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Correlations of W boson mass fits from m_T, p_T^l and p_T^ν fits

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Abstract

We study the statistical correlations between m_T, p_T^l and p_T^ν mass fits for 2 fb^{-1} W mass analysis using 100 million PYTHIA events. The m_T-p_T^l, m_T-p_T^ν and p_T^l-p_T^ν correlation coefficients for $W \to \mu\nu$ channel are found to be 0.672 ± 0.028, 0.658 ± 0.028 and 0.255 ± 0.047. The corresponding correlation coefficients for $W \to e\nu$ channel are found to be 0.709 ± 0.025, 0.694 ± 0.026 and 0.307 ± 0.045. The reliability of the errors on correlation coefficients is established by using Monte Carlo simulations.

1 Introduction

We extract the W boson mass m_W from three kinematic distributions: transverse mass m_T, transverse momentum of charged lepton p_T^l and transverse momentum of neutrino p_T^ν. Since the three distributions are constructed with the same W candidate events, the fitted mass values are thus statistically correlated. We present in this study a way to estimate the correlation coefficients ρ and the associated errors σ_ρ between m_T, p_T^l and p_T^ν.

2 Correlations between mass fits in $W \to \mu\nu$

We obtain the correlations between m_T, p_T^μ and p_T^ν fits by using large Monte Carlo samples. We use PYTHIA[1] Version 6.208 to generate $W \to \mu\nu$ events for this study. A vector file containing 100 M $W \to \mu\nu$ events are generated at each of the three W mass input values at 80 GeV, 80.45 GeV and 81 GeV. The two vector files generated at $m_W = 80$ GeV and $m_W = 81$ GeV are used to construct fitting templates, while the vector file generated at $m_W = 80.45$ GeV is splitted into 400 sub-samples with equal statistics to form pseudo-data files.

Each of the 400 psedo-data files are then fitted against the templates. The 400 groups of m_T, p_T^μ and p_T^ν fitting results are used to estimate the population correlation coefficient ρ by constructing the sample correlation coefficient r:

$$r_{XY} = \frac{E(XY) - E(X)E(Y)}{s_X s_Y} \tag{1}$$

$$= \frac{\frac{1}{n}\sum_{i=1}^{n} x_i y_i - \bar{x}\bar{y}}{\sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}\sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right)}} \quad , \tag{2}$$

where x_i (y_i) is the i^{th} value of fit type X (Y), S_X (S_Y) is the sample variance of fit type X (Y), $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$; n is the number of $(x_i,\ y_i)$ pairs. The 1σ error on ρ_{XY} can be estimated using Fisher's z transformation on the sample correlation coefficient r_{XY}. The Fisher's z transformation is given by [2]:

$$z = \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right) \tag{3}$$

where z is approximately normally distributed with mean $\frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right)$ and standard error $\frac{1}{\sqrt{n-3}}$. Thus we have

$$z^{+\sigma_z} \approx \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right) + \frac{1}{\sqrt{n-3}} \tag{4}$$

$$z^{-\sigma_z} \approx \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right) - \frac{1}{\sqrt{n-3}} \quad , \tag{5}$$

which lead to

$$r^{+\sigma_r} = r + \sigma_r \approx \frac{e^{2(z^{+\sigma_z})} - 1}{e^{2(z^{+\sigma_z})} + 1} \tag{6}$$

$$r^{-\sigma_r} = r - \sigma_r \approx \frac{e^{2(z^{-\sigma_z})} - 1}{e^{2(z^{-\sigma_z})} + 1} \quad . \tag{7}$$

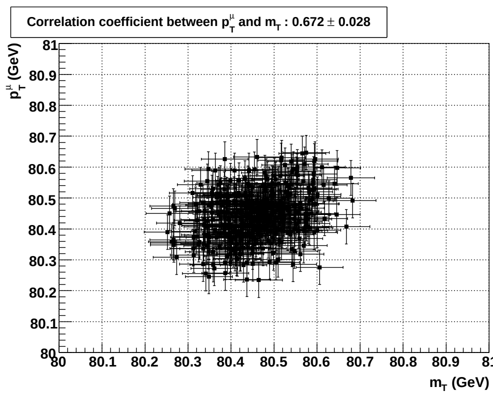
We symmetrize the $+1\sigma$ and -1σ to get

$$\sigma_\rho \approx \sigma_r \approx \frac{r^{+\sigma_r} - r^{-\sigma_r}}{2} \quad . \tag{8}$$

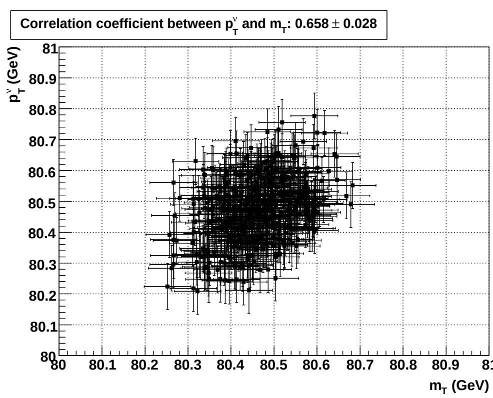
Figure 1 shows the correlations among m_T, p_T^μ and p_T^ν fits. The correlation coefficients are summarized in Table 1.

2.1 Monte Carlo study of σ_r

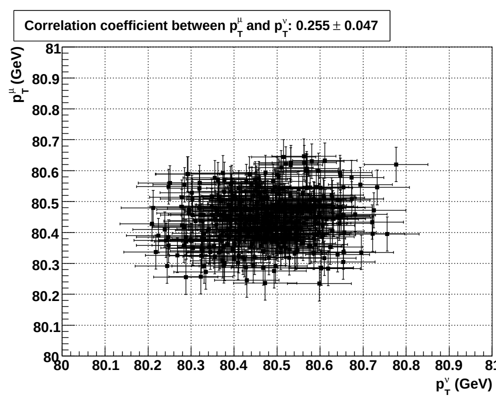
We use Monte Carlo simulation to cross-check the errors on the correlation coefficients calculated from Fisher's z transformation. For ease of explanation, we use random



(a) Correlation between m_T and p_T^μ fits.



(b) Correlation between m_T and p_T^ν fits.



(c) Correlation between p_T^μ and p_T^ν fits.

Figure 1: Correlations between m_T, p_T^μ and p_T^ν fits in $W \to \mu\nu$ channel.

Table 1: Statistical correlations between m_T, p_T^μ and p_T^ν fits in $W \to \mu\nu$ channel.

$W \to \mu\nu$ fit variable	Correlation Coefficient
m_T vs. p_T^μ	0.672 ± 0.028
m_T vs. p_T^ν	0.658 ± 0.028
p_T^μ vs. p_T^ν	0.255 ± 0.047

variables x_1, x_2, x_3 to represent m_T, p_T^μ and p_T^ν fits which we want to generate from simulation. The 1σ statistical errors on x_1, x_2 and x_3 are set to be the same as those obtained from data fits, i.e., $\sigma_1 = 0.015$ GeV, $\sigma_2 = 0.017$ GeV, and $\sigma_3 = 0.021$ GeV. The correlation coefficients among x_1, x_2 and x_3 are set to be the numbers shown in

Table 2: Comparison of two methods to estimate errors on correlation coefficients between m_T, p_T^μ and p_T^ν fits in $W \to \mu\nu$ channel.

$W \to \mu\nu$ fit variable	Error on r (Fisher)	Error on r (MC)
m_T vs. p_T^μ	0.028	0.027
m_T vs. p_T^ν	0.028	0.027
p_T^μ vs. p_T^ν	0.047	0.043

Table 1 with $r_{1,2} = r_{2,1} = 0.672$, $r_{1,3} = r_{3,1} = 0.658$ and $r_{2,3} = r_{3,2} = 0.255$. From the above information, the covariance matrix $\mathbf{\Sigma}$ can be constructed as

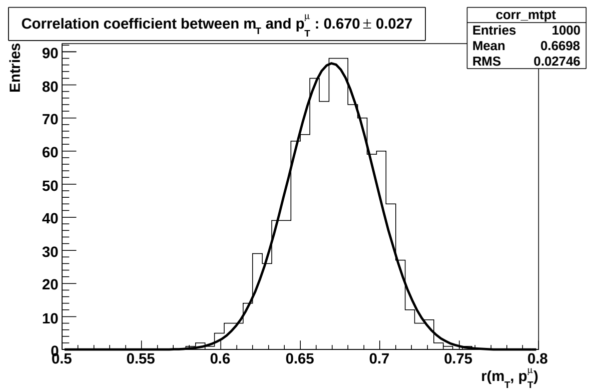
$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1^2 & r_{1,2}\sigma_1\sigma_2 & r_{1,3}\sigma_1\sigma_3 \\ r_{2,1}\sigma_1\sigma_2 & \sigma_2^2 & r_{2,3}\sigma_2\sigma_3 \\ r_{3,1}\sigma_1\sigma_3 & r_{3,2}\sigma_2\sigma_3 & \sigma_3^2 \end{pmatrix} .$$

Correlated random variables x_i ($i = 1, 2, 3$) can be generated from independent random variables z_i ($i = 1, 2, 3$) using a matrix decomposition method called Singular Value Decomposition[3] (SVD) according to the formula:

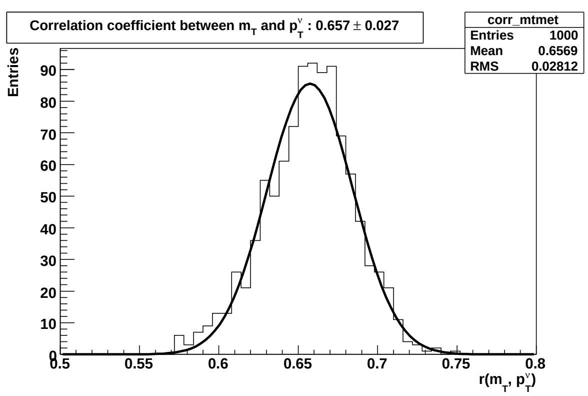
$$\mathbf{X} = \mathbf{M} + (\mathbf{U}\mathbf{D^{1/2}})\mathbf{Z} \quad , \tag{9}$$

where $\mathbf{X}$ is a vector of random variables x_i ($i = 1, \cdots, n$), $\mathbf{M}$ is a vector of m_i ($i = 1, \cdots, n$) with m_i equal to $\mathrm{E}(x_i)$, the mean value of x_i. $\mathbf{Z}$ is a vector of n independent N(0,1) random variables; $\mathbf{U}$ is the matrix of n eigenvectors obtained from covariance matrix $\mathbf{\Sigma}$, with the i-th column of $\mathbf{U}$ be the eigenvector correponding to the i-th eigenvalue λ_i from $\mathbf{\Sigma}$; $\mathbf{D}$ is a $n \times n$ diagonal matrix with diagonal elements to be the eigenvalues (λ_1, λ_2, $\cdots$, λ_n) of covariance matrix $\mathbf{\Sigma}$. $\mathbf{D^{1/2}}$ is a diagonal matrix with diagonal elements $\sqrt{\lambda_1}$, $\sqrt{\lambda_2}$, $\cdots$, $\sqrt{\lambda_n}$.

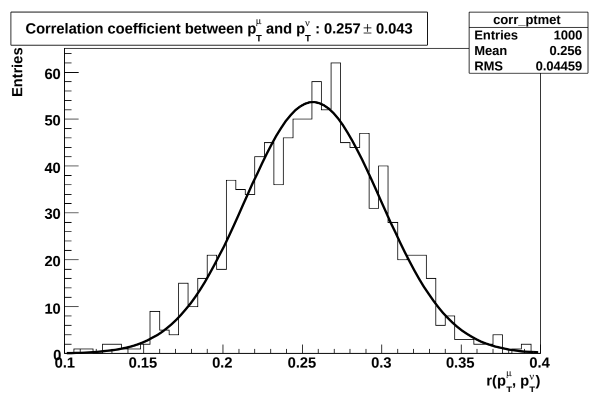
In our study with $n = 3$, we assume the expected value from all three fits are the same by requiring $m_i = E(x_i) = 80.45$ GeV ($i = 1, 2, 3$). Once 400 groups of x_1, x_2, x_3 values are simulated using the SVD method, we then use Eqn. (1) to calculate r_{XY}. Take m_T (x_1) and p_T^μ (x_2) as an example, 400 (x_1, x_2) pairs are used to obtain one value of $r_{1,2}$. We then repeat this process 1000 times to get a distribution of r_{12}, as illustrated in Figure 2 (a). The standard deviation of the Gaussian fit gives an estimate of the error on the correlation coefficient between x_1 and x_2. The same procedure is adopted to estimate the errors on other correlation coefficients (see Figure 2 (b) and (c)). Table 2 summarizes simulated errors on correlation coefficients and compares them with the errors calculated using Fisher's z transformation. We conclude the obtained errors on correlation coefficients using Fisher's z transformation are reliable.



(a) Error on m_T-and-p_T^μ correlation coefficient.



(b) Error on m_T-and-p_T^ν correlation coefficient.



(c) Error on p_T^μ-and-p_T^ν correlation coefficient.

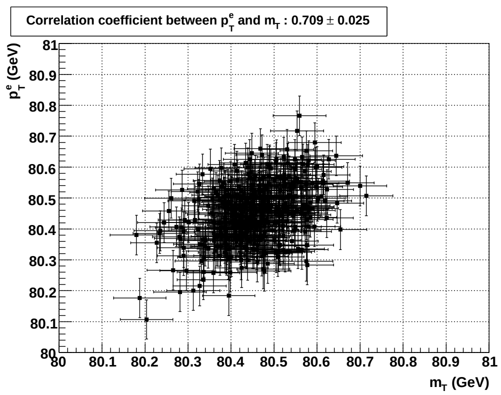
Figure 2: Monte Carlo study of the errors on correlation coefficients between m_T, p_T^μ and p_T^ν fits in $W \to \mu\nu$ channel.

3 Correlations between mass fits in $W \to e\nu$

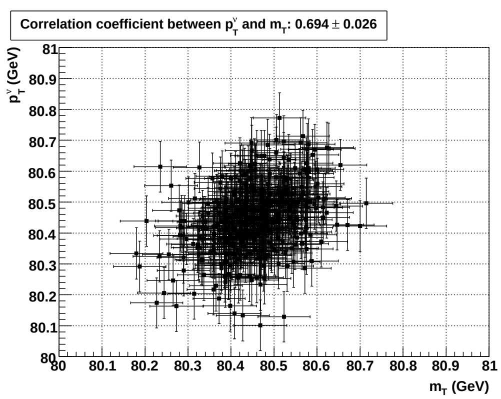
The same procedure is adopted in studying the correlations between m_W mass fits in $W \to e\nu$ channel. The scatter plots are shown in Figure 3 and results are summarized in 3.

4 Summary

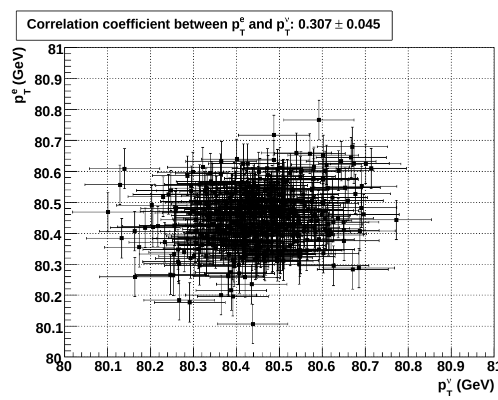
The correlation coefficients on W mass fits from m_T, p_T^l and p_T^ν for 2 fb^{-1} W mass analysis are obtained using large Monte Carlo samples. We find the correlation coefficients in m_T-p_T^l, m_T-p_T^ν and p_T^l-p_T^ν to be 0.672 ± 0.028, 0.658 ± 0.028 and 0.255 ± 0.047



(a) Correlation between m_T and p_T^e fits.



(b) Correlation between m_T and p_T^ν fits.



(c) Correlation between p_T^e and p_T^ν fits.

Figure 3: Correlations between m_T, p_T^e and p_T^ν fits in $W \to e\nu$ channel.

Table 3: Statistical correlations between m_T, p_T^e and p_T^ν fits in $W \to e\nu$ channel.

$W \to e\nu$ fit variable	Correlation Coefficient
m_T vs. p_T^e	0.709 ± 0.025
m_T vs. p_T^ν	0.694 ± 0.026
p_T^e vs. p_T^ν	0.307 ± 0.045

for $W \to \mu\nu$ channel, while 0.709 ± 0.025, 0.694 ± 0.026 and 0.307 ± 0.045 for $W \to e\nu$ channel.

References

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[2] R. Fisher, Biometrika **10**:507-521 (1915); R. Fisher, Metron **1**:3-32 (1921).

[3] G. Strang, Introduction to Linear Algebra, 3rd edition, Wellesley-Cambridge Press (2003).

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