Optimum number of parameters for np interaction using reference potential approach

Ayushi Awasthi and O.S.K.S Sastri[1*]
[1] *Department of Physics and Astronomical Sciences, Central University of Himachal Pradesh, Dharamshala, 176215,H.P., Bharat(India)*

Introduction

NucleonNucleon (NN) interaction is always divided into three parts: long range part (r $\geq$ 2fm), intermediate range (1fm $\leq$ r $\leq$ 2fm) and short range (r $\leq$ 1fm) [1]. The phenomenological potentials like Nijmegen groups (NijmI, NijmII, Reid93) [2], CD Bonn [3] and Argonne v18 potential [4] are used to explain the observed n-p scattering phase shifts by considering various internal interactions between nucleons. Among these potentials the Reid93 , CD Bonn and Argonne v18 are three of the best realistic potentials that have been successfully used in offshell calculations for light nuclei. The Reid93 potential has 50 parameters which are parametrized with χ^2/N_{data} = 1.03. The Argonne v18 potential has 40 model parameters giving $\chi^2/N_{data} = 1.09$ for 4301 pp and np data in the energies from 0-350 MeV while CD Bonn potential has 43 free parameters. Recently, our group has utilised the reference potential approach to construct inverse potentials for n-p scattering [5]. We have considered Morse, with three model parameters, as a reference potential and obtained total interaction potentials for all the 18 partial waves by considering the latest data of scattering phase shifts (SPS) from Perez et al, of Granada group [6]. Hence, a total 54 parameters were optimised to explain the scattering phase shifts for 18 partial waves, way higher than those of the other realistic potentials. On closer observation, we found that for 10 of the 18 states, one of the parameters was 0.01, which could have been, may be 0. So in this work, by utilising the fact that *"Positive phase shifts have an attractive potential and negative phase shifts result in repulsive potential"*, we have re-optimized the model parameters, using only either the attractive or the repulsive part of Morse function, based on the nature of phase shifts of the considered states.

*Electronic address: sastri.osks@hpcu.ac.in

Methodology

The phase equation, of Ricatti type first order non-linear differential equation, is given as [5]

$$\frac{d\delta_\ell(k,r)}{dr} = -\frac{U(r)}{k}\Big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr)\Big]^2 \quad (1)$$

where $U(r) = V(r)/(\hbar^2/2\mu)$ & $k = \sqrt{E_{c.m}/(\hbar^2/2\mu)}$ and $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Ricatti-Bessel and Ricatti-Neumann functions of order ℓ.
Utilising reference potential approach, we have considered as input, part of Morse function as follows:

$$V(r) = \begin{cases} V_0 e^{-2r/a_m}, & \forall \text{ -ve SPS} \\ -2V_0 e^{-r/a_m}, & \forall \text{ +ve SPS} \end{cases}$$

This is the zeroth approximation for obtaining the empirical model of interaction. Here, model parameters V_0 and a_m represents potential strength , and range of the potential in MeV and fm^{-1} respectively.

Results and Discussion

For *np* system, the value of $\hbar^2/2\mu$ = 41.47 MeVfm^2 and centre of mass energy $E_{c.m.}$ is

TABLE I: Re-optimized Model Parameters for Channels with Positive and Negative SPS; States in bold indicate an attractive potential.

States	Complete Morse [5]				**Attractive**/ Repulsive Morse		
	V_0	r_m	a_m	MSE	V_0	a_m	MSE
$^1P_1$	0.0100	5.4424	1.0156	1.5678	474.9935	0.9809	1.3431
$^3P_1$	0.0100	4.5138	0.7781	0.8324	1166.3660	0.7543	0.7223
$^1D_2$	**131.3019**	**0.0100**	**0.5256**	**0.0259**	**120.9522**	**0.5369**	**0.0224**
$^3D_1$	0.0100	7.4007	1.4034	0.2735	409.8140	1.3444	0.1725
$^1F_3$	0.0100	7.645	1.8772	0.0561	38.3705	1.6944	0.0305
$^3F_3$	0.0100	8.8072	2.4413	0.0248	15.8048	2.0826	0.0257
$^1G_4$	**20.3905**	**0.0100**	**0.6728**	**0.0024**	**18.9481**	**0.6851**	**0.0022**
$^3G_3$	0.0100	6.8456	1.4855	0.0089	121.1211	1.3527	0.0022
$^3G_4$	**59.2187**	**0.0100**	**0.7862**	**0.0149**	**53.6988**	**0.8056**	**0.0117**
$^3H_4$	**23.2448**	**0.0100**	**0.6931**	**0.0002**	**22.7834**	**0.6971**	**0.0002**

related to laboratory energy $E_{\ell ab}$, by relation for non-relativistic kinematics: $E_{c.m.} = 0.5E_{\ell ab}$ From our recent work [5], we have observed that we don't need complete Morse potentials for $^1P_1$,$^3P_1$,$^1D_2$,$^3D_1$,$^1F_3$,$^3F_3$,$^1G_4$, $^3G_3$,$^3G_4$,$^3H_4$ states. The optimized model parameters are shown in Table I. From this table, it is clearly seen that parameter V_0 i.e. depth of Morse potential comes out to be 0.01 for $^1P_1$,$^3P_1$,$^3D_1$,$^1F_3$,$^3F_3$ and $^3G_3$. For states $^1D_2$,$^1G_4$,$^3G_4$ and $^3H_4$, the parameter r_m i.e. equilibrium distance, is 0.01. So, one can infer that three parameter phenomenological potential is not required for these states. From table I, it is shown that the states which are highlighted in bold indicate an equilibrium distance that is nearly zero, suggesting that the potential is predominantly attractive. Conversely, the states that are not in bold, with potential depths nearly equal to zero, imply that the potential should exhibit a repulsive nature Hence, we have considered only attractive part of Morse function for those states having positive phase shifts and repulsive part for those having negative phase shifts. That is, only 20 parameters are to be optimized for explaining the scattering phase shifts of these 10 states. We have optimised the model parameters by minimising Mean Square Error (MSE). From Table I, one can observe that MSE with two parameters Morse function is less than that obtained with 3 parameters. To conclude, one needs only 44 parameters for obtaining total interaction potentials for n-p system. That is, if one were to build the internal interactions by considering various aspects such as spin, isospin, spin-orbit, etc., then 44 parameter phenomenological potential should be sufficient for explaining observed scattering phase shifts completely.

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