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Lessons and complications from gravitationally induced entanglement

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Abstract. We critically review arguments depicting entanglement generated between exclusively gravitationally interacting particles as evidence for the necessity of a quantized gravitational field. For proposed experiments which are supposed to witness this gravitationally induced entanglement, we discuss the implications of a positive result and defend the possibility of a semiclassical theory of quantum matter on a classical spacetime which cannot be conclusively ruled out by witnessing entanglement. For thought experiments that resolve apparent causality issues with quantum systems entangled via the Newtonian gravitational interaction by considering a quantized gravitational field, we demonstrate that quantization of gravitational degrees of freedom is not only unnecessary but may result in remaining paradoxical behavior, unless a previously disregarded decoherence channel is taken into consideration.

1. Introduction

Despite tremendous effort and progress in the detailed understanding of the various approaches to a quantum theory of the gravitational interaction, just as little is known about which way is the proper way to quantize gravity as it has been with the discovery of the principles of quantum field theory about a century ago. This lack of progress is often attributed to the absence of concrete predictions for feasible experiments from candidate theories of quantum gravity. Hopes for at least some guidance from observation have been mostly focused on generic features, such as the effects of a minimal length scale [1] or modified uncertainty relations [2, 3], which are not linked to any specific model but a common feature of a variety of different approaches.

In the same vein, driven by recent progress in laboratory quantum technology, there has been a renewed interest in experiments aimed at probing the question whether the gravitational field must be quantized at all. From early on, beginning with discussions between Feynman, Rosenfeld, and others at the 1957 Chapel Hill Conference [4], there has been a plethora of theoretical arguments for the necessity to quantize gravity, alongside similarly sophisticated refutations of these very arguments [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] (for a recent review see also [20, 21]). In the light of these combined arguments, the question whether gravity must be quantized remains without a definitive answer, although a consistent coupling of quantum matter to classical gravity [21, 22] faces technical difficulties that might be comparable to those faced by quantum gravity approaches. Earlier proposals [16, 23, 24] for such laboratory experiments to establish the necessity of quantizing the gravitational field focused on testing the specific model of the Schrödinger–Newton equations [25, 11, 26, 27], which results from

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coupling quantum matter to a classical spacetime via Einstein's equations sourced by the mean stress-energy in the quantum state of the matter fields.

Recently, there has been great interest in the idea that more generic conclusions could be drawn from the study of gravitationally induced entanglement. This idea has been sparked by the Kafri–Taylor–Milburn [28] toy model, where the Newtonian gravitational force between two particles is considered an effect of local operations with classical communication (LOCC). Interactions of this kind are known not to induce entanglement in initially separable quantum states, leading to the proposal [29, 30] that experimental evidence of entanglement between particles which only interact through the Newtonian gravitational forces between them would ascertain the 'quantumness' of the gravitational field. We critically assess the scope of such claims in section 2.

On a related note, it has been shown by Belenchia et al. [17] that an apparent paradox [31] allowing for faster-than-light signaling via the entanglement induced by the Coulombian or Newtonian forces between quantum particles is resolved by taking the quantum field character of the electromagnetic or gravitational interaction into account. However, as important as it is to understand how consistency is achieved in the electromagnetic case—and as curious as one may find the fact that similar arguments could be applied to gravity were it indeed a perturbatively defined quantum field theory at low energies—the only rigorous conclusion to be drawn from this argument is that quantization of the gravitational field would be a sufficient condition to resolve the paradox, not a necessary one. A similar case has been made by Rosenfeld [6] in response to the interpretation of his work with Bohr [32] as alleged evidence for the need to quantize the electromagnetic field—and analogies drawn for gravity. Notably, the paradox is resolved in a trivial and—as we argue in section 3—more robust way if gravity remains *unquantized*.

2. On laboratory experiments and the need to quantize gravity

A 'classical channel' interaction assumes a many-particle interaction potential $\sum_{i\neq j} V_i(\hat{x}_j)$, where each V_i is a 'classical' potential determined through measurements of the i-th particle. For such an interaction, an initially separable state $\Psi = \bigotimes_i \psi_i$ would remain separable, contrary to a typical quantum interaction potential $\sum_{i\neq j} V(|\hat{x}_i - \hat{x}_j|)$ which induces entanglement. A driving reason to consider this type of interaction as a model for the Newtonian gravitational force [28] is that it permits a consistent dynamical law for the density operator—and therefore an operational interpretation of quantum mechanics. This is in contrast to the Schrödinger–Newton model, for which finding a consistent model of the measurement process poses a challenge. Hence, despite being experimentally refuted [33] itself, the Kafri-Taylor-Milburn model has inspired different approaches how to couple quantum matter to classical gravity:

Tilloy and Diósi [34] propose a model of classical Newtonian gravity source by a mass density which is the signal of weak measurements. This model can also be interpreted in the context of collapse models [35] where the stochastic localization events become the source of the gravitational potential. This model is akin to the Schrödinger–Newton model in that it provides a joint dynamical evolution law for a quantum state Ψ for matter and a classical gravitational potential Φ. Unlike the deterministic Schrödinger–Newton model, however, this evolution is stochastic and can thereby maintain a closed, linear evolution law for the matter density operator.

Oppenheim [22] proposes a general framework for quantum-classical hybrid theories which applied to the Hamiltonian formulation of general relativity promises a consistent coupling of classical gravity and quantum matter. The model provides a deterministic evolution law for a hybrid 'classical-quantum' state $\hat{\varrho}(z)$, a subnormalized positive Hilbert space operator on each classical phase-space point z, from which both classical probability densities in phase space and quantum probabilities can be derived. Hence, like the Schrödinger evolution law in Hilbert space, a probability interpretation is necessary—not only for the quantum matter but also for the spacetime degrees of freedom. In this sense, the model can be seen as a fundamentally stochastic one, similar to quantum theory, or an effective stochastic description of a yet unknown underlying theory, comparable to classical statistical mechanics.

All of these models, including the Schrödinger–Newton model, predict no entanglement between gravitationally interacting particles, whereas perturbative quantum gravity predicts such entanglement in complete analogy to electrodynamics [36]. Hence, experiments confirming entanglement [29] would provide clear evidence against these specific semiclassical models. On the other hand, all four of these semiclassical models make further definitive predictions: the Kafri–Taylor–Milburn model has already been experimentall refuted [33], the proposals [23, 24] for testing the Schrödinger–Newton model are arguably closer to technological feasibility than witnessing entanglement, and the Tilloy–Diósi and Oppenheim models predict unaccounted decoherence or heating effects which are already being actively investigated in order to put bounds on collapse models [37, 38, 39].

What, then, is the additional benefit of testing gravitationally induced entanglement? If one believes the claims of its proponents, a huge one: namely 'that if entanglement is observed, then *all* classical models for gravity, obeying our general principles, are ruled out.' [40] This claim has been contested [41, 42, 43] as putting too tight constraints on the allowed 'classical' models, namely by a restriction to the LOCC perspective. To illustrate the loophole remaining due to this restriction, in [44] we propose a class of simple toy models that explicitly demonstrates a semiclassical model in which entanglement *is* induced through the gravitational interaction.

These proposed models are based on the de Broglie–Bohm theory where, in addition to the N-particle wave function Ψ one has the N particle coordinates $\vec{q}_i$, satisfying the guiding equation

$$\frac{\mathrm{d}\vec{q}_i(t)}{\mathrm{d}t} = \frac{\hbar}{m_i}\,\mathrm{Im}\left(\frac{\nabla_i\Psi(t;\vec{r}_1,\ldots,\vec{r}_N)}{\Psi(t;\vec{r}_1,\ldots,\vec{r}_N)}\right)\Bigg|_{\vec{r}_1=\vec{q}_1(t),\ldots,\vec{r}_N=\vec{q}_N(t)}\,. \tag{1}$$

Now, the wave function is expected to follow a Schrödinger equation including a Newtonian gravitational potential. The distinguishing features of the various models are the different sources of said potential, i.e. the mass density ρ in the Poisson equation $\nabla^2 V = 4\pi G m\rho$ for the potential energy:

(i) In the Schrödinger–Newton model, the gravitational mass density is determined by the marginal distributions of the N particles:

$$\rho(t,\vec{r}) = \sum_i m_i \int \mathrm{d}^3r_1\cdots\mathrm{d}^3r_{i-1}\mathrm{d}^3r_{i+1}\cdots\mathrm{d}^3r_N|\Psi(t;\vec{r}_1,\ldots,\vec{r}_{i-1},\vec{r},\vec{r}_{i+1},\ldots,\vec{r}_N)|^2\,. \tag{2}$$

(ii) In the model by Tilloy and Diósi, the potential is sourced by the *signal* ρ, defined via

$$\rho\,\mathrm{d}t = \langle\hat{\varrho}\rangle_t\,\mathrm{d}t + \mathrm{d}W_t\,, \tag{3}$$

where $\hat{\varrho}$ is the mass density operator (smeared over some width σ in order to avoid divergences) and W_t a spatially correlated white-noise term, entering the stochastic Schrödinger equation [45] for the quantum state (with decoherence rate γ):

$$\mathrm{d}\psi_t = -\frac{\mathrm{i}}{\hbar}\hat{H}\psi_t\mathrm{d}t - \frac{\gamma}{2\hbar^2}\left(\hat{\varrho}-\langle\hat{\varrho}\rangle_t\right)^2\psi_t\mathrm{d}t + \frac{2\gamma}{\hbar}\left(\hat{\varrho}-\langle\hat{\varrho}\rangle_t\right)\psi_t\mathrm{d}W_t\,. \tag{4}$$

(iii) In the Newtonian limit of perturbative quantum gravity, the gravitational potential energy does *not* satisfy a Poisson equation but instead is an operator valued function

$$\hat{V} = -\frac{G}{2}\int \mathrm{d}^3u\,\mathrm{d}^3v\,\frac{\hat{\varrho}(\vec{u})\hat{\varrho}(\vec{v})}{|\vec{u}-\vec{v}|}\,. \tag{5}$$

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(iv) Finally, in de Broglie–Bohm theory the canonical choice would be to simply implement the same potential (5) in the Hamiltonian evolution of the wave function. However, the existence of the 'hidden' variables $\vec{q}_i$ leaves us with the alternative to implement a *classical* gravitational mass density sourced by the Bohmian trajectories:

$$\rho(t, \vec{r}) = \sum_i m_i \delta(\vec{r} - \vec{q}_i(t)) \,. \tag{6}$$

Together with the Schrödinger equation and the guiding equation (1), the mass density (6) provides a consistent nonrelativistic model for the gravitational interaction between fundamental particles which agrees with the Newtonian limit of quantum gravity (and thus the classical Newtonian theory) in the classical limit. The crucial observation is that this model has a semiclassical interpretation: the Bohmian particles can be understood as sourcing the curvature of a *classical* spacetime on which both the wave function and the particle coordinates are defined. Despite this semiclassical nature, this Bohmian trajectory sourced Newtonian interaction *will* result in entanglement between particles, providing an explicit counter-example to the claim cited above.

There are two caveats to be considered: Firstly, the model as defined above, when applied to two interacting particles, results in a relative phase that is twice that predicted by the quantum potential (5). Whereas the resulting difference in predictions for entanglement witnesses [29] could be seen as a feature making the model testable, this phase prediction contradicts the *observed* gravitational phases in the homogeneous field of the Earth [46]. It can be easily remedied by adding to the Hamiltonian a term proportional to identity operator on Hilbert space that depends solely on the trajectories $\vec{q}_i$, although at the cost of rendering the interpretation in terms of wave functions and trajectories on a classical spacetime less natural.

Second and more importantly, the interpretation in terms of a classical spacetime sourced by the trajectories assumes that the model is the nonrelativistic limit of a well-defined general relativistic theory. Considering the tremendous difficulties faced by relativistic generalizations of Bohmian mechanics, this may justifiably raise concerns. Note, however, that the same problem in principle affects *all* models listed (although possibly with different severity): The Schrödinger–Newton model can be understood as the limit of the semiclassical Einstein equations, but it cannot be defined consistent with observations unless a proper objective dynamical law for the wave-function collapse can be found [20, 21]. Similarly, the Tilloy–Diósi model is based on collapse models, whose relativistic generalization [47] is not fully understood. Finally, perturbative quantum gravity, due to its nonrenormalizability [48] is generally understood as the low energy limit of some more fundamental theory, but the existence of a consistent and predictive theory with this limit has not yet been rigorously proven.

The class of toy models we propose interpolates between the Bohmian trajectory sourced gravitational mass density (6) and the Schrödinger–Newton model (2) by taking the integrals $\int \mathrm{d}^3 r_i$ in equation (2) not over the whole configuration space $\mathbb{R}^3$ but only over a sphere of radius R around the Bohmian coordinate $\vec{q}_i(t)$. The resulting models are parameterized by this radius R, where the Schrödinger–Newton model is recovered in the limit $R \to \infty$ and the Bohmian trajectory model in the limit $R \to 0$. It can then be shown [44] that for any finite value of R one can always find an entanglement witness that confirms a nonzero entanglement between two particles due to the gravitational interaction. The amount of entanglement (as measured by the deviation of the witness function from the classically allowed values) is suppressed exponentially for large R.

In conclusion, claims that witnessing gravitationally induced entanglement would rule out *any* conceivable coupling of quantum matter to classical general relativity are evidently wrong. Nonetheless, a positive outcome of such an experiment would conclusively rule out the few explicit models in existence, as well as put even tighter constraints on the possibility of consistent

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semiclassical models. In this regard, experimental tests of gravitationally induced entanglement would complement direct tests of quantum gravity alternatives [16, 26, 23, 24, 33, 39], although there are good reasons [49] to believe that—comparable resources provided—they are also more technologically challenging than those direct tests.

3. On thought experiments in the Galilean sector of quantized gravity

Somewhat orthogonal to the efforts towards an experimental confirmation of the quantization of the gravitational field, outlined in the previous section, are purely theoretical consistency arguments based on thought experiments. Assume Alice and Bob, located at distant places, both possess a single quantum particle. Alice's particle at position x_A is the source of a Coulomb type potential to which Bob's particle at position x_B is sensitive, i.e. there is a force $F \sim |x_A - x_B|^{-2}$ acting on Bob's particle. The shift δx of Bob's particle position due to this force after some fixed time t_B is a strictly monotonous function of x_A, such that Bob acquires *which-way information* about Alice's particle, provided a sufficient resolution for the shift δx. Now, an essential consequence of quantum mechanics is the complementarity principle. If Alice performs an interferometry experiment with her particle, interferometric visiblity will be lost as which-way information about the particle's position can be gained. Assuming that Alice's particle is readily prepared in a superposition state at time $t = 0$, followed by the time interval $[0, t_B]$ during which Bob can choose freely to conduct or not to conduct his experiment in order to acquire which-way information about Alice's particle, Alice reunites the possible particle trajectories with the intent to create an interference pattern during the time interval $[t_B, t_A + t_B]$. As long as the total time $t_A + t_B$ is shorter than the time a light signal would take from Alice to Bob, Bob can send a binary message faster than light.

This apparent paradox has been discussed by Mari et al. [31]. Belenchia et al. [17] demonstrate how the paradoxical signaling is prohibited if the interaction between Alice's and Bob's particles is due to a quantized field. Simply put, the existence of vacuum fluctuations puts a limit on the best possible resolution which requires a minimum time t_B on Bob's side, whereas on Alice's side the experiment must be performed sufficiently slow in order to not already decohere the interference pattern by emitting radiation. Jointly, these constraints are such that both in the electromagnetic and the gravitational case they can prevent the possibility of faster-than-light signalling. Note that in the gravitational case, whereas these conditions *can* be justified from the quantization of the gravitational field, such an interpretation is by no means required; they can also be understood as a consequence of classical radiation [19, 20].

Quantum entanglement, though not explicitly mentioned in the above description, again plays a crucial role. With Alice's particle initially in a superposition state $|\psi\rangle \sim |x_A\rangle + |x'_A\rangle$, the above argument presupposes that the interaction is a quantum potential $\hat{V}(x) \sim |\hat{x}_A - x|^{-1}$ and the final state an entangled state $|\Psi\rangle \sim |x_A\rangle \otimes |x_B\rangle + |x'_A\rangle \otimes |x'_B\rangle$. If the potential instead is of the semiclassical type that results in a separable state $|\Psi\rangle \sim (|x_A\rangle + |x'_A\rangle) \otimes |x_B\rangle$, Bob cannot gain any which-way information through the position of his particle. It is, therefore, somewhat misleading to argue that the resolution of the faster-than-light signaling paradox 'provides support for the view that (linearized) gravity should have a quantum field description' [17]. Field quantization is a *sufficient* property for consistency but not a *necessary* one.

The proposal for experimental tests of gravity induced entanglement and the thought experiment outlined above have in common that they do not require many assumptions about the gravitational interaction beyond a certain analogy to the electromagnetic field, which exhibits itself in the identical form of the Newtonian gravitational interaction compared to the Coulomb interaction in the Schrödinger equation for the electromagnetic Galilean limit. As long as gravity works 'just like electromagnetism', there is no reason to doubt the positive outcome of entanglement witnessing experiments [29], and field quantization seems a perfectly valid argument to resolve the apparent paradoxes [17]. We know, however, that this analogy between gravitational and electromagnetic forces must have limitations:

(i) On the level of the classical theories, there are some fundamental differences between general relativity and Maxwell's theory. Not only are Maxwell's equations linear in contrast to the nonlinear Einstein's equations; Maxwell's theory also defines a linear connection on a line bundle, whereas the Levi–Civita connection in general relativity is the *affine* connection for the tangent bundle of spacetime. Crucially, this implies that for gravity—contrary to electrodynamics—there is no meaningful definition of a 'vacuum state' upon which a quantum field theory could be built [21].

(ii) Although a perturbative quantum field theory around a fixed background spacetime can formally be defined, quite similar to the quantization of electrodynamics, in the gravitational case this theory is known to be nonrenormalizable [48].

Hence, we propose that more promising lessons might be learned from such thought experiments which do not have a simple electromagnetic analogy.

One such effect with no immediate analogy is the universal coupling of gravity to all forms of energy. This has already been exploited for the suggestion of time-dilation related dephasing [50, 51] where it has been proposed that a nonrelativistic quantum system with internal degrees of freedom, subject to a gravitational potential $\Phi(x)$ including lowest order relativistic corrections, should be described by a Hamiltonian

$$\hat{H} = \hat{H}_{\text{cm}} + \hat{H}_{\text{int}} + \left(m + \hat{H}_{\text{int}}\right) \Phi(\hat{x}_{\text{cm}}) \,, \tag{7}$$

where $\hat{H}_{\text{cm}}$ and $\hat{x}_{\text{cm}}$ are the Hamiltonian and position operators acting on the center-of-mass Hilbert space and $\hat{H}_{\text{int}}$ the Hamiltonian for the internal dynamics. Notably, a derivation of the Hamiltonian (7) from quantum field theory in curved spacetime is lacking with some more rigorous arguments for the conditions under which it may hold given recently [52, 53]. If correct, the term $\sim \hat{H}_{\text{int}}\Phi(\hat{x}_{\text{cm}})$ in equation (7) suggests entanglement between the internal and external degrees of freedom which can result in the loss of interferometric visibility.

Following the idea to focus on situations where the analogy to electromagnetism breaks down, we can combine the consequences of the Hamiltonian (7) with the thought experiment described before. Instead of a free fall experiment, Bob could sense the gravitational field sourced by Alice's particle using an interferometer himself. Instead of the displacement δx, he would then measure the COW [46] phase shift due to gravity. It has been shown by Rydving et al. [19] that the limitations from field quantization remain the same: instead of δx needing to be at least a Planck length in order to be resolvable, the interferometric fringe spacing must be at least a Planck length. In the context of equation (7) it is worth noting, however, that for a typical interferometric path with short acceleration phases splitting and recombining the trajectories with a long period of free flight, the constraint on fringe spacing depends on the dynamics during the acceleration phases, whereas the gravitational phases are determined during the free flight phase. Thus, it seems, the constraint preventing faster-than-light signaling could be overcome could one increase the mass during the free flight phase only.

The Hamiltonian (7) allows for exactly that [54]: after an initial acceleration with the rest mass m photons of energy $E \gg mc^2$ are absorbed and emitted just before entering the final acceleration phase. Does this imply that faster-than-light signaling is possible? Of course not. Firstly, the Hamiltonian (7), not being rigorously derived from a fundamental relativistic theory, could simply be inapplicable to the situation at hand. Secondly, if gravity was correctly described by a semiclassical theory and not induce entanglement, the paradoxic situation would be easily resolved.

Assuming both equation (7) and gravitationally induced entanglement to be correct models, the paradox can still be resolved. To achieve this, the comparison to electrodynamics is not helpful: a corresponding thought experiment where charge is dynamically changed is not feasible in the same way. Remembering, however, that the incoming photons carry energy and therefore gravitate, intuition suggests that the absorption and emission of these photons at the location of the particle correspond to an accelerated energy distribution which should emit gravitational waves. Rough estimates based on dimensional analysis suggest that the which-way information carried by these gravitational waves could be just enough to decohere the interference pattern in those situations that would otherwise allow faster-than-light signaling. This reasoning allows us to predict a novel type of decoherence effect: decoherence due to the emission of gravitational waves during the (otherwise coherent!) absorption or stimulated emission of photons by a massive body.

A better theoretical justification of this proposed decoherence effect is currently a work in progress. The most important lesson to be learned from this example, in our opinion, is that—unexpected experimental results such as a confirmation of the Schrödinger–Newton model or a falsification of gravitationally induced entanglement aside—the most promising thought experiments that could guide us to a proper theory of gravitating quantum matter are those without analogy in the realm of electromagnetism.

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