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CAVITY DIMENSIONS FOR CONSTANT GRADIENT ACCELERATOR SECTIONS

$\tau = 0.57$, $2\pi/3$-MODE

Objective:

This note gives a tabulation of the cavity dimensions for two designs of constant gradient accelerator sections using disk-loaded waveguide. The two designs correspond to two different disk-thicknesses: t = 0.230 in. and t = 0.120 in.

Theory:

R.B. Neal has shown[1] that a constant gradient accelerator section can be obtained by tapering the cavity dimensions as a function of length so as to cause the group velocity to decrease linearly according to

$$\frac{v_g}{c} = \frac{\omega\ell(1 - z/\ell\,(1 - e^{-2\tau})\ )}{Qc\ (1 - e^{-2\tau})}$$

where

ω is the frequency

ℓ is the length of the section

c is the velocity of light

$\tau = I\ell$ is the attenuation per section

Q is the quality factor

In designing the constant gradient accelerator sections, the following parameters were chosen:

Operating mode: $2\pi/3$ phase shift per cavity

d = distance between disk center lines = 1.378 in. = 3.5 cm

N = number of cavities = 86

ℓ = 86 × 3.5 cm = 301.00 cm = 118.508 inches

f = 2856 Mc/sec

$\omega = 1.79448 \times 10^{10}$ cycles × 2π

$\tau = I\ell = 0.57$ nepers

Q = 13,000

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[1] R.B. Neal, ML Report 513, page 16.

For these parameters, the group velocity must vary linearly between

$$\frac{v_g}{c} = 0.0204 \text{ and } \frac{v_g}{c} = 0.0065$$

Empirical Design:

The cavity dimensions tabulated in this note were obtained experimentally from test cells using uniform 6-cavity sets (5 full cavities and two half-cavities). The group velocity was obtained using Robson's[2] Fourier analysis.

For the case $t = 0.230$ in, 13 experimental sets were used yielding 13 sets of points. For the case $t = 0.120$ in., 8 experimental sets were used, yielding 8 sets of points.

All experimental data were corrected for errors due to frequency variances from set to set (never exceeding 2Mc/sec) and variances in h (see below).

The experimental sets of points were fed into a Burroughs 220 computer program. The data for the two cases were smoothed by respectively using fourth and sixth order polynomials and by minimizing least square deviations.

Cavity Dimensions:

Tables I and II give the cavity dimensions for the respective disk thicknesses. The desired variation in group velocity is obtained by tapering the diameter $2a$ of the disk-hole aperture. The inside cavity diameter $2b$ is tapered to keep the frequency constant for the condition $v_p = c$ (phase velocity equal to velocity of light). The shape of the disk-aperture is kept constant and it is standardized to the dimensions shown in Fig. 1. The value of h is tabulated for two different steel ball diameters for $t = 0.230$ in. and for only one ball diameter for $t = 0.120$ in. It is obtained by calculation from dimension $2a$ as shown in Fig. 1. Figs. 2 and 3 show the variations of the tabulated dimensions.

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[2] P.N. Robson, "Fourier Series Representation of Dispersion Curves For Circular Corrugated Waveguides Used in Traveling Wave Linear Accelerators," Metropolitan-Vickers Limited, Research Report No. 5105, November, 1956.

Conventions:

Cavities are numbered from 0 to 85. Cavities 0 and 85 are coupler cavities. The dimension 2a for a certain value of v_g/c always applies to the output end of a particular cavity. Thus:

$(2a)_0$ for cavity 0 (z = 0 to 3.5 cm) at z = 3.5 cm corresponds to $(v_g/c)_0 = 0.02024$.

$(2a)_{85}$ for cavity 85 (z = 297.5 to 301 cm) at z = 301 cm corresponds to $(v_g/c)_{85} = 0.00650$

To obtain the desired frequency, the experimental tests were done at: f = 2855.5 Mc/sec, 78°F, 42% RH.

The dimension 2b given for a certain cavity applies to the input end of that cavity and corresponds to the group velocity at that z-coordinate. Thus:

$(2b)_0$ for cavity 0 (z = 0 to 3.5 cm) at z = 0 cm corresponds to $v_g/c = 0.02040$.

$(2b)_{85}$ for cavity 85 (z = 297.5 to 301 cm) at z = 297.5 corresponds to $v_g/c = 0.00666$.

TABLE I

DIMENSIONS OF CONSTANT GRADIENT ACCELERATOR SECTION

(t = 0.230 in.)

$2\pi/3$ MODE, 86 CAVITIES, $\tau = 0.57$

Note: The dimensions of the coupler cavities (Nos. 0 and 85) are given assuming no coupling-iris aperture. To correct for this perturbation, reduce:
2b of cavity No. 0 by approximately 0.050 in.
2b of cavity No. 85 by approximately 0.040 in.

DISK THICKNESS = 0.230 in.

CAVITY	2A	2B	H	GROUP VEL. v_g/c	CAVITY	2A	2B	H	GROUP VEL. v_g/c
0	1.0314	3.2843	.80105	.0202383	43	.91505	3.2533	.60939	.0132883
1	1.0290	3.2836	.80331	.0200767	44	.91192	3.2526	.61326	.0131267
2	1.0266	3.2829	.80557	.0199151	45	.90877	3.2518	.61708	.0129651
3	1.0242	3.2822	.80783	.0197534	46	.90560	3.2510	.62087	.0128034
4	1.0218	3.2815	.81008	.0195918	47	.90239	3.2502	.62461	.0126418
5	1.0194	3.2809	.81232	.0194302	48	.89917	3.2495	.62832	.0124802
6	1.0170	3.2802	.81457	.0192686	49	.89592	3.2487	.63199	.0123186
7	1.0145	3.2795	.81681	.0191069	50	.89264	3.2479	.63563	.0121569
8	1.0121	3.2788	.81904	.0189453	51	.88934	3.2471	.63923	.0119953
9	1.0096	3.2781	.82127	.0187837	52	.88601	3.2463	.64280	.0118337
10	1.0071	3.2774	.82349	.0186220	53	.88265	3.2455	.64635	.0116720
11	1.0046	3.2767	.82571	.0184604	54	.87926	3.2447	.64987	.0115104
12	1.0020	3.2760	.82793	.0182988	55	.87584	3.2439	.65336	.0113488
13	.99955	3.2753	.83014	.0181372	56	.87240	3.2431	.65682	.0111872
14	.99698	3.2746	.83235	.0179755	57	.86892	3.2423	.66027	.0110255
15	.99441	3.2739	.83455	.0178139	58	.86542	3.2415	.66369	.0108639
16	.99181	3.2731	.83675	.0176523	59	.86188	3.2407	.66708	.0107023
17	.98920	3.2724	.83895	.0174906	60	.85831	3.2399	.67046	.0105406
18	.98658	3.2717	.84114	.0173290	61	.85471	3.2391	.67382	.0103790
19	.98393	3.2710	.84332	.0171674	62	.85108	3.2383	.67716	.0102174
20	.98127	3.2703	.84551	.0170058	63	.84741	3.2374	.68049	.0100558
21	.97860	3.2696	.84768	.0168441	64	.84370	3.2366	.68379	.0098941
22	.97590	3.2689	.84986	.0166825	65	.83997	3.2358	.68708	.0097325
23	.97319	3.2681	.85203	.0165209	66	.83619	3.2350	.69036	.0095709
24	.97047	3.2674	.85420	.0163593	67	.83238	3.2341	.69362	.0094093
25	.96772	3.2667	.85636	.0161976	68	.82854	3.2333	.69687	.0092476
26	.96496	3.2660	.85852	.0160360	69	.82465	3.2325	.70010	.0090860
27	.96218	3.2652	.86068	.0158744	70	.82073	3.2317	.70332	.0089244
28	.95938	3.2645	.86284	.0157127	71	.81676	3.2308	.70654	.0087627
29	.95656	3.2638	.86499	.0155511	72	.81276	3.2300	.70974	.0086011
30	.95373	3.2631	.86714	.0153895	73	.80871	3.2292	.71293	.0084395
31	.95087	3.2623	.86928	.0152279	74	.80462	3.2283	.71610	.0082779
32	.94800	3.2616	.87143	.0150662	75	.80049	3.2275	.71928	.0081162
33	.94511	3.2608	.87357	.0149046	76	.79632	3.2267	.72244	.0079546
34	.94220	3.2601	.87571	.0147430	77	.79210	3.2258	.72559	.0077930
35	.93926	3.2594	.87785	.0145813	78	.78784	3.2250	.72874	.0076313
36	.93631	3.2586	.87998	.0144197	79	.78353	3.2242	.73187	.0074697
37	.93334	3.2579	.88212	.0142581	80	.77917	3.2233	.73500	.0073081
38	.93035	3.2571	.58927	.0140965	81	.77477	3.2225	.73813	.0071465
39	.92733	3.2564	.59341	.0139348	82	.77032	3.2217	.74124	.0069848
40	.92429	3.2556	.59748	.0137732	83	.76582	3.2209	.74436	.0068232
41	.92124	3.2548	.60150	.0136116	84	.76126	3.2201	.74746	.0066616
42	.91815	3.2541	.60547	.0134500	85	.75666	3.2193	.75056	.0065000

h_{II} spans cavities 33–36; h_I spans cavities 37–42.

All dimensions in inches, fifth significant figure to be disregarded.

TABLE II

DIMENSIONS OF CONSTANT GRADIENT ACCELERATOR SECTION

(t = 0.120 in.)

$2\pi/3$ MODE, 86 CAVITIES, $\tau = 0.57$

Note: The dimensions of the coupler cavities (Nos. 0 and 85) are given assuming no coupling-iris aperture. To correct for this perturbation, reduce:
2b of cavity No. 0 by approximately 0.040 in.
2b of cavity No. 85 by approximately 0.030 in.

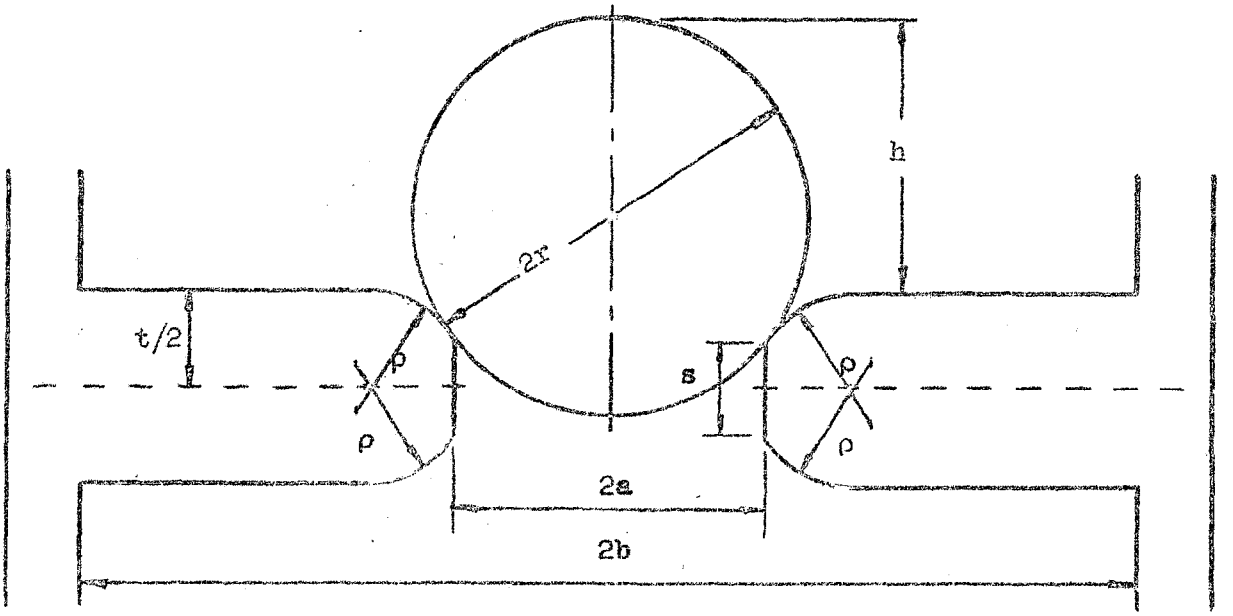
DISK THICKNESS = 0.120 in.

CAVITY	2A	2B	H	GROUP VEL. v_g/c	CAVITY	2A	2B	H	GROUP VEL. v_g/c
0	.93315	3.2475	.63569	.0202383	43	.81994	3.2229	.74681	.0132883
1	.93081	3.2469	.63876	.0200767	44	.81702	3.2223	.74901	.0131267
2	.92846	3.2464	.64181	.0199151	45	.81409	3.2217	.75120	.0129651
3	.92609	3.2458	.64482	.0197534	46	.81113	3.2212	.75339	.0128034
4	.92370	3.2453	.64781	.0195918	47	.80814	3.2206	.75558	.0126418
5	.92129	3.2447	.65077	.0194302	48	.80512	3.2200	.75777	.0124802
6	.91887	3.2442	.65370	.0192686	49	.80208	3.2194	.75996	.0123186
7	.91642	3.2436	.65661	.0191069	50	.79900	3.2188	.76215	.0121569
8	.91396	3.2431	.65950	.0189453	51	.79588	3.2182	.76434	.0119953
9	.91149	3.2425	.66236	.0187837	52	.79273	3.2176	.76654	.0118337
10	.90899	3.2419	.66520	.0186220	53	.78954	3.2170	.76874	.0116720
11	.90648	3.2414	.66802	.0184604	54	.78632	3.2164	.77094	.0115104
12	.90395	3.2408	.67082	.0182988	55	.78304	3.2158	.77315	.0113488
13	.90140	3.2402	.67359	.0181372	56	.77973	3.2152	.77537	.0111872
14	.89884	3.2396	.67634	.0179755	57	.77637	3.2146	.77759	.0110255
15	.89626	3.2391	.67907	.0178139	58	.77297	3.2140	.77982	.0108639
16	.89367	3.2385	.68177	.0176523	59	.76951	3.2133	.78206	.0107023
17	.89107	3.2379	.68445	.0174906	60	.76601	3.2127	.78431	.0105406
18	.88846	3.2373	.68711	.0173290	61	.76246	3.2121	.78657	.0103790
19	.88583	3.2368	.68974	.0171674	62	.75885	3.2115	.78883	.0102174
20	.88319	3.2362	.69236	.0170058	63	.75520	3.2108	.79111	.0100558
21	.88054	3.2356	.69494	.0168441	64	.75149	3.2102	.79339	.0098941
22	.87789	3.2350	.69751	.0166825	65	.74773	3.2095	.79568	.0097325
23	.87522	3.2344	.70005	.0165209	66	.74392	3.2089	.79797	.0095709
24	.87254	3.2339	.70257	.0163593	67	.74006	3.2082	.80027	.0094093
25	.86986	3.2333	.70507	.0161976	68	.73615	3.2076	.80257	.0092476
26	.86717	3.2327	.70754	.0160360	69	.73219	3.2069	.80488	.0090860
27	.86447	3.2321	.70999	.0158744	70	.72819	3.2063	.80719	.0089244
28	.86176	3.2315	.71242	.0157127	71	.72414	3.2056	.80950	.0087627
29	.85905	3.2310	.71483	.0155511	72	.72004	3.2049	.81181	.0086011
30	.85632	3.2304	.71722	.0153895	73	.71591	3.2043	.81411	.0084395
31	.85359	3.2298	.71959	.0152279	74	.71175	3.2036	.81641	.0082779
32	.85085	3.2292	.72194	.0150662	75	.70756	3.2030	.81870	.0081162
33	.84811	3.2287	.72427	.0149046	76	.70333	3.2023	.82097	.0079546
34	.84535	3.2281	.72659	.0147430	77	.69910	3.2016	.82324	.0077930
35	.84258	3.2275	.72888	.0145813	78	.69484	3.2010	.82548	.0076313
36	.83980	3.2269	.73117	.0144197	79	.69058	3.2003	.82770	.0074697
37	.83701	3.2264	.73343	.0142581	80	.68633	3.1997	.82990	.0073081
38	.83420	3.2258	.73569	.0140965	81	.68208	3.1991	.83207	.0071465
39	.83138	3.2252	.73793	.0139348	82	.67785	3.1984	.83421	.0069848
40	.82855	3.2246	.74016	.0137732	83	.67365	3.1978	.83631	.0068232
41	.82570	3.2241	.74239	.0136116	84	.66949	3.1972	.83836	.0066616
42	.82283	3.2235	.74460	.0134500	85	.66539	3.1966	.84037	.0065000

All dimensions in inches, fifth significant figure to be disregarded.

FIGURE I

SCHEMATIC DRAWING OF CAVITY SHOWING DEFINITION OF SYMBOLS AND CONVENTIONS



t	.230 in.	.120 in.
s	.031 in	.031 in.
ρ	.1215 in.	.0625 in.
$2r_{II}$	1 3/16 in.	-
$2r_{I}$	1 in.	1 in.
Disks measured with r_{II}	0 - 37	-
Disks measured with r_{I}	38 - 85	0 - 85

$$h = r - \rho + \sqrt{(r + \rho)^2 - (a + m)^2}$$

$$m^2 = \rho^2 - (s/2 + \rho - t/2)^2$$

FIGURE 2

Variations of 2b, 2a and s a Function of Cavity Number

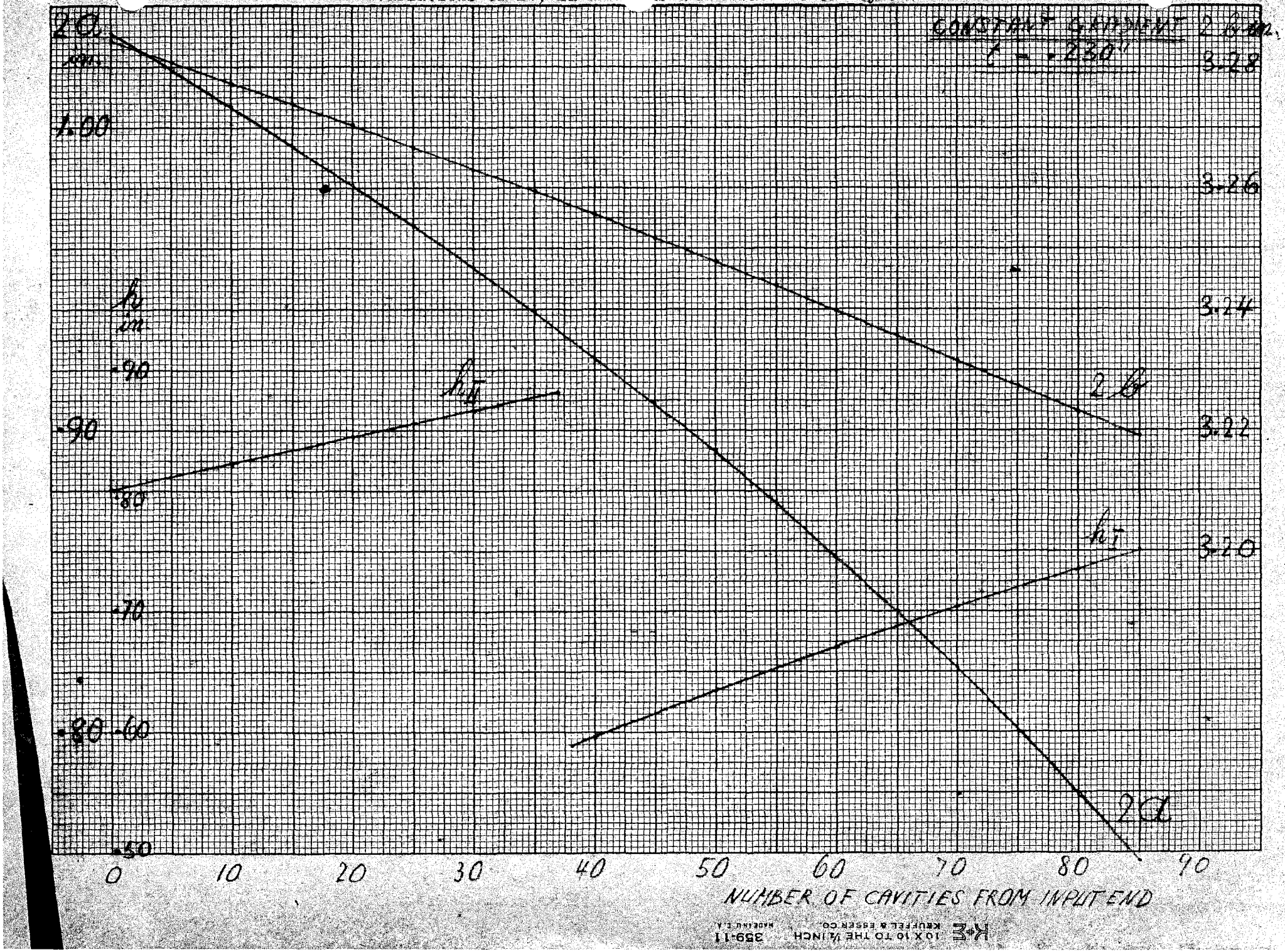


FIGURE 3

Variations of 2b, 2a and h as a Function of Cavity Number