Particle Accelerators
1984 Vol. 16 pp. 13–17
0031-2460/84/1602–0013$10.00/0

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Printed in the United States of America

CONTRIBUTION OF THE SURFACE PLASMON TO ENERGY LOSSES BY ELECTRONS IN A CYLINDRICAL CHANNEL†

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(Received October 20, 1983; in revised form January 25, 1984)

The surface-plasmon contribution to the energy losses experienced by an electron beam in a cylindrical channel in a metal is derived. The work is motivated in part by considerations relevant to an electron ring accelerator. The nonrelativistic limit is used for convenience in order to display the effect in a simpler manner. A relativistic derivation previously given by Merkel, which includes only ohmic losses, may be improved upon with the methods shown here. We include surface-plasmon energy losses in the nonrelativistic limit, but it is well known that surface-plasmon excitation increases with increasing beam energy well into the keV regime. The results obtained here can be generalized to the relativistic case quite easily and show how the optical properties of the metal surrounding the channel affect the energy losses.

A charged particle traveling near a surface experiences a potential arising from the polarization it induces in the medium. The interaction between the charged particle and the induced field produces the energy loss of the particle. The energy loss for a charged ring traveling in a cylindrical metal shell was derived previously by Merkel[1] in the consideration of ohmic loss in a metal shell. In particular, Merkel was interested in examining energy losses to the walls in an electron ring accelerator. A more general version of the energy loss of charged particles in a cylindrical channel, which includes the excitation of surface plasmons, is given here. However, in order to display the effect most simply, the nonrelativistic limit is used. The results demonstrate that the charged particle has maximum energy loss when the surface-plasmon dispersion relation is satisfied in the formula for the stopping power. While our results do not directly yield the energy losses in an electron ring accelerator, they do show that a detailed calculation for that case should include more than simple ohmic losses.

We consider a charge q traveling with velocity $v \ll c$ paraxially in a cylindrical channel of radius a surrounded by a metal with a local dielectric function $\epsilon(\omega)$. Without losing generality, the charge q can be set to travel in the x–z plane (see Fig. 1). The quasi-electrostatic potential inside and outside the channel can be solved from the equations

$$\nabla^2 \Phi_{\text{in}} = -\frac{4\pi q}{r}\,\delta(r - r_0)\,\delta(\phi)\,\delta(z - vt), \tag{1}$$

† Work sponsored by the Office of Health and Environmental Research, U.S. Department of Energy, under contract W-7405-eng-26 with Union Carbide Corporation.
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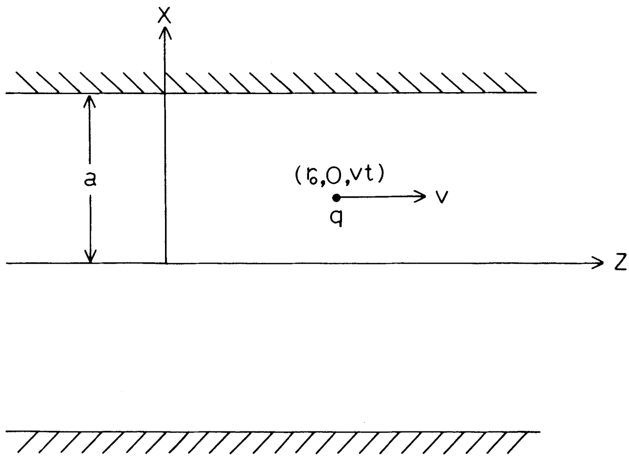

FIGURE 1 Schematic of configuration.

and
$$\nabla^2\Phi_{\rm out} = 0, \tag{2}$$

where we have chosen a cylindrical coordinate system (r, ϕ, z), and the coordinates of the charge are $(r_0, 0, vt)$.

The solutions of Eqs. (1) and (2) are

$$\Phi_{\rm in} = \Phi_G + \sum_{m=0}^{\infty} (2 - \delta_m{}^0)\cos(m\phi) \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, e^{ikz} A_m(k, t) K_m(|k|a) I_m(|k|r), \tag{3a}$$

with

$$\Phi_G = 2q \sum_{m=0}^{\infty} (2 - \delta_m{}^0)\cos(m\phi) \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, e^{ik(z-vt)} L_m(|k|r, |k|r_0), \tag{3b}$$

and

$$\Phi_{\rm out} = \sum_{m=0}^{\infty} (2 - \delta_m{}^0)\cos(m\phi) \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, e^{ikz} B_m(k, t) I_m(|k|a) K_m(|k|r), \tag{4}$$

where A_m and B_m are coefficients that are determined by the boundary conditions, K_m and I_m are modified Bessel functions, and L_m is defined as

$$L_m(|k|r, |k|r_0) = K_m(|k|r) I_m(|k|r_0)\theta(r - r_0) + K_m(|k|r_0) I_m(|k|r)\theta(r_0 - r), \tag{5}$$

which satisfies

$$\frac{d^2L_m}{dr^2} + \frac{1}{r}\frac{dL_m}{dr} - \left(k^2 + \frac{m^2}{r^2}\right)L_m = -\frac{1}{r}\,\delta(r - r_0).$$

The function $\theta(x)$ that appears in Eq. (5) is the Heaviside step function and is defined as

$$\theta(x) = \begin{cases} 1 & \text{for } x > 0, \\ 0 & \text{for } x < 0. \end{cases}$$

We note that I_m and K_m are finite at the origin and infinity, respectively.

The potential is continuous at the surface of the cylinder, so we have

$$\Phi_{\text{in}}|_{r=a} = \Phi_{\text{out}}|_{r=a}. \tag{6}$$

The coefficient $A_m(k, t)$ can be obtained from the boundary condition given in Eq. (6):

$$A_m(k, t) = B_m(k, t) - 2qe^{-ikvt}I_m(|k|r_0)/I_m(|k|a). \tag{7}$$

The dielectric function $\epsilon(\omega)$ is a function of angular frequency ω. Thus the second boundary condition of continuity of the electric displacement vector becomes

$$-\left.\frac{\partial\tilde{\Phi}_{\text{in}}}{\partial r}\right|_{r=a} = -\epsilon(\omega)\left.\frac{\partial\tilde{\Phi}_{\text{out}}}{\partial r}\right|_{r=a}, \tag{8}$$

where $\tilde{\Phi}_{\text{in}}$ and $\tilde{\Phi}_{\text{out}}$ are the Fourier potentials, given as

$$\tilde{\Phi}_{\substack{\text{in}\\\text{out}}} = \int_{-\infty}^{\infty} dt\, \Phi_{\substack{\text{in}\\\text{out}}}\, e^{i\omega t}.$$

From Eq. (8), we obtain the Fourier transform of the coefficient B_m as

$$\begin{aligned}\tilde{B}_m(k, \omega) = {} & 4\pi q\, \delta(\omega - kv)I_m(|k|r_0)\{(|k|a)\,[I_m(|k|a)]\,[K_m(|k|a)I_m'(|k|a) \\ & - \epsilon(\omega)I_m(|k|a)K_m'(|k|a)]\}^{-1},\end{aligned} \tag{9}$$

and $\tilde{A}_m(k, \omega)$ follows then from Eq. (7). The prime appearing in Eq. (9) represents the derivative with respect to the argument.

The homogeneous portion of the potential inside the cylinder is

$$\Phi_0 = \sum_{m=0}^{\infty} (2 - \delta_m{}^0)\cos(m\phi) \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, e^{ikz} A_m(k, t)K_m(|k|a)I_m(|k|r). \tag{10}$$

Using Eqs. (7) and (9), we get the Fourier transform of Φ_0

$$\begin{aligned}\tilde{\Phi}_0 = {} & 4\pi q \sum_{m=0}^{\infty} (2 - \delta_m{}^0)\cos(m\phi) \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, \delta(\omega - kv)e^{ikz} \\ & \times \{[I_m(|k|r_0)K_m(|k|a)I_m(|k|r)]/I_m(|k|a)\} \\ & \times \{(|k|a)^{-1}[K_m(|k|a)I_m'(|k|a) - \epsilon(\omega)I_m(|k|a)K_m'(|k|a)]^{-1} - 1\}.\end{aligned} \tag{11}$$

Now, taking the inverse Fourier transform of $\tilde{\Phi}_0$, we obtain

$$\Phi_0 = 2q \sum_{m=0}^{\infty} (2 - \delta_m{}^0)\cos(m\phi) \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik(z-vt)}$$
$$\times \{[I_m(|k|r_0)K_m(|k|a)I_m(|k|r)]/I_m(|k|a)\}$$
$$\times \{(|k|a)^{-1}[K_m(|k|a)I_m'(|k|a) - \epsilon(kv)I_m(|k|a)K_m(|k|a)]^{-1} - 1\}. \quad (12)$$

The stopping power for the charged particle is

$$\frac{dW}{dz} = -q\frac{d\Phi_0}{dz}\bigg|_{r=r_0,\phi=0,z=vt}$$
$$= -(4q^2/a) \sum_{m=0}^{\infty} (2 - \delta_m{}^0) \int_{-\infty}^{\infty} \frac{dk}{2\pi} \epsilon_2(kv)I_m{}^2(|k|r_0)F_m(|k|a), \quad (13)$$

where

$$F_m(|k|a) = K_m(|k|a)K_m'(|k|a)/\{[G_m(|k|a)]^2 + [\epsilon_2(kv)I_m(|k|a)K_m'(|k|a)]^2\},$$

with

$$G_m(|k|a) = K_m(|k|a)I_m'(|k|a) - \epsilon_1(kv)I_m(|k|a)K_m'(|k|a),$$

where ϵ_1 and ϵ_2 are the real and imaginary part of ϵ, respectively. We note that the charged particle has maximum energy loss if the relation is satisfied

$$G_m(|k|a) = 0. \quad (14)$$

The dispersion relation, Eq. (14), is satisfied at the surface-plasmon frequencies $\omega_m(|k|a)$ characteristic of the mode, the material, and the geometry. Given Eq. (14), one may use optical data for the dielectric function of the material to obtain the contribution of the energy loss peak.

In the nonrelativistic limit described above, the planar limit is obtained at distances from the surface at which the energy-loss probability is not negligible unless the cylinder radius is less than the order of 10 nm. In this event, Eq. (13) simplifies considerably, as found by Echenique and Pendry.[2]

In order to compare the surface plasmon losses and ohmic losses, one may write in the planar limit Eq. (13) as

$$\frac{dW}{dz} = \frac{2q^2}{\pi v^2} \int_0^{\infty} d\omega\, \omega K_0(2\omega z_0/v)\mathrm{Im}\left[\frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 1}\right], \quad (15)$$

where z_0 is the distance from the surface. Then the ohmic losses produce a stopping power equal to

$$\frac{dW}{dz} = \frac{q^2 v}{2\pi\sigma(2z_0)^3}, \quad (16)$$

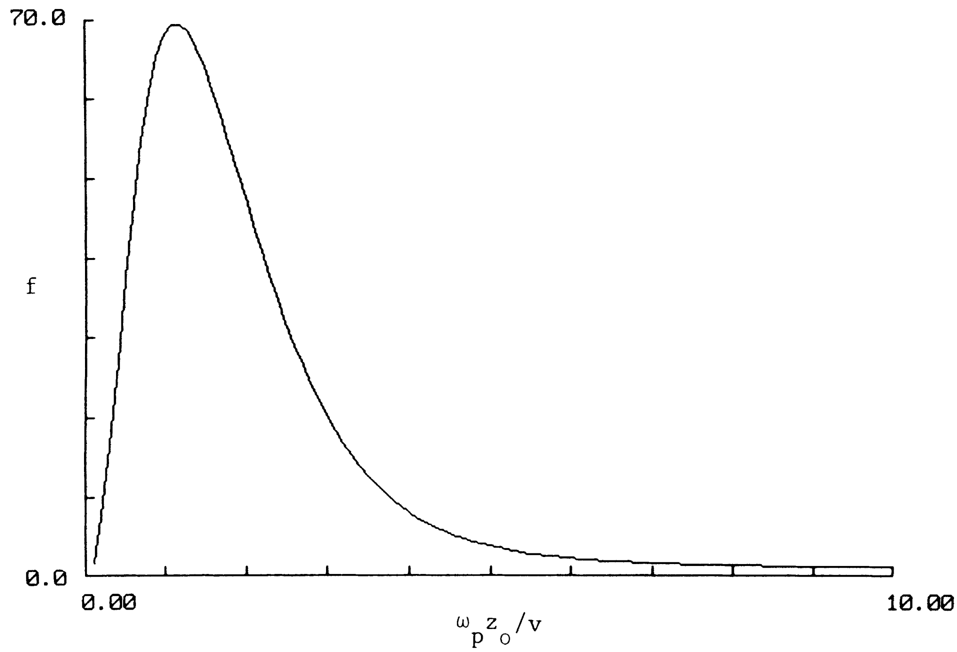

FIGURE 2 Plot of the ratio of the surface plasmon losses and ohmic losses versus $\omega_p z_0/v$.

where σ is the dc conductivity of the metal. The ratio of these results is

$$f = 4\sigma(2z_0/v)^3 \int_0^\infty d\omega\, \omega K_0(2\omega z_0/v)\mathrm{Im}\left[\frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 1}\right]. \tag{17}$$

Using the optical data table given by Hagemann *et al.*[3] for silver, we can calculate the integral in Eq. (17). A graph of f versus $\omega_p z_0/v$, where $\omega_p = 0.1367$ a.u. = plasma frequency for silver, in Hartree atomic units for the case of a silver surface is shown in Fig. 2.

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