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Free energy and direct photon emission at finite chemical potential

S. Somorendro Singh

Department of Physics and Astrophysics, University of Delhi, Delhi, India

E-mail: sssingh@physics.du.ac.in

Abstract. We investigate the evolution of free energy and direct photon production from quark-gluon plasma (QGP) considering finite chemical potential. The evolution of QGP formation at the chemical potential is done through finite value of quark mass. The evolution rate is found to be decreasing with chemical potential. We further study the direct photon emission from the fireball of such QGP and found the result to be increasing function of chemical potential in all the channels of photon production. It also shows enhancement of photon emission in comparison to the other theoretical calculation of direct photon productions.

1. Introduction

The research on the ultra-relativistic heavy-ion collisions indicate the evolution of strongly interacting matter called quark-gluon plasma (QGP) [1]. The matter exists for a short life in the process of transformation from a confined to a deconfined state. This short period of existence as a deconfined matter in the early evolution is believed to be in the form of a super-fluid matter. Due to the complicated nature of the state, the investigation on this matter has become a core issue in the present scenarios of heavy ion reactions. Many theoretical and experimental researcher have a keen view on search of evolution of the early universe. There are many theoretical and experimental published works on QGP and the reports of these works are obtained through the path from Relativistic Heavy Ion-collider (RHIC) and Large Hadron Collider(LHC) [2, 3]. The lattice QCD calculation is one method for prediction of such matter and they also inform the presence of occurring such matter (QGP) at very high temperature [4] and at very high nuclear density. In fact, experiments at SPS/CERN, RHIC/BNL and LHC/CERN have claimed for the creation of such situation at very high temperature for the study of QGP [5]. The formation of QGP in these experiments is done through the central collision of two massive nuclei, resulting the product of many particles subsequently bringing the entire information about the collision zone. So many theorist have modeled to create the existence of QGP formation. They also studied the phase structure of QGP-hadron phase transition. Likewise, we use a simple statistical model to construct the free energy evolution of QGP adopting the mean field potential between the interacting particles. After the evolution of the QGP fireball we consider all related processes for the production of particles from these processes. These produced particles consist of a large number of various elements varying from the heavy particles to light elements called dileptons and photons [6 − 10]. The light particles produced are considered to be the most probable signals for the formation of QGP. As they interact electromagnetically they carry the entire information of the reaction zone

to the detector and give the fundamental information for the formation of QGP. So photons and dileptons are assumed to be the most promising and exciting signal in the formation of QGP.

Moreover, the present experimental programs at RHIC/BNL and LHC/CERN have proved the main goal of its experimental pursuit in the research of QGP with the higher beam energies. These active programs will provide clear ideas in the experimental outcomes and in the analysis of theoretical modeling too. In addition to this, the transverse momentum distribution function of direct photon is significant in the photon induced reaction at the same energies $\sqrt{s}$ [11] for the momentum greater than 1.5 GeV/c in the central collision. It implies the importance of direct photon production dependence on the space time evolution scenarios of the finite QGP formation. So, there are many probable outcomes of direct photon and dilepton production from the finite baryonic density. Among the calculations at finite baryonic density, it is Dumitru et al.'s photon production at finite baryonic density and subsequently Strickland's calculation using the Jüttner distribution function [12 − 14]. Later on there are works on dilepton and photon production at finite baryonic potential from Hammon and Bass et al. [15, 16] etc. Even at the present experiments the consideration of finite chemical potential is taken into account at FAIR experiments as an example of CBM (compact baryon matter). On these information, we focus the photon radiation directly from the thermalized quark-gluon plasma at $T = 0.57$ GeV which at last freeze out to hadronic matter with the production of latent heat, that again reprocess heating and cooling into process of hadronization as pions at around temperature $T = 0.15$ GeV [17].

In this present work, we investigate our simple statistical model of strongly interacting matter of quark-gluon plasma at finite baryonic density incorporating the parametrized momentum factor in the quark mass and study free energy evolution at finite chemical potential. Then we study the photon production at these finite chemical potential. In most calculations we found that the photon production is done through the massless quark. Due to this massless quark we obtain infrared divergence in the production rate. So we use the finite value of quark mass for removal the infrared (IR) divergence. The finite value of quark mass is taken as the effective quark mass and its value is obtained through the momentum cut off defined as [18]:

$$m_q^2 = \frac{1}{3}\frac{8\pi}{(33-2n_f)}\frac{T^2}{\ln(1+\frac{p^2}{\Lambda^2})} \tag{1}$$

where $p = (\frac{2\pi\gamma^3 T^6 \Lambda^6}{33-2n_f})^{1/12}$ is known as low momentum cut off with quark flavor n_f. γ is defined as root mean square, equal to $\sqrt{\frac{2}{\gamma_q^2}+\frac{2}{\gamma_g^2}}$ with $\gamma_g = a\gamma_q$. γ_q is equal to 1/6 with the suitable value of a searched in an ad-hoc fashion to fit the free energy evolution of the QGP with the stable droplet size [19]. So we calculate the photon radiation at finite baryonic density at the finite temperature for the quark flavor n_f. The calculation of photon production is performed at the temperature $T = 0.57$ GeV. At last we compare the result with our earlier results without chemical potential and other theoretical works.

Thus, we organize the paper as: In section 2 we study a brief idea of free energy evolution of QGP at finite chemical potential. In section 3, we present the photon radiation from QGP at finite baryonic density. In last section 4, we give the results and conclusions.

2. Free energy evolution of QGP at finite baryon density

The free energies of QGP evolution can be obtained through the density of states of the non-interacting fermions and bosons at the finite baryonic density. Here fermions are contributed by quarks and antiquarks and bosons are represented by gluons, which are considered to be active constituent particles of QGP system. So the free energy of these particles is defined as follows [20]:

$$F_i = \mp T g_i \int dp \rho_i(p) \ln[1 \pm e^{-(\sqrt{m_i^2+p^2}-\mu)/T}], \tag{2}$$

where g_i is degeneracy factor and $\rho_i(p)$ is density of states of the constituent particles of the QGP, which we define below. To calculate the density of state, we first use the thermal Hamiltonian process of thermal dependent quark mass. The thermal Hamiltonian is composed of the unperturbed Hamiltonian part and the interaction potential between the constituent particles. This interaction potential represents the mean field potential in the phase space presentation. Now the Hamiltonian is obtained as [21]:

$$H(p,T) = H_o(p,T) + \frac{1}{2p}\gamma_{q,g}g^2(p)T^2 \tag{3}$$

where first term, $H_o(p,T)$ is unperturbed Hamiltonian and second one is the effective mean field potential between the particles in which $g^2(p) = 4\pi\alpha_s$ with QCD coupling constant of α_s defined as

$$\alpha_s = \frac{4\pi}{(33-2n_f)\ln(1+\frac{p^2}{\Lambda^2})}. \tag{4}$$

Now the density of state of quarks and gluons is constructed by the method of Ramanathan and Bethe et al [22] using the interacting mean field potential obtained earlier. It is obtained after analytical calculation using their model.

$$\rho(p) = \frac{v^2\gamma^3T^6}{4\pi^2p^3}g^4(p)[\frac{dg(p)}{dp} - \frac{g^2(p)}{2p}]. \tag{5}$$

So at last we obtain the free energies of quarks, antiquarks and gluons plugging this density of states in Eqn.2. Besides these free energy, there is pion free energy and the Weyl [23] surface energy which confine the system instead of using Bag energy. They are represented as follows

$$F_\pi = (3T/2\pi^2)v\int_0^\infty p^2\ln[1-e^{-\sqrt{(m_\pi^2+p^2)}/T}]dp. \tag{6}$$

and

$$F_{surface} = \gamma R^2\int\frac{T}{p^2}\delta(p-T)dp \tag{7}$$

where R is the size of the droplets. Now, the total free energy of the QGP can be computed as sum of all the relevant energies in the system.

$$F_{total} = \sum_i F_i \tag{8}$$

where i denotes each contribution of the particles stated above.

3. Photon production from QGP at finite baryon density

The photon production from QGP at finite baryon density is very interesting theoretical problem. There are a lot of studies for the calculation of photon production at finite temperature and baryon density. In most of the studies, the production rate of photon and dilepton are measured in three classified regions. In first measurement the photon spectra is obtained at the low transverse momentum which are mainly produced due to the soft hadronic decay. In second region, the spectra is studied in the intermediate transverse momentum and subsequently, in higher transverse momentum, which is mainly contributed by the strongly interacting QGP. Here we focus on a baryon rich density and thermalized soup of QGP system after the Big Bang process. In the process the system takes a longer time compared to the time scale associated with the photon production. Moreover the production rate for the coupling parameter $\alpha_s << 1$ [24], turns out to be slow expansion near the equilibrium temperature. Depending on

such issues, we estimate our work of photon production at finite baryon density considering the QCD quark antiquark annihilation process and Compton process. Due to rich baryon density the number of quark contents in the system is large enough to interact among themselves and other particles. The quark-antiquark annihilation and QCD Compton are the dominant processes for the production of photon from thermalized and baryon rich system of quarks and gluons. The production rate through the annihilation process $q\bar{q} \rightarrow \gamma g$, at thermal equilibrium temperature $T = 0.57$ GeV is calculated as:

$$\begin{aligned} E\frac{dN^a}{d^3pd^4x} &= \frac{16f_q(p)}{(2\pi)^6 4E}\sum_{f=1}^{N_f}\int dsdEf_{\bar{q}}(E_{\bar{q}}) \\ &\times [1+f_g(E_{\bar{q}})]\sqrt{s(s-4m^2)}\sigma_{q\bar{q}\rightarrow\gamma g}, \end{aligned} \tag{9}$$

where N_f is number of quark flavors with the Jüttner distribution functions. The Jüttner distributions for quarks, anti-quarks and gluons are defined as:

$$f_{q(\bar{q})} = \frac{\lambda_{q(\bar{q})}e^{\pm\mu/T}}{e^{E/T}+1} \quad and \quad f_g = \frac{\lambda_g}{e^{E/T}-1}. \tag{10}$$

We substitute the distribution functions in the above relation and obtain photon radiation rate at the finite baryon density through annihilation and it is same with other theoretical works of Ref. [13, 14, 25 − 29] :

$$\begin{aligned} E\frac{dN^a}{d^3pd^4x} &= \frac{2\alpha\alpha_s}{\pi^4}\lambda_Q^2T^2e^{-E/T}\sum_f e_f^2 \\ &\times [ln(\frac{4ET}{k_c^2}) - C_{euler} - 1 - \ln(n)], \end{aligned} \tag{11}$$

where $\lambda_{Q(\bar{Q})} = \lambda_{q(\bar{q})}e^{\pm\mu/T}$ and $k_c^2 = 2m_q^2$.

In the similar line we perform one loop calculation of Compton process $q(\bar{q})g \rightarrow q(\bar{q})\gamma$ in the finite baryon density as:

$$\begin{aligned} E\frac{dN^c}{d^3pd^4x} &= \frac{8N_\epsilon f_q(p)}{(2\pi)^5}\sum_{f=1}^{N_f}\int dsdE_gf_g(E_g) \\ &\times [1-f_q(E_g)](s-m^2)\sigma_{gq\rightarrow\gamma q}. \end{aligned} \tag{12}$$

Again we substitute the distribution functions of quark, anti-quark and gluon in Compton process expression. Finally we obtain photon radiation rate as [13, 14, 25 − 29]:

$$\begin{aligned} E\frac{dN^c}{d^3pd^4x} &= \frac{2\alpha\alpha_s}{\pi^4}\lambda_Q\lambda_gT^2e^{-E/T}\sum_f e_f^2 \\ &\times [\ln(\frac{4ET}{k_c^2}) - C_{Euler} + 1/2 - \ln(n)], \end{aligned} \tag{13}$$

where $C_{Euler} = 0.577216$. Similarly, the production rates of photons due to two loops order of AWS and Bremsstrahlung processes are given as [28, 29]:

$$\begin{aligned} E\frac{dN}{d^3pd^4x} &= \frac{2N_cn_f}{3\pi^5}\alpha\alpha_s\{\frac{2}{5}\lambda_q^3 + \frac{3}{5}\lambda_g\lambda_q^2\} \\ &\times \sum_f e_f^2TE[J_T - J_L], \end{aligned} \tag{14}$$

and

$$E\frac{dN}{d^3pd^4x} = \frac{2N_cn_f}{\pi^5}\alpha\alpha_s\{\frac{4}{7}\lambda_q^2+\frac{3}{7}\lambda_g\lambda_q\} \times \sum_f e_f^2T^2e^{-E/T}[J_T-J_L]\ln(2) \quad (15)$$

where N_c is color degree of freedom. J_L and J_T values are taken as 1.13 and 1.20 for $n_f = 3$.

Further we integrate the total photon rate over the space time history of the collision to obtain the total photon spectrum for all the channels. It is expressed as [14, 29, 30]:

$$\frac{dN}{d^2p_Tdy} = \int d^4x(E\frac{dN}{d^3pd^4x}) = Q\int_{\tau_0}^{\tau_f}\tau d\tau\int dy(E\frac{dN}{d^3pd^4x}) \quad (16)$$

where τ is the initial and final value of time evolution. We take rapidity $y_{nuc} = \pm 5.3$ corresponding to RHIC scale. The transverse cross-section of the considered nuclei is taken as $Q \sim 180fm^2$. p_T is the photon transverse momentum. Then the quantity on the right hand side is defined in the centre-of-mass system with the photon energy $E = p_Tcosh(y' - y)$. With the values of rapidity and p_T, we get the total photon spectrum shown in figures.

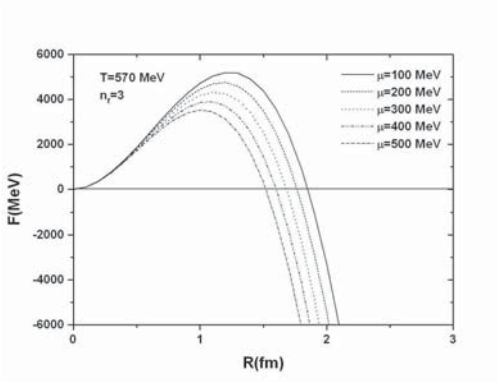

Figure 1. Free energy changes with the size of droplet for various value of chemical potential at temperature $T = 0.57$ GeV with $n_f = 3$.

4. Results and Conclusions

In the results and conclusions we represent the updated and extended calculations of free energy evolution and photon radiation through the finite chemical potential. The calculation is performed for quark flavor $n_f = 3$. The formation/ evolution of QGP droplet is shown in Fig. 1 for quark flavor $n_f = 3$. In Fig. 1, the droplet size of QGP formation is found to vary from $R = 2.0$ to 1.5 fm for the chemical potential varying from $\mu = 100 - 500$ MeV. It means that increase of chemical potential decreases the droplet size. It is just opposite pattern to the the case of finite temperature. In case of finite temperature, the size of droplet is increased with respect to the increase in temperature, which indicates good output in respect of phase structure. In QCD phase structure the phase transition with the chemical potential is just opposite to the temperature, which indicates the good output in terms of phase structure.

In Fig. 2 we show the photon emission rate at the initial temperature $T = 0.57$ GeV through the annihilation process for quark flavor $n_f = 3$. The emission rates are found to be increasing function with the chemical potential μ. The increase in the emission rate is highly effected by the temperature as well as the chemical potential of the system. It is found to be very large in the production at very high temperature and chemical potential. In Fig. 3 we again show the photon emission at temperature $T = 0.57$ GeV through the Compton mechanism. In this

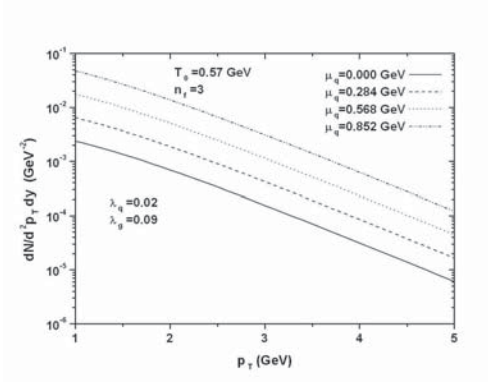

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Figure 2. The photon emission rate through ann. at thermal temperature $T = 0.57$ GeV with the transverse momentum for $n_f = 3$.

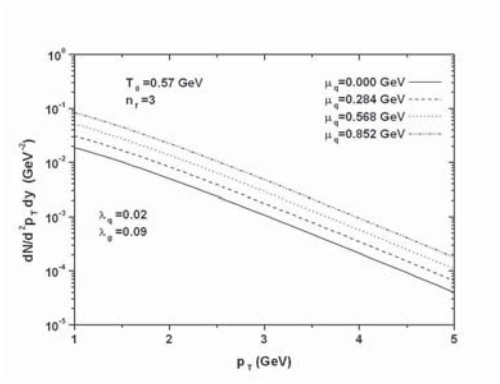

Figure 3. The photon emission rate through Compton at thermal temperature $T = 0.57$ GeV for $n_f = 3$.

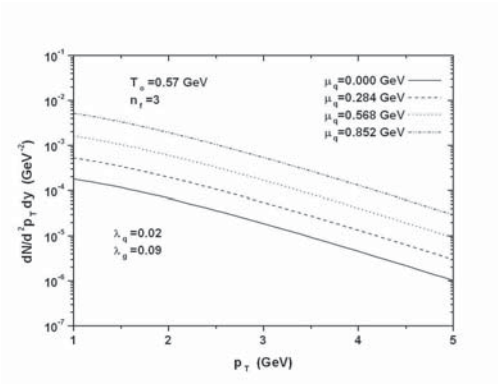

Figure 4. The photon emission rate through AWS process at thermal temperature $T = 0.57$ GeV for $n_f = 3$.

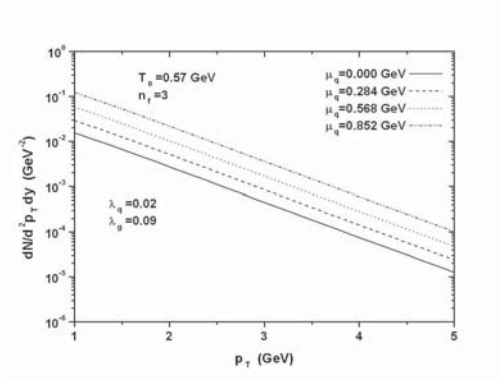

Figure 5. The photon emission rate through Bremsstrahlung process at thermal temperature $T = 0.57$ GeV for $n_f = 3$.

channel too the production rate is high in accordance with the chemical potential. The large production is found in the case of Compton process in comparison to the annihilation process for the same value of chemical potential. So the Compton shows better outcomes in terms of production rates [14, 26, 27, 28].

Now we study the production rate of AWS channel. The photon productions rate is shown in Fig. 4. It shows that production rates follow same pattern as before. Yet AWS channels produce less photons in comparison with the earlier channels. In last process, we do the Bremsstrahlung process which is shown in Fig. 5. Photon production due to this process is increased with the increased chemical potential, and again in this process the production rate is still high compared to the AWS channel and other two channels above. This implies that in all the processes/channels, our model has large advantage in the photon production rate particularly Compton and Bremsstralung processes. So the production rate of our model with the flavor $n_f = 3$ have improvement from other works in these channels. It means that our result is dominant over the production rate of Ref.[14, 27, 28]. Finally, we conclude that the evolution of the fireball through the parametrization factor decrease the size of droplet formation in this

finite chemical potential. It shows that the evolution of fireball is suppressed in comparison with the evolution of fireball at the finite temperature. The suppression may be due to large presence of quark, antiquark and specially due to strange quark particles. Even though it is not encouraged in the evolution of the plasma, the condensed matter of the system enhance the interaction between the particles and therefore enhance the production rates. Overall the results show that the calculation of photon production as a function of photon transverse momentum incorporating chemical potential and the factor in the quark mass give the improved results in the photon yield products from the earlier results without the chemical potential [31]. Thus, the consideration of parametrization factor in the quark mass has important role in the evolution as well as in the photon measurements of the high energy heavy ion collisions in finite chemical potential.

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References

[1] Harris J W and Müller B 1996 *Ann. Rev. Nucl. Sci.* **46** 71; Geiger K, Müller B 1992 *Nucl. Phys. B* **369** 600.
[2] Arsene I *et al*. 2005 *Nucl. Phys. A* **757** 1; Back B B *et al*. 2005 *ibid A* **757** 28.
[3] Adams J *et al*. 2005 *ibid A* **757** 102; Adcox K *et al*. 2005 *ibid A* **757** 184.
[4] Karsch F and Laermann E 2004 *QGP*, ed R C Hwa and X N Wang (World Scientific, Singapore).
[5] Proc. of Quark Matter'01 2002 *Nucl. Phys. A* **698**.
[6] Shuryak E V 1978 *Phys. Lett. B 78* **150**; 1980 *Phys. Rep.* **61** 71; Kajantie K and Miettinen H I 1981 *Z. Phys. C* **9** 341; Domikos Gand Goldman J 1981 *Phys. Rev. D* **23** 203.
[7] Mclerran L and Toimela 1985 *Phys. Rev. D* **31** 545.
[8] Gale C and Kapusta J I 1991 *Nucl. Phys. B* **357** 65.
[9] Peitzmann T and Thoma M H 2002 *Phys. Rep.* **364** 175.
[10] Cassing W and Bratkovskaya E L 1999 *Phys. Rep.* **308** 65; Aurenche P (*Preprint* hep/0201011).
[11] Wang S Y and Boyanovsky D 2001 *Phys. Rev. D* **63** 051702(R); Boyanovsky D and de Vega H J 2003 *Phys. Rev. D* **68** 065018.
[12] Dumitru A, Rischke D H, Schönfed Th, Winckelmann L, Stöker H and Greiner W 1993 *Phys. Rev. Lett.* **70** 1860; Dumitru *et al*. 1993 *Mod. Phys. Lett. A* **8** 1291.
[13] Strickland M 1994 *Phys. Lett. B* **331** 245; Biro T S, Doorn E V, Müller B, Thomas M H and Wang X N, 1993 *Phys. Rev. C* **48** 1275.
[14] Dutta D, Mohanty A K, Kumar K and Choudhury R K 2000 *Phys. Rev. C* **61** 064911.
[15] Hammon N *et al*. 1999 *Phys. Rev. C* **61** 014901.
[16] Bass S A *et al*. 2003 *Phys. Rev. Lett.* **91** 052302; Ze-Jun He *et al*. 2005 *C Phys. Lett.* **22** 1350.
[17] Srivastava D K and Sinha B 1994 *Phys. Rev. Lett.* **73** 2421.
[18] Kumar Y and Singh S S 2012 *Can. J. Phys.* **90** 955.
[19] Ramanathan R, Mathur Y K, Gupta K K and Jha A K 2004 *Phys. Rev. C* **70** 061408; Ramanathan R, Gupta K K, Jha A K and Singh S S 2007 *Pram. J. Phys.* **68** 757.
[20] Neerguard G and Madsen J 1999 *Phys. Rev. D* **60** 054011; Mustafa M G, Srivastava D K and Sinha B 1998 *E. Phys. J. C* **5** 711.
[21] Peshier A, Kämpfer B, Pavlenko O P and Soff G 1994 *Phys. Lett. B* **337** 235.
[22] Bethe H A 1937 *Rev. Mod. Phys.* **9** 69; Fermi E 1928 *Z Phys.* **48** 73; Thomas L H 1927 *Proc. Cambridge Philos. Soc.* **23** 542.
[23] Weyl H 1911 *Nachr. Akad.* Wiss Gottingen 110 .
[24] Moore G D (*Preprint* hep.ph/0403169); V.Ruuskanen 1991 *Quark-Gluon Plasma* , ed R C Hwa (World Scientific, Singapore).
[25] Kampfer B and Pavlenko O P 1994 *Z. Phys. C* **62** 491.
[26] He Z, Long J, Ma Y and Ma G 2005 *C. Phys. Lett.* **22** 1350.
[27] Long J, He Z, Ma Y and Liu B 2006 *Nucl. Phys. A* **766** 201
[28] Mustafa M G and Thoma M H 2000 *Phys. Rev. C* **62** 014902.
[29] Long J, He Z, Ma Y and Liu B 2005 *Phys. Rev. C* **72** 064907.

[30] Steffan F D and Thoma M H 2001 *Phys. Lett. B* **510** 98; Traxler C T and Thoma M H 1996 *Phys. Rev. C* **53** 1348.
[31] Singh S S and Kumar Y Accepted in *Int. J. Mod. Phys. A*

Corrigendum: Free energy and direct photon emission at finite chemical potential

S. Somorendro Singh

Department of Physics and Astrophysics, University of Delhi, Delhi, India

E-mail: sssingh@physics.du.ac.in

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S. Somorendro Singh and Y. Kumar

Department of Physics and Astrophysics, University of Delhi, Delhi, India

E-mail: sssingh@physics.du.ac.in ; yogesh.du81@gmail.com