

# Multi-Connected Cosmologies

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## Abstract

The global topology of the universe is a fundamental question in cosmology which has often been overlooked in the literature, except by some pioneering authors (Ellis 1971; Sokolov and Schvartsman, 1974; Gott, 1980; Fang and Sato 1983; Fagundes, 1985). In this talk we review some elements of topology useful for the classification of spaces, we discuss the observational methods to test multi-connected "small" universes and we argue that the only test which can firmly establish a non-trivial topology is the statistical analysis of separation distances between luminous objects in the universal covering space.

## 1 Introduction

The purpose of relativistic cosmology is to deduce from Einstein's field equations some physically plausible models of the universe as a whole. However Einstein equations describe only local properties of spacetime; they do not fix global features such as the finiteness, the shape, the volume - in other words the topology of spacetime. Thus to a given metric element correspond several topologically distinct universe models with the same dynamics.

In some sense topology plays to differential geometry the same essential role as quantum theory to classical physics : both lead from continuous to the discrete, and at their levels relationships are more global and less local. It is the reason why the problem of topology of spacetime is now much discussed in the framework of quantum gravity and quantum cosmology. For

instance, the spontaneous birth of the universe from quantum fluctuations requires the Universe to have compact spacelike hypersurfaces, and the closure of space is often considered as a necessary condition in quantum theory of gravity.

However the topology of spacetime also enters in a fundamental way in *classical* general relativity. Until now, the most realistic universe models are the spatially homogeneous, isotropic Friedmann–Lemaître models (hereafter denoted FL), which admit spatial sections of the spherical, euclidean or hyperbolic type according when the curvature is positive, zero or negative. We use to read in most of the literature that the spherical model has a finite volume whereas the euclidean and hyperbolic models have infinite volumes. In such a conservative view the question whether the space is finite or infinite (one of the oldest cosmological problems going back to almost twenty-five centuries) would be reduced to the estimation of the average mass-density of the universe: above the critical density the space would be closed, whereas below and at the critical density the space would be necessarily open, infinite (current astronomical observations seem to favor this case). This statement is correct only when we arbitrarily assume that the spatial topology is that of the corresponding singly-connected covering spaces. As we shall recall below, there are multi-connected, spatially finite topologies with zero or negative curvature, that satisfy the gravitational field equations as well as the singly-connected FL solutions.

The motivation of this review comes from the fact that many cosmologists are surprisingly unaware of how topology and cosmology fit together to provide new highlights in universe models. Except section 4.3.3. which presents new results, much of what is said below is contained in some form in previous work, and is widely developed in Lachièze-Rey and Luminet (1994).

## 2 Basic elements of topology

From considerations about causality, the study of the global structure of spacetime can be reduced to that of its spacelike sections. The determination of all the possible forms of the universe then reduces to the problem of the topological classification of three-dimensional riemannian manifolds. In the following we deal with complete lorentzian globally hyperbolic 4-

manifolds without boundary, admitting orientable spacelike hypersurfaces.

Let us first recall some very elementary notions of topology (see, for instance, Massey 1987). The topological properties of a manifold are those which remain insensitive to continuous deformations. A manifold  $M$  is *singly-connected* if every loop is homotopic to a point, that is, can be continuously shrunk to a point. If not, the manifold is said to be *multi-connected*. The study of homotopic loops is a way of detecting holes or handles in  $M$ . Loops equivalent under homotopy can be endowed with a group structure: this first homotopy group is called the *fundamental group*, independant on the base point. It is a topological invariant of the manifold. Multi-connectedness implies non-unicity of geodesics lines between two any points and is thus the deep origin of the multiplication of images in multi-connected cosmologies.

In 2 dimensions it is often easy to visualize surfaces by embedding them in 3 dimensional space  $R^3$ , but it is not always possible. Mathematicians have found more efficient representations of surfaces, which can be generalized to spaces of arbitrary dimension. For instance, it is well known that any compact surface with  $n$  holes can be represented by a  $2n$ -edges convex polygon, with edges identified by pairs. More generally, the *fundamental polyhedron* is a representation of the largest singly-connected domain of a manifold. It is necessarily convex, with a finite number of faces, and faces are identified by pairs. The displacements carrying a face of the polyhedron to its homologous face are the generators of the so-called *holonomy group*  $\Gamma$ , which is isometric to the fundamental group. The holonomy group is discontinuous and its generators have no fixed point (except the identity). This last property is very restrictive (it excludes for instance the rotations), and allows to classify all the possible groups of holonomy leaving invariant the fundamental polyhedron.

The configuration formed by the fundamental polyhedron  $F$  and its images  $\gamma F$ , ( $\gamma \in \Gamma$ ) is a regular tiling of the so-called *universal covering space*, each image  $\gamma F$  being a cell of the tiling. A singly-connected space is obviously identical to its universal covering space, whereas a multi-connected space is the quotient of the universal covering space by the holonomy group.

These notions are well exemplified by the three-dimensional simple torus (often referred to as the *hypertorus*). Such a space is obtained by identi-

ying opposite faces of a parallelepiped in euclidean space  $x = x + L_1$ ,  $y = y + L_2$ ,  $z = z + L_3$ . The hypertorus is thus a locally euclidean space with constant zero curvature. However its topology is completely different from that of  $R^3$  since the hypertorus is compact, with a finite volume equal to  $L_1 \times L_2 \times L_3$ . Its universal covering space is  $R^3$ , its fundamental polyhedron is a parallelepiped and its holonomy group is generated by the three independant translations along the edges of the parallelepiped. Now let us imagine we have luminous bodies and we are immersed in such a space. The light emitted by our back crosses the face of the parallelepiped behind us and reappears on the opposite face in front of us; therefore, looking forward we can see our back (as in the static Einstein's universe model). The same phenomenon arises if we watch on the right (we see our left profile) or upwards (we see the bottom of our feet). In fact, as light propagates in all directions, we would observe an infinity of ghost images of a same body viewed under all angles, the images been distributed in a network of parallelepipeds extending in all directions, giving the visual impression of infinite space, although the real space is closed.

The method for classifying the admissible topologies of a manifold is to determine the universal covering space and the fundamental polyhedron, and to calculate the holonomy group acting on the polyhedron. However such a characterization is still incomplete excepted in some restricted cases such as 2-dimensional closed surfaces, 3-dimensional flat and spherical spaces.

A particularly important case for application to realistic cosmologies is that of locally homogeneous and isotropic 3 - dimensional riemannian manifolds, *i.e.* admitting one of the three geometries of constant curvature. Any space of constant curvature  $M$  can thus be expressed as the quotient  $M = \tilde{M}/\Gamma$  where the universal covering space  $\tilde{M}$  is either the euclidean space  $R^3$  if  $K = 0$ , the 3-sphere  $S^3$  if  $K > 0$  or the hyperbolic 3-space  $H^3$  if  $K < 0$ , and  $\Gamma$  is a discrete subgroup of isometries without fixed point of  $\tilde{M}$ .

### 3 Classification of 3 - dimensional riemannian spaces with constant curvature

#### 3.1 Three-dimensional euclidean space forms

The generators of the holonomy group are the identity, the translations, the glide reflections and the helicoidal motions occurring in various combinations. They generate 18 distinct types of locally euclidean spaces (Wolf 1984; also Ellis, 1971). Eight forms are open (non compact), ten are closed (compact).

The compact models are visualised by identifying appropriate faces of fundamental polyhedra. Six of them are orientable.

When the fundamental polyhedron is a parallelepiped, the space forms are :

- $T_1$  (opposite faces identified by translations. The resulting space is the hypertorus)
- $T_2$  (opposite faces identified, one pair being rotated by angle  $\pi$ )
- $T_3$  (opposite faces identified, one pair being rotated by  $\pi/2$ )
- $T_4$  (opposite faces identified, all three pairs being rotated by  $\pi$ )

The fundamental polyhedron can also be an hexagonal network translated orthogonally to the plane. The corresponding space forms are :

- $T_5$  (opposite faces identified, the top face being rotated by an angle  $2\pi/3$  with respect to the bottom face)
- $T_6$  (opposite faces identified, the top face being rotated by an angle  $\pi/3$  with respect to the bottom face)

#### 3.2 Three-dimensional spherical space forms

All the three-manifolds of constant positive curvature, admitting the compact  $S^3$  as universal covering space, are necessarily compact.

The holonomy group is a subgroup of  $SO(3)$ . Wolf (1984) gives an explicit description of each admissible finite subgroup  $\Gamma$  of  $SO(3)$ . All the homogeneous space forms of constant curvature are then given by

- $S^3$  (the singly-connected 3-sphere)
- $P^3$  (the three-dimensional projective space, obtained by identification of diametrically opposite points on  $S^3$ )
- $S^3/Z_p$ , ( $p > 2$ ), where  $Z_p$  is the cyclic group of order  $p$
- $S^3/D_m$ , ( $m > 2$ ), where  $D_m$  is the dihedral group of order  $2m$
- $S^3/T$ ,  $S^3/O$  and  $S^3/I$ , where  $T, O, I$  are respectively the symmetry groups of the tetrahedron (4 vertices, 6 edges, 4 faces), of order 12; of the octahedron (6 vertices, 12 edges, 8 faces), of order 24; of the isocahedron (12 vertices, 30 edges, 20 faces), of order 60.

They are in infinite number due to parameters  $m$  and  $n$ . The volume of a spherical 3-space  $M = S^3/\Gamma$  is simply  $vol(M) = 2\pi^2/|\Gamma|$ , where  $|\Gamma|$  is the order of the group  $\Gamma$  ( $2\pi^2$  is the volume of the 3-sphere of constant curvature  $+1$ ). Thus for topologically complicated 3-manifolds, the order  $|\Gamma|$  becomes large and  $vol(M)$  is small.

### 3.3 Three-dimensional hyperbolic space forms

The universal covering is  $H^3$ , whose fundamental group is isometric to  $SL(2, C)$ . The classification of three-dimensional manifolds is yet far from being fully understood. Most of the progress in the comprehension of hyperbolic manifolds is due to Thurston, 1979 (for a recent report see Benedetti and Petronio, 1991). The Mostow theorem illustrates an essential difference between 2-dimensional hyperbolic geometry and higher dimensions: while a surface of genus  $\geq 2$  supports uncountably many non equivalent hyperbolic structures, for  $n \geq 3$  a connected oriented  $n$ -dimensional manifold supports *at most one* hyperbolic structure. It follows that geometric invariants such as the volume or the lengths of closed geodesics are also topological invariants. It is the reason why the tentative classification of closed hyperbolic manifolds is based on increasing volumes, starting from an absolute minimum (we refer the interested reader to Matveev and Fomenko, 1988).

Since there is no general classification available, some particular examples may be given. Among compact models, the Seifert-Weber manifold is

obtained with a fundamental group generated by a translation and a rotation by 108 degrees; the fundamental polyhedron is a dodecahedron with opposite pentagonal faces fitted together after twisting by  $3/10^{th}$  turn.

In the Löbell model (studied by Gott (1980) in a cosmological context), the fundamental polyhedron is a 14 faces polyhedron, two faces of which being regular rectangular hexagones and the 12 others rectangular regular pentagones.

Best (1971) mentions closed hyperbolic manifolds where the fundamental polyhedron is a regular hyperbolic isocahedron in  $H^3$ . They have been studied in details by Fagundes (1989) in a cosmological context.

## 4 Observing a multi-connected Universe

### 4.1 Small Universes

In cosmology the space-time manifold is described by the metric

$$ds^2 = c^2 dt^2 - R(t)^2 d\sigma^2,$$

where  $R(t)$  is the scale factor, which may be chosen equal to the spatial curvature radius for non spatially flat models. The quadratic form  $R(t)^2 d\sigma^2$  denotes the metric of space at the cosmic time  $t$ , which remains homothetic to itself. The metric of comoving space is  $R_0^2 d\sigma^2$ ,  $R_0 = R(t_0)$  being the present value of the scale factor. A multi-connected model and the associated singly-connected model have exactly the same kinematics and dynamics: in particular the scale factor  $R(t)$  and its evolution are identical. In fact most of the characteristics of the singly-connected FL models are preserved when we turn to the possible multi-connected FL models.

All cosmic information comes through light-rays, i.e., null geodesics of space-time. The observer lies at one end, an event (emission of radiation from the source) at the other. The set  $S$  of measurable quantities characterizes the geodesics, and not the cosmic event itself. In a singly-connected model, there is a one-to-one correspondance between  $S$  and the position of the emitting object in real space. For instance, the larger its redshift, the further away the object lies in real space. In a multi-connected model, such a correspondance does not hold in general. As we have seen, the



multi-connectedness of space implies multiplicity of geodesics between any two points, for instance a cosmic source and an observer. An image with larger  $z$  may correspond to an object closer in real space. To an unique object in space are in general associated many images with different redshifts (the one of smallest redshift is called the "real image"; the other ones are called *ghosts*). We can thus speak of a "topological lens" effect. The difference with usual gravitational lenses is that the topological lens is no more localized in space, so that the ghost images of a given original source are distributed all over the sky both in direction and in apparent distance. The universal covering space is the "observer's world", which may differ drastically from the real world.

In a multi-connected universe model, the spatial scales associated to the fundamental polyhedron play a role. Let us call  $\alpha$  the smallest length associated to the fundamental polyhedron. The ratio  $\alpha/R_0$  can only take specified values in a non flat space : when  $k > 0$  (resp.  $k < 0$ ), the geometry imposes a maximum (resp. minimum) value for it, whereas it remains arbitrary in flat space. The dimensions of the fundamental cell involve another scale  $\beta$ , the maximum length inscriptible in the fundamental cell (for instance, the diagonal of the parallelepipedic cell in the case of the hypertorus.  $\beta$  is also the maximum distance between 2 images of the same object belonging to adjacent cells.

Directly observable effects of multi-connectedness are expected only if the relevant topological scales  $\alpha$  or  $\beta$  are smaller than the horizon size. Here we will concentrate on such "small universes" (Ellis, 1971), where the space sections of the universe are compact with size of the order of a few 100 Mpc. Of course, the smaller the basic cell, the easier it will be to observe topological effects.

By construction, up to a distance equal to  $\alpha/2$ , there are only original images and no ghost image. Between  $\alpha/2$  and  $\beta/2$  there is a mixture of original and ghost images, beyond  $\beta/2$  there are only ghosts. The number of potentially visible ghosts, and their characteristics, depend on the cosmic parameters and on the topology of the model considered. There is a priori as much potential ghosts than cells in the universal covering space. Their number is finite in the case of positive constant spatial curvature, infinite otherwise. However, only ghosts nearer than the horizon can be seen, so that their number is reduced to that of the cells in the observable

universe. Moreover, in practice, we cannot observe beyond a magnitude or redshift cutoff, depending on the type of objects (galaxies, clusters, ...), the instrumentation, and so on.

To get a qualitative idea of the multiplication of images in small multi-connected cosmology, let assume an Einstein-de Sitter model (FL model with zero curvature) with the topology of the hypertorus. If the Hubble constant is  $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , the size  $\alpha$  of the fundamental cell is 1000 Mpc and the redshift cutoff  $z = 1000$  (corresponding to the time of emission of the cosmic microwave background), then the multiplication factor is 7000 ! If  $\alpha$  is reduced to 600 Mpc and  $z$  to 1, the multiplication factor is still about 170.

Could such a huge "cosmic illusion" escape to observation ? The answer is not obvious. The only way to prove that our universe is multiconnected would be to show, ponctually or statistically, the existence of ghost images of some object, or of some specific recognizable configuration in space. In the following we present different methods which have been, or which can be applied to this purpose. Observational tests are naturally attached to the two kinds of cosmic sources : the discrete (galaxies, clusters, ...) and the diffuse (cosmic microwave background, hereafter CMB) sources.

## 4.2 Images of recognizable peculiar objects

### 4.2.1 Ghost images of individual galaxies

A fascinating possibility of non trivial topologies lies in their ability to provide us with images of our own Galaxy. The number, the distances and orientations of the ghost images of the Milky Way depend on the topology. The nearest images must be at a distance  $\alpha$ . In the universal covering space, a sphere of radius  $R$  contains between  $(\frac{2R}{\alpha})^k$  and  $(\frac{2R}{\beta})^k$  images, where  $k$  is the number of principal directions. The fact that we do not see any image of our galaxy up to a distance  $d$  allows to exclude topologies with  $\alpha < 2d$ . The maximum distance  $d$  up to which we would be able to recognize our galaxy has been discussed by Sokolov and Shvartsman (1974). They deduce that  $d > 7.5 h^{-1} = 7.5 \text{ CAMpc}$ , implying  $\alpha > 15 h^{-1} \text{ Mpc}$  (see also Fagundes & Wichoski, 1987). The interesting fact about our Galaxy is that it lies automatically on all the principal directions. Thus the proper comoving distances of its ghost images are exactly quantized

(entire multiples of the distance of the first ghost in this direction), with a spatial resolution depending on our proper motion. But there is no simple mean to decide which galaxy may be a ghost of our own Galaxy, at some distance, at some moment in the past, and under some inclination angle which are not known. Thus it is almost hopeless to recognize a ghost image of our own Galaxy in the sky and to prove in this way that we live in a multi-connected model.

It could be also interesting, in principle, to search for two or more galaxies which may be ghosts images of a same real one. However, galaxies are so numerous that there is no chance to decide if a galaxy in the sky is a ghost image of another one. From a statistical point of view, the spatial coverage of galaxies does not extend far enough to allow interesting studies. Sokolov and Schvartsman (1974) deduced from the study of giant galaxies that  $\alpha > 20$  Mpc, but, given the fact that arbitrary galaxies do not lie along principal directions, they seem even less interesting than the Milky Way to test multi-connectedness.

#### 4.2.2 Ghost images of galaxy clusters

Sokolov and Schvartsman (1974) proposed to extend the preceding method to clusters of galaxies. Their lifetime seems to be sufficiently long to guarantee an appearance almost constant during the time necessary for light to cross a small Universe. Although the Virgo cluster – the nearest of the Abell clusters – appears as an interesting candidate, it is not very rich and not easy to recognize. The Coma cluster – the more prominent in our neighborhood, and also the best studied – appears therefore more interesting. Coma is about  $70 h^{-1}$  Mpc from us, in the direction of the north galactic pole. Estimating that a ghost image of Coma could not have escaped detection at a distance lower than  $140 h^{-1}$  Mpc, *i.e.* in a sphere of radius  $70 h^{-1}$  Mpc centered on the cluster itself, Gott (1980) deduced  $\beta > 140 h^{-1}$  Mpc, and  $\alpha \geq 60 h^{-1}$  Mpc. Since Coma is much richer than the other close Abell clusters, one cannot expect stronger constraints from other individual clusters.

Sokolov and Schvartsman (1974) tried to establish other constraints from the study of catalogs of clusters. They considered the Abell catalog, containing 2712 rich clusters and that of Zwicky containing 9730 clusters of all types. Both are limited at redshifts  $\approx 0.2$ , corresponding to about

$600 h^{-1}$  Mpc, covering only the northern half of space around us. They concluded that rich clusters detected up to this distance must be originals since the closer clusters are poorer, so that they cannot be identical objects in a more recent stage of evolution. For a hypertorus model, they concluded that  $\beta > 600 h^{-1}$  Mpc.

#### 4.2.3 Ghost images of superclusters

Beyond clusters, the next step in the hierarchy of cosmic scales is that of superclusters. Gott (1980) remarked that the Serpens – Virgo region, containing several rich clusters, constitutes the most prominent large structure, at a mean distance of  $280 h^{-1}$  Mpc. Arguing that there is no image of this structure nearest to us, at least in the directions covered by the Shane–Wirtanen survey, he pointed that there is no image closer than  $200 h^{-1}$  Mpc to the source itself. He deduced that  $\beta > 400 h^{-1}$  Mpc. These conclusions do not differ very much from those of Sokolov and Schvartsman. Certainly it would be interesting to search for our local supercluster, or other recognized superclusters, but our view of the large scale structure faraway is presently too imprecise. Gott (1980) performed numerical simulations of a hypertorus universe. He considered a cubic cell with  $\alpha = 27.5$  Mpc, so that  $\beta = 50$  Mpc. Real galaxies were disposed thanks to a numerical code (providing a pattern of clustering and a correlation function in agreement with observations in the nearby universe) in the original cell. Ghost images were then calculated in the universal covering space, to simulate the appearance of the universe, as it would be seen from a randomly selected point. He concluded that the multiple images of rich clusters could not have escaped detection and that a survey up to magnitude 14 was able to exclude values  $\beta < 25$  Mpc (for the case of a torus). Scaling argument allowed him to conclude that the corresponding limit for the Shane–Wirtanen count survey was  $\beta \approx 330$  Mpc, not too far from the 400 Mpc that he gave from the Serpens–Virgo supercluster.

#### 4.2.4 Ghost images of quasars

Quasars seem potentially interesting to test the topology, because of their strong intrinsic luminosities implying that they occupy a large volume of the observable universe. But the quasar phenomenon is short-lived compared

to the expected time necessary for a photon to turn around a small multi-connected universe. Typically, with lifetimes less than  $\approx 10^8$  yrs, quasars could be only used to test models smaller than 200 *Mpc*, a value already excluded from other observations.

### 4.3 Statistical Effects

#### 4.3.1 Periodicity of redshifts distribution

There have been some claims of periodicity in the observed distribution of redshifts. References can be found for instance in Fang (1990), beginning with a paper of Burbidge (1968). Fang *et al.* (1982) claimed an observed periodicity in the quantity  $w = \log(1 + z)$ . Chu and Zhu (1989) also reported a periodicity in the redshift distribution of the *Ly*  $\alpha$  absorbing clouds. These effects are generally interpreted as selection effects, but some authors argue more drastically for a non cosmological origin of the redshifts. A multiply-connected topology has also been invoked (Fang and Sato 1983, Fang and Mo 1987, Fang 1990).

In a multi-connected cosmology, a given original object gives rise to a potential number of ghosts approximatively equal to the volume of the observable universe (inside horizon) divided by the volume of the fundamental polyhedron. Along a direction where a ghost is observed, other ghosts also lie with their (comoving) distances periodically distributed in the universal covering space. However, along any direction, only a very limited number, if any, of these potential ghosts would really be visible. This is especially true for quasars because of their short life-time. Also, for a given object, the periodicities differ along different lines-of-sight. And in fact, the expected periodicity is not in  $z$ , but in proper comoving distance. For all these reasons, no global redshift-periodicity is expected in multi-connected small universes, as already pointed out by Ellis and Schreiber (1987) and confirmed by numerical simulations (Lachièze-Rey, Lehoucq and Luminet 1994). As Fang (1990) already noticed, a peak in the redshift distribution only appears if one or several objects are present very close to the observer (*i.e.* near the center of the fundamental polyhedron). The whole signal comes from the ghosts of these peculiar objects, but there is no global effect and the question of periodicity in redshifts reduces to the search for ghosts of nearby objects.

#### 4.3.2 The question of discordant redshifts

Although the cosmological interpretation for the redshifts of galaxies, quasars and distant objects is widely accepted, there are still some claims that observed associations of cosmic objects cannot be explained in the framework of standard big-bang cosmology (Burbidge, 1981). Such an association is defined as a statistically significative reunion of cosmic objects – galaxies and/or quasars – in the same *projected* region of the sky, although they have very different redshifts. It was thus advocated that such configurations were highly improbable in the standard FL models (with cosmological redshifts). However Fagundes (1985 & 1989) emphasized that such situations are naturally expected in the framework of a multi-connected small universe, without abandoning the standard interpretation of redshifts. The explanation comes from the fact that, in multi-connected models, the relation between redshift and distance is modified. Apparent associations would be due to an accumulation of different ghost images of a same physical source, in a given direction of the sky.

Fagundes did not present a complete quantitative discussion but he rather showed, within the choice of a peculiar multi-connected hyperbolic model, the possibility of CA accumulations of ghost images along the same line-of sight, which can possibly account for the discordant associations.

#### 4.3.3 Periodicity of separation distances

Lachièze-Rey, Lehoucq and Luminet (1994) present numerical simulations showing that, independently of a peculiar topology, the only observable periodicity in a multi-connected small universe lies in the separation distances between the sources in the universal covering space. Their idea exploits the simple fact that the displacements carrying a ghost image to another ghost image of the object is an isometry in the universal covering space (for instance a translation). They consider an Einstein-de Sitter model with the four possible compact topologies with a parallelepipedic cell  $T_1, \dots, T_4$  presented in section 3.1. They fill the fundamental cell with real galaxies and calculate the ghost images in the universal covering space, to simulate the appearance of the observable universe. To give some numerical examples, with a Hubble constant equal to  $75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , the

horizon distance is 8000 Mpc; with the size of the cell equal to  $2500\text{Mpc}$ , there are only originals up to a redshift  $z = 0.40$  and only ghosts beyond  $z = 0.88$ . Assuming a cut-off at  $z_{max} = 4$  (corresponding to the present observations of the most distant quasars), the multiplication factor is about 25. For instance, with 50 original galaxies in the cell, about 1200 ghost images are generated. Then they calculate all the separation distances between the pairs of points and trace the histogram of separations. If the galaxies are randomly distributed within the cell, the histogram of separations distances exhibits strong peaks superimposed on a Poisson background distribution (see figure 1). The peaks correspond to repetition lengths related to the size of the fundamental cell. Their presence reveals unambiguously the multi-connectedness of space. Moreover, the relative positions and amplitudes of the peaks characterize in a unique way the topological type: different topologies generate different recognizable peak structures. The next step will be to apply this test to the observed distribution of sources (galaxy clusters), but until now the available catalogs are not complete enough to draw a positive evidence.

## 4.4 The Cosmic Microwave Background

### 4.4.1 An early Homogenization of the Universe ?

A standing cosmological problem is why the Universe appears, through the CMB, so homogeneous. Although this homogeneity is a postulated property of the standard FL models (with or without inflation), it has often been asked how a smooth universe could have settled down given the fact that the concerned parts of the Universe were causally disconnected (the so-called "horizon" paradox). Gott (1980) examined the possibility that the observed isotropy of the CMB and the present global homogeneity of the Universe both resulted from an homogenizing process in a multiconnected Universe. If the Universe was small then the CMB would be a collection of duplicates of a same region, which would explain its apparent isotropy on some angular scales. Gott showed that the thermalization of the CMB is not possible in a non flat multi-connected universe, given the geometrical constraints and the value of  $R_0$  imposed by observations, and that in a spatially flat (hypertorus) model the identification length should be less than 400 Mpc in order to allow this "topological" smoothing occur.

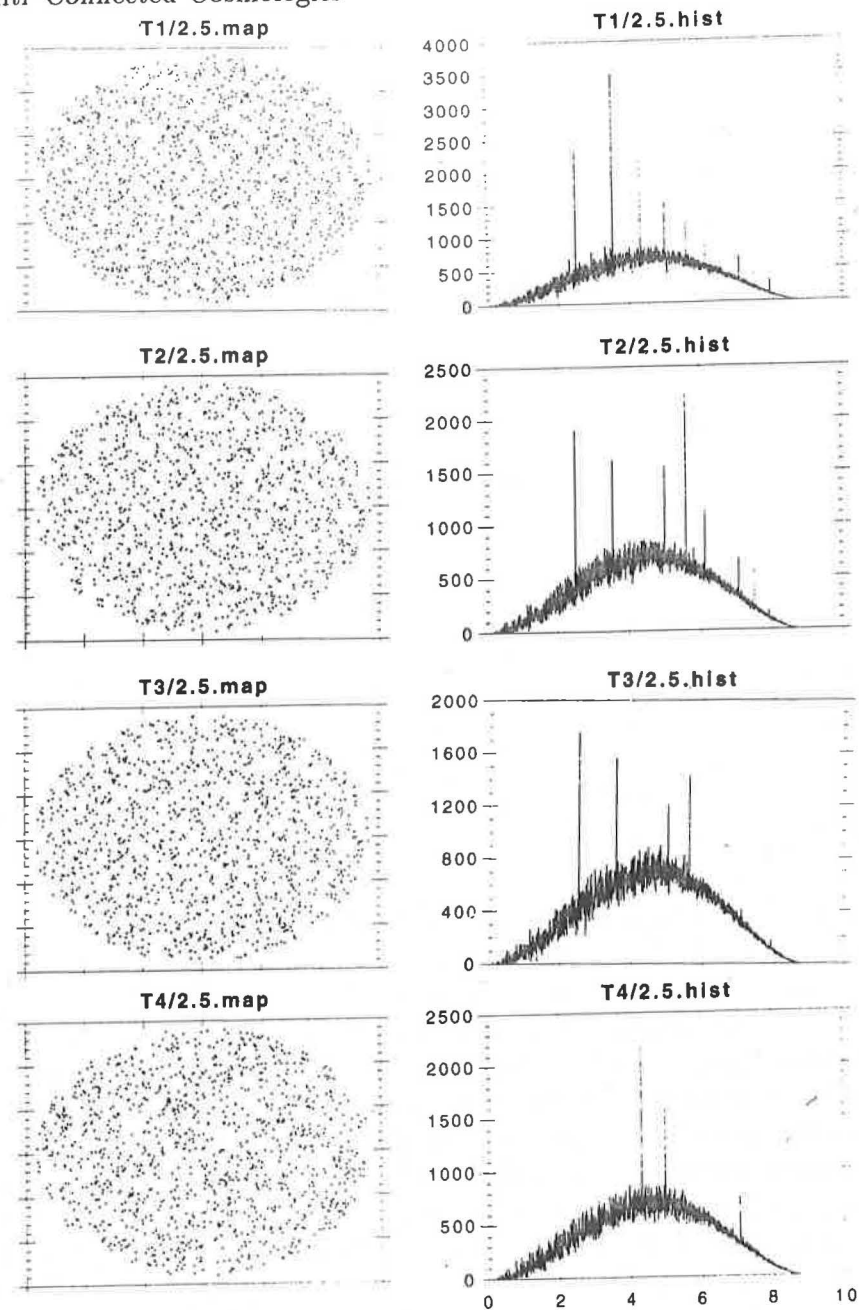


Figure 1. (see caption at the end of paper)



#### 4.4.2 CMB anisotropies in a multi-connected cosmology

The search for anisotropies in the CMB is an old story. The dipole anisotropy is now well interpreted as the Doppler shift due to the motion of the Earth with respect to the last scattering surface. The existence, magnitude, and interpretation of this effect remain exactly identical in a multi-connected model. Beside the dipole, the only firmly detected anisotropy comes from the result of the DMR instrument on the *COBE* satellite, at a very low level  $\delta T/T \approx 10^{-5}$ , and at large angular scales  $\theta \geq 7$  deg (Smoot et. al. 1992). In particular the quadrupole component (corresponding to  $\theta = 90$  deg) appears very low.

If space is multiconnected with a characteristic length  $\alpha$  smaller than the horizon at recombination, the relation between angular anisotropies of the CMB and the spatial fluctuation spectrum present at the recombination is modified. The last scattering surface is a spherical surface in observational space, centered on the observer, and of radius comparable to the present horizon. Observing the CMB in directions separated by an angle  $\theta$  is equivalent to observe a comoving length  $L(\theta)$  in the universal covering space – the observer's world (for instance,  $L \approx 500 Mpc$  for  $\theta = 6$  deg). In the case of a small multi-connected model, large values of  $\theta$  may correspond to values of  $L$  comparable to, or greater than  $\alpha$ .

Stevens et al. (1993) and Starobinsky (1993) have analyzed the effects of multiconnectedness on the CMB anisotropies. In a multi-connected model, since space is finite and closed, at least in some directions, only a restricted collection of wavevectors  $k$  are allowed. In the simplest case of a hypertorus with sides  $\alpha_x, \alpha_y, \alpha_z$ , for instance, the allowed vectors have components  $k_x = 2\pi n_x/\alpha_x, k_y = 2\pi n_y/\alpha_y$ , and  $k_z = 2\pi n_z/\alpha_z$ , where  $n_x, n_y, n_z$  takes entire values, so that the sum is reduced to these values.

They consider the simplest case where  $\alpha_x = \alpha_y = \alpha_z = \alpha$  and the value  $n = 1$ , corresponding to the Harrison-Zeldovich spectrum, as predicted for instance from inflation (which, in a multi-connected model, could occur with the same conditions than in a singly-connected one). They normalize the calculations to the value observed by COBE at the angular scale  $\theta = 18$  deg. With the density fluctuation spectrum so determined (slope and normalization) they are able to estimate the statistics of the temperature anisotropies for any value of  $\alpha$ . They conclude that the results can fit the COBE observations only for  $\alpha > 2\,400 h^{-1} Mpc$  (comparable to the

horizon size of  $3\,000 h^{-1} Mpc$  with their hypotheses).

They also consider the other models with zero curvature and cubic cell  $\mathcal{T}_1, \dots, \mathcal{T}_4$ , and obtained comparable limits. They conclude that, if the Universe is spatially flat, if the COBE anisotropies are intrinsic and due to the Sachs-Wolfe effect, with a negligible variance, and if the density fluctuation spectrum at recombination has a gaussian statistics with a scale invariant ( $n = 1$ ) spectrum, then the above multi-connected cosmologies are ruled out by the COBE observations.

To briefly comment their result, let us just remark that it relies on a set of hypotheses related to models of large scale structure formation, for which there is presently no satisfactory agreement. In particular, the formerly favorite cold dark matter model, with  $\Omega = 1, n = 1$ , etc., has now serious troubles. Thus, if we are ready to consider the possibility of a small multi-connected universe, it would not seem reasonable to rule it out from requirements involving a particular class of galaxy formation models, which have not obtained presently a solid confirmation.

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### Figure caption

Figure 1: Left : Appearance of the sky (in equal area projection) in a multi-connected Einstein-de Sitter universe with topologies  $\mathcal{T}_1, \dots, \mathcal{T}_4$ . The cell size is  $2500 Mpc$  and 50 galaxies are randomly distributed in the original cell. The redshift cut-off is  $z = 4$ . No information about multi-connectedness appears. Right : Histogram of separation distances between all the pairs of images. The peaks reveal repetition scales related to the size of the fundamental cell. All the information about multi-connectedness and the topological type appears.