Table 2

Nonstrange mesons (masses in GeV)

Particle \ Wave function	Ψ_1	Ψ_2	Ψ_3
$(q\bar{q})_{J=1}=\rho$	1.623	1.572	1.578
$(q\bar{q})_{J=0}=\pi$	1.484	1.289	1.578
$(s\bar{s})_{J=1}=\phi$	1.753	1.710	1.765
$(s\bar{s})_{J=0}$	1.653	1.522	1.765

Table 3

Strange mesons (masses in GeV)

$(q\bar{s})_{J=0}=K$	1.560	1.417	1.66
$(q\bar{s})_{J=1}=K^*$	1.670	1.637	1.67

So there is no place for ρ'(1250) in the bag model proposed here, as distinct from the "naive" bag model which had been used in [7] . However, the existence of ρ'(1250) is not firmly established and needs further experimental check.

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THE POTENTIAL AND QUASIPOTENTIAL MODELS FOR BOUND QUARKS

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The idea that quarks "inside" the hadrons are bounded by a simple potential is as old as the idea about quarks themselves. It is based on simple and conventional physical concepts and is attractive to many physicists. This viewpoint was first used by N.N.Bogolubov et al.[1,2].

The discovery of new particles and their interpretation as bound states of the charmed quarks[3] stimulated the creation of a "new wave" of papers, in which different simple potentials describing the interaction between quarks are used. The most popular is the linear potential

$$V = gr - V_0 \qquad (1)$$

where r is a relative distance between quarks.

This potential is suggested by gauge theories of quark confinement and by exact soluble models in the 2-dimensional quantum electrodynamics.

The spectroscopy which arises here is in satisfactory agreement with the experimental data on the mesonic (and baryonic) masses. Of course, all such models are the only first approximation to the real picture. The consistent solution of this problem should comprise both the quantum effects and all other attributes of the interaction of quarks.

The main part of this talk is devoted to the application of the quasipotential (q.p.) approach[4]. It is connected directly with quantum field theory and it is strictly relativistic. In fact, we shall employ the version of the q.p. approach given by Kadyshevsky[5]. But before passing to the q.p. analysis of hadronic spectra we should mention the papers devoted to investigation of this spectra on the basis of linear and other simple potentials[6].

The q.p. equations for the wave function and the scattering amplitude given in[5] are absolute with respect to the geometry of the momentum space and could be obtained from the non-relativistic Schroedinger and Lippmann-Schwinger equations by changing the non-relativistic (Euclidean) expressions for energy, volume element, etc., by their relativistic (non-Euclidean) analogs. This fact allows us to pass to the relativistic r-representation using the expansion over matrix elements of the unitary irreducible representations of the Lorentz group[7]. The kernel of this relativistic Fourier transformation has the form[x]

$$\xi(\vec{q},\vec{r}) = \left(ch\chi_q - (\vec{n}\vec{n}_q)\, sh\chi_q\right)^{-1-ir} \qquad (2)$$

$\vec{r}=r\cdot\vec{n}$, $\vec{q}=\vec{n}_q\cdot q$ (the relative momentum of quarks in the c.m.s.) $\vec{n}^2=\vec{n}_q^2=1$. Here r is the eigenvalue of the Casimir operator of the Lorentz group $(0<r<\infty)$.

In the nonrelativistic limit

$$\xi(\vec{q},\vec{r}) \to e^{i\vec{q}\vec{r}} \qquad (3)$$

where $\vec{r}$ is a usual 3-vector.

The equation for the relativistic wave function $\Psi_q(\vec{r})$ in $\vec{r}$ representation is the finite-difference equation

$$\left[H_0 - 2E_q + V(\vec{r},E_q)\right]\Psi_q(\vec{r}) = 0 \qquad (4)$$

where $V(\vec{r},E_q)$ is the quasipotential and H_0 has the form

$$H_0 = 2\, ch\, i\frac{\partial}{\partial r} + \frac{2i}{r}\, sh\, i\frac{\partial}{\partial r} - \frac{\Delta_{\theta,\varphi}}{r^2} e^{i\frac{\partial}{\partial r}} \qquad (5)$$

This equation was successfully applied to the analysis of the $c\bar{c}$ system (ψ-particle) in the case of linear potential in paper[8]. The supplementary series of the Lorentz group representation was used in[9].

There is a number of difficulties connected with the finite-difference character of the q.p. equation. The main of them is the problem of boundary conditions.

We suggest a version of q.p. equation which corresponds to the second order differential equation in $\vec{r}$-representation[10]. We write the denominator in the Lippman-Schwinger equation in the form

$$\frac{1}{E_q - E_k + i\varepsilon} = \frac{1}{\frac{S^2(q,0)}{2} - \frac{S^2(k,0)}{2} + i\varepsilon} \qquad (6)$$

where $S(q,0)$ is the Euclidean distance between the point $\vec{q}$ and the origin in the flat non-relativistic momentum space. Passing now to the relativistic q.p. equation we must change

$$S(q,0) = \sqrt{\vec{q}^2} \to \chi_q = \ln\left(E_q + \sqrt{E_q^2 - 1}\right) \qquad (7)$$

where χ_q is the distance in the Lobachevsky momentum space, or rapidity.

The q.p. equation is

$$A(\vec{p},\vec{q}) = -\frac{1}{4\pi}V(\vec{p},\vec{q};E_q) + \frac{1}{2\pi}\frac{\chi_q}{sh\chi_q}\int\frac{V(\vec{p},\vec{k})\,d\Omega_k\,A(\vec{k},\vec{q})}{\chi_q^2 - \chi_k^2 + i\varepsilon} \qquad (8)$$

The equation for the radial wave function in $\vec{r}$-representation is

$$\left[\frac{d^2}{dr^2} + \chi_q^2 - R_\ell(r,\chi_q) + V(r,\chi_q)\right]\varphi_{q\ell}(r) = 0 \qquad (9)$$

where $R_\ell(r,\chi_q)$ is a specific centrifugal term[10] which in the non-relativistic limit goes over to the usual one

$$R_\ell(r,\chi_q) \to \frac{\ell(\ell+1)}{r^2} \qquad (10)$$

$$R_0 = 0. \qquad (11)$$

For linear potential (1) and $\ell=0$ the solution is the Airy function:

$$\varphi_{n0}(r) = \left(\frac{g\chi_n}{sh\chi_n}\right)Ai\left[\left(\frac{g\chi_n}{sh\chi_n}\right)^{1/3}\left(r - \frac{V_0}{g} - \frac{\chi_n sh\chi_n}{g}\right)\right] \qquad (12)$$

n is the principal quantum number. The spectrum of rapidities is given by the condition $\varphi_n(0)=0$, i.e., by zeros of Airy function. For $\ell\neq 0$ we have no analytic solutions.

Using the lowest mesonic masses and leptonic width (the colour is taken into account) as an input parameter, we made the computer calculations of radial and orbital excitations in $p\bar{p}$, $n\bar{n}$, $\lambda\bar{\lambda}$ and $c\bar{c}$ systems. The results are in good agreement with the data on mesonic spectroscopy[10]. For example the first radial excitations in the $c\bar{c}$ system take the following values:

$3.09 \sim \psi(3.095)$; $3.68 \sim \psi'(3.68)$;
$4.16 \sim \psi''(4.15)$; $4.57 \sim \psi'''(4.41)$.

The detailed results of computations are presented in[10].

We investigated also the case of extremely heavy quarks, which is suggested by the field-

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[x] In what follows we consider the system $q\bar{q}$ i.e., the system of particles with equal masses. We employ the system of units in which $\hbar=m=c=1$, where m is the mass of quark.

theoretical scheme in which the momenta of quanta off the mass shell belong to the de-Sitter space[11-13]:

$$\frac{1}{\ell_0^2} p_4^2 + p_0^2 - \vec{p}^2 = \frac{1}{\ell_0^2} \qquad (13)$$

Following paper[14] we identify ℓ_0 with the length scale arising in the weak interaction theory:

$$\ell_0 = \sqrt{\frac{G_F}{\hbar c}} \cong 6\cdot 10^{-4} fm$$
$$M = \frac{\hbar}{\ell_0 c} \cong 300\, GeV \qquad (14)$$

The quanta with the mass M (the maximons), play in the theory with the de-Sitter momentum space the principal role[x].

The attractive idea is to identify the quark with maximon. In such a case these particles, originating due to the properties of the geometry of the momentum space, are at the same time the fundamental constituents of hadrons.

We consider the model of ψ-mesons represented as the bound states of the quarks C and $\bar{C}$ with mass M. With slight modifications of the arguments of ref.[12] we obtain the q.p. equation which does not differ in form from (9), but includes the rapidity χ_q of another geometrical nature:

$$q_4 = Mc\, ch\chi_q \qquad (15)$$

The masses μ of all known resonances are much smaller than M, that is $\chi_q \cong i\frac{\pi}{2}$ for low-laying excitations. It is easy to see that $V_0 \cong \frac{\pi}{2}M$, and the absolute value $2|\vec{p}|$ of the relative momentum equals

$$2|\vec{p}| \cong 2Mc \qquad (16)$$

within the accuracy $(\mu/M)^2$. Thus, the quarks motion has the relativistic character in this case.

Taking masses of ψ(3.095) and ψ'(3.686) as input parameters we obtain in this case also the higher excitations in the $C\bar{C}$ system, which could be identified with known ψ''(4.15) and ψ'''(4.40) states.

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[x] The mass M is the limiting mass of the virtual quanta[11-13]. The term "maximon" is taken from paper[15].

The knowledge of the relativistic bound state wave function permits one to obtain further information about its structure. For example, the computation of the values of mean-square radius $\langle r^2\rangle^{1/2}$ of the composite system $\psi(c\bar{c})$ yields

$$\langle r^2\rangle^{1/2} \cong 0.05\, fm \qquad (17)$$

It is remarkable that uncertainty relation in the relativistic $\vec{r}$-space[x]

$$\langle r^2\rangle^{1/2}\langle \Delta\chi\rangle \cong \ell_0 \qquad (18)$$

results in the correct estimate for $\langle r^2\rangle^{1/2}$.

It follows from (16) that the mean value of rapidity equals $i\pi/2$ within accurate of $(\mu/M)^2$. Hence

$$\mu/M = ch\, i(\pi/2 - \Delta\chi) = sh\Delta\chi \cong \Delta\chi \qquad (19)$$

Substituting (19) into (18) we obtain the estimation

$$\langle r^2\rangle^{1/2} \cong \ell_0 \frac{M}{\mu} \cong 0.06 \qquad (20)$$

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[x] The rapidity χ and the relativistic relative distance r are canonically conjugated in the sense of the relativistic Fourier transformation[7]. This relation is essentially used in the relativistic scheme describing the data on high energy hadron-hadron scattering[16].

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NULL-PLANE QUANTIZATION AND QUASIPOTENTIAL EQUATION FOR COMPOSITE PARTICLES

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Methods of investigation of relativistic bound systems are known from the time of creation of quantum field theory. At present, (as always, of course) the problem is to develop more simpler and economical ways for dealing with bound states.

We are inclined to think that the null-plane /1/ quantum field theory is to be more adapted to the problems under consideration because, if we eliminate the $p^+=0$ modes, null-plane canonical commutation relations have simplest, Fock, representation even in the presence of interaction /2/ . Bound state wave functions at the equal $\tau = x^0 + x^3$ "times" for constituents are maximally close to nonrelativistic expressions without the transition to the infinite momentum frame /3,4/.

Bound state problem for two spinless particles on the one null-plane was considered in /5/ using Tamm-Dancoff approximation and in /6/ - on the basis of DGS spectral representation.

From our point of view, the most successive approach to the bound state problem in relativistic quantum field theory is a quasipotential method /7/ . Equal-time quasipotential method had been successfully applied in many investigations /8/ . Null-plane quasipotential equation was considered in papers /9/ for spinless particles and in /10,11/ - for spinorial particles.

In this report we shall discuss the main features of null-plane quasipotential approach and give an application to the asymptotic behaviour of composite particle form factors at large $\vec{p}_\perp$.

Taking into account that the $p^+=0$ modes may be eliminated /12/ from the Fock-space with the help of second class constraints /13/ it