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Generation of vector boson masses in Electroweak Model without dynamical symmetry breaking

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Abstract. The vector boson masses are generated by transformation of the *free* (without any potential term) Lagrangian of the Electroweak Model from Cartesian coordinates to a coordinates on the sphere S_3, which is defined by the gauge invariant quadratic form $\phi^\dagger\phi = \rho^2$ in the matter field space Φ_2. This transformation corresponds to transition from linear representation of the gauge group in the space Φ_2 to its nonlinear representation in the space of functions on S_3. Such modified Electroweak Model keep all experimentally verified fields of the standard Electroweak Model and does not include massive scalar field (Higgs boson), if the sphere radius does not depend on the space-time coordinates $\rho = R = const$. The concept of generation masses for vector bosons in Electroweak Model by transformation to radial coordinates is further developed in context of nonlinearly realized gauge groups, as well as in context of nonlinear sigma models. The limiting case of the modified Electroweak Model which corresponds to the contracted gauge group is discussed.

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1. Introduction

The Standard Electroweak Model based on gauge group $SU(2) \times U(1)$ gives a good description of electroweak processes. The massive vector bosons predicted by the model was experimentally observed and have the masses $m_W = 80GeV$ for charged W-boson and $m_Z = 91GeV$ for neutral Z-boson. One of the unsolved problems is the origin of the vector bosons masses. In the standard formulation these masses are arisen as a result of spontaneous symmetry breaking vie Higgs mechanism which include three steps: 1) the potential of the self-acting scalar field ϕ of the special form $V(\phi) = \mu^2\bar{\phi}\phi + \lambda(\bar{\phi}\phi)^2$ is introduced *by hand* in the Lagrangian; 2) its minimal values are considered for imaginary mass $\mu^2 < 0$ and are interpreted as degenerate vacuum; 3) one of the gauge equivalent vacuum is fixed and then all fields are regarded in the neighborhood of this vacuum.

Sufficiently artificial Higgs mechanism with its imaginary bare mass is a naive relativistic analog of the phenomenological description of superconductivity [1]. Therefore there are a serious doubt whether electroweak symmetry is broken by such a Higgs mechanism, or by something else. The existence of the Higgs boson is not yet experimentally verified. One expect that the future experiments on LHC will given definite answer on the question: does Higgs boson really exist or not. The emergence of large number Higgsless models [2]–[6] was stimulated by difficulties with Higgs boson. These models are mainly based on extra dimensions of different

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types or larger gauge groups. A finite electroweak model without a Higgs particle which is used a regularized quantum field theory [7],[8] was developed in [6].

The simple mechanism for generation of the vector boson masses in Electroweak Model was recently suggested [9]. It is based on the fact that the quadratic form $\phi^\dagger\phi = R^2$ in the matter fields space Φ_2 is invariant with respect to gauge transformations. This form define the 3-dimensional sphere S_3 of the radius R in the space Φ_2 which is C_2 or R_4 if real components are counted. The vector boson masses are automatically generated by the *free* (without any potential term) Lagrangian on the sphere S_3 and are expressed through the sphere radius R, so there is no need in special mechanism of spontaneous symmetry breaking. Higgs boson field does not presented in the model. The free Lagrangian on the sphere S_3 can be obtained as well from standard Electroweak Lagrangian by transformation of the Cartesian coordinates in Φ_2 to a some coordinates on the sphere S_3. This transformation corresponds to the transition from linear to nonlinear representation of the gauge group in the space of functions on S_3. The fermion Lagrangian of the Standard Electroweak Model are modified by replacing of the fields ϕ with the restricted on the quadratic form fields in such a way that its second order terms provide the electron mass and neutrino remain massless.

One of the important ingredient of the Electroweak Model is the simple group $SU(2)$. More then fifty years it is well known in physics the notion of group contraction [10], i.e. limit operation, which transforms, for example, a simple or semisimple group to a non-semisimple one. From general point of view for better understanding of a physical system it is useful to investigate its properties for limiting values of their physical parameters. In particular, for a gauge model one of the similar limiting case corresponds to a model with contracted gauge group. The gauge theories for non-semisimple groups which Lie algebras admit invariant non-degenerate metrics was considered in [11],[12]. In the present paper the limiting case of the modified Electroweak Model which corresponds to the contracted gauge group $SU(2;j) \times U(1)$ is regarded. We find transformation properties of gauge and matter fields under contractions. After that we obtain the Lagrangian of the contracted model from the noncontracted one by the substitution of the transformed fields. When contraction parameter tends to zero $j \to 0$ or takes nilpotent value $j = \iota$ the field space is fibered in such a way that electromagnetic, Z-boson and electron fields are in the base whereas charged W-bosons and neutrino fields are in the fiber. Within framework of the limit model the base fields can be interpreted as an external ones with respect to the fiber fields in the sense that the fiber fields do not effect on the base fields. The field interactions are simplified under contraction.

2. Modified Electroweak Model

The bosonic part of the Electroweak Model is given by the sum [13]

$$L_B = L_A + L_\phi. \tag{1}$$

where

$$L_A = \frac{1}{8g^2}\mathrm{Tr}(F_{\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2 = -\frac{1}{4}[(F^1_{\mu\nu})^2 + (F^2_{\mu\nu})^2 + (F^3_{\mu\nu})^2] - \frac{1}{4}(B_{\mu\nu})^2 \tag{2}$$

is the gauge fields Lagrangian for $SU(2) \times U(1)$ group. The gauge fields are

$$A_\mu(x) = -ig\sum_{k=1}^{3} T_k A^k_\mu(x) = -i\frac{g}{2}\begin{pmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{pmatrix},$$

$$B_\mu(x) = -ig'YB_\mu(x) = -i\frac{g'}{2}\begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix}, \tag{3}$$

where g, g' are coupling constants, $T_k = \frac{1}{2}\tau_k$, with τ_k being Pauli matrices, take their values in Lie algebras $su(2)$, $u(1)$ respectively. The stress tensors look as follows

$$F_{\mu\nu}(x) = \mathcal{F}_{\mu\nu}(x) + [A_\mu(x), A_\nu(x)], \quad \mathcal{F}_{\mu\nu} = \partial_\mu A_\mu - \partial_\nu A_\mu, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{4}$$

The second term in (1)

$$L_\phi = \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi \tag{5}$$

is the *free* matter fields Lagrangian. Here D_μ are the covariant derivatives

$$D_\mu\phi = \partial_\mu\phi - ig\left(\sum_{k=1}^{3} T_k A_\mu^k\right)\phi - ig'YB_\mu\phi \tag{6}$$

of the fields ϕ from the space C_2 of fundamental representation of $SU(2)$.

For $\Omega \in SU(2)$, $e^{i\omega} \in U(1)$ the gauge transformations of the fields are as follows

$$\phi^\Omega = \Omega\phi, \quad \phi^\omega = e^{i\omega}\phi,$$
$$A_\mu^\Omega(x) = \Omega A_\mu(x)\Omega^{-1} - \partial_\mu\Omega\cdot\Omega^{-1}, \quad A_\mu^\omega(x) = A_\mu(x),$$
$$B_\mu^\Omega = B_\mu, \quad B_\mu^\omega = B_\mu - 2\partial_\mu\omega \tag{7}$$

and Lagrangian (1) is invariant under $SU(2)\times U(1)$ gauge group.

The Lagrangian $L_B = L_A + L_\phi$ describe massless fields. In a standard approach to generate mass terms for vector bosons the Higgs mechanism is used. We have proposed [9] new mechanism for generation masses of vector bosons in Electroweak Model which is based on the topological idea of the restriction of the free bosonic Lagrangian $L_B = L_A + L_\phi$ from the whole noncompact space R_4 to the compact sphere S_3. The complex two dimensional matter fields space C_2 can be regarded as four dimensional real Euclidean space R_4 if real components are counted. The quadratic form

$$\phi^\dagger\phi = \phi_1^*\phi_1 + \phi_2^*\phi_2 = R^2 \tag{8}$$

is invariant with respect to $SU(2)\times U(1)$ group and define the three dimensional sphere S_3 in C_2. Therefore the restriction of the Electroweak Model on the sphere S_3 will be the gauge model with the same gauge symmetry. Similar restriction (8) was appeared in a unified conformal model for fundamental interactions [14] as a consequence of the gauge fixing freedom connected with the local conformal symmetry group.

Let us introduce the real fields r, $\bar\psi = (\psi_1, \psi_2, \psi_3)$ by the equations

$$\phi_1 = r(\psi_2 + i\psi_1), \quad \phi_2 = r(1 + i\psi_3), \tag{9}$$

then quadratic form (8) is written as

$$r^2(1 + \bar\psi^2) = R^2, \tag{10}$$

where $\bar\psi^2 = \psi_1^2 + \psi_2^2 + \psi_3^2$, and define the sphere S_3 of the radius $R > 0$ in the space R_4. Three independent real fields $\bar\psi$ form intrinsic Beltrami coordinate system on S_3 with the following metric tensor

$$g_{kk}(\bar\psi) = \frac{1 + \bar\psi^2 - \psi_k^2}{(1 + \bar\psi^2)^2}, \quad g_{kl}(\bar\psi) = \frac{-\psi_k\psi_l}{(1 + \bar\psi^2)^2}, \; k, l = 1, 2, 3. \tag{11}$$

Let us define the *free* matter fields Lagrangian L_ϕ with the help of the metric tensor of the spherical space S_3 in the form

$$L_\psi = \frac{R^2}{2}\sum_{k,l=1}^{3} g_{kl} D_\mu\psi_k D_\mu\psi_l = \frac{R^2\left[(1+\bar{\psi}^2)(D_\mu\bar{\psi})^2 - (\bar{\psi}, D_\mu\bar{\psi})^2\right]}{2(1+\bar{\psi}^2)^2}. \tag{12}$$

The covariant derivatives are obtained with the help of the standard expressions (6), using now the nonlinear representations of generators for the algebras $su(2)$, $u(1)$ in the space S_3 [9]

$$T_1\bar{\psi} = \frac{i}{2}\begin{pmatrix} -(1+\psi_1^2) \\ \psi_3 - \psi_1\psi_2 \\ -(\psi_2+\psi_1\psi_3) \end{pmatrix}, \quad T_2\bar{\psi} = \frac{i}{2}\begin{pmatrix} -(\psi_3+\psi_1\psi_2) \\ -(1+\psi_2^2) \\ \psi_1 - \psi_2\psi_3 \end{pmatrix},$$

$$T_3\bar{\psi} = \frac{i}{2}\begin{pmatrix} -\psi_2+\psi_1\psi_3 \\ \psi_1+\psi_2\psi_3 \\ 1+\psi_3^2 \end{pmatrix}, \quad Y\bar{\psi} = \frac{i}{2}\begin{pmatrix} -(\psi_2+\psi_1\psi_3) \\ \psi_1-\psi_2\psi_3 \\ -(1+\psi_3^2) \end{pmatrix} \tag{13}$$

and are as follows:

$$D_\mu\psi_1 = \partial_\mu\psi_1 + \frac{g}{2}\left[-(1+\psi_1^2)A_\mu^1 - (\psi_3+\psi_1\psi_2)A_\mu^2 - (\psi_2-\psi_1\psi_3)A_\mu^3\right] - \frac{g'}{2}(\psi_2+\psi_1\psi_3)B_\mu,$$

$$D_\mu\psi_2 = \partial_\mu\psi_2 + \frac{g}{2}\left[(\psi_3-\psi_1\psi_2)A_\mu^1 - (1+\psi_2^2)A_\mu^2 + (\psi_1+\psi_2\psi_3)A_\mu^3\right] + \frac{g'}{2}(\psi_1-\psi_2\psi_3)B_\mu,$$

$$D_\mu\psi_3 = \partial_\mu\psi_3 + \frac{g}{2}\left[-(\psi_2+\psi_1\psi_3)A_\mu^1 + (\psi_1-\psi_2\psi_3)A_\mu^2 + (1+\psi_3^2)A_\mu^3\right] - \frac{g'}{2}(1+\psi_3^2)B_\mu. \tag{14}$$

Let us stress that Lagrangian (12) can be obtain from the Lagrangian (5) by the transformation from the Cartesian coordinates in Euclidean space R_4 to the Beltrami coordinates on the sphere S_3, when sphere radius does not depend on space-time variables. The gauge fields Lagrangian (2) does not depend on the fields ϕ and therefore remains unchanged. So the full Lagrangian L_B is given by the sum of (2) and (12).

The particle content of the model is described by the second order part of the full Lagrangian. The ground state of L_B correspond to zero fields. For small fields in the neighborhood of ground state, the second order part of L_ψ is written as

$$L_\psi^{(2)} = \frac{R^2}{2}\sum_{k=1}^{3}\left[(D_\mu\psi_k)^{(1)}\right]^2, \tag{15}$$

where linear terms $(D_\mu\psi_k)^{(1)}$ in covariant derivatives have the form

$$(D_\mu\psi_1)^{(1)} = -\frac{g}{2}\left(A_\mu^1 - \frac{2}{g}\partial_\mu\psi_1\right) = -\frac{g}{2}\hat{A}_\mu^1,$$

$$(D_\mu\psi_2)^{(1)} = -\frac{g}{2}\left(A_\mu^2 - \frac{2}{g}\partial_\mu\psi_2\right) = -\frac{g}{2}\hat{A}_\mu^2,$$

$$(D_\mu\psi_3)^{(1)} = \partial_\mu\psi_3 + \frac{1}{2}(gA_\mu^3 - g'B_\mu) = \frac{1}{2}\sqrt{g^2+g'^2}Z_\mu.$$

For the new fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(\hat{A}_\mu^1 \mp i\hat{A}_\mu^2\right), \quad (W_\mu^-)^* = W_\mu^+$$

$$Z_\mu = \frac{gA_\mu^3 - g'B_\mu + 2\partial_\mu\psi_3}{\sqrt{g^2+g'^2}}, \quad A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}} \tag{16}$$

the quadratic part of the full Lagrangian

$$L_B^{(2)} = L_A^{(2)} + L_\psi^{(2)} =$$

$$= -\frac{1}{2}\mathcal{W}_{\mu\nu}^+\mathcal{W}_{\mu\nu}^- + m_W^2 W_\mu^+ W_\mu^- - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu} - \frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}_{\mu\nu} + \frac{m_Z^2}{2}Z_\mu Z_\mu, \tag{17}$$

where $\mathcal{X}_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ for $X_\mu = W_\mu^\pm, Z_\mu$, describes massive vector fields $W_\mu^\pm$ with identical mass $m_W = \frac{1}{2}gR$ (W-bosons), massless vector field A_μ (photon) and massive vector field Z_μ, $m_Z = \frac{R}{2}\sqrt{g^2+g'^2}$ (Z-boson). In other words it describes all the experimentally verified parts of the Electroweak Model, but does not include the scalar Higgs field. The particle masses are identical to those of the Standard Model and are expressed through the free parameter R of the model, which is now interpreted as the curvature radius of the spherical matter field space S_3.

The fermion Lagrangian of the standard Electroweek Model is taken in the form [13]

$$L_F = L_l^\dagger i\tilde{\tau}_\mu D_\mu L_l + e_r^\dagger i\tau_\mu D_\mu e_r - h_e[e_r^\dagger(\phi^\dagger L_l) + (L_l^\dagger\phi)e_r], \tag{18}$$

where $L_l = \begin{pmatrix} e_l \\ \nu_{e,l} \end{pmatrix}$ is the $SU(2)$-doublet, e_r the $SU(2)$-singlet, h_e is constant and e_r, e_l, ν_e are two component Lorentz spinor. Here τ_μ are Pauli matrices, $\tau_0 = \tilde{\tau}_0 = \mathbf{1}$, $\tilde{\tau}_k = -\tau_k$. The covariant derivatives $D_\mu L_l$ are given by (6) with L_l instead of ϕ and $D_\mu e_r = (\partial_\mu + ig'B_\mu)e_r$. The convolution on the inner indices of $SU(2)$-doublet is denoted by $(\phi^\dagger L_l)$.

The matter field ϕ appears in Lagrangian (18) only in mass terms. After expression ϕ through $\bar{\psi}$ the mass terms are rewritten in the form

$$h_e[e_r^\dagger(\phi^\dagger L_l) + (L_l^\dagger\phi)e_r] = \frac{h_e R}{\sqrt{1+\bar{\psi}^2}}\left\{e_r^\dagger e_l + e_l^\dagger e_r + \right.$$

$$\left. + i\psi_3\left(e_l^\dagger e_r - e_r^\dagger e_l\right) + i\left[\psi_1\left(\nu_{e,l}^\dagger e_r - e_r^\dagger\nu_{e,l}\right) + i\psi_2\left(\nu_{e,l}^\dagger e_r + e_r^\dagger\nu_{e,l}\right)\right]\right\}. \tag{19}$$

Its second order terms $h_e R\left(e_r^\dagger e_l^- + e_l^{-\dagger} e_r\right)$ provide the electron mass $m_e = h_e R$, and neutrino remain massless.

The topological idea with compact matter field space S_3 instead of noncompact one R_4 was used and developed in several papers [15]–[20]. The transformation of the free Lagrangian from Cartesian to radial coordinates $R_+ \times S_3$ in R_4 was regarded in [15], where the sphere S_3 was parametrized by elements of $SU(2)$ groups. When sphere radius depend on space-time coordinates the real positive massless scalar field is presented in the model.

We are interested in parametrization of S_3 with $R = const$ by elements of $SU(2)$ groups, so write ϕ as

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = R\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = R\begin{pmatrix} \chi_1 & -\chi_2^* \\ \chi_2 & \chi_1^* \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv Rh\varphi_0, \tag{20}$$

then from (8) it follows that

$$\chi^\dagger\chi = \chi_1^*\chi_1 + \chi_2^*\chi_2 = 1 \tag{21}$$

and the matrix h is unimodular $\det h = 1$ and unitary $h^\dagger h = 1$. So the vector $\chi = h\varphi_0 \in S_3$ or the matrix $h \in SU(2)$ defines the coordinates on the sphere (21). The matrix h transforms under the gauge transformations as

$$h^\Omega = \Omega h, \quad h^\omega = \begin{pmatrix} e^{i\omega}\chi_1 & -e^{-i\omega}\chi_2^* \\ e^{i\omega}\chi_2 & e^{-i\omega}\chi_1^* \end{pmatrix} = he^{i\omega\tau_3}, \tag{22}$$

where $\tau_3 = \mathrm{diag}(1, -1)$.

The matter fields Lagrangian (5) takes the form

$$L_\phi = \frac{1}{2}\frac{R^2}{4}\left(\sqrt{g^2+g'^2}\right)^2 (Z_\mu)^2 + \frac{R^2 g^2}{4} W_\mu^+ W_\mu^-, \tag{23}$$

where the new fields are introduced

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right), \quad Z_\mu = \frac{gW_\mu^3 + g'B_\mu}{\sqrt{g^2+g'^2}}, \quad A_\mu = \frac{g'W_\mu^3 - gB_\mu}{\sqrt{g^2+g'^2}},$$

$$W_\mu(x) = h^\dagger A_\mu(x) h + h^\dagger \partial_\mu h = -i\frac{g}{2}\sum_{k=1}^{3} W_\mu^k \tau_k. \tag{24}$$

These fields are invariant under $SU(2)$ transformations: $X^\Omega = X, \quad X = W_\mu^\pm, Z_\mu, A_\mu$ and are transforms under those of $U(1)$ as

$$\left(W_\mu^\pm\right)^\omega = e^{\mp 2i\omega} W_\mu^\pm, \quad Z_\mu^\omega = Z_\mu, \quad A_\mu^\omega = A_\mu + \frac{2}{e}\partial_\mu \omega, \quad e = \frac{gg'}{\sqrt{g^2+g'^2}}. \tag{25}$$

The stress tensors of the fields $A_\mu(x)$ and $W_\mu(x)$ (24) are connected by

$$W_{\mu\nu}(x) = h^\dagger F_{\mu\nu}(x) h \tag{26}$$

therefore $\mathrm{tr}(F_{\mu\nu}(x))^2 = \mathrm{tr}(W_{\mu\nu}(x))^2$and the gauge fields Lagrangian (2) can be written as

$$L_A = -\frac{1}{4}\left[(W_{\mu\nu}^1)^2 + (W_{\mu\nu}^2)^2\right] - \frac{1}{4}\left[(W_{\mu\nu}^3)^2 + (B_{\mu\nu})^2\right]. \tag{27}$$

Then the bosonic Lagrangian (1) assumes the form

$$L_B = -\frac{1}{2}\mathcal{W}_{\mu\nu}^+ \mathcal{W}_{\mu\nu}^- + m_W^2 W_\mu^+ W_\mu^- - \frac{1}{4}(\mathcal{F}_{\mu\nu})^2 - \frac{1}{4}(\mathcal{Z}_{\mu\nu})^2 + \frac{1}{2}m_Z^2 (Z_\mu)^2 +$$
$$+ \frac{igP - g^2 S - (g\mathcal{Z}_{\mu\nu} + g'\mathcal{F}_{\mu\nu})H_{\mu\nu}}{2\sqrt{g^2+g'^2}} - \frac{1}{4}(H_{\mu\nu})^2, \tag{28}$$

where terms with third and fourth powers in fields are as follows

$$P = \mathcal{W}_{\mu\nu}^+\left[W_\mu^-(gZ_\nu + g'A_\nu) - W_\nu^-(gZ_\mu + g'A_\mu)\right] -$$
$$-\mathcal{W}_{\mu\nu}^-\left[W_\mu^+(gZ_\nu + g'A_\nu) - W_\nu^+(gZ_\mu + g'A_\mu)\right],$$
$$S = \left[W_\mu^+(gZ_\nu + g'A_\nu) - W_\nu^+(gZ_\mu + g'A_\mu)\right]\left[W_\mu^-(gZ_\nu + g'A_\nu) - W_\nu^-(gZ_\mu + g'A_\mu)\right],$$
$$H_{\mu\nu} = -ig\left(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+\right). \tag{29}$$

The second order terms of (28) coinside with thouse of (17), i.e. the particle contents of the model are identical. This demonstrate that the generation vector bosons masses do not depend on the choice of the specific coordinate system on S_3, but has topological origin. The parametrization of the sphere S_3 by the elements of $SU(2)$ group leads to more simple expressions for higher order terms. The Lagrangian (28) depend only on $SU(2)$-invariant fields, so the local $SU(2)$ symmetry is factored out of it. The concept of generation masses for vector bosons in Electroweak Model by transformation to radial coordinates is further developed in [16] and [17].

The Electroweak Model based on the nonlinearly realized $SU(2) \times U(1)$ gauge groups was suggested in [18],[19]. The $SU(2)$ matrix Ω and the matter field Φ are taken in the form

$$\Omega = \frac{1}{v}\left(\phi_0 + i\sum_{k=1}^{3}\tau_k\phi_k\right), \quad \Phi = \left(\begin{array}{c} i\phi_1 + \phi_2 \\ \phi_0 - i\phi_3 \end{array}\right), \tag{30}$$

where $\phi_0 = r, \phi_k = r\psi_k, k = 1,2,3, v = R$ in our notations (8)–(10). The nonlinearity of the representation comes from the constraint

$$\Omega^\dagger\Omega = 1 \Rightarrow \phi_0^2 + \bar{\phi}^2 = v^2, \tag{31}$$

which is the same as (11). The quantization of this model was consistently defined in the perturbative loop-wise expansion and satisfies Physical Unitarity.

The nonlinear partial-trace σ-model on G/H, which provides mass terms to the intermediate vector bosons associated with the quotient G/H and remain those of H massless, was developed in [20]. An infinite-dimensional symmetry, with non-trivial Noether invariants, which ensures quantum integrability of the model in a non-canonical quantization scheme was found. For $G = SU(2) \times U(1)$ and $H = U_{em}(1)$ this model gives Higgsless alternative to the Standard Model with a partial trace on a quotient manifold $G/H = SU(2) \cong S_3$.

3. Contraction of special unitary group $SU(2;j)$

Let us regard complex space $\Phi_2(j)$ with hermitian form

$$\phi^\dagger(j)\phi(j) = |\phi_1|^2 + j^2|\phi_2|^2, \tag{32}$$

where $\phi(j) = \left(\begin{array}{c} \phi_1 \\ j\phi_2 \end{array}\right) \in \Phi_2(j),\ \phi^\dagger(j) = (\phi_1^*, j\phi_2^*)$ and star * denotes complex conjugation. The contraction parameter takes two values: $j = 1, \iota$, where $\iota \neq 0$ is nilpotent unit $\iota^2 = 0$. For $j = \iota$ the space $\Phi_2(\iota)$ is a fibered space with projection $pr : \{\phi_1, \phi_2\} \to \{\phi_1\}$, which has 1D base $\{\phi_1\}$ and 1D fiber $\{\phi_2\}$.

The group $SU(2;j)$ is defined as a transformation group of $\Phi_2(j)$, which keep invariant this hermitian form, i.e.

$$\phi'(j) = \left(\begin{array}{c} \phi_1' \\ j\phi_2' \end{array}\right) = \left(\begin{array}{cc} \alpha & j\beta \\ -j\beta^* & \alpha^* \end{array}\right)\left(\begin{array}{c} \phi_1 \\ j\phi_2 \end{array}\right) = u(j)\phi(j),$$

$$\det u(j) = |\alpha|^2 + j^2|\beta|^2 = 1, \quad u(j)u^\dagger(j) = 1. \tag{33}$$

Generators of the Lie algebra $su(2;j)$ in the fundamental representation are easily obtained

$$T_1(j) = j\frac{1}{2}\tau_1, \quad T_2(j) = j\frac{1}{2}\tau_2, \quad T_3(j) = \frac{1}{2}\tau_3,$$

and are subject of commutation relations

$$[T_1, T_2] = -j^2T_3, \quad [T_3, T_1] = -T_2, \quad [T_2, T_3] = -T_1.$$

They form the general element of $su(2;j)$

$$T(j) = \sum_{k=1}^{3} a_kT_k(j) = \frac{i}{2}\left(\begin{array}{cc} a_3 & j(a_1 - ia_2) \\ j(a_1 + ia_2) & -a_3 \end{array}\right) = -T^\dagger(j),$$

which can be obtained from that of $su(2)$ by substitution

$$a_1 \to ja_1, \ \ a_2 \to ja_2, \ \ a_3 \to a_3. \tag{34}$$

For $j=\iota$, it follows from (33) that $\det u(\iota) = |\alpha|^2 = 1$, i.e. $\alpha = e^{i\varphi}$, therefore

$$u(\iota) = \begin{pmatrix} e^{i\varphi} & \iota\beta \\ -\iota\beta^* & e^{-i\varphi} \end{pmatrix} \in SU(2;\iota), \ \ \beta \in C.$$

The simple group $SU(2)$ is contracted to the non-semisimple group $SU(2;\iota)$, which is isomorphic to the Euclid group $E(2)$.

The actions of the unitary group $U(1)$ with generator $Y = \frac{i}{2}\mathbf{1}$ and the electromagnetic subgroup $U(1)_{em}$ with generator $Q = Y + T_3$ in the fibered space $\Phi_2(\iota)$ are given by the same matrices

$$u(\beta) = e^{\beta Y} = \begin{pmatrix} e^{i\frac{\beta}{2}} & 0 \\ 0 & e^{i\frac{\beta}{2}} \end{pmatrix}, \quad u_{em}(\gamma) = e^{\gamma Q} = \begin{pmatrix} e^{i\gamma} & 0 \\ 0 & 1 \end{pmatrix},$$

as on the space Φ_2.

4. Electroweak Model for contracted gauge group $SU(2;j) \times U(1)$

The space $\Phi_2(j)$ can be obtained from Φ_2 by substitution $\phi_2 \to j\phi_2$, which induces the substitution (34) for Lie algebra $su(2)$ parameters. As far as the gauge fields take their values in Lie algebra, we can substitute gauge fields

$$W_\mu^\pm \to jW_\mu^\pm, \quad Z_\mu \to Z_\mu, \quad A_\mu \to A_\mu. \tag{35}$$

After that the bosonic Lagrangian (28) assumes the form

$$L_B(j) = -\frac{1}{4}(\mathcal{F}_{\mu\nu})^2 - \frac{1}{4}(\mathcal{Z}_{\mu\nu})^2 + \frac{1}{2}m_Z^2\,(Z_\mu)^2 +$$

$$+j^2\left\{-\frac{1}{2}\mathcal{W}_{\mu\nu}^+\mathcal{W}_{\mu\nu}^- + m_W^2 W_\mu^+ W_\mu^- + \frac{igP - g^2 S - (g\mathcal{Z}_{\mu\nu} + g'\mathcal{A}_{\mu\nu})H_{\mu\nu}}{2\sqrt{g^2+g'^2}}\right\} - j^4\frac{1}{4}(H_{\mu\nu})^2 =$$

$$= L_b + j^2 L_f + j^4 L_h. \tag{36}$$

When contraction parameter takes nilpotent value $j=\iota$ or tends to zero $j^2 \to 0$, then the contribution L_f of W-bosons fields to the Lagrangian (36) will be small in comparison with the contribution L_b of Z-boson and electromagnetic fields. The term L_h being fourth order in j can be neglected. In other words the limit Lagrangian L_b includes Z-boson and electromagnetic fields and charded W-bosons fields does not effect on these fields. The part L_f form a new Lagrangian for W-bosons fields and their interactions with other fields. The appearance of two Lagrangians L_b and L_f for the limit model is in correspondence with two hermitian forms of fibered space $\Phi_2(\iota)$, which are invariant under the action of contracted gauge group $SU(2;\iota)$. Electromagnetic and Z-boson fields can be regarded as external ones with respect to the W-bosons fields.

In mathematical language the field space $\left\{A_\mu, Z_\mu, W_\mu^\pm\right\}$ is fibered after contraction $j=\iota$ to the base $\{A_\mu, Z_\mu\}$ and the fiber $\left\{W_\mu^\pm\right\}$. (In order to avoid terminological misunderstanding let us stress that we have in view locally trivial fibering, which is defined by the projection $pr: \left\{A_\mu, Z_\mu, W_\mu^\pm\right\} \to \{A_\mu, Z_\mu\}$ in the field space. This fibering is understood in the context of semi-Riemannian geometry [22] and has nothing to do with the principal fiber bundle.) Then L_b

in (36) presents Lagrangian in the base and L_f is Lagrangian in the fiber. In general, properties of a fiber are depend on a points of a base and not the contrary. In this sense fields in the base can be interpreted as external ones with respect to fields in the fiber.

Let us note that field interactions in contracted model are more simple as compared with the standard Electroweak Model due to nullification of some terms. For example, the last term $j^4 L_h$ in (36) disappears as having fourth order in $j \to 0$.

When the gauge group $SU(2)$ is contracted to $SU(2;j)$ and the matter field is fibered to $\phi(j)$ the same take place with doublet L_l in (18), namely, the first component e_l does not changed, but the second component is multiplied by contraction parameter: $\nu_{e,l} \to j\nu_{e,l}$. Then the mass terms (19) are rewritten in the form

$$h_e[e_r^\dagger(\phi^\dagger(j)L_l(j)) + (L_l^\dagger(j)\phi(j))e_r] = \frac{h_e R}{\sqrt{1+\bar{\psi}^2(j)}} \left\{ e_r^\dagger e_l + e_l^\dagger e_r + \right.$$

$$\left. + i\psi_3 \left(e_l^\dagger e_r - e_r^\dagger e_l \right) + ij^2 \left[\psi_1 \left(\nu_{e,l}^\dagger e_r - e_r^\dagger \nu_{e,l} \right) + i\psi_2 \left(\nu_{e,l}^\dagger e_r + e_r^\dagger \nu_{e,l} \right) \right] \right\}, \qquad (37)$$

where the $SU(2)$-singlet e_r does not transforms under contraction. For nilpotent value of the contraction parameter $j = \iota$ the fermion Lagrangian (18) is also split on electron part in the base and neutrino part in the fiber, i.e. in the limit model electron field is external one relative to neutrino field. This fact may probably explain why neutrino interaction with the matter is so weak. The mass terms (37) for $j = \iota$ are

$$h_e[e_r^\dagger(\phi^\dagger(\iota)L_l(\iota)) + (L_l^\dagger(\iota)\phi(\iota))e_r] = \frac{h_e R}{\sqrt{1+\psi_3^2}} \left[e_r^\dagger e_l^- + e_l^{-\dagger} e_r + i\psi_3 \left(e_l^{-\dagger} e_r - e_r^\dagger e_l^- \right) \right]. \qquad (38)$$

Its second order terms $h_e R \left(e_r^\dagger e_l^- + e_l^{-\dagger} e_r \right)$ provide the electron mass $m_e = h_e R$ and neutrino remain massless in exactly the same way as for the Standard Model.

5. Conclusions

The topological mechanism for generation of vector boson masses in the Electroweak Model is discussed. Vector boson masses are automatically generated by transformation of the free Lagrangian from the noncompact R_4 matter fields space to the compact sphere S_3. This model describes all experimentally observed fields and does not include the (up to now unobserved) scalar Higgs field. W- and Z-boson masses are expressed through the parameter R by the same formulas as in the standard case. The free parameter R of the model is now interpreted as the curvature radius of the spherical matter fields space. The development of this topological idea in different aspects, such as transformation to radial coordinates or nonlinearly realized gauge group or nonlinear partial-trace sigma-model, is briefly reviewed.

The limiting case of the modified Electroweak Model which corresponds to the contracted gauge group $SU(2;\iota) \times U(1)$ is presented. The masses of particles involved in the Electroweak Model remain the same under contraction, but field interactions are simplified. Electromagnetic and Z-boson fields can be regarded as external fields with respect to the W-boson fields, i.e. Z-boson and electromagnetic fields can act upon W-boson fields, but not on the contrary. In much the same way in the fermion sector electron field is external one relative to neutrino field.

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