15th International Particle Accelerator Conference,Nashville, TN JACoW Publishing
ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-MOPC76

TRANSVERSELY DRIVEN COHERENT BEAM OSCILLATIONS IN THE EIC ELECTRON STORAGE RING*

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Abstract

We study coherent transverse beam oscillations in the EIC electron storage ring (ESR), to specify the tolerance for high-frequency ripple of the dipole magnet power supplies. To avoid unacceptable proton emittance growth from the oscillating beam-beam kick from the electrons, the amplitude of these oscillations at the proton betatron frequency needs to be limited to about 1e-4 fraction of the beam size at the interaction point. We show that the oscillations potentially caused by the ESR magnet dipole power supply ripple could be substantial, but still tolerable, if we account for the eddy current shielding in the vacuum chamber.

INTRODUCTION

The ESR has very tight tolerances for the beam position and size stability at the interaction point (IP). The oscillations at the proton betatron frequency and its harmonics are the most dangerous because they could lead to unacceptable proton emittance growth from the oscillating beam-beam kick from the electrons at the amplitude of the positional oscillations as low as 10^{-4} of the rms beam size [1, 2].

These oscillations may have many different causes, including the dipole power supply (PS) ripple, the phase noise in the main [3] or crab RF systems [4], and some collective instabilities (e.g. [1]). The dipole PS is our focus here.

A dipole rippling at low frequency causes closed orbit oscillations. However, if the ripple frequency is close to the fractional part of the betatron tune or its harmonics, then, for the same ripple amplitude, beam oscillations of much larger magnitude around the (fixed) closed orbit can be resonantly excited. This effect is the main subject of this paper.

For the ESR revolution frequency $f_0 = 78.2$ kHz and the lowest possible value of the fractional part of the betatron tune of ~0.1, these oscillations can be caused by driving sources in the range of ~[8-40] kHz (the frequencies above $f_0/2$ are folded back for once-per-turn beam sampling).

For the usual switching type DC power supplies, narrow-band switching frequency harmonics as well as some wide-band noise could be present in this frequency range.

In this paper, we present an analytical study of the effects of these two types of noise. To make the problem tractable, we make several simplifying assumptions and conservative approximations so that our conclusions can be used to specify the ESR dipole PS high-frequency ripple and noise with some safety margin.

MAXIMUM AMPLITUDE AT THE IP

There are generally two types of fast oscillations in DC magnets, predictable baseline shifts due to things like power supply switching and 60 Hz harmonics ripple, and true noise. The net field at a given location in the vacuum chamber is then written

$$B(t) = B_0 + B_p(t) + B_n(t). \quad (1)$$

The net fluctuating field is $B_1(s,t) = B_p + B_n$, where s is the longitudinal Serret-Frenet coordinate which updates by the circumference, C, each turn.

Consider horizontal motion of a single electron bunch subjected to dipole field errors

$$\frac{d^2x}{ds^2} + K(s)x = \frac{B_1(s,t)}{(B\rho)} - \frac{2}{s_d}\frac{dx}{ds}, \quad (2)$$

where $(B\rho)$ is the magnetic rigidity, $s_d = c\tau_d$ is the total damping distance and $K(s)$ is the net focusing. The solution is

$$x(s) = \int_0^s ds_1 \frac{B_1(s_1,t_1)}{(B\rho)}\sqrt{\beta(s)\beta(s_1)}\sin[\psi(s)-\psi(s_1)] \times \exp[(s_1-s)/s_d]. \quad (3)$$

Since $s_d \gg C$, the details of beta function weighting have been ignored. The motion of the electron bunch is due to the summation of many independent kicks. On a given turn the motion is basically a free oscillation, so we can understand the statistics of the process by focusing on a fixed location in the ring. For simplicity assume that we have $s = 0$ at this location and that $d\beta/ds = 0$ there. Then

$$(x + i\beta x')_{n+1} = (x + i\beta x')_n \exp(-i2\pi\nu - C/s_d) + F(n),$$

$$F(n) = \int_0^C ds_1 \frac{B_1(s_1, s_1/c + nT_0)}{(B\rho)}\sqrt{\beta(0)\beta(s_1)}\, i e^{-i(\psi(s_1)-2\pi\nu)},$$

where $\psi(0) = 0$, ν is the betatron tune, $T_0 = C/c$ is the revolution period.

Now suppose there are M independent magnet strings. An exact representation appears difficult but to a reasonable approximation take

$$\frac{B_1(s,t)}{(B\rho)} = \sum_{k=1}^{M} b_k(s)n_k(t-\tau(s)), \quad (4)$$

where $b_k(s) = b_k(s + C)$ describes the spatial extent of the string and $\tau(s) = \tau(s + C)$ describes the effect of voltage travel time within the magnet string. At this point, we

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ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-MOPC76

temporarily ignore the B_p term in Eq. (1) and consider true noise with unit standard deviation and correlation function $\langle n_k(t_1)n_k(t_2)\rangle = \rho_k(t_1 - t_2)$. Since everything is linear, we may write $F(n) = \sum_{1\le k\le M} F_k(n)$ with

$$F_k(n) = \int_0^C ds_1 b_k(s_1) n_k(nT_0 + s_1/c - \tau(s_1))\sqrt{\beta(s)\beta(s_1)} \times i\exp[-i(\psi(s_1) - 2\pi\nu)]. \quad (5)$$

The $F_k(n)$s are discrete random variables. Since the change in amplitude will involve many thousands of kicks we may assume the noise is Gaussian so that only the rms value and the frequency spectrum are needed.

Without more information, it is difficult to evaluate these expressions. If we assume the worst case where all errors add coherently then

$$F(n) = \sum_k F_k(n) \approx 2\pi\sqrt{\bar{\beta}\beta_0}\frac{\delta B(n)}{\bar{B}}, \quad (6)$$

where the overbar denotes the ring average. The spectrum associated with $F(n)$ is

$$S_F(\phi) = \lim_{N\to\infty}\langle\frac{1}{N}\left|\sum_{n=1}^{N} F(n)e^{in\phi}\right|^2\rangle. \quad (7)$$

While $F(n)$ is complex, the error with taking it to be real is small since it takes many kicks to change the amplitude appreciably.

A significant danger of these electron oscillations is that they will drive emittance growth in the hadron beam. Let x_n denote the time series of electron centroid offsets at the IP. The proton un-normalized rms emittance will grow in a random walk according to [1]

$$\frac{d\epsilon}{dn} = \frac{(4\pi\Delta\nu_{bb})^2}{2\beta^*}\sum_{m=-\infty}^{\infty}\langle x_{m+n}x_n\rangle\cos(2\pi m\nu_p), \quad (8)$$

where $\Delta\nu_{bb}$ is the beam-beam tune shift, β^* is the proton beta function at the IP and ν_p is the proton tune. The horizontal emittance of the protons is of order 10^{-8} m. Assume this doubles in 10 hours. The beam-beam tune shift is limited to 0.015. The horizontal $\beta^* \sim 80$ cm. Taken together this implies the sum on the right of Eq. (8) is 1.6×10^{-16} m^2. For white noise this implies $\langle x_{m+n}x_n\rangle = \delta_{m,0}\langle x^2\rangle = \delta_{m,0}(0.00012\sigma)^2$ with σ the horizontal rms size.

More sophisticated estimates using multiparticle tracking for various noise models give a similar constraint [2]. Below this will be rounded off to

$$x_{\rm rms} = 10^{-4}\sigma. \quad (9)$$

MAXIMUM DIPOLE NOISE AND RIPPLE

Maximum Random Noise

To calculate $\langle x_{m+n}x_n\rangle$ we assume $F(n)$ is white noise and we will also assume $\nu = \nu_p$. Assume that s_d corresponds to many turns. All the sums can be done in closed form and to a good approximation one finds

$$\sum_{m=-\infty}^{\infty}\langle x_{m+n}x_n\rangle\cos(2\pi m\nu_p) = \frac{\langle F^2\rangle}{4}\left(\frac{s_d}{C}\right)^2. \quad (10)$$

For actual noise one makes the replacement $\langle F^2\rangle \rightarrow S_F(2\pi\nu)$.

To proceed we take a worst-case estimate for F. Suppose all the dipole errors add in phase. Then one has

$$\langle F^2\rangle \approx 4\pi^2\bar{\beta}\beta^* S_{\delta B/B}(2\pi\nu). \quad (11)$$

Combining everything one finds

$$(10^{-4}\sigma)^2 = \pi^2\bar{\beta}\beta^* S_{\delta B/B}(2\pi\nu)\left(\frac{s_d}{C}\right)^2. \quad (12)$$

We postpone numerical estimates for the maximum allowable noise until after we discuss the maximum ripple.

Maximum Ripple

Here we assume a sinusoidally-varying field ripple that causes resonant transverse oscillations when the frequency of the B_p term in Eq. (1), is close to the betatron tune. This problem was treated before, e.g. for AC-dipoles in hadron rings or for the swept-sine excitation tune measurement in electron rings. We will follow [5] below.

First, assume a single thin dipole at $s = 0$ with the kick

$$\theta(t) = \theta_0\cos(\omega t + \phi). \quad (13)$$

Also, assume that the damping rate, $\alpha = 1/\tau_d$, comes from the synchrotron radiation as well as from the betatron tune spread

$$\alpha = \alpha_R + \delta\omega_\beta, \quad (14)$$

where the betatron frequency distribution is taken to be Lorentzian with the half-width $\delta\omega_\beta$. The resulting positional oscillations in complex notation are given by

$$x(s,t) = -\frac{\theta_0}{4}\sqrt{\beta(0)}\sqrt{\beta(s)}e^{i(\omega t+\phi)}e^{-i(\omega-i\alpha)s/c}e^{i\alpha T_0/2} \times(\pm)\frac{e^{\pm i\psi(s)}e^{i\pi(\nu_d\mp\nu_\beta)}}{\sin[\pi(\nu_d\mp\nu_\beta) - i\alpha T_0/2]}, \quad (15)$$

where $\psi(s)$ is the phase advance, $\psi(0) = 0$, $\nu_d = \omega T_0/(2\pi)$ is the driving tune, and summation over $(\pm)$ is assumed.

At resonance, $\nu_d - \nu_\beta = n$, for some integer n, this simplifies to

$$x(s,t) = \frac{\theta_0\sqrt{\beta(0)}\sqrt{\beta(s)}}{4\sin(i\alpha T_0/2)}e^{i(\omega t+\phi-(\omega-i\alpha)\frac{s}{c}+\alpha\frac{T_0}{2}+\psi(s))}. \quad (16)$$

Taking the real part, and assuming that $\alpha T_0 \ll 1$, we get the peak amplitude of the oscillations in physical space

$$\hat{x}(s) = \frac{\theta_0\sqrt{\beta(0)}\sqrt{\beta(s)}}{2\alpha T_0}. \quad (17)$$

From linearity, the same relation holds between the rms amplitude, $x_{\rm rms}(s)$, and the rms kick, $\theta_{0,\rm rms}$.

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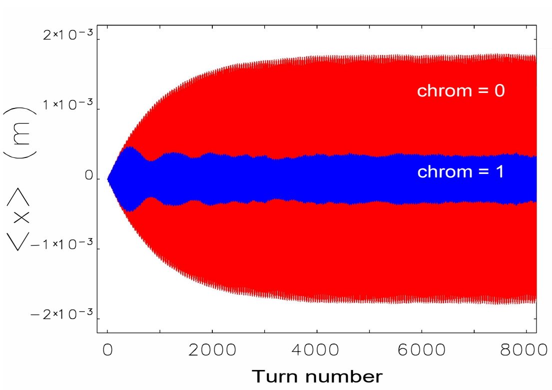

Figure 1: Centroid of the bunch driven at ν_x.

We found good agreement between Eq. (17) and Elegant [6] tracking simulations. For instance, for the parameters used for the red trace in Fig. 1 ($E = 18$ GeV, $\alpha T_0 = 1/1000$ at zero chromaticity and no beam-beam, $\theta_0 = 1$ μrad, $\beta(0) = 32$ m, $\beta(s) = 0.4$ m), Eq. (17) gives the peak value of $\hat{x}(s) = 1.8$ mm, virtually the same as in Fig. 1.

Note that for the ESR the coherent damping mainly comes from the tune spread due to beam-beam interaction. While the beam-beam parameters $\xi_{x,y}$ vary between different colliding configurations, a 100-turn damping, $\alpha T_0 = 1/100$, is a reasonable conservative choice to be used below.

To add the kicks from multiple dipoles driven at the same frequency, one can sum up the contributions from each as given by Eq. (16). In contrast to the single-kick case, Eq. (17), the resulting amplitude will additionally depend on the betatron phase advances, the unknown electrical phases (through $e^{i\phi}$ factors), and the magnet s-locations.

In the worst case, the contributions from all rippling dipoles, each given by Eq. (16), will add in phase at the IP. Dividing the resulting rms kick by the total bend angle and using Eq. (9), we get for the rms field ripple,

$$\delta B/B = \frac{\theta_{0,\mathrm{rms}}}{2\pi} = 10^{-4}\sigma\frac{\alpha T_0}{\pi\sqrt{\bar{\beta}}\sqrt{\beta^*}}. \qquad (18)$$

HIGH-FREQUENCY DIPOLE PS SPECS

We first note that Eq. (18) is identical to Eq. (12) with replacements $\delta B/B = \sqrt{S_{\delta B/B}}$ and $(s_d/C)\,\alpha T_0 = 1$. The fact that both estimates— one assuming stochastic noise and the other sinusoidally driven motions—yield the same result, instills confidence in our comprehensive consideration of noise contributions of any type. Below we will use "ripple" to denote "noise and ripple" for short, because the two were found equivalent.

Substituting $\sigma = 100$ μm, $\beta^* = 0.4$ m, and $\bar{\beta} = 30$ m in Eq. (18) we finally get $\delta B/B = 9.2\text{e-}12$ rms.

According to this very conservative estimate, 1 ppm PS current ripple at[1] ν_x is acceptable, as long as 5 more orders of magnitude of attenuation are coming from elsewhere.

[1] More accurately, we must consider a band of frequencies which fall within the beam-beam tune-spread, so we take this bandwidth to be $\Delta\nu$=0.01.

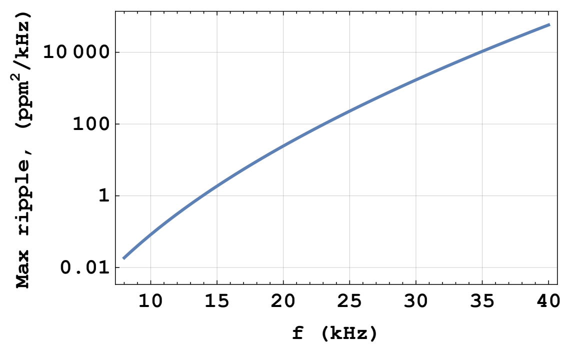

Figure 2: Maximum rms current ripple for ESR dipole PS.

According to the conservative estimates in [7,8], this amount of attenuation does occur at frequencies exceeding ~20 kHz due to eddy currents in the vacuum chamber.

In reality, even if all dipoles are put on the same PS, their high-frequency ripples do not add in phase (see Eq. (16)), so at least $N_d^{-1/2}$ additional cancellation can be safely assumed. Here N_d ~700 is the total number of dipoles. With this assumption, and taking the relevant width of the tune line to be on the order of the beam-beam parameter, $\Delta\nu = 0.01$, the maximum power spectral density of the field ripple becomes

$$P_{\delta B/B} = \frac{T_0}{\Delta\nu}\left(10^{-4}\sigma\sqrt{N_d}\frac{\alpha T_0}{\pi\sqrt{\bar{\beta}\beta^*}}\right)^2. \qquad (19)$$

Dividing this expression by the attenuation factor due to eddy currents in the vacuum chamber (see [7, 8]) and accounting for the 5/18 factor for the ESR energy variation we get the maximum allowable power spectral density for the dipole PS current ripple, $P_{\delta I/I_{\max}}$, plotted in Fig. 2. This ripple is normalized to $I_{\max}$ - the maximum operating PS current which occurs at $E = 18$ GeV.

CONCLUSION

The high-frequency dipole field ripple specifications for the ESR were derived, requiring $10^{-4}\sigma$ rms positional stability at the IP in the frequency range [8-40] kHz. We assumed the worst-case scenario of an external perturbation exciting the electron beam at the betatron tune. The field ripple specifications were propagated to the dipole PS current ripple, taking credit for the attenuation in the vacuum chamber. If the PS switching frequency exceeds ~20 kHz, the high-frequency ripple specification does not appear to be overly restrictive.

We emphasize that the conclusions above only apply to the resonant excitation, which may occur above ~8 kHz. Low-frequency orbit and beam size oscillations at the IP require a different analysis, which results in very tight magnet PS specifications (see e.g. [9]) and may also force us to consider some fast beam-based feedbacks at the IP. This work is still ongoing.

ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-MOPC76

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