p-2,1) \times SO(2) \times SO(2) \times SO(7-p)$ symmetry group. This seems to be justified for static objects or in the probe limit but it is possible that the dynamics of the

[13] Recall that here we restrict ourselves to $1 \leq p \leq 6$. It thus might be possible to write down fully localized solutions for $p=7$ but such setups are better described in F-theory[107].

D-branes (or the dynamics of O-planes at strong coupling) could break this symmetry group. It would therefore be interesting to see whether one can relax the requirement of this large unbroken symmetry group. Here let us note that[99] discusses the D3/D5-brane intersection, where the D5 brane extends along $x^0, x^1, x^2, x^3, x^4, x^5$ and the D3-brane along x^0, x^1, x^2, x^6. In this case the authors only assume the presence of an $SO(2,1)$ Poincaré group and an $SO(3)$-symmetry group in the mutually transverse x^7, x^8, x^9 directions. While solving the equations of motion they discover the necessary presence of an extra $SO(3)$-symmetry acting on the x^3, x^4, x^5 directions, before they find that no localized solution exists. Thus, it is conceivable that our result might still hold even if we were to give up the $SO(2) \times SO(2)$ symmetry and/or allow for a non-diagonal metric along the corresponding directions. It would be interesting to check this explicitly in particular given that the orbifold blow ups discussed recently in[97] would break this $SO(2) \times SO(2)$ symmetry. In our setup one could glue in a $\mathbb{P}^1$ to remove the orbifold singularity and this would correspond to giving a non-zero vev to the Kähler modulus that controls its size. However, it is unclear to us that this would happen dynamically and what could fix the scale of a non-zero vev for the Kähler modulus.

It is a far stretch to go from our setup of two intersecting sources in flat space to a full compactification of 10d supergravity like the massive type IIA flux compactifications discussed in[6,84]. However, we note here that the two papers[86,87] only worked to first order in the sources, i.e., in our language to the first order in the $\tilde{\mu}_p$. This means that contradictions like equations (9.15) or (9.23) above that are quadratic would not be visible when working at linear order. It would be therefore of great importance to extend the work of [86,87] to higher order. Already in the simplest case of a toroidal compactification we note that the preserved symmetry group gets dramatically reduced and it is conceivable that then localized solutions exist.

The key concept throughout this thesis is the important role played by O-planes and D-branes. First, their charge gave us more freedom in choice of fluxes. We saw that the tadpole condition would require the fluxes to cancel in the absence of O-planes and D-branes. Their presence allowed us to find, for example, the solutions described in subsection 6.1. As the basis of the central question being asked in the second part of this thesis, O-planes and D-branes were central to those sections.

A Details of the dual type IIA models

A.1 Kähler potential

As mentioned in the text the way in which one derives the formula for the Kähler potential is by using mirror symmetry. The usual formula for the Kähler potential in Type IIA flux compactifications on Calabi-Yau manifolds is [108]

$$K = -\log\left[\frac{4}{3}\int J \wedge J \wedge J\right] - 2\log\left[2\int \mathrm{Re}\,(C\Omega_3) \wedge \star\mathrm{Re}\,(C\Omega_3)\right] , \quad \text{(A.1)}$$

where J is the Kähler form and Ω the holomorphic 3-form. The volume is given by $\mathrm{vol}_6 = \frac{1}{6}\int J \wedge J \wedge J$. The so-called 4d dilaton is defined via $e^D = e^\phi/\sqrt{\mathrm{vol}_6}$ and $C\overline{C}\int \Omega \wedge \overline{\Omega} = e^{-2D}$. The supergravity fields are introduced by expanding the complexified Kähler form and the complexified holomorphic 3-form[108]

$$J_c = B_2 + \mathrm{i}J = \sum_{a=1}^{h^{(1,1)}} T^a \omega_a \,, \tag{A.2}$$

$$\Omega_c = C_3 + 2\mathrm{i}\mathrm{Re}\,(C\Omega_3) = S\alpha_0 + \sum_{k=1}^{h^{(2,1)}} U^k \alpha_k \,. \tag{A.3}$$

When $h^{(2,1)} = 0$ there are no complex structure moduli. We can always write the volume in terms of the triple intersection number $\kappa_{abc} = \int \omega_a \wedge \omega_b \wedge \omega_c$ of the Calabi-Yau manifold, which leads (up to a constant) to the Kähler potential

$$K = -\log\left[\frac{\mathrm{i}}{6}\kappa_{abc}\left(T^a - \overline{T}^a\right)\left(T^b - \overline{T}^b\right)\left(T^c - \overline{T}^c\right)\right] - 4\log\left[-\frac{\mathrm{i}}{2\sqrt{2}}\left(S - \overline{S}\right)\right] . \tag{A.4}$$

Mirror symmetry simply exchanges the $h^{1,1}$ Kähler moduli T^a with the $h^{2,1}$ complex structure moduli U^k. Since we have no complex structure moduli the mirror dual Kähler potential is the one given above in equation (3.10), if one restricts to the torus bulk moduli and sets them all equal[9]. The superpotential can be derived in the same way but was also argued for directly in type IIB in[8].

A.2 KK towers

In this section, following the original work[6], we quickly review how to derive the KK scale in type IIA flux compactifications. Using mirror symmetry we can then derive the mass scale for a light tower in the non-geometric type IIB flux compactifications discussed in this paper. As on the type IIA side, this is not proven to be always the lightest tower but no other lighter tower is expected to arise in the type IIA side, so presumably the same is true on the type IIB side. Also, our infinite families of AdS vacua are all consistent with the refined AdS distance conjecture[24,36], which means that this is likely the relevant tower of massive states.

The KK scale in type IIA flux compactifications is controlled by the internal volumes of 2-cycles, $\text{Im}(T^a)$. In the isotropic limit where we set the three bulk 2-cycles of the torus equal we will simply use $\text{Im}(T)$ to describe this volume. So, we know that m_{KK}^2 scales like $1/\text{Im}(T)$. Compactifying from 10d to 4d and then going to 4d Einstein frame introduces an extra factor and the correct KK scale is given by

$$m_{KK}^2 \sim \frac{1}{\text{vol}_6 e^{-2\phi} \text{Im}(T)} = \frac{1}{(\text{Im}(S))^2 \text{Im}(T)} \,. \tag{A.5}$$

Again using mirror symmetry, we find a dual massive tower with masses that scale like

$$m_{\text{tower}}^2 \sim \frac{1}{(\text{Im}S)^2 \, \text{Im}(U)} \,. \tag{A.6}$$

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Vita

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