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On interactions of massless spin 3 and scalar fields

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Abstract Using new approach for the deformation procedure in the case of reducible gauge theories (Lavrov in Eur Phys J C 82:429, 2022), it is shown that in the model of massless spin 3 fields and a real scalar field local cubic vertices invariant under initial gauge transformations do not exist while local quartic gauge-invariant vertices can be constructed.

1 Introduction

The Batalin–Vilkovisky (BV) formalism [1,2] being the most powerful covariant quantization method plays the very important role in solving the deformation problem which is considered as a way in construction of suitable interactions among fields. The reason is related with embedding the problem into solutions to the classical master-equation of the BV-formalism. Standard approach [3,4] operates infinite number of equations appeared in expansion of the solution with respect to the deformation parameter. Then the system of these equations is analyzed in each order with respect to the deformation parameter using cohomological methods. Quite recently, the new method to find solutions of the deformation procedure within the BV-formalism has been proposed [5–7].[1] The method is based on the invariance of the classical master-equation under anticanonical transformations acting in minimal antisymplectic space of the BV-formalism. The anticanonical transformations (for recent developments see [12,13]) by itself present an effective tool in studying principal properties of general gauge theories. So, the gauge fixing procedure and the gauge dependence of Green functions can be described in terms of special anticanonical transformations observed firstly in [14]. Moreover, the anticanonical transformations admit to describe the renormalization procedure of general gauge theories [14] that, in its turn, allowed to introduce new understanding the renormalization problem beyond the usual index arguments [15]. Here, we meet with a new face of anticanonical transformations in the solution of the deformation problem, which make it possible to represent solutions in an explicit and closed form.

In the present paper, we apply the new method [5–7] to construct suitable interactions between massless spin 3 and scalar fields as the result of deformation of initial free action. The initial action is sum of the Fronsdal action for massless spin 3 fields [16] and the action for a free real scalar field. The choice of initial model allows to use the BV-formalism and the method [7] directly because the model belongs to the class of reducible gauge theories. In the case of massless integer spin $s > 3$ fields, the direct application of the BV-formalism is impossible due to the traceless conditions on fields. Fortunately, the situation is not so hopeless because there exist unconstrained formulations for higher spin fields [17–19] which make possible application of the BV-formalism and new method in high spin theory. In the case under consideration we study cubic and quartic vertices containing one spin 3 field and two or three scalar fields using special anticanonical transformations acting non-trivially in the sector of spin 3 fields only. It is shown that the gauge algebra does not deform under proposed transformations. It has the following consequence: (a) the cubic vertices appearing as a local part of the deformed action disappears under requirement of the gauge invariance, (b) the local quartic vertices invariant under the gauge transformations do exist. In particular, it means appearance for the first time in quantum field theory an explicit and consistent local gauge model of interacting fields with spin $s > 2$.

The paper is organized as follows. In Sect. 2, a short presentation of new approach to the deformation procedure in the BV-formalism for reducible gauge theories is given. In Sect. 3, the Fronsdal action for spin 3 fields as a gauge action

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[1] Analog of this approach in canonical formalism for dynamical systems with constraints within the Batalin–Fradkin–Vilkovisky formalism [8–10] is known as well [11].

of the first-stage reducible theory is discussed. In Sect. 4, deformations of free model of massless spin 3 and scalar fields responsible for local cubic and quartic vertices are studied. In Sect. 5, we summarize the results.

We use the DeWitt's condensed notations [20] and employ the symbols $\varepsilon(A)$ for the Grassmann parity and $\mathrm{gh}(A)$ for the ghost number respectively. The right and left functional derivatives are marked by special symbols "$\leftarrow$" and "$\rightarrow$" respectively. Arguments of any functional are enclosed in square brackets [ ], and arguments of any function are enclosed in parentheses, ( ). The partial right derivative of a function $F(A)$ with respect to field A^i is denoted as $F_{,i}(A)$.

2 Deformations in BV-formalism

In this section, we consider in short main statements of the new approach [5–7] in solving the deformation problem within the BV-formalism. Starting point of the BV-formalism is a gauge theory of the fields $A = \{A^i\}$ with Grassmann parities $\varepsilon(A^i) = \varepsilon_i$ and ghost numbers $\mathrm{gh}(A^i) = 0$. The theory is described by an initial action $S_0[A]$. It is assumed that the action is invariant under the gauge transformations[2]

$$S_{0,i}[A]R^i_\alpha(A) = 0, \quad \delta A^i = R^i_\alpha(A)\xi^\alpha, \quad \alpha = 1, 2, \ldots, m \tag{1}$$

where $R^i_\alpha(A)$ ($\varepsilon(R^i_\alpha(A)) = \varepsilon_i + \varepsilon_\alpha$, $\mathrm{gh}(R^i_\alpha(A)) = 0$) are gauge generators, and gauge parameters ξ^α ($\varepsilon(\xi^\alpha) = \varepsilon_\alpha$) are arbitrary functions of space-time coordinates. It is assumed that the fields $A = \{A^i\}$ are linear independent with respect to the index i however, in general, the generators $R^i_\alpha(A)$ may be linear dependent with respect to index α. Linear dependence of $R^i_\alpha(A)$ implies that the matrix $R^i_\alpha(A)$ has at the extremals $S_{0,j}[A] = 0$ zero-eigenvalue eigenvectors $Z^\alpha_{\alpha_1} = Z^\alpha_{\alpha_1}(A)$, such that

$$R^i_\alpha(A)Z^\alpha_{\alpha_1}(A) = S_{0,j}[A]K^{ji}_{\alpha_1}(A), \quad \alpha_1 = 1, \ldots, m_1, \tag{2}$$

and the number $\varepsilon_{\alpha_1} = 0, 1$ can be found in such a way that $\varepsilon(Z^\alpha_{\alpha_1}) = \varepsilon_\alpha + \varepsilon_{\alpha_1}$. Matrices $K^{ij}_{\alpha_1} = K^{ij}_{\alpha_1}(A)$ can be chosen to possess the properties:

$$K^{ij}_{\alpha_1} = -(-1)^{\varepsilon_i\varepsilon_j}K^{ji}_{\alpha_1}, \quad \varepsilon(K^{ji}_{\alpha_1}) = \varepsilon_i + \varepsilon_j + \varepsilon_{\alpha_1}.$$

Here, we restrict ourself to the case of first-stage reducibility when the set of zero-eigenvalue eigenvectors $Z^\alpha_{\alpha_1}$ is linear independent with respect to the index α_1.

[2] To simplify presentation of all relations containing the right functional derivative of functional $S[A]$ with respect to field A^i we will use the symbol $S_{,i}[A]$.

In general, the generators $R^i_\alpha(A)$ satisfy the following relations

$$R^i_{\alpha,j}(A)R^j_\beta(A) - (-1)^{\varepsilon_\alpha\varepsilon_\beta}R^i_{\beta,j}(A)R^j_\alpha(A) = -R^i_\gamma(A)F^\gamma_{\alpha\beta}(A) - S_{0,j}[A]M^{ji}_{\alpha\beta}(A), \tag{3}$$

where $F^\gamma_{\alpha\beta}(A) = F^\gamma_{\alpha\beta}$ ($\varepsilon(F^\gamma_{\alpha\beta}) = \varepsilon_\alpha + \varepsilon_\beta + \varepsilon_\gamma$, $\mathrm{gh}(F^\gamma_{\alpha\beta}) = 0$) are the structure coefficients which may depend on the fields A^i. They obey the following symmetry properties $F^\gamma_{\alpha\beta} = -(-1)^{\varepsilon_\alpha\varepsilon_\beta}F^\gamma_{\beta\alpha}$, and $M^{ij}_{\alpha\beta}(A) = M^{ij}_{\alpha\beta}$ satisfy the conditions $M^{ij}_{\alpha\beta} = -(-1)^{\varepsilon_i\varepsilon_j}M^{ji}_{\alpha\beta} = -(-1)^{\varepsilon_\alpha\varepsilon_\beta}M^{ij}_{\beta\alpha}$. If $M^{ij}_{\alpha\beta} = 0$ then the gauge algebra is called closed.

According to main statements of papers [5–7], the deformation procedure for general gauge theories can be described in terms of the set of generating functions $h^i(A)$ of anticanonical transformations depending on initial fields A^i only and having the same transformation properties and quantum numbers as for A^i. The deformed action, $\widetilde{S}[A]$, is defined by the relation

$$\widetilde{S}[A] = S_0[A + h(A)] \tag{4}$$

being invariant under the deformed gauge transformation

$$\widetilde{S}_{,i}[A]\widetilde{R}^i_\alpha(A) = 0, \quad \widetilde{\delta}A = \widetilde{R}^i_\alpha(A)\xi^\alpha, \tag{5}$$

where $\widetilde{R}^i_\alpha(A)$ are the deformed gauge generators,

$$\widetilde{R}^i_\alpha(A) = (M^{-1}(A))^i_{\ j}R^j_\alpha(A + h(A)). \tag{6}$$

Here, the matrix $(M^{-1}(A))^i_{\ j}$ is inverse to

$$M^i_{\ j}(A) = \delta^i_{\ j} + h^i_{\ ,j}(A), \tag{7}$$

The deformed gauge generators on extremals of the deformed action are linear dependent,

$$\widetilde{R}^i_\alpha(A)\widetilde{Z}^\alpha_{\alpha_1}(A) = \widetilde{S}_{,j}[A]\widetilde{K}^{ji}_{\alpha_1}(A), \tag{8}$$

where the functions $\widetilde{Z}^\alpha_{\alpha_1}(A)$ and $\widetilde{K}^{ji}_{\alpha_1}(A)$ are

$$\widetilde{Z}^\alpha_{\alpha_1}(A) = Z^\alpha_{\alpha_1}(A + h(A)),$$
$$\widetilde{K}^{ji}_{\alpha_1}(A) = -(M^{-1}(A))^j_{\ l}(M^{-1}(A))^i_{\ k} \times K^{kl}_{\alpha_1}(A + h(A))(-1)^{\varepsilon_l\varepsilon_i}. \tag{9}$$

The deformed gauge generators satisfy the relations

$$\widetilde{R}^i_{\alpha,j}(A)\widetilde{R}^j_\beta(A) - (-1)^{\varepsilon_\alpha\varepsilon_\beta}\widetilde{R}^i_{\beta,j}(A)\widetilde{R}^j_\alpha(A) = -\widetilde{R}^i_\gamma(A)\widetilde{F}^\gamma_{\alpha\beta}(A) - \widetilde{S}_{,j}[A]\widetilde{M}^{ji}_{\alpha\beta}(A), \tag{10}$$

where $\widetilde{F}^\gamma_{\alpha\beta}(A) = F^\gamma_{\alpha\beta}(A+h(A))$ and $\widetilde{M}^{ji}_{\alpha\beta}(A) = -(M^{-1}(A))^j_{\ l}(M^{-1}(A))^i_{\ k}M^{kl}_{\alpha\beta}(A+h(A))(-1)^{\varepsilon_l\varepsilon_i}$. Therefore, the deformed theory looks like as the gauge theory of the first-stage reducibility similar to the initial one. It is remarkable fact that the deformation of an initial gauge theory is described in the explicit and closed form with the help of generating functions $h^i(A)$ of special anticanonical transformations in the BV-formalism. Moreover, all arbitrariness in the deformation procedure is controlled by the generating functions $h^i(A)$ only. Any choice of these functions guarantees that the deformed action will be invariant under the corresponding deformed gauge transformations.

Deformation of initial action looks like as the shift of argument $A \to A + h(A)$. Such kind of transformations is trivial in the case of local function $h(A)$. To obtain non-trivial deformations, the function $h(A)$ must be non-local. It means the non-locality of the deformed action. It may happen that there exists a local sector of the deformed action. If this local part of the deformed action will be invariant under local part of the deformed gauge generators then the deformation procedure creates a consistent local gauge-invariant theory. Namely, in paper [5] it was exactly shown that the Yang–Mills theory has been reproduced by special non-local deformation of free vector field model.

3 Fronsdal action for massless spin 3 fields

The Fronsdal action for massless spin 3 fields has the form

$$S_0[\varphi] = \int dx \Big[\varphi^{\mu\nu\rho}(x)\Box\varphi_{\mu\nu\rho}(x) -3\eta_{\mu\nu}\eta^{\rho\sigma}\varphi^{\mu\nu\delta}(x)\Box\varphi_{\rho\sigma\delta}(x) -\frac{3}{2}\varphi_{\mu\nu\lambda}\eta^{\mu\nu}\partial^{\lambda}\partial_{\alpha}\varphi^{\alpha\beta\gamma}\eta_{\beta\gamma} -3\varphi^{\mu\rho\sigma}(x)\partial_{\mu}\partial^{\nu}\varphi_{\nu\rho\sigma}(x) +6\eta_{\mu\nu}\varphi^{\mu\nu\delta}(x)\partial^{\rho}\partial^{\sigma}\varphi_{\rho\sigma\delta}(x)\Big], \tag{11}$$

where $\varphi^{\mu\nu\lambda}$ is completely symmetric third rank tensor, $\Box$ is the D'Alembertian, $\Box = \partial_\mu\partial^\mu$, and $\eta_{\mu\nu}$ is the metric tensor of flat Minkowski space of the dimension d. The action (11) is invariant under the gauge transformations[3]

$$\delta\varphi^{\mu\nu\lambda} = \partial^{(\mu}\xi^{\nu\lambda)} = R^{\mu\nu\lambda}_{\rho\sigma}\xi^{\rho\sigma} \tag{12}$$

when gauge parameters $\xi^{\mu\nu}$ are subjected to the traceless condition,

$$\eta_{\mu\nu}\xi^{\mu\nu} = 0. \tag{13}$$

The gauge generators, $R^{\mu\nu\lambda}_{\rho\sigma} = \partial^{(\mu}\delta^{\nu}_{\rho}\delta^{\lambda)}_{\sigma}$, do not dependent on fields. They are symmetric in lower indices, $R^{\mu\nu\lambda}_{\rho\sigma} = R^{\mu\nu\lambda}_{\sigma\varrho}$, and, therefore, satisfy the relations

$$R^{\mu\nu\lambda}_{\rho\sigma}Z^{\rho\sigma}_{\ell} = 0, \quad Z^{\rho\sigma}_{\ell} = -Z^{\sigma\rho}_{\ell}, \quad \ell = 1, 2, \ldots, \frac{d(d-1)}{2}, \tag{14}$$

with antisymmetric matrices $Z^{\rho\sigma}_{\ell}$ which are linear independent with respect to index ℓ. We assume that $Z^{\rho\sigma}_{\ell}$ are Lorentz second-rank tensor with respect to upper indices. Therefore, in the BV-formalism we have to consider the case $s = 3$ as a gauge theory with first-stage reducible algebra specified by additional restrictions, $F^{\gamma}_{\alpha\beta}(A) = 0$, $M^{ij}_{\alpha\beta}(A) = 0$, $K^{ij}_{\alpha_1}(A) = 0$. The zero-eigenvalue eigenvectors, $Z^{\rho\sigma}_{\ell}$, do not depend on fields. In particular, it means that they do not change in the deformation process.

4 Deformations of initial action

We are going to study a possibility in construction of interactions of massless spin 3 fields with a real scalar field $\phi = \phi(x)$. We assume that the initial action has the form

$$S_0[\varphi, \phi] = S_0[\varphi] + S_0[\phi], \tag{15}$$

where $S_0[\varphi]$ is defined in (11) and $S_0[\phi]$ is the action of a free real scalar field,

$$S_0[\phi] = \int dx \frac{1}{2}\big[\partial_\mu\phi\, \partial^\mu\phi - m^2\phi^2\big]. \tag{16}$$

The action (15) is invariant under the following gauge transformations

$$\delta\varphi^{\mu\nu\lambda} = \partial^{(\mu}\xi^{\nu\lambda)}, \quad \delta\phi = 0, \quad \eta_{\mu\nu}\xi^{\mu\nu} = 0, \tag{17}$$

and, therefore, belongs to the class of first-stage reducible theories. Relations with notations used in the Sect. 2 are

$$A^i = (\varphi^{\mu\nu\lambda}, \phi), \quad R^i_\alpha(A) = (R^{\mu\nu\lambda}_{\rho\sigma}, 0), \quad Z^{\alpha}_{\alpha_1}(A) = (Z^{\rho\sigma}_{\ell}, 0), \quad \xi^\alpha = (\xi^{\rho\sigma}, 0), \; F^{\gamma}_{\alpha\beta}(A) = 0, \quad M^{ij}_{\alpha\beta}(A) = 0, \quad K^{ij}_{\alpha_1}(A) = 0. \tag{18}$$

Now, we consider possible deformations of initial action using the procedure which is ruled out by the generating functions $h^i(A)$. Here, we restrict ourself by the case of anticanonical transformations acting effectively in the sector of fields $\varphi^{\mu\nu\lambda}$ of the initial theory. It means the following structure of generating functions $h^i(A) = (h^{\mu\nu\lambda}(\varphi, \phi), 0)$. In construction of suitable generating functions $h^{\mu\nu\lambda} = (h^{\mu\nu\lambda}(\varphi, \phi)$, we have to take into account the dimensions of quantities involved in the initial action $S_0[\varphi, \phi]$,

$$\dim(\varphi^{\mu\nu\lambda}) = \dim(\phi) = \frac{d-2}{2}, \quad \dim(\xi^{\mu\nu}) = \frac{d-4}{2}, \quad \dim(\partial_\mu) = 1, \quad \dim(\Box) = 2. \tag{19}$$

The generating function $h^{\mu\nu\lambda}$ should be symmetric and non-local with the dimension equals to $(d-2)/2$. The non-locality

[3] The symbol $(\cdots)$ means the cycle permutation of indexes involved.

will be achieved by using the operator $1/\Box$.[4] To construct cubic vertices $\sim \varphi\phi\phi$, the $h^{\mu\nu\lambda}$ should be at least quadratic in fields ϕ. The tensor structure of $h^{\mu\nu\lambda}$ is obeyed by using partial derivatives ∂_μ and the metric tensor $\eta_{\mu\nu}$. The minimal number of derivatives equals to 3. Therefore, the more general form of $h^{\mu\nu\lambda} = h^{\mu\nu\lambda}(\phi)$ satisfying requirements listed above reads

$$\begin{aligned} h^{\mu\nu\lambda} = a_0 \frac{1}{\Box}\big(&c_1 \partial^\mu\partial^\nu\partial^\lambda\phi\;\phi + c_2 \partial^{(\mu}\partial^\nu\phi\;\partial^{\lambda)}\phi \\ &+ c_3 \eta^{(\mu\nu}\Box\partial^{\lambda)}\phi\;\phi + c_4 \Box\phi\;\eta^{(\mu\nu}\partial^{\lambda)}\phi \\ &+ c_5 \eta^{(\mu\nu}\partial^{\lambda)}\partial_\sigma\phi\;\partial^\sigma\phi\big), \end{aligned} \tag{20}$$

where a_0 is the coupling constant with $\dim(a_0) = -d/2$ and $c_i, \quad i = 1.2, \ldots, 5$ are arbitrary dimensionless constants. Local part of the deformed action has the form

$$S_{loc}[\varphi,\phi] = S_0[\varphi,\phi] + S_{1\,loc}[\varphi,\phi] \tag{21}$$

where $S_{1\,loc} = S_{1\,loc}[\varphi,\phi]$,

$$\begin{aligned} S_{1\,loc} = 2a_0 \int dx \varphi_{\mu\nu\lambda}\Big[&c_1 \partial^\mu\partial^\nu\partial^\lambda\phi\;\phi + c_2\partial^{(\mu}\partial^\nu\phi\;\partial^{\lambda)}\phi \\ &-\big(c_1 + c_3(d+1)\big)\eta^{(\mu\nu}\Box\partial^{\lambda)}\phi\;\phi \\ &-\big(c_2 + c_4(d+1)\big)\Box\phi\;\eta^{(\mu\nu}\partial^{\lambda)}\phi \\ &-\big(2c_2 + c_5(d+1)\big)\eta^{(\mu\nu}\partial^{\lambda)}\partial_\sigma\phi\;\partial^\sigma\phi\Big], \end{aligned} \tag{22}$$

corresponds to possible cubic vertices. Due to the special structure of generating functions $h^i(A) = (h^{\mu\nu\lambda}(\phi), 0)$, the matrix $(M^{-1}(A))^i{}_j$ can be found in the explicit form

$$(M^{-1}(A))^i{}_j = \begin{pmatrix} E^{\mu\nu\lambda}_{\rho\sigma\gamma} & -h^{\mu\nu\lambda}(\phi)\overleftarrow{\partial}_\phi \\ 0 & 1 \end{pmatrix}, \tag{23}$$

where $E^{\mu\nu\lambda}_{\rho\sigma\gamma}$ are elements of the unit matrix in the space of third rank symmetric tensors. Therefore, the deformed gauge generators coincide with initial ones, $\widetilde{R}^i_\alpha(A) = R^i_\alpha(A)$.

Consider the variation of $S_{1\,loc}$ under the gauge transformations (17). We have

$$\begin{aligned} \delta S_{1\,loc} = -6a_0 \int dx \xi_{\nu\lambda}\Big[&-(c_1 + 2c_3(d+1))\Box\partial^\nu\partial^\lambda\phi\;\phi \\ &-2c_4(d+1)\Box\partial^\nu\phi\;\partial^\lambda\phi \\ &+(c_1 - 3c_2 - 2c_5(d+1))\partial^\mu\partial^\nu\partial^\lambda\phi\;\partial_\mu\phi \\ &-2(c_2 + c_5(d+1))\partial^\lambda\partial^\mu\phi\;\partial_\mu\partial^\nu\phi \\ &-(c_2 + 2c_4(d+1))\Box\phi\;\partial^\nu\partial^\lambda\phi\Big]. \end{aligned} \tag{24}$$

[4] It seems quite natural to control non-locality with the help of operator $1/\Box$ because the kinematic part of free models contains terms $A\Box A$. Existence of other forms of the non-locality leading to a local part of the deformed action remains open.

The system of algebraic equations appears

$$\begin{aligned} &c_1 + 2c_3(d+1) = 0, \quad c_1 - 3c_2 - 2c_5(d+1) = 0, \\ &c_4(d+1) = 0, \end{aligned} \tag{25}$$

$$c_2 + c_5(d+1) = 0, \quad c_2 + 2c_4(d+1) = 0 \tag{26}$$

as consequence of requirement $\delta S_{1loc} = 0$. This system has the solution

$$c_i = 0, \quad i = 1, 2, \ldots, 5, \tag{27}$$

which leads to the important conclusion

$$S_{1\,loc}[\varphi,\phi] = 0. \tag{28}$$

Therefore, we have proved impossibility in the theory of massless spin 3 and scalar fields to construct cubic vertices $\varphi\phi\phi$ being invariant under original gauge transformations (17).

In connection with the obtained result, a natural question arises: Do gauge invariant quartic vertices $\varphi\phi\phi\phi$ in the theory under consideration exist? Repeating the main arguments that led to the construction of the generating function (20) of the anticanonical transformation, the most general form of the generating function $h^{\mu\nu\lambda}$ with three derivatives responsible for the generation of quartic vertices reads

$$\begin{aligned} h^{\mu\nu\lambda} = a_1\frac{1}{\Box}\big[&c_1\partial^\mu\partial^\nu\partial^\lambda\phi\;\phi^2 + c_2\partial^{(\mu}\partial^\nu\phi\;\partial^{\lambda)}\phi\;\phi \\ &+c_3\partial^\mu\phi\;\partial^\nu\phi\;\partial^\lambda\phi + c_4\eta^{(\mu\nu}\partial^{\lambda)}\Box\phi\;\phi^2 \\ &+c_5\eta^{(\mu\nu}\partial^{\lambda)}\partial_\sigma\phi\;\partial^\sigma\phi\;\phi + c_6\eta^{(\mu\nu}\partial^{\lambda)}\phi\;\partial_\sigma\phi\;\partial^\sigma\phi \\ &+c_7\Box\phi\;\eta^{(\mu\nu}\partial^{\lambda)}\phi\;\phi\big], \end{aligned} \tag{29}$$

where a_1 is a coupling constant with $\dim(a_1) = -(d-1)$ and $c_i, \quad i = 1, 2, \ldots, 7$ are arbitrary dimensionless constants. For the local addition, $S_{2\,loc} = S_{2\,loc}[\varphi,\phi]$, to the initial action (15), we obtain

$$\begin{aligned} S_{2\,loc} = 2a_1\int dx\varphi_{\mu\nu\lambda}\Big[&c_1\partial^\mu\partial^\nu\partial^\lambda\phi\;\phi^2 + c_2\partial^{(\mu}\partial^\nu\phi\;\partial^{\lambda)} \\ &\phi\;\phi + c_3\partial^\mu\phi\;\partial^\nu\phi\;\partial^\lambda\phi \\ &-(c_1 + c_4(d+1))\eta^{(\mu\nu}\partial^{\lambda)}\Box\phi\;\phi^2 \\ &-(2c_2 + c_5(d+1))\eta^{(\mu\nu}\partial^{\lambda)}\partial_\sigma\phi\;\partial^\sigma\phi\;\phi \\ &-(c_3 + c_6(d+1))\eta^{(\mu\nu}\partial^{\lambda)}\phi\;\partial_\sigma\phi\;\partial^\sigma\phi \\ &-(c_2 + c_7(d+1))\Box\phi\;\eta^{(\mu\nu}\partial^{\lambda)}\phi\;\phi\Big]. \end{aligned} \tag{30}$$

Notice that as in previous case, the gauge generators do not transform under anticanonical transformations generated by functions (29).

Omitting the algebraic manipulations, we find the variation $S_{2\,loc}$ under gauge transformations (17) in the form

$$\delta S_{2\,loc} = 6a_1\int dx\xi_{\nu\lambda}\Big[(c_1+2c_4(d+1))\partial^\nu\partial^\lambda\Box\phi\,\phi^2 -(2c_1-3c_2-2c_5(d+1))\partial^\sigma\partial^\nu\partial^\lambda\phi\,\partial_\sigma\phi\,\phi +2(2c_1+2c_4(d+1)+c_7(d+1))\Box\partial^\nu\phi\,\partial^\lambda\phi\,\phi +2(c_2+c_5(d+1))\partial^\sigma\partial^\nu\phi\,\partial^\lambda\partial_\sigma\phi\,\phi +(c_2+2c_7(d+1))\partial^\nu\partial^\lambda\phi\,\Box\phi\,\phi +2(c_2+c_3+(c_5+2c_6)(d+1))\partial^\sigma\partial^\nu\phi\,\partial^\lambda\phi\,\partial_\sigma\phi -(c_2-2c_3-2c_6(d+1))\partial^\nu\partial^\lambda\phi\,\partial_\sigma\phi\partial^\sigma\phi -(c_3-2c_2-2c_7(d+1))\Box\phi\,\partial^\nu\phi\,\partial^\lambda\phi\Big]. \quad (31)$$

Note that the system of algebraic equations

$$c_1+2c_4(d+1)=0,\quad 2c_1-3c_2-2c_5(d+1)=0,\quad 2c_1+2c_4(d+1)+c_7(d+1)=0,\quad c_2+c_5(d+1)=0,\quad c_2+2c_7(d+1)=0,\quad c_2+c_3+(c_5+2c_6)(d+1)=0,\quad c_2-2c_3-2c_6(d+1)=0,\quad c_3-2c_2-2c_7(d+1)=0 \quad (32)$$

has non-trivial solution

$$c_2=c_3=2c_1,\quad c_4=-\frac{1}{2(d+1)}c_1,\quad c_5=-\frac{2}{d+1}c_1,\quad c_6=c_7=-\frac{1}{d+1}c_1. \quad (33)$$

Therefore, the functional

$$S_{2\,loc}=2a_1\int dx\varphi_{\mu\nu\lambda}\Big[\partial^\mu\partial^\nu\partial^\lambda\phi\,\phi^2+2\partial^{(\mu}\partial^\nu\phi\,\partial^{\lambda)}\phi\,\phi +2\partial^\mu\phi\,\partial^\nu\phi\,\partial^\lambda\phi-\frac{1}{2}\eta^{(\mu\nu}\partial^{\lambda)}\Box\phi\,\phi^2 -2\eta^{(\mu\nu}\partial^{\lambda)}\partial_\sigma\phi\,\partial^\sigma\phi\,\phi-\eta^{(\mu\nu}\partial^{\lambda)}\phi\,\partial_\sigma\phi\,\partial^\sigma\phi -\Box\phi\,\eta^{(\mu\nu}\partial^{\lambda)}\phi\,\phi\Big] \quad (34)$$

is gauge invariant,

$$\delta S_{2\,loc}=0, \quad (35)$$

and presents the quartic vertices.

The local action

$$S[\varphi,\phi]=S_0[\varphi,\phi]+S_{2\,loc}[\varphi,\phi] \quad (36)$$

describes the model of interacting $\varphi^{\mu\nu\lambda}$ and ϕ fields which is invariant under gauge transformations (17) and belongs to the class of first-stage reducible theories. In fact, the model (36) is the first new example of local gauge theory constructed with using the new method [5–7]. Moreover, in quantum field theory the action (36) presents the first consistent local gauge model of interacting fields with integer spin $s>2$.

5 Discussion

In the present paper, the new approach [5–7] to the deformation procedure [3,4] has been applied to construct local cubic and quartic vertices for massless spin 3 and scalar fields. It was shown that non-local anticanonical transformations acting non-trivial only in the sector of spin 3 fields forbid existence of local cubic vertices being invariant under original gauge transformations while local gauge-invariant quartic vertices do exist. In fact, for the first time, the new gauge model of interacting fields including fields with integer spin $s>2$ was explicitly constructed. Let us note that reproducing the obtained result in the standard Noether procedure adopted for construction of interactions in the theory of higher spin fields [3,4] is not possible without its essential reformulation. Consider in more details relations between the standard Noether procedure and the new method. From the standard Noether procedure, the new method looks formally like as an explicit summation of infinite Taylor series in powers of the deformation parameter g. Indeed, the deformed action $S=S[A]$ and deformed gauge transformation $\delta_\xi A=R(A)\xi$ are presented as

$$S=S_0+gS_1+g^2S_2+\cdots,\quad \delta_\xi A=\delta^{(0)}_\xi A+g\delta^{(1)}_\xi A+g^2\delta^{(2)}_\xi A+\cdots, \quad (37)$$

where g is the deformation parameter, $S_0=S_0[A]$ and $\delta^{(0)}_\xi A=R^{(0)}(A)\xi$ are the initial gauge action and original gauge transformations, respectively. The standard Noether procedure to the deformation means the infinite set of coupled relations appearing as consequence of the invariance of the deformed action under deformed gauge transformations, $\delta_\xi S=0$,

$$\delta^{(1)}_\xi S_0+\delta^{(0)}_\xi S_1=0,\quad \delta^{(2)}_\xi S_0+\delta^{(1)}_\xi S_1+\delta^{(0)}_\xi S_2=0, \quad (38)$$

and so on. In the new approach, firstly, an explicit form both the deformed action and deformed gauge symmetry are found and, secondly, arbitrariness in solutions is established. It is proved that the deformation is described by special anticanonical transformations in the minimal antisymplectic space of the BV formalism. The anticanonical transformations by itself are defined by generating function $h(A)$ in a number equal to original fields A. In terms of $h(A)$ the deformed action $S=S[A]$ reads

$$S[A]=S_0[A+gh(A)] \quad (39)$$

while the deformed gauge transformations have the following closed form

$$\delta_\xi A=M^{-1}(A)R^{(0)}(A+gh(A))\xi, \quad (40)$$

where $M^{-1}(A)$ is the matrix inverse to $M(A)=I+gh(A)\overleftarrow{\partial}_A$. The evident advantage of new method follows from the two above statements. All process of deformation is controlled by one function $h(A)$ of anticanonical transformation which is unique arbitrariness appearing in the new method. For any anticanonical transformation the deformed action obeys invariance under corresponding deformed gauge transformation, $\delta_\xi S[A] = 0$. Now, solutions to the standard Noether procedure read

$$\begin{aligned}
S_1[A] &= S_0[A]\overleftarrow{\partial}_A h(A),\\
S_2[A] &= \frac{1}{2}S_0[A]\big(\overleftarrow{\partial}_A\big)^2 h^2(A), \ldots,\\
\delta^{(1)}_\xi A &= -h(A)\overleftarrow{\partial}_A R^{(0)}(A)\xi + R^{(0)}(A)\overleftarrow{\partial}_A h(A)\xi,\\
\delta^{(2)}_\xi A &= \big(h(A)\overleftarrow{\partial}_A\big)^2 R^{(0)}(A)\xi - \big(h(A)\overleftarrow{\partial}_A\big)\\
&\quad\times\big(R^{(0)}(A)\overleftarrow{\partial}_A h(A)\big)\xi\\
&\quad+\frac{1}{2}R^{(0)}(A)\big(\overleftarrow{\partial}_A h(A)\big)^2\xi, \ldots .
\end{aligned} \tag{41}$$

For deformation of free models one has the following specifications

$$S_n[A] = 0,\ n = 3, 4, \ldots, \quad R^{(0)}(A)\overleftarrow{\partial}_A h(A)\xi = 0, \tag{42}$$

so that $\delta^{(n)}_\xi A = \big(h(A)\overleftarrow{\partial}_A\big)^n R^{(0)}(A)\xi,\ n = 2, 3, \ldots$. For anticanonical transformations used in the present paper there exists additional restriction $h(A)\overleftarrow{\partial}_A = 0$ leading to the absence of deformation for the original gauge symmetry. From above presentation, it follows evident advantages of the new method. Note that in recent studies of the high spin theory expansions of the deformed action and symmetry are considered not in the deformation parameter but in fields [21–24]). Relation between such Noether procedure and new method corresponds to expansion of generating function $h(A)$ in the above equations (41), (42),

$$h(A) = h_2(A) + h_3(A) + \cdots , \tag{43}$$

where $h_k(A)$, $(k = 2, 3, \ldots ,)$ is a function of k-th order in fields providing then by corresponding reorganization of Taylor series. In particular, the triviality of deformed gauge symmetry claimed in [22,23]) when cubic vertices are vanished is formulated in the new method as

$$h(A)\overleftarrow{\partial}_A R^{(0)}(A)\xi = 0, \quad R^{(0)}(A)\overleftarrow{\partial}_A h(A)\xi = 0, \tag{44}$$

without any reference to cubic vertices.

On the first sight, it seems that the obtained result (28) contradicts with the statement of paper [21][5] that the gauge invariant cubic vertices $\varphi\phi\phi$ in the theory of massless spin 3 and a pair of scalars ϕ^i, $i = 1, 2$ can be constructed. The corresponding action $S_{1Zin} = S_{1Zin}[\varphi, \phi]$ in [21] has the form

$$\begin{aligned}
S_{1Zin} &= \frac{a_0}{6}\varepsilon^{ij}\varphi_{\mu\nu\lambda}\Big[-2\partial^\mu\partial^\nu\phi_i\ \partial^\lambda\phi_j + 2\eta^{\nu\lambda}\Box\phi_i\ \partial^\mu\phi_j\\
&\quad+\eta^{\nu\lambda}\partial^\mu\partial_\sigma\phi_i\ \partial^\sigma\phi_j\Big].
\end{aligned} \tag{45}$$

Indeed, this action is not invariant under the gauge transformations (17),

$$\begin{aligned}
\delta S_{1Zin} &= -\frac{a_0}{6}\varepsilon^{ij}\xi_{\nu\lambda}\Big[-2\partial^\sigma\partial^\nu\phi_i\ \partial^\lambda\partial_\sigma\phi_j - 2\partial^\nu\partial^\lambda\phi_i\ \Box\phi_j\\
&\quad+4\Box\phi_i\ \partial^\nu\partial^\lambda\phi_j\Big] \neq 0.
\end{aligned} \tag{46}$$

But it needs to have in mind that the cubic vertices for the initial free model of massless scalar fields and massless spin 3 fields were studied using the standard Noether's procedure when scalar fields were subjected to gauge transformations too. Then, the gauge invariance of the theory using action (45) is supported in the linear approximation only [21]. In the approach applied in Sect. 4 it corresponds to anticanonical transformations when the generating function responsible for change of massless scalar field ϕ $(m = 0)$ is not equal to zero, $h \neq 0$. In its turn, it will lead to non-trivial deformations of scalar fields and gauge algebra. So, there is no any contradictions between the result of [21] and the result obtained in the present paper because these tasks are different in their formulation.

We can analyze the interactions of massless spin 3 field with the pair of scalars ϕ^i, $i = 1, 2$ within the new approach leading to the cubic vertex which will be anti-symmetric on them. The most general form of generating function containing three partial derivatives reads

$$\begin{aligned}
h^{\mu\nu\lambda} &= a_0\frac{1}{\Box}\varepsilon^{ij}\big(c_1\partial^\mu\partial^\nu\partial^\lambda\phi_i\ \phi_j + c_2\partial^{(\mu}\partial^\nu\phi_i\ \partial^{\lambda)}\phi_j\\
&\quad+c_3\eta^{(\mu\nu}\Box\partial^{\lambda)}\phi_i\ \phi_j\\
&\quad+c_4\Box\phi_i\ \eta^{(\mu\nu}\partial^{\lambda)}\phi_j + c_5\eta^{(\mu\nu}\partial^{\lambda)}\partial_\sigma\phi_i\ \partial^\sigma\phi_j\big),
\end{aligned} \tag{47}$$

where a_0 is the coupling constant with $\dim(a_0) = -d/2$ and c_i, $i = 1.2, \ldots, 5$ are arbitrary dimensionless constants. Then, the local addition to the initial action (15) has the form

$$\begin{aligned}
S_{1\,loc} &= 2a_0\varphi_{\mu\nu\lambda}\varepsilon^{ij}\Big[c_1\partial^\mu\partial^\nu\partial^\lambda\phi_i\ \phi_j + c_2\partial^{(\mu}\partial^\nu\phi_i\ \partial^{\lambda)}\phi_j\\
&\quad-\big(c_1 + c_3(d+1)\big)\eta^{(\mu\nu}\Box\partial^{\lambda)}\phi_i\ \phi_j\\
&\quad-\big(c_2 + c_4(d+1)\big)\Box\phi_i\ \eta^{(\mu\nu}\partial^{\lambda)}\phi_j\\
&\quad-\big(2c_2 + c_5(d+1)\big)\eta^{(\mu\nu}\partial^{\lambda)}\partial_\sigma\phi_i\ \partial^\sigma\phi_j\Big],
\end{aligned} \tag{48}$$

Due to the fact that the gauge algebra does not deform under anticanonical transformations with the generating function (47), the functional (48) must be invariant under original gauge transformations, $\delta S_{1\,loc} = 0$, $\delta\varphi_{\mu\nu\lambda} = \partial_{(\mu}\xi_{\nu\lambda)}$, $\eta_{\mu\nu}\xi^{\mu\nu} = 0$, $\delta\phi_i = 0$, $i = 1, 2$. Analysis of this requirement leads to the result $S_{1\,loc} = 0$. We can also discuss the situation with quartic vertices, $\sim \varphi\phi\phi\phi$, in the case under consideration. To construct such interactions, in

[5] Notice that in [25] $N = 2$ supersymmetric generalization of cubic vertices containing in bosonic sector interactions of spin 0 field with massless integer spin s fields has been considered.

general, one has to use the generating functions $h^{\mu\nu\lambda}$ in the sector of fields $\varphi^{\mu\nu\lambda}$ and h^i in the sector of fields ϕ^i. In turn, the structure of generating functions should be as follows $h^{\mu\nu\lambda} \sim \varepsilon^{jk}\partial^\mu\partial^\nu\partial^\lambda\phi_i\phi_j\phi_k$ and $h^i \sim \varepsilon^{jk}\partial_\mu\partial_\nu\partial_\lambda\varphi^{\mu\nu\lambda}\phi_j\phi_k$ that is contrary to the tensor properties for these functions. Therefore, quartic vertices are forbidden as well.

Quite recently, it was shown that non-trivial quartic vertices describing interactions among massless spin 3 field, $\varphi^{\mu\nu\lambda}$, and massive vector, A_μ, and real scalar, ϕ, fields of the form $\sim \varphi A\phi\phi$ can be constructed [26]. These vertices are invariant under original gauge transformations, similar to the case considered in Sect. 4.

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