Master formula for one-loop renormalization of bosonic SMEFT operators

Gerhard Buchalla[a], Alejandro Celis[a], Claudius Krause[b] and Jan-Niklas Toelstede[a,c]

[a]Ludwig-Maximilians-Universität München, Fakultät für Physik,
Arnold Sommerfeld Center for Theoretical Physics, D–80333 München, Germany

[b]Theoretical Physics Department, Fermi National Accelerator Laboratory,
Batavia, IL, 60510, USA

[c]Physik Department T31, James-Franck-Straße 1,
Technische Universität München,
D–85748 Garching, Germany

Abstract

Using background-field method and super-heat-kernel expansion, we derive a master formula for the one-loop UV divergences of the bosonic dimension-6 operators in Standard Model Effective Field Theory (SMEFT). This approach reduces the calculation of all the UV divergences to algebraic manipulations. Using this formula we corroborate results in the literature for the one-loop anomalous dimension matrix of SMEFT obtained via diagrammatic methods, considering contributions from the operators $X^3, \phi^6, \phi^4 D^2, X^2 \phi^2$ of the Warsaw basis. The formula is derived in a general way and can be applied to other quantum field theories as well.

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1 Introduction

In the background field method, the one-loop effective action for a general quantum field theory involving fermions and bosons can be written elegantly in terms of the super-determinant of a fluctuation operator [1–3]. This formulation is particularly suitable for the calculation of the ultraviolet (UV) one-loop divergences of the theory, as the problem is reduced to algebraic manipulations in which gauge invariance for the background fields is manifest during all stages of the computation [4]. These methods have been used in the one-loop renormalization of effective theories for QCD at low energies [5, 6] and, more recently, for effective theories of the Higgs sector [7].

When the fluctuation operator can be cast in the minimal form $D_\mu D^\mu + Y$, the one-loop divergences are expressed with a compact formula (in terms of the standard expression for the second Seeley–DeWitt coefficient [1–3]). Renormalizable gauge theories fall in this category if an appropriate choice is made for the gauge fixing of the fluctuating fields [8]. When the fluctuation operator is non-minimal, obtaining a closed expression for the one-loop divergences is a much more challenging task. It is not hard to find these situations. For instance, gauge theories with a general R_ξ-gauge fixing for the fluctuating fields will give non-minimal operators. A calculation of this type has been done within the context of chiral perturbation theory in [9].

In this work we are interested in the one-loop renormalization of the Standard Model Effective Field Theory (SMEFT) at the level of dimension-6 operators [10, 11]. The complete derivation of the one-loop anomalous dimension matrix in this case was accomplished via diagrammatic methods in [12–15]. We present here a first step in the program of performing this calculation relying on functional methods. Using background field method and super-heat-kernel expansion, we derive a master formula for the one-loop UV divergences of all the operators of the type $X^3, \phi^6, \phi^4 D^2, X^2 \phi^2$ from the Warsaw basis [11]. Some of these operators have multiple derivatives and introduce non-minimal structures in the bosonic part of the fluctuation operator. We will present the master formula in general terms as it can be useful within other quantum field theories. The results presented in this article have also been discussed in the thesis [16].

Using the master formula we provide a non-trivial cross-check of the results in [12–14], finding agreement with the renormalization group equations (RGEs) when taking into account contributions from the operators $X^3, \phi^6, \phi^4 D^2, X^2 \phi^2$. Such cross-checks are relevant given the phenomenological impact of renormalization group effects when analyzing scenarios of physics beyond the Standard Model (SM) [14, 17, 18].

The article is organized as follows. In Sec. 2 we review the super-heat-kernel approach to the one-loop effective action and derive a master formula for the UV divergences. In Sec. 3 we provide the context for the application of the master formula to SMEFT. The derivation of the RGEs from the one-loop divergences is discussed in Sec. 4. In Sec. 5 we give details on our calculation of divergences and RGEs for the bosonic dimension-6 operators of SMEFT. We summarize the results in Sec. 6 and conclude in Sec. 7. App. A contains details on the real representation of the Higgs field. App. B summarizes the building blocks of the master formula and the beta functions of the SM at dimension 4.

2 Master formula

2.1 One-loop effective action

We consider a quantum field theory with real boson fields (ϕ) and spin-1/2 fermion fields (ψ). In the background field method, fields are expanded in terms of a background field that satisfies the classical equations of motion and a fluctuating field: $\phi \to \phi + \xi$, $\psi \to \psi + \eta$, $\bar{\psi} \to \bar{\psi} + \bar{\eta}$ [4]. We assume that the action of the theory expanded to second order in the fluctuations can be written as

$$S^{(2)} = \int d^d x \left[-\frac{1}{2}\xi^T A \xi + \bar{\eta} B \eta + \bar{\eta}\Gamma\xi + \xi^T \bar{\Gamma}\eta \right] . \tag{1}$$

We work in d-dimensional Minkowski space with $d = 4 - 2\epsilon$, following the usual prescription in dimensional regularization. The differential operators A, B and the fermionic functions $\Gamma, \bar{\Gamma}$ depend on the background fields. One-loop corrections to the effective action are given by [1–3]

$$\Gamma^{1\text{L}}_{\text{eff}}[\phi, \psi, \bar{\psi}] = \frac{i}{2}\text{Str} \ln K , \tag{2}$$

with

$$K = \begin{pmatrix} A & \bar{\Gamma} & -\Gamma^T \\ -\bar{\Gamma}^T & 0 & B^T \\ \Gamma & -B & 0 \end{pmatrix} . \tag{3}$$

Here Str stands for supertrace. For a review of supermatrix algebra see [19]. We will distinguish supertraces with and without integration over Minkowski space $\text{Str}\, O = \int d^d x\, \text{str} \langle x|O|x\rangle$. It is useful to write the one-loop effective action in the form [3]

$$\Gamma^{1\text{L}}_{\text{eff}}[\phi, \psi, \bar{\psi}] \equiv \frac{i}{2}\text{Str} \ln \Delta = \frac{i}{2}\text{Str} \ln \begin{pmatrix} A & \sqrt{2}i\bar{\Gamma}\gamma_5 B\gamma_5 \\ \sqrt{2}i\Gamma & B\gamma_5 B\gamma_5 \end{pmatrix} , \tag{4}$$

which applies in the present case, where operators are at most bilinear in fermion fields. When the differential operator takes the form $A = (\partial^\mu + N^\mu)(\partial_\mu + N_\mu) + M$ and $B = i\slashed{\partial} - F$, it is possible to cast the fluctuation operator Δ into the minimal form

$$\Delta = D^\mu D_\mu + Y , \tag{5}$$

with $D_\mu = \partial_\mu + X_\mu$

$$X_\mu = \begin{pmatrix} N_\mu & \frac{1}{\sqrt{2}}\bar{\Gamma}\gamma_\mu \\ 0 & \frac{i}{2}(F\gamma_\mu - \gamma_\mu F_5) \end{pmatrix} ,$$

$$Y = \begin{pmatrix} M & -\frac{1}{\sqrt{2}}\bar{\Gamma}(\overleftarrow{\slashed{\partial}} - \slashed{N} + \frac{i}{2}\gamma_\mu F\gamma^\mu) \\ \sqrt{2}i\Gamma & \frac{1}{4}(F\gamma_\mu F\gamma^\mu + \gamma_\mu F_5\gamma^\mu F_5 - \gamma_\mu F_5 F\gamma^\mu) - \frac{i}{2}\partial_\mu(F\gamma^\mu + \gamma^\mu F_5) \end{pmatrix} . \tag{6}$$

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Here we have defined $F_5 = \gamma_5 F \gamma_5$. Furthermore, we will assume that F can be decomposed as

$$F = r P_R + l P_L + R_\mu \gamma^\mu P_R + L_\mu \gamma^\mu P_L , \tag{7}$$

with $P_{L,R}$ the chiral projectors.

2.2 Minimal operator

For the operator (5), the one-loop divergences at the level of the Lagrangian ($\Gamma_{\text{eff}}^{1L} = \int d^d x \, \mathcal{L}^1$) are given by the usual formula [1–3]

$$\mathcal{L}_{\text{div}}^1(\Delta) = \frac{1}{16\pi^2(4-d)} \text{str}\left(\frac{1}{12} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} Y^2\right) , \tag{8}$$

with $X_{\mu\nu} = [D_\mu, D_\nu] = \partial_\mu X_\nu - \partial_\nu X_\mu + [X_\mu, X_\nu]$. Eq. (8) can be developed further by evaluating the supertrace and by performing the trace over Dirac indices. This leads to the well known 't Hooft formula [8]

$$\mathcal{L}_{\text{div}}^1(\Delta) = \frac{1}{16\pi^2(4-d)} \text{tr}\bigg(\frac{1}{12} N_{\mu\nu} N^{\mu\nu} + \frac{1}{2} M^2 + i\bar{\Gamma} \slashed{D} \Gamma$$

$$+ 2(D_\mu l)(D^\mu r) - 2(lr)^2 - \frac{1}{3} R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} L_{\mu\nu} L^{\mu\nu}\bigg) . \tag{9}$$

Here we use the definitions

$$D_\mu \Gamma \equiv \partial_\mu \Gamma + N_\mu \Gamma - \frac{i}{2} F \gamma_\mu \Gamma , \quad N_{\mu\nu} \equiv \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu] ,$$

$$D_\mu r \equiv \partial_\mu r + i L_\mu r - i r R_\mu , \qquad R_{\mu\nu} \equiv \partial_\mu R_\nu - \partial_\nu R_\mu + i[R_\mu, R_\nu] , \tag{10}$$

$$D_\mu l \equiv \partial_\mu l + i R_\mu l - i l L_\mu , \qquad L_{\mu\nu} \equiv \partial_\mu L_\nu - \partial_\nu L_\mu + i[L_\mu, L_\nu] .$$

2.3 Extension for non-minimal operator

We will consider a fluctuation operator that receives the following corrections in the bosonic block of the supermatrix

$$\Delta' = \Delta + \begin{pmatrix} c_B + 2b_B^\mu D_\mu + a_B^{\mu\nu} D_\mu D_\nu & 0 \\ 0 & 0 \end{pmatrix} . \tag{11}$$

The operator Δ is assumed to be written in the minimal form (5). Here $D_\mu = \partial_\mu + N_\mu$ is the covariant derivative in the bosonic sector, c_B and $a_B^{\mu\nu} = a_B^{\nu\mu}$ are symmetric in the field indices while b_B^μ is antisymmetric. The terms c_B and b_B^μ can be absorbed into M and N^μ of Δ, however it will be useful to keep these corrections separate. The correction

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$a_B^{\mu\nu} D_\mu D_\nu$ introduces a non-minimal structure in Δ', similar to what occurs in a gauge theory with a generic R_ξ gauge fixing term. We treat the corrections $(c_B, b_B^\mu, a_B^{\mu\nu})$ as first-order perturbations of Δ, which is sufficient for the one-loop renormalization of SMEFT at the level of dimension-6 operators. This feature simplifies considerably the calculation.

In order to derive the one-loop UV divergences using the super-heat-kernel method, it is convenient to write Δ' in the form

$$\Delta' = \Delta + c + 2b^\mu D_\mu + a^{\mu\nu} D_\mu D_\nu \,, \tag{12}$$

where now the covariant derivates are acting in superspace $D_\mu = \partial_\mu + X_\mu$, with X_μ defined in (6). The new terms $(c, b^\mu, a^{\mu\nu})$ are given by

$$c = \begin{pmatrix} c_B & \sqrt{2}\bar{\Gamma}\slashed{b}_B - \frac{1}{\sqrt{2}}(\partial_\mu\bar{\Gamma})a_B^{\mu\nu}\gamma_\nu + \frac{1}{\sqrt{2}}\bar{\Gamma}N_\mu a_B^{\mu\nu}\gamma_\nu + \frac{1}{\sqrt{2}}\bar{\Gamma}a_B^{\mu\nu}\gamma_\mu X_{22\nu} \\ 0 & 0 \end{pmatrix} \,,$$

$$b^\mu = \begin{pmatrix} b_B^\mu & -\frac{1}{\sqrt{2}}\bar{\Gamma}a_B^{\mu\nu}\gamma_\nu \\ 0 & 0 \end{pmatrix} \,, \tag{13}$$

$$a^{\mu\nu} = \begin{pmatrix} a_B^{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \,.$$

Here a, b and c are supermatrices and $X_{22\nu}$ is the lower right entry of X_ν.

The one-loop UV divergences associated with c and b^μ can be obtained from Eq. (8) after a redefinition of Y and X_μ, keeping terms linear in c and b^μ. We focus in the following on the corrections due to $a^{\mu\nu}$. To evaluate the divergences from $a^{\mu\nu}$ we use the integral representation (dropping an irrelevant constant)

$$\text{Str} \ln \Delta' = -\int d^d x \; \text{str} \int_0^\infty \frac{d\tau}{\tau} \langle x|e^{-\tau\Delta'}|x\rangle \,, \tag{14}$$

and implement the heat-kernel expansion [20, 21]

$$\langle x|e^{-\tau\Delta'}|x\rangle \equiv \frac{i}{(4\pi\tau)^{\frac{d}{2}}} \sum_{n=0}^\infty a_n(x,x)\tau^n \,. \tag{15}$$

The one-loop UV divergences are contained in the hermitean part of the second Seeley–DeWitt coefficient [21]

$$\mathcal{L}_{\text{div}}^1(\Delta') = \frac{1}{2(4\pi)^2(4-d)}\text{str}\left(a_2(x,x) + a_2(x,x)^\dagger\right) \,. \tag{16}$$

The latter can be obtained from (14) by going to momentum space and extracting the coefficient of τ^2:

$$a_2(x,x) = -i\int \frac{d^d p}{\pi^{\frac{d}{2}}} e^{p^2} e^{-\left(\tau\Delta + \sqrt{\tau}2ip^\mu D_\mu + \tau a^{\mu\nu}D_\mu D_\nu + \sqrt{\tau}a^{\mu\nu}2ip_\mu D_\nu - a^{\mu\nu}p_\mu p_\nu\right)}\bigg|_{\tau^2} \,. \tag{17}$$

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Rearranging terms in the final result, and including the contributions from b and c, we get the following one-loop UV divergences from Δ'

$$\mathcal{L}^1_{\text{div}}(\Delta') = \mathcal{L}^1_{\text{div}}(\Delta) + \frac{1}{2}\frac{1}{(4\pi)^2(4-d)}\text{str}\left(cY + \frac{1}{3}X_{\mu\nu}[D^\mu, b^\nu] - Y[D_\mu, b^\mu]\right.$$

$$- \frac{1}{24}a^\lambda_{\ \lambda}X_{\mu\nu}X^{\mu\nu} + \frac{1}{6}a^{\mu\nu}X_{\mu\lambda}X_\nu^{\ \lambda} + \frac{1}{6}X_{\mu\lambda}[D_\nu, [D^\lambda, a^{\mu\nu}]] \tag{18}$$

$$\left.- \frac{1}{4}a^\lambda_{\ \lambda}Y^2 + \frac{1}{3}Y[D_\mu, [D_\nu, a^{\mu\nu}]] - \frac{1}{12}Y[D_\mu, [D^\mu, a^\lambda_{\ \lambda}]] + \text{h.c.}\right).$$

The term $\mathcal{L}^1_{\text{div}}(\Delta)$ represents the divergences associated with the minimal operator Δ in (12), these are given by (8). The remaining terms contain the corrections introduced by $(c_B, b^\mu_B, a^{\mu\nu}_B)$.

We can simplify the result by evaluating the supertrace and arrive at a master formula for $\delta\mathcal{L}^1_{\text{div}} \equiv \mathcal{L}^1_{\text{div}}(\Delta') - \mathcal{L}^1_{\text{div}}(\Delta)$:

$$\delta\mathcal{L}^1_{\text{div}} = \frac{1}{(4\pi)^2(4-d)}\text{tr}\left(c_B M + \frac{1}{3}N_{\mu\nu}[D^\mu, b^\nu_B] + i\bar{\Gamma}\slashed{b}_B\Gamma - \frac{1}{6}\bar{\Gamma}i\overset{\leftrightarrow}{\mathcal{D}}_{a_B}\Gamma\right.$$

$$- \frac{1}{24}a^\lambda_{B\lambda}N_{\mu\nu}N^{\mu\nu} + \frac{1}{6}a^{\mu\nu}_B N_{\mu\lambda}N_\nu^{\ \lambda} + \frac{1}{6}N_{\mu\lambda}[D_\nu, [D^\lambda, a^{\mu\nu}_B]] \tag{19}$$

$$\left.- \frac{1}{4}a^\lambda_{B\lambda}M^2 - \frac{1}{12}M[D_\mu, [D^\mu, a^\lambda_{B\lambda}]] + \frac{1}{3}M[D_\mu, [D_\nu, a^{\mu\nu}_B]]\right).$$

Here we use $N_{\mu\nu} = \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu]$ and

$$\bar{\Gamma}i\overset{\leftrightarrow}{\mathcal{D}}_{a_B}\Gamma \equiv \left(i\bar{\Gamma}a^{\mu\nu}_B\gamma^\lambda(\partial^\rho + N^\rho)\Gamma - i\bar{\Gamma}\gamma^\lambda(\overset{\leftarrow}{\partial^\rho} - N^\rho)a^{\mu\nu}_B\Gamma\right)(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho})$$

$$+ \frac{1}{2}\bar{\Gamma}a^{\mu\nu}_B\gamma^\lambda F\gamma^\rho\Gamma(g_{\mu\nu}g_{\lambda\rho} + 2g_{\mu\lambda}g_{\nu\rho}). \tag{20}$$

To derive (19) we assumed that $a^{\mu\nu}_B$ and N^μ have a block diagonal structure with respect to the bosonic variables from spin-0 and spin-1 fields.

In the particular case $a^{\mu\nu}_B = a_B g^{\mu\nu}$, the master formula (19) simplifies considerably

$$\delta\mathcal{L}^1_{\text{div}} = \frac{1}{(4\pi)^2(4-d)}\text{tr}\left(c_B M + \frac{1}{3}N_{\mu\nu}[D^\mu, b^\nu_B] + i\bar{\Gamma}\slashed{b}_B\Gamma\right.$$

$$\left.- a_B M^2 - \frac{1}{2}\bar{\Gamma}a_B i\slashed{D}\Gamma + \frac{1}{2}\bar{\Gamma}i\overset{\leftarrow}{\slashed{D}}a_B\Gamma\right), \tag{21}$$

with the covariant derivatives on fermions Γ, $\bar{\Gamma}$

$$i\slashed{D} = i\slashed{\partial} + i\slashed{N} + \frac{1}{2}\gamma_\mu F\gamma^\mu \qquad \text{and} \qquad i\overset{\leftarrow}{\slashed{D}} = i\overset{\leftarrow}{\slashed{\partial}} - i\slashed{N} - \frac{1}{2}\gamma_\mu F\gamma^\mu. \tag{22}$$

3 Application to SMEFT

Assuming that the SM degrees of freedom are the only ones present at energies below a certain high-energy scale $\Lambda \gg M_W$ where new dynamics enters, we can parametrize physics at low energies ($E \ll \Lambda$) by the effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right) . \tag{23}$$

Here $\mathcal{L}_{\text{SM}}$ is the renormalizable SM Lagrangian, $Q_k^{(5,6)}$ are higher-dimensional operators of dimensions 5 and 6, $C_k^{(5,6)}$ are their Wilson coefficients. At dimension 5 there is only the so-called Weinberg operator. A non-redundant basis of dimension-6 operators was defined in [11].

We denote the SU(3) and SU(2) generators by $T^A = \frac{1}{2}\lambda^A$ and $\tau^a = \frac{1}{2}\sigma^a$, where λ^A and σ^a are the Gell-Mann and Pauli matrices, respectively. The Higgs field is parametrized in terms of 4 real components φ_i:

$$H \equiv \frac{1}{\sqrt{2}}\left(\widetilde{\phi}, \phi\right) = i\tau^i \varphi_i . \tag{24}$$

Here ϕ is the Higgs doublet and $\widetilde{\phi} = i\sigma_2 \phi^*$ its conjugate, $i \in \{0,1,2,3\}$ and we defined $\tau^0 = -\frac{i}{2}\mathbf{1}$. Using the real representation for the Higgs field we can write the SM Lagrangian as (our sign conventions for covariant derivatives and field strengths coincide with [11])

$$\begin{aligned}
\mathcal{L}_{\text{SM}} = &-\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&+ \frac{1}{2}(D_\mu\varphi)_i(D^\mu\varphi)_i + \frac{m^2}{2}\varphi_i\varphi_i - \frac{\lambda}{8}(\varphi_i\varphi_i)^2 \\
&+ \bar{\psi}i\slashed{D}\psi - \left(\bar{\psi}\sqrt{2}H\mathcal{Y}P_R\psi + \text{h.c.}\right) .
\end{aligned} \tag{25}$$

Fermions have been collected into $\psi = (u,d,\nu,e)^T$, where every component is a Dirac spinor. The Yukawa matrices are grouped into $\mathcal{Y} = \text{diag}(\mathcal{Y}_u, \mathcal{Y}_d, \mathcal{Y}_\nu, \mathcal{Y}_e)$. We will take $\mathcal{Y}_\nu = 0$ in the following. More details about the real representation of the Higgs field used here are provided in App. A. We expand the fields around their background component as

$$\begin{aligned}
G^{A\mu} &\to G^{A\mu} + \alpha^{A\mu} , & \varphi_i &\to \varphi_i + \xi_i , \\
W^{a\mu} &\to W^{a\mu} + \omega^{a\mu} , & \psi &\to \psi + \eta , \\
B^\mu &\to B^\mu + \beta^\mu , & \bar{\psi} &\to \bar{\psi} + \bar{\eta} .
\end{aligned} \tag{26}$$

The operator Δ in (12) will contain the fluctuations of the SM Lagrangian while the corrections generated by the dimension-6 SMEFT operators will be encoded in $(c_B, b_B^\mu, a_B^{\mu\nu})$.

Table 1: *Dimension-6 bosonic operators of the SMEFT from the Warsaw basis [11]. Here* $\tilde{X}_{\mu\nu} \equiv 1/2\,\epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$, $\epsilon_{0123} = +1$.

	X^3		$X^2\phi^2$
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\phi G}$	$\phi^\dagger\phi\,G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\phi B}$	$\phi^\dagger\phi\,B_{\mu\nu} B^{\mu\nu}$
Q_W	$\epsilon^{abc}W_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$	$Q_{\phi W}$	$\phi^\dagger\phi\,W_{\mu\nu}^a W^{a\mu\nu}$
$Q_{\widetilde{W}}$	$\epsilon^{abc}\widetilde{W}_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$	$Q_{\phi WB}$	$\phi^\dagger\sigma^a\phi\,W_{\mu\nu}^a B^{\mu\nu}$
	ϕ^6	$Q_{\phi\widetilde{G}}$	$\phi^\dagger\phi\,\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_ϕ	$(\phi^\dagger\phi)^3$	$Q_{\phi\widetilde{B}}$	$\phi^\dagger\phi\,\widetilde{B}_{\mu\nu} B^{\mu\nu}$
	$\phi^4 D^2$	$Q_{\phi\widetilde{W}}$	$\phi^\dagger\phi\,\widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$
$Q_{\phi\Box}$	$(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$	$Q_{\phi\widetilde{W}B}$	$\phi^\dagger\sigma^a\phi\,\widetilde{W}_{\mu\nu}^a B^{\mu\nu}$
$Q_{\phi D}$	$(\phi^\dagger D^\mu\phi)^*(\phi^\dagger D_\mu\phi)$		

We group the bosonic fluctuations into $(\alpha^{A\alpha}, \omega^{a\lambda}, \beta^\sigma, \xi_i)$. To write the bosonic kinetic term as in (1) we redefine the gauge field fluctuations $(\alpha^{A\mu}, \omega^{a\mu}, \beta^\mu) \to i(\alpha^{A\mu}, \omega^{a\mu}, \beta^\mu)$. The SM fluctuation operator written in minimal form is given in App. B.

The master formula (19) can be applied to all the SMEFT operators of the type [11]

$$X^3,\ \phi^6,\ \phi^4 D^2,\ X^2\phi^2\,. \tag{27}$$

See Table 1 for a detailed list. These give rise to a fluctuation operator that can be cast in the form (11). Using (19) we calculate the one-loop UV divergences generated by these operators, and evaluate the corresponding contributions to the renormalization group equations.

Under renormalization the operators in Table 1 not only mix among each other, but also into dimension-6 operators of the classes $\psi^2\phi^3$, $\psi^2 X\phi$ and $\psi^2\phi^2 D$, in the notation of [11]. Using the representation ψ for the SM fermions, introduced above, these operators can be expressed in a compact form. The operators in the class $\psi^2\phi^3$ can be combined into a single term:

$$(\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}HC_{\psi\phi}\psi_R + \text{h.c.}\,, \tag{28}$$

where, as in the case of the Yukawa matrices, the Wilson coefficients have been collected into a matrix in flavour space as $C_{\psi\phi} = \text{diag}(C_{u\phi}, C_{d\phi}, 0, C_{e\phi})$. The same scheme can be applied to the $\psi^2 X\phi$ operators. The resulting structures and flavour matrices are given

by

$$\bar{\psi}\sigma^{\mu\nu}G^A_{\mu\nu}T^A\sqrt{2}HC_{\psi G}\psi \ , \qquad C_{\psi G} = \mathrm{diag}\left(C_{uG}, C_{dG}, 0, 0\right) \ ,$$

$$\bar{\psi}\sigma^{\mu\nu}W^a_{\mu\nu}\sigma^a\sqrt{2}HC_{\psi W}\psi \ , \qquad C_{\psi W} = \mathrm{diag}\left(C_{uW}, C_{dW}, 0, C_{eW}\right) \ , \qquad (29)$$

$$\bar{\psi}\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}HC_{\psi B}\psi \ , \qquad C_{\psi B} = \mathrm{diag}\left(C_{uB}, C_{dB}, 0, C_{eB}\right) \ .$$

Finally, using the real representation of the covariant derivatives in appendix A, we can reduce the operators of the remaining class $\psi^2\phi^2 D$ to the form

$$-2i\bar{\psi}_R\gamma^\mu C_{\phi\psi 2}\varphi(\tau^1 t_R^1 + \tau^2 t_R^2)D_\mu\varphi\psi_R \ , \qquad C_{\phi\psi 2} = \mathrm{diag}(C_{\phi ud}, C_{\phi du}, 0, 0) \ ,$$

$$2i(\varphi t_R^3 D_\mu\varphi)\bar{\psi}_R\gamma^\mu C_{\phi\psi}\psi_R \ , \qquad C_{\phi\psi} = \mathrm{diag}(C_{\phi u}, C_{\phi d}, 0, C_{\phi e}) \ ,$$

$$2i(\varphi t_R^3 D_\mu\varphi)\bar{\psi}_L\gamma^\mu C^{(1)}_{\phi\psi}\psi_L \ , \qquad C^{(1)}_{\phi\psi} = \mathrm{diag}(C^{(1)}_{\phi q}, C^{(1)}_{\phi l}) \ , \qquad (30)$$

$$2i(\varphi t_L^a D_\mu\varphi)\bar{\psi}_L\sigma^a\gamma^\mu C^{(3)}_{\phi\psi}\psi_L \ , \qquad C^{(3)}_{\phi\psi} = \mathrm{diag}(C^{(3)}_{\phi q}, C^{(3)}_{\phi l}) \ .$$

The coefficient matrices are chosen such that the corresponding operators are hermitian. For this, we had to generalize the first operator that contains ϕud.

In the following sections we discuss the renormalization group equations for SMEFT and present the calculation of the one-loop UV divergences for the bosonic dimension-6 operators. We find agreement with the renormalization group equations presented in [12–14].[1]

4   Renormalization group equations

We next present a concise derivation of the one-loop renormalization group equations of dimension-6 operators in SMEFT. The results will allow us to convert the UV divergences of the one-loop corrections into the beta functions for the operator coefficients.

The coefficients and operators of the dimension-6 Lagrangian in (23) are renormalized as

$$C_i^{(0)}Q_i^{(0)} = Z_{ij}C_j Z_F^{(i)}Q_i = C_iQ_i + Q_i(Z_F^{(i)}Z_{ij} - \delta_{ij})C_j \,. \qquad (31)$$

The factor $Z_F^{(i)}$ collects the field-renormalization constants coming from the fields of the operator Q_i. A summation over i and j is understood. It will be convenient to define

$$Z_{ij} = \delta_{ij} + \frac{K_{ij}}{32\pi^2\epsilon} \,, \qquad Z_F^{(i)} = 1 + \frac{K_{(i)}}{32\pi^2\epsilon} \qquad (32)$$

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[1]We take into account the errata in [22].

for the one-loop Z factors in minimal subtraction. Eq. (31) then implies for the counterterm Lagrangian

$$32\pi^2\epsilon\,\mathcal{L}_{\text{CT}} = Q_i\left(K_{(i)}\delta_{ij} + K_{ij}\right)C_j\,\frac{1}{\Lambda^2} \equiv -32\pi^2\epsilon\,\delta\mathcal{L}_{\text{div}}^1\,, \tag{33}$$

which is identical, up to a sign, to the divergent part of the one-loop effective Lagrangian $\delta\mathcal{L}_{\text{div}}^1$, to be computed with the methods discussed in section 2.

In order to relate the coefficients K_{ij} in (33) to the renormalization-group evolution of the C_j, we note that

$$C_i^{(0)} = \mu^{n_{(i)}\epsilon}Z_{ij}C_j\,. \tag{34}$$

Here the scale μ makes explicit the mass dimension $n_{(i)}\epsilon$ of the unrenormalized coefficient $C_i^{(0)}$. Z_{ij} and C_j are dimensionless. Differentiating (34) with respect to $t \equiv \ln\mu$, using (32) and working to one-loop accuracy, we find for the beta functions of the operator coefficients

$$\beta_i \equiv 16\pi^2\frac{dC_i}{dt} = -\frac{1}{2\epsilon}\frac{dK_{ij}}{dt}C_j + \frac{n_{(j)} - n_{(i)}}{2}K_{ij}C_j\,. \tag{35}$$

This expression can be simplified[2] using the concept of *chiral dimensions*, which keeps track of the loop order of terms in a quantum field theory [24].[3] Consider an operator containing a number of field-strength factors $X_{\mu\nu}$, scalar fields ϕ, fermions ψ and derivatives D, schematically

$$Q = X_{\mu\nu}^{N_X}\phi^{N_\phi}D^{N_D}\psi^{N_F}\,. \tag{36}$$

In $4 - 2\epsilon$ space-time dimensions this operator has *canonical dimension*

$$d(Q) = N_X(2 - \epsilon) + N_\phi(1 - \epsilon) + N_D + N_F\left(\frac{3}{2} - \epsilon\right)$$

$$= 2N_X + N_\phi + N_D + \frac{3}{2}N_F - (N_X + N_\phi + N_F)\epsilon\,. \tag{37}$$

In 4 space-time dimensions the canonical dimension is $d_0(Q) = 2N_X + N_\phi + N_D + 3/2N_F$, constrained to be $d_0(Q) = 6$ for the case at hand. Since the Lagrangian term $C^{(0)}Q^{(0)}/\Lambda^2$ has canonical dimension $4 - 2\epsilon$, it follows that

$$d(C^{(0)}) = (N_X + N_\phi + N_F - 2)\epsilon \tag{38}$$

for the coefficient $C^{(0)}$ of this operator. Using the *chiral dimension* of Q in (36),

$$\chi(Q) = N_X + N_D + \frac{1}{2}N_F\,, \tag{39}$$

we have $d(C^{(0)}) = (d_0(Q) - \chi(Q) - 2)\epsilon = (4 - \chi(Q))\epsilon$ and therefore

$$n_{(i)} = 4 - \chi_i \tag{40}$$

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[2]An equivalent discussion in terms of NDA weights has been given in [23].

[3]The chiral dimension is 0 for a boson field, and 1 for a weak coupling, a derivative, or a fermion bilinear. The loop order L of a term is related to its total chiral dimension χ through $\chi = 2L + 2$ [24].

for the parameter $n_{(i)}$ defined in (34). The canonical dimension of the coefficient $C^{(0)}$ can thus be expressed through the chiral dimension χ_i of the corresponding dimension-6 operator.

Inspecting (33), we note that the one-loop corrections of an operator Q_j yield the divergent terms

$$\Delta\mathcal{L}_{\text{div}}[Q_j] = -\frac{1}{32\pi^2\epsilon} \sum_i Q_i \left(K_{(i)}\, \delta_{ij} + K_{ij}\right) \frac{C_j}{\Lambda^2} \,. \tag{41}$$

The chiral dimension of this expression equals the chiral dimension of Q_j increased by 2, since it results from the insertion of Q_j into one-loop diagrams. Therefore

$$\chi_i + \chi(K_{(i)}\, \delta_{ij} + K_{ij}) = \chi_j + 2 \,. \tag{42}$$

It follows that

$$\chi(K_{ij}) = \chi_j - \chi_i + 2 \,. \tag{43}$$

For $i = j$ we also have $\chi(K_{(j)}) = \chi(K_{jj}) = 2$.

Since, by definition, fields and derivatives belong to the operators Q_i, the chiral dimension of K_{ij} can only arise from weak couplings. Eq. (43) implies that the coefficient K_{ij} is proportional to $\chi_j - \chi_i + 2$ weak couplings κ, which may be gauge or Yukawa couplings, or $\sqrt{\lambda}$. Each of those fulfills $d\kappa/d\ln t = -\epsilon\kappa + \mathcal{O}(1/16\pi^2)$. We then conclude that

$$\frac{d}{dt}K_{ij} = (\chi_i - \chi_j - 2)\epsilon\, K_{ij} \,. \tag{44}$$

Inserting (40) and (44) into (35), we finally obtain

$$\beta_i = K_{ij}C_j \,. \tag{45}$$

This means that the contributions to the beta function for coefficient C_i from the insertions of operator Q_j can be simply read off from the term K_{ij} in the counterterm Lagrangian. For the mixing among coefficients of dimension-6 operators with $i \neq j$, (45) can be applied immediately. In the case of $i = j$, (33) shows that $K_{(i)}$ has to be subtracted from the coefficient of the divergence $(K_{(i)} + K_{ii})$ to obtain the beta function entry K_{ii}. The $K_{(i)}$ are determined by the renormalization constants of the fields composing the operator Q_i. The renormalization factors needed for the bosonic operators are

$$Z_{fW} = 1 + \frac{g^2}{32\pi^2\epsilon}\left(\frac{44}{3} - \frac{2}{3}(N_c + 1)f - \frac{1}{3}\right) = 1 + \frac{g^2}{32\pi^2\epsilon}\frac{19}{3}$$

$$Z_{fB} = 1 - \frac{g'^2}{32\pi^2\epsilon}\left(\left(\frac{22N_c}{27} + 2\right)f + \frac{1}{3}\right) = 1 - \frac{g'^2}{32\pi^2\epsilon}\frac{41}{3}$$

$$Z_{fG} = 1 + \frac{g_s^2}{32\pi^2\epsilon}\frac{22N_c - 4N_f}{3} = 1 + \frac{g_s^2}{32\pi^2\epsilon}14$$

$$Z_{f\phi} = 1 + \frac{6g^2 + 2g'^2 - 2\gamma_\phi}{32\pi^2\epsilon} \tag{46}$$

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where $N_c = 3$, $N_f = 6$ and $f = 3$ denotes the number of colours, quark flavours and fermion families, respectively, and

$$\gamma_\phi = N_c \operatorname{tr}(\mathcal{Y}_u^\dagger \mathcal{Y}_u + \mathcal{Y}_d^\dagger \mathcal{Y}_d) + \operatorname{tr}(\mathcal{Y}_e^\dagger \mathcal{Y}_e) \equiv \langle \mathcal{Y}^\dagger \mathcal{Y} \rangle. \tag{47}$$

Using (46), we find the coefficients $K_{(i)}$ for the bosonic operators in Table 1:

$$K_{(G)} = K_{(\tilde{G})} = 21 g_s^2, \qquad K_{(W)} = K_{(\tilde{W})} = \frac{19}{2} g^2$$

$$K_{(\phi)} = 6(3g^2 + g'^2 - \gamma_\phi), \qquad K_{(\phi\square)} = K_{(\phi D)} = 4(3g^2 + g'^2 - \gamma_\phi)$$

$$K_{(\phi G)} = K_{(\phi\tilde{G})} = 14 g_s^2 + 6g^2 + 2g'^2 - 2\gamma_\phi$$

$$K_{(\phi W)} = K_{(\phi\tilde{W})} = \frac{37}{3} g^2 + 2g'^2 - 2\gamma_\phi$$

$$K_{(\phi B)} = K_{(\phi\tilde{B})} = 6g^2 - \frac{35}{3} g'^2 - 2\gamma_\phi$$

$$K_{(\phi WB)} = K_{(\phi\tilde{W}B)} = \frac{55}{6} g^2 - \frac{29}{6} g'^2 - 2\gamma_\phi. \tag{48}$$

Due to the presence of the mass parameter m in the leading-order Lagrangian, a dimension-6 operator $Q_j \equiv Q_j^{(6)}$ can also mix into dimension-4 operators $Q_i^{(4)}$. Such terms are generated from the one-loop corrections to Q_j in the form $m^2 Q_i^{(4)}$, which may formally be viewed as a dimension-6 operator. The master formula for the beta functions in (45) also applies in this case, once the normalization of the coupling associated with $Q_i^{(4)}$ has been properly taken into account. In particular, from the divergent one-loop corrections to Q_j proportional to dimension-4 terms,

$$32\pi^2 \epsilon \, \Delta\mathcal{L}_{\text{div}}^{(4)}[Q_j] = -\sum_i m^2 Q_i^{(4)} K_{ij} \frac{C_j}{\Lambda^2}, \tag{49}$$

we find the contribution to the beta function

$$\beta_i^{(4)} \supseteq \frac{m^2}{\Lambda^2} K_{ij} C_j \tag{50}$$

for $i = m^2$, λ, $\mathcal{Y}_{rs}$, where

$$Q_{m^2}^{(4)} = \phi^\dagger \phi, \qquad Q_\lambda^{(4)} = -\frac{1}{2}(\phi^\dagger \phi)^2, \qquad Q_{\mathcal{Y}_{rs}}^{(4)} = -\sqrt{2}\bar{\psi}_L^r H \psi_R^s. \tag{51}$$

From the gauge-kinetic terms

$$Q_{g_X}^{(4)} = X_{\mu\nu}^a X^{a\mu\nu} \tag{52}$$

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the beta function of the corresponding gauge coupling g_X receives the contribution

$$\beta_{g_X} \supseteq 2g_X \frac{m^2}{\Lambda^2} K_{g_X j} C_j \, . \tag{53}$$

Alternatively, we can apply the equations of motion of the scalar to set $Z_{f\phi} = 1$. For consistency, these have to be applied at the dimension-6 level, generating additional contributions:

$$\frac{1}{2} D_\mu \varphi_i D_\mu \varphi^i (-6g^2 - 2g'^2 + 2\gamma_\phi) \frac{1}{16\pi^2\epsilon} \rightarrow \frac{1}{16\pi^2\epsilon} (3g^2 + g'^2 - \gamma_\phi) \frac{\delta}{\delta\varphi} (\mathcal{L}_4 + \mathcal{L}_6), \tag{54}$$

yielding the same beta functions as the previously discussed procedure in all cases.

5 Bosonic operators

5.1 Renormalization of the operator class X^3

We begin our computation[4] by calculating the one-loop renormalization of the X^3-operators. The four operators of this class contain the field strengths of the gauge groups $SU(3)_C$ and $SU(2)_L$. To apply our algorithm, it is useful to distinguish operators with and without dual field strength. The group structure is the same in both cases, and we only have to work out the fluctuation matrices once. The divergences and the renormalization are then obtained for the two cases in an analogous way.

5.1.1 G- and W-operators

We start with the operator[5]

$$Q_G = f^{ABC} G^A_{\mu\nu} G^B_{\nu\lambda} G^C_{\lambda\mu} \, . \tag{55}$$

The symmetrized fluctuation matrices from (11) are given by

$$c_{(A\alpha)(B\beta)} = 6g_s f^{ABC} f^{CDE} G^D_{\alpha\lambda} G^E_{\lambda\beta} + 6g_s f^{ADC} f^{BEC} G^D_{\alpha\lambda} G^E_{\lambda\beta}$$

$$+ 3g_s f^{ADC} f^{BEC} G^D_{\mu\nu} G^E_{\mu\nu} g_{\alpha\beta} \, ,$$

$$b^\lambda_{(A\alpha)(B\beta)} = \frac{3}{2} f^{ABC} \left(D_\alpha G_{\beta\lambda} + D_\beta G_{\alpha\lambda} + g^{\alpha\lambda} D_\mu G_{\mu\beta} + g^{\beta\lambda} D_\mu G_{\mu\alpha} \right)^C \tag{56}$$

$$- 3f^{ABC} (D_\mu G^{\mu\lambda})^C g_{\alpha\beta} \, ,$$

$$a^{\mu\nu}_{(A\alpha)(B\beta)} = 3f^{ABC} \left(g^{\mu\alpha} G_{\nu\beta} + g^{\nu\alpha} G_{\mu\beta} - g^{\mu\beta} G_{\nu\alpha} - g^{\nu\beta} G_{\mu\alpha} - 2G_{\alpha\beta} g^{\mu\nu} \right)^C \, .$$

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[4]For cross-checks of our calculations, the programs FeynCalc [25, 26] and Mathematica [27] proved useful, as well as the compilation of formulas in [28].

[5]Throughout this chapter, we will sometimes drop the distinction of upper and lower Lorentz indices for notational convenience.

Here and in the following, the matrices a_B, b_B, c_B from (11) will simply be denoted as a, b, c. Since we are concerned with purely bosonic operators, no confusion can arise in the present context. Only the non-zero entries of these matrices are quoted explicitly.

Divergences and RGEs from G

We evaluate the constituents of the master formula (19) for the G-operator:

$$\text{tr } cM = -18g_s^2 C_3^{\text{ad}} f^{ABC} G_{\mu\nu}^A G_{\nu\lambda}^B G_{\lambda\mu}^C ,$$

$$-\frac{1}{3}\text{tr } [D^\mu, N_{\mu\nu}]b^\nu = -2g_s C_3^{\text{ad}} (D_\mu G^{\mu\lambda})^A (D^\nu G_{\nu\lambda})^A ,$$

$$-\frac{1}{4}\text{tr } a_\lambda^\lambda M^2 = 6g_s^2 C_3^{\text{ad}} f^{ABC} G_{\mu\nu}^A G_{\nu\lambda}^B G_{\lambda\mu}^C ,$$

$$-\frac{1}{12}\text{tr } a_\lambda^\lambda[D_\mu, [D^\mu, M]] = 2g_s C_3^{\text{ad}} G_{\mu\nu}^A (D_\lambda D^\lambda G^{\mu\nu})^A , \tag{57}$$

$$+\frac{1}{3}\text{tr } a^{\mu\nu}[D_\mu, [D_\nu, M]] = 4g_s^2 C_3^{\text{ad}} f^{ABC} G_{\mu\nu}^A G_{\nu\lambda}^B G_{\lambda\mu}^C ,$$

$$\text{tr } i\bar{\Gamma}\slashed{D}\Gamma = 6g_s C_3^{\text{ad}} (D_\mu G^{\mu\lambda})^A (D^\nu G_{\nu\lambda})^A ,$$

$$-\frac{1}{6}\text{tr } \bar{\Gamma} i\overset{\leftrightarrow}{\mathcal{D}}_a\Gamma = -9g_s^2 \bar{q}_L \sigma^{\mu\nu} G_{\mu\nu}^A T^A \sqrt{2}H\mathcal{Y}q_R + \text{h.c.}$$

To obtain the first equality we used the identity $f^{ADE}f^{BEF}f^{CFD} = C_3^{\text{ad}}/2 f^{ABC}$ for the structure constants, where $C_N^{\text{ad}} = N$. Building blocks that vanish during the calculation are not explicitly listed. Summing the terms and using the equations of motion, we find the divergent Lagrangian

$$32\pi^2 \epsilon \mathcal{L}_G^{\text{div}} = \frac{C_G}{\Lambda^2}\bigg(-36g_s^2 f^{ABC} G_{\mu\nu}^A G_{\nu\lambda}^B G_{\lambda\mu}^C$$
$$-9g_s^2 \bar{q}_L \sigma^{\mu\nu} G_{\mu\nu}^A T^A \sqrt{2}H\mathcal{Y}q_R + \text{h.c.}\bigg) . \tag{58}$$

Using the results of Sec. 4, the contributions to the beta functions induced by these divergences are given by

$$\beta_G \supseteq 15g_s^2 C_G , \qquad \beta_{qG} \supseteq 9g_s^2 C_G \mathcal{Y}_q . \tag{59}$$

Divergences and RGEs from W

The W-operator is

$$Q_W = \epsilon^{abc} W_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c . \tag{60}$$

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This operator is analogous to (55), and we simply adapt the fluctuations (56) to the case of $SU(2)$. For this operator, the terms in the master formula then become

$$\text{tr } cM = -18g^2 C_2^{\text{ad}} \epsilon^{abc} W_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c + 3g^3 C_2^{\text{ad}} (\phi^\dagger \phi) W_{\mu\nu}^a W^{a\mu\nu} ,$$

$$-\frac{1}{3}\text{tr } [D^\mu, N_{\mu\nu}]b^\lambda = -2g C_2^{\text{ad}} (D_\mu W^{\mu\lambda})^a (D^\nu W_{\nu\lambda})^a ,$$

$$-\frac{1}{4}\text{tr } a_\lambda^\lambda M^2 = 6g^2 C_2^{\text{ad}} \epsilon^{abc} W_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c + 6g^3 C_2^{\text{ad}} (\phi^\dagger \phi) W_{\mu\nu}^a W^{a\mu\nu}$$

$$- 6i g^2 C_2^{\text{ad}} W_{\mu\nu}^a (D^\mu \varphi) t_L^a (D^\nu \varphi) ,$$

$$-\frac{1}{12}\text{tr } a^\lambda[D_\mu, [D^\mu, M]] = 2g C_2^{\text{ad}} W_{\mu\nu}^a (D_\lambda D^\lambda W^{\mu\nu})^a ,$$

$$\frac{1}{3}\text{tr } a^{\mu\nu}[D_\mu, [D_\nu, M]] = 4g^2 C_2^{\text{ad}} \epsilon^{abc} W_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c ,$$

$$\text{tr } i\bar{\Gamma}\slashed{D}\Gamma = 6g C_2^{\text{ad}} (D_\mu W^{\mu\lambda})^a (D^\nu W_{\nu\lambda})^a$$

$$- 6i g C_2^{\text{ad}} (D_\mu W^{\mu\nu})^a (\varphi t_L^a D_\nu \varphi) .$$

(61)

In total, we find the divergent Lagrangian

$$32\pi^2 \epsilon \mathcal{L}_W^{\text{div}} = \frac{C_W}{\Lambda^2}\Bigg(-24g^2 \epsilon^{abc} W_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c + 15g^3 (\phi^\dagger \phi) W_{\mu\nu}^a W^{a\mu\nu}$$

$$- 3g^2 g' (\phi^\dagger \sigma^a \phi) W_{\mu\nu}^a B^{\mu\nu} \Bigg) .$$

(62)

This gives rise to the contributions

$$\beta_W \supseteq \frac{29}{2} g^2 C_W , \qquad \beta_{\phi W} \supseteq -15 g^3 C_W , \qquad \beta_{\phi W B} \supseteq 3g^2 g' C_W .$$ (63)

5.1.2 $\widetilde{G}$- and $\widetilde{W}$-operators

The operators with dual field strength read

$$Q_{\widetilde{G}} = f^{ABC} \widetilde{G}_{\mu\nu}^A G_{\nu\lambda}^B G_{\lambda\mu}^C .$$ (64)

To work out the fluctuation Lagrangian, it is useful to relate the different kinds of tensors by

$$\epsilon^{\alpha\beta\mu\nu} G_{\nu\lambda}^A = g^{\alpha\lambda} \widetilde{G}_{\mu\beta}^A - g^{\beta\lambda} \widetilde{G}_{\mu\alpha}^A - g^{\mu\lambda} \widetilde{G}_{\alpha\beta}^A .$$ (65)

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We find fluctuation matrices similar to (56), namely

$$c_{(A\alpha)(B\beta)} = 3g_s f^{ADC} f^{CBE} (\widetilde{G}^D_{\alpha\mu} G^E_{\mu\beta} + G^D_{\alpha\mu} \widetilde{G}^E_{\mu\beta}) + 3g_s f^{ADC} f^{CBE} \widetilde{G}^D_{\mu\nu} G^{E\mu\nu} g_{\alpha\beta}$$

$$+ 3f^{ABC} (D_\mu D^\mu \widetilde{G}_{\alpha\beta})^C ,$$

$$b^\lambda_{(A\alpha)(B\beta)} = \frac{3}{2} f^{ABC} (D_\alpha \widetilde{G}_{\beta\lambda} + D_\beta \widetilde{G}_{\alpha\lambda})^C ,$$

$$a^{\mu\nu}_{(A\alpha)(B\beta)} = 3f^{ABC} (g^{\mu\alpha} \widetilde{G}_{\nu\beta} + g^{\nu\alpha} \widetilde{G}_{\mu\beta} - g^{\mu\beta} \widetilde{G}_{\nu\alpha} - g^{\nu\beta} \widetilde{G}_{\mu\alpha} - 2\widetilde{G}_{\alpha\beta} g^{\mu\nu})^C .$$

(66)

Note that several terms vanish due to the Bianchi identity, which implies $(D_\mu \widetilde{G}^{\mu\nu})^A = 0$. The computation simplifies in comparison with the G- and W-operators.

Divergences and RGEs from $\widetilde{G}$

For (66), the non-zero parts of the master formula are

$$\text{tr } cM = -18g_s^2 C_3^{\text{ad}} f^{ABC} \widetilde{G}^A_{\mu\nu} G^B_{\nu\lambda} G^C_{\lambda\mu} ,$$

$$-\frac{1}{4} \text{tr } a^\lambda_\lambda M^2 = 6g_s^2 C_3^{\text{ad}} f^{ABC} \widetilde{G}^A_{\mu\nu} G^B_{\nu\lambda} G^C_{\lambda\mu} ,$$

$$-\frac{1}{12} \text{tr } a^\lambda_\lambda [D_\mu, [D^\mu, M]] = -4g_s^2 C_3^{\text{ad}} f^{ABC} \widetilde{G}^A_{\mu\nu} G^B_{\nu\lambda} G^C_{\lambda\mu} ,$$

(67)

$$\frac{1}{3} \text{tr } a^{\mu\nu} [D_\mu, [D_\nu, M]] = 4g_s^2 C_3^{\text{ad}} f^{ABC} \widetilde{G}^A_{\mu\nu} G^B_{\nu\lambda} G^C_{\lambda\mu} ,$$

$$-\frac{1}{6} \text{tr } \bar{\Gamma} i \overleftrightarrow{\mathcal{D}}_a \Gamma = -9ig_s^2 \bar{q}_L \sigma^{\mu\nu} G^A_{\mu\nu} T^A \sqrt{2} H \mathcal{Y} q_R + \text{h.c.}$$

We sum the terms and obtain

$$32\pi^2 \epsilon \mathcal{L}^{\text{div}}_{\widetilde{G}} = \frac{C_{\widetilde{G}}}{\Lambda^2} \bigg(-36g_s^2 f^{ABC} \widetilde{G}^A_{\mu\nu} G^B_{\nu\lambda} G^C_{\lambda\mu}$$

$$-9ig_s^2 \bar{q}_L \sigma^{\mu\nu} G^A_{\mu\nu} T^A \sqrt{2} H \mathcal{Y} q_R + \text{h.c.} \bigg) .$$

(68)

In this case, the divergences result in

$$\beta_{\widetilde{G}} \supseteq 15g_s^2 C_{\widetilde{G}} , \qquad \beta_{qG} \supseteq 9ig_s^2 C_{\widetilde{G}} \mathcal{Y}_q .$$

(69)

Divergences and RGEs from $\widetilde{W}$

The last operator of this class is

$$Q_{\widetilde{W}} = \epsilon^{abc} \widetilde{W}^a_{\mu\nu} W^b_{\nu\lambda} W^c_{\lambda\mu} .$$

(70)

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Again, we translate the fluctuations from (66) to $SU(2)$ and find the divergent pieces

$$\text{tr } cM = -18g^2 C_2^{\text{ad}} \epsilon^{abc} \widetilde{W}_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c + 3g^3 C_2^{\text{ad}}(\phi^\dagger\phi)\widetilde{W}_{\mu\nu}^a W^{\mu\nu} ,$$

$$-\frac{1}{4}\text{tr } a_\lambda^\lambda M^2 = 6g^2 C_2^{\text{ad}} \epsilon^{abc} \widetilde{W}_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c + \frac{9}{2}g^3 C_2^{\text{ad}}(\phi^\dagger\phi)\widetilde{W}_{\mu\nu}^a W^{a\mu\nu} ,$$

$$-\frac{3}{2}g^2 g' C_2^{\text{ad}}(\phi^\dagger\sigma^a\phi)\widetilde{W}_{\mu\nu}^a B^{\mu\nu} , \tag{71}$$

$$-\frac{1}{12}\text{tr } a^\lambda[D_\mu,[D^\mu,M]] = -4g^2 C_2^{\text{ad}} \epsilon^{abc} \widetilde{W}_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c ,$$

$$\frac{1}{3}\text{tr } a^{\mu\nu}[D_\mu,[D_\nu,M]] = 4g^2 C_2^{\text{ad}} \epsilon^{abc} \widetilde{W}_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c .$$

The terms combine to the divergent Lagrangian

$$32\pi^2\epsilon\mathcal{L}_{\widetilde{W}}^{\text{div}} = \frac{C_{\widetilde{W}}}{\Lambda^2}\left(-24g^2\epsilon^{abc}\widetilde{W}_{\mu\nu}^a W_{\nu\lambda}^b W_{\lambda\mu}^c + 15g^3(\phi^\dagger\phi)\widetilde{W}_{\mu\nu}^a W^{a\mu\nu} \right.$$
$$\left. -3g^2 g'(\phi^\dagger\sigma^a\phi)\widetilde{W}_{\mu\nu}^a B^{\mu\nu} \right) . \tag{72}$$

The divergences lead to the renormalization group contributions

$$\beta_{\widetilde{W}} \supseteq \frac{29}{2}g^2 C_{\widetilde{W}} , \qquad \beta_{\phi\widetilde{W}} \supseteq -15g^3 C_{\widetilde{W}} , \qquad \beta_{\phi\widetilde{W}B} \supseteq 3g^2 g' C_{\widetilde{W}} . \tag{73}$$

5.2 Renormalization of the operator class $X^2\phi^2$

We next consider the operators of the class $X^2\phi^2$. We again divide the computation into several steps, in a similar way as in the discussion of the class X^3.

5.2.1 ϕX-operators

We first consider the operator

$$Q_{\phi G} = (\phi^\dagger\phi)G_{\mu\nu}^A G^{A\mu\nu} . \tag{74}$$

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We work out the fluctuation matrices and find ($\varphi^2 \equiv \varphi_i \varphi_i$)

$$c_{(A\alpha)(B\beta)} = -3g_s f^{ABC} G^C_{\alpha\beta} \varphi^2 + \delta^{AB} g_{\alpha\beta} \Box(\varphi^2) - \frac{1}{2} \delta^{AB} \{\partial_\alpha, \partial_\beta\}(\varphi^2) \;,$$

$$c_{(A\alpha)i} = 2i\varphi_i (D^\mu G_{\mu\alpha})^A + 2i(D^\mu \varphi)_i G^A_{\mu\alpha} \;,$$

$$c_{ij} = -G^A_{\mu\nu} G^{A\mu\nu} \delta_{ij} \;,$$

$$b^\mu_{(A\alpha)(B\beta)} = \frac{1}{2} \delta^{AB} g_{\mu\alpha} \partial_\beta(\varphi^2) - \frac{1}{2} \delta^{AB} g_{\mu\beta} \partial_\alpha(\varphi^2) \;,$$

$$b^\mu_{(A\alpha)i} = 2i G^A_{\mu\alpha} \varphi_i \;,$$

$$a^{\mu\nu}_{(A\alpha)(B\beta)} = -\delta^{AB} S^{\alpha\beta\mu\nu} \varphi^2 \;.$$

(75)

In the last entry, we defined the tensor $S^{\alpha\beta\mu\nu} \equiv 2g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}$, which is symmetric under the exchange of $\alpha \leftrightarrow \beta$ and $\mu \leftrightarrow \nu$.

Divergences and RGEs from ϕG

For the fluctuations above, the various terms of the master formula become

$$\mathrm{tr}\, cM = -\left(6\lambda + 12g_s^2 C_3^{\mathrm{ad}} + \frac{3}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger \phi) G^A_{\mu\nu} G^{A\mu\nu}$$

$$+ 4m^2 G^A_{\mu\nu} G^{A\mu\nu} \;,$$

$$-\frac{1}{4}\mathrm{tr}\, a^\lambda_\lambda M^2 = 12g_s^2 C_3^{\mathrm{ad}}(\phi^\dagger \phi) G^A_{\mu\nu} G^{A\mu\nu} \;,$$

$$-\frac{1}{24}\mathrm{tr}\, a^\lambda_\lambda N_{\mu\nu} N^{\mu\nu} = -2g_s^2 C_3^{\mathrm{ad}}(\phi^\dagger \phi) G^A_{\mu\nu} G^{A\mu\nu} \;,$$

$$\frac{1}{6}\mathrm{tr}\, a^{\mu\nu} N_{\mu\lambda} N^{\nu\lambda} = 2g_s^2 C_3^{\mathrm{ad}}(\phi^\dagger \phi) G^A_{\mu\nu} G^{A\mu\nu} \;,$$

$$i\bar{\Gamma}\slashed{b}\Gamma = 4g_s \bar{q}_L \sigma^{\mu\nu} G^A_{\mu\nu} T^A \sqrt{2} H \mathcal{Y} q_R + \mathrm{h.c.} \;,$$

$$-\frac{1}{6}\mathrm{tr}\, \bar{\Gamma} i \overleftrightarrow{\mathcal{D}}_a \Gamma = -24g_s^2 C_3^F(\phi^\dagger \phi) \bar{q}_L \sqrt{2} H \mathcal{Y} q_R + \mathrm{h.c.}$$

(76)

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We observe that terms proportional to C_3^{ad} cancel in the sum. From the remaining terms we obtain the divergent Lagrangian

$$32\pi^2\epsilon\mathcal{L}_{\phi G}^{\mathrm{div}} = \frac{C_{\phi G}}{\Lambda^2}\Bigg(-\left(6\lambda + \frac{3}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)G_{\mu\nu}^A G^{A\mu\nu} + 4m^2 G_{\mu\nu}^A G^{A\mu\nu}$$
$$-32g_s^2(\phi^\dagger\phi)\bar{q}_L\sqrt{2}H\mathcal{Y}q_R + 4g_s\bar{q}_L\sigma^{\mu\nu}G_{\mu\nu}^A T^A\sqrt{2}H\mathcal{Y}q_R + \mathrm{h.c.}\Bigg)\,. \tag{77}$$

The coefficient of operator ϕG then contributes to the beta functions

$$\beta_{\phi G} \supseteq \left(6\lambda - 14g_s^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2 + 2\gamma_\phi\right)C_{\phi G}\,, \qquad \beta_{q\phi} \supseteq 32g_s^2\mathcal{Y}_q C_{\phi G}$$
$$\beta_{g_s} \supseteq -8g_s\frac{m^2}{\Lambda^2}C_{\phi G}\,, \qquad\qquad\qquad\qquad \beta_{qG} \supseteq -4g_s\mathcal{Y}_q C_{\phi G}\,. \tag{78}$$

Divergences and RGEs from ϕW

In a similar way we treat the ϕW-operator

$$Q_{\phi W} = (\phi^\dagger\phi)W_{\mu\nu}^a W^{a\mu\nu}\,. \tag{79}$$

The fluctuation matrices can be adapted from (75). In the case of $SU(2)$ they yield

$$\mathrm{tr}\ cM = -\left(6\lambda + \frac{7}{2}g^2 + 12g^2 C_2^{\mathrm{ad}} + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)W_{\mu\nu}^a W^{a\mu\nu}$$
$$+ 4m^2 W_{\mu\nu}^a W^{a\mu\nu} - 2gg'(\phi^\dagger\sigma^a\phi)W_{\mu\nu}^a B^{\mu\nu}$$
$$+ 9g^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi)\,,$$
$$-\frac{1}{4}\mathrm{tr}\ a_\lambda^\lambda M^2 = 12g^2 C_2^{\mathrm{ad}}(\phi^\dagger\phi)W_{\mu\nu}^a W^{a\mu\nu} + 18m^2 g^2(\phi^\dagger\phi)^2$$
$$- \left(18\lambda g^2 - 9g^4 - 3g^2 g'^2\right)(\phi^\dagger\phi)^3 - 9g^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$$
$$- 9g^2(\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}H\mathcal{Y}\psi_R + \mathrm{h.c.}\,,$$
$$-\frac{1}{24}\mathrm{tr}\ a_\lambda^\lambda N_{\mu\nu}N^{\mu\nu} = -2g^2 C_2^{\mathrm{ad}}(\phi^\dagger\phi)W_{\mu\nu}^a W^{a\mu\nu}\,,$$
$$\frac{1}{6}\mathrm{tr}\ a^{\mu\nu}N_{\mu\lambda}N^{\nu\lambda} = 2g^2 C_2^{\mathrm{ad}}(\phi^\dagger\phi)W_{\mu\nu}^a W^{a\mu\nu}\,,$$
$$-\frac{1}{12}\mathrm{tr}\ a^\lambda_\lambda[D_\mu,[D^\mu,M]] = 6g^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi)\,,$$
$$\frac{1}{3}\mathrm{tr}\ a^{\mu\nu}[D_\mu,[D_\nu,M]] = -6g^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi)\,,$$
$$\mathrm{tr}\ i\bar{\Gamma}\slashed{\partial}\Gamma = g\bar{\psi}_L\sigma^{\mu\nu}W_{\mu\nu}^a\sigma^a\sqrt{2}H\mathcal{Y}\psi_R + \mathrm{h.c.} \tag{80}$$

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The divergent Lagrangian then reads

$$32\pi^2\epsilon\mathcal{L}_{\phi W}^{\text{div}} = \frac{C_{\phi W}}{\Lambda^2}\Bigg(-\left(6\lambda + \frac{7}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)W_{\mu\nu}^a W^{a\mu\nu} + 4m^2 W_{\mu\nu}^a W^{a\mu\nu}$$

$$-2gg'(\phi^\dagger\sigma^a\phi)W_{\mu\nu}^a B^{\mu\nu} - \left(18\lambda g^2 - 9g^4 - 3g^2 g'^2\right)(\phi^\dagger\phi)^3$$

$$+18m^2 g^2(\phi^\dagger\phi)^2 - 9g^2(\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}H\mathcal{Y}\psi_R$$

$$+ g\bar{\psi}_L\sigma^{\mu\nu}W_{\mu\nu}^a\sigma^a\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c.}\Bigg) .$$

(81)

Finally, we find the RGE contributions

$$\beta_{\phi W} \supseteq \left(6\lambda - \frac{53}{6}g^2 - \frac{3}{2}g'^2 + 2\gamma_\phi\right)C_{\phi W} , \qquad \beta_{\psi\phi} \supseteq 9g^2\mathcal{Y}C_{\phi W} ,$$

$$\beta_{\phi WB} \supseteq 2gg'C_{\phi W} , \qquad\qquad\qquad \beta_{\psi W} \supseteq -g\mathcal{Y}C_{\phi W} ,$$

$$\beta_\phi \supseteq \left(18\lambda g^2 - 9g^4 - 3g^2 g'^2\right)C_{\phi W} , \qquad \beta_g \supseteq -8g\frac{m^2}{\Lambda^2}C_{\phi W} ,$$

$$\beta_\lambda \supseteq 36g^2\frac{m^2}{\Lambda^2}C_{\phi W} .$$

(82)

Divergences and RGEs from ϕB

The last operator of the class ϕX is

$$Q_{\phi B} = (\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu} .$$

(83)

The fluctuations in (75), converted to $U(1)$, lead to

$$\text{tr } cM = -\left(6\lambda + \frac{3}{2}g^2 + \frac{5}{2}g'^2\right)(\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu} + 4m^2 B_{\mu\nu}B^{\mu\nu}$$

$$- 2gg'(\phi^\dagger\sigma^a\phi)W^a_{\mu\nu}B^{\mu\nu} + 3g'^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi) ,$$

$$-\frac{1}{4}\text{tr } a^\lambda_\lambda M^2 = -\left(6\lambda g'^2 - 3g^2 g'^2 - 3g'^4\right)(\phi^\dagger\phi)^3 - 3g'^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$$

$$+ 6m^2 g'^2(\phi^\dagger\phi)^2 - 3g'^2(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c.} ,$$

$$-\frac{1}{12}\text{tr } a^\lambda_\lambda[D_\mu, [D^\mu, M] = 2g'^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi) ,$$

$$\frac{1}{3}\text{tr } a^{\mu\nu}[D_\mu, [D_\nu, M]] = -2g'^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi) ,$$

$$i\bar\Gamma\slashed{b}\Gamma = 2g'\bar\psi_L\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}(Y_L + Y_R)\psi_R + \text{h.c.} ,$$

$$-\frac{1}{6}\text{tr } \bar\Gamma i\overset{\leftrightarrow}{\mathcal{D}}_a\Gamma = -24g'^2(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}Y_L Y_R\psi_R + \text{h.c.}$$

(84)

Summing all the terms, we obtain

$$32\pi^2\epsilon\mathcal{L}^{\text{div}}_{\phi B} = \frac{C_{\phi B}}{\Lambda^2}\Bigg(-\left(6\lambda + \frac{3}{2}g^2 + \frac{5}{2}g'^2\right)(\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu} + 4m^2 B_{\mu\nu}B^{\mu\nu}$$

$$- 2gg'(\phi^\dagger\sigma^a\phi)W^a_{\mu\nu}B^{\mu\nu} - \left(6\lambda g'^2 - 3g^2 g'^2 - 3g'^4\right)(\phi^\dagger\phi)^3$$

$$+ 6m^2 g'^2(\phi^\dagger\phi)^2 - 12g'^2(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}(Y_L^2 + Y_R^2)\psi_R$$

$$+ 2g'\bar\psi_L\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}(Y_L + Y_R)\psi_R + \text{h.c.}\Bigg) .$$

(85)

The resulting contributions to the RGEs are

$$\beta_{\phi B} \supseteq \left(6\lambda - \frac{9}{2}g^2 + \frac{85}{6}g'^2 + 2\gamma_\phi\right)C_{\phi B} , \qquad \beta_{\psi\phi} \supseteq 12g'^2(Y_L^2 + Y_R^2)\mathcal{Y}C_{\phi B} ,$$

$$\beta_{\phi WB} \supseteq 2gg'C_{\phi B} , \qquad \beta_{\psi B} \supseteq -2g'(Y_L + Y_R)\mathcal{Y}C_{\phi B} ,$$

$$\beta_\phi \supseteq \left(6\lambda g'^2 - 3g^2 g'^2 - 3g'^4\right)C_{\phi B} , \qquad \beta_{g'} \supseteq -8g'\frac{m^2}{\Lambda^2}C_{\phi B} ,$$

$$\beta_\lambda \supseteq 12g'^2\frac{m^2}{\Lambda^2}C_{\phi B} .$$

(86)

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5.2.2 $\phi\widetilde{X}$-operators

The operators of the class $\phi\widetilde{X}$ mix into CP-violating operators that we have not considered yet. We therefore introduce a Lagrangian

$$\mathcal{L}_\theta = \frac{\theta_s g_s^2}{32\pi^2}\widetilde{G}^A_{\mu\nu}G^{A\mu\nu} + \frac{\theta g^2}{32\pi^2}\widetilde{W}^a_{\mu\nu}W^{a\mu\nu} + \frac{\theta' g'^2}{32\pi^2}\widetilde{B}_{\mu\nu}B^{\mu\nu} \tag{87}$$

Although these operators correspond to total derivatives and play no role in perturbation theory, they are related to non-perturbative effects. For the beta functions, we will use (87) as reference.

The prototype for operator class $\phi\widetilde{X}$ is

$$Q_{\phi\widetilde{G}} = \phi^\dagger\phi \; \widetilde{G}^A_{\mu\nu}G^{A\mu\nu} \; . \tag{88}$$

In this case, the fluctuations are given by the four non-trivial entries

$$c_{ij} = -\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}\delta_{ij} \; , \qquad b^\lambda_{(A\alpha)(B\beta)} = \epsilon^{\lambda\alpha\beta\mu}\partial_\mu(\varphi^2)\delta^{AB} \; ,$$

$$c_{(A\alpha)i} = 2i(D^\mu\varphi)_i\widetilde{G}^A_{\mu\alpha} \; , \qquad b^\lambda_{(A\alpha)i} = 2i\widetilde{G}^A_{\lambda\alpha}\varphi_i \; . \tag{89}$$

Divergences and RGE for $\phi\widetilde{G}$

From the fluctuations (89) we obtain

$$\text{tr } cM = -\left(6\lambda + \frac{3}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)\widetilde{G}^A_{\mu\nu}G^{A\mu\nu} + 4m^2\widetilde{G}^A_{\mu\nu}G^{A\mu\nu} \; ,$$

$$\text{tr } i\bar{\Gamma}\slashed{D}\Gamma = -32ig_s^2(\phi^\dagger\phi)\bar{q}_L\sqrt{2}H\mathcal{Y}q_R + 4ig_s\bar{q}_L\sigma^{\mu\nu}G^A_{\mu\nu}T^A\sqrt{2}H\mathcal{Y}q_R + \text{h.c.} \tag{90}$$

The divergent Lagrangian is then

$$32\pi^2\epsilon\mathcal{L}^{\text{div}}_{\phi\widetilde{G}} = \frac{C_{\phi\widetilde{G}}}{\Lambda^2}\Bigg(-\left(6\lambda + \frac{3}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)\widetilde{G}^A_{\mu\nu}G^{A\mu\nu} + 4m^2\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$$

$$- 32ig_s^2(\phi^\dagger\phi)\bar{q}_L\sqrt{2}H\mathcal{Y}q_R + 4ig_s\bar{q}_L\sigma^{\mu\nu}G^A_{\mu\nu}T^A\sqrt{2}H\mathcal{Y}q_R + \text{h.c.}\Bigg) \; . \tag{91}$$

The first term in $\text{tr } cM$ gives a self-renormalization of $Q_{\phi\widetilde{G}}$, while the second term renormalizes the QCD θ term $\mathcal{L}_{\theta_s} = \theta_s g_s^2/(32\pi^2)\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$. The two contributions in $\text{tr } i\bar{\Gamma}\slashed{D}\Gamma$ already correspond to operators of the Warsaw basis. The term $(\phi^\dagger\phi)\bar{q}_L\sqrt{2}H\mathcal{Y}q_R$ corresponds to the operators $Q_{u\phi} = (\phi^\dagger\phi)(\bar{q}_Lu_R\widetilde{\phi})$ and $Q_{d\phi} = (\phi^\dagger\phi)(\bar{q}_Ld_R\phi)$. The last piece $\bar{q}_L\sigma^{\mu\nu}G^A_{\mu\nu}T^A\sqrt{2}H\mathcal{Y}q_R$ corresponds to the operators $Q_{uG} = (\bar{q}_L\sigma^{\mu\nu}T^Au_R)\widetilde{\phi}G^A_{\mu\nu}$ and $Q_{dG} = (\bar{q}_L\sigma^{\mu\nu}T^Ad_R)\phi G^A_{\mu\nu}$.

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We deduce the renormalization group contributions

$$\beta_{\phi\widetilde{G}} \supseteq \left(6\lambda - 14g_s^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2 + 2\gamma_\phi\right)C_{\phi\widetilde{G}}, \qquad \beta_{q\phi} \supseteq 32ig_s^2\mathcal{Y}_q C_{\phi\widetilde{G}},$$

$$\beta_{\theta_s} \supseteq -\frac{128\pi^2}{g_s^2}\frac{m^2}{\Lambda^2}C_{\phi\widetilde{G}} \qquad\qquad\qquad \beta_{qG} \supseteq -4ig_s\mathcal{Y}_q C_{\phi\widetilde{G}}. \tag{92}$$

Divergences and RGEs from $\phi\widetilde{W}$

The next operator we consider is

$$Q_{\phi\widetilde{W}} = (\phi^\dagger\phi)\widetilde{W}_{\mu\nu}^a W^{a\mu\nu}. \tag{93}$$

The $SU(2)$-analogy of (89) leads to

$$\operatorname{tr} cM = -\left(6\lambda + \frac{7}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)\widetilde{W}_{\mu\nu}^a W^{a\mu\nu} + 4m^2\widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$$

$$- 2gg'(\phi^\dagger\sigma^a\phi)\widetilde{W}_{\mu\nu}^a B^{\mu\nu}, \tag{94}$$

$$\operatorname{tr} i\bar{\Gamma}\slashed{D}\Gamma = -9ig^2(\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}H\mathcal{Y}\psi_R + ig\bar{\psi}_L\sigma^{\mu\nu}W_{\mu\nu}^a\sigma^a\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c.}$$

Adding these results, we obtain

$$32\pi^2\epsilon\mathcal{L}_{\phi\widetilde{W}}^{\text{div}} = \frac{C_{\phi\widetilde{W}}}{\Lambda^2}\Bigg(-\left(6\lambda + \frac{7}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)\widetilde{W}_{\mu\nu}^a W^{a\mu\nu} + 4m^2\widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$$

$$- 2gg'(\phi^\dagger\sigma^a\phi)\widetilde{W}_{\mu\nu}^a B^{\mu\nu} \tag{95}$$

$$- 9ig^2(\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}H\mathcal{Y}\psi_R + ig\bar{\psi}_L\sigma^{\mu\nu}W_{\mu\nu}^a\sigma^a\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c.}\Bigg).$$

The contributions to the beta functions are

$$\beta_{\phi\widetilde{W}} \supseteq \left(6\lambda - \frac{53}{6}g^2 - \frac{3}{2}g'^2 + 2\gamma_\phi\right)C_{\phi\widetilde{W}}, \qquad \beta_{\psi\phi} \supseteq 9ig^2\mathcal{Y}C_{\phi\widetilde{W}},$$

$$\beta_{\phi\widetilde{W}B} \supseteq 2gg'C_{\phi\widetilde{W}}, \qquad\qquad\qquad \beta_{\psi W} \supseteq -ig\mathcal{Y}C_{\phi\widetilde{W}}, \tag{96}$$

$$\beta_\theta \supseteq -\frac{128\pi^2}{g^2}\frac{m^2}{\Lambda^2}C_{\phi\widetilde{W}}.$$

Divergences and RGEs from $\phi\widetilde{B}$

The last operator of this class is

$$Q_{\phi\widetilde{B}} = (\phi^\dagger\phi)\widetilde{B}_{\mu\nu}B^{\mu\nu}. \tag{97}$$

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We translate the fluctuations (89) to the case of $U(1)$ and find

$$\operatorname{tr} cM = - \left(6\lambda + \frac{3}{2}g^2 + \frac{5}{2}g'^2 \right)(\phi^\dagger\phi)\widetilde{B}_{\mu\nu}B^{\mu\nu} + 4m^2\widetilde{B}_{\mu\nu}B^{\mu\nu}$$

$$- 2gg'(\phi^\dagger\sigma^a\phi)\widetilde{W}^a_{\mu\nu}B^{\mu\nu} \, ,$$

$$\operatorname{tr} i\bar{\Gamma}\slashed{D}\Gamma = - 12ig'^2(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}(Y_L^2 + Y_R^2)\psi_R$$

$$+ 2ig'\bar\psi_L\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}(Y_L + Y_R)\psi_R + \text{h.c.}$$

(98)

These results add up to the divergent Lagrangian

$$32\pi^2\epsilon\mathcal{L}^{\text{div}}_{\phi\widetilde{B}} = \frac{C_{\phi\widetilde{B}}}{\Lambda^2}\Bigg(- \left(6\lambda + \frac{3}{2}g^2 + \frac{5}{2}g'^2 \right)(\phi^\dagger\phi)\widetilde{B}_{\mu\nu}B^{\mu\nu} + 4m^2\widetilde{B}_{\mu\nu}B^{\mu\nu}$$

$$- 2gg'(\phi^\dagger\sigma^a\phi)\widetilde{W}^a_{\mu\nu}B^{\mu\nu} - 12ig'^2(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}(Y_L^2 + Y_R^2)\psi_R \quad (99)$$

$$+ 2ig'\bar\psi_L\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}(Y_L + Y_R)\psi_R + \text{h.c.} \Bigg) \, .$$

We infer the following contributions to the RGEs

$$\beta_{\phi\widetilde{B}} \supseteq \left(6\lambda - \frac{9}{2}g^2 + \frac{85}{6}g'^2 + 2\gamma_\phi \right)C_{\phi\widetilde{B}} \, , \qquad \beta_{\psi\phi} \supseteq 12ig'^2(Y_L^2 + Y_R^2)\mathcal{Y}C_{\phi\widetilde{B}} \, ,$$

$$\beta_{\phi\widetilde{W}B} \supseteq 2gg'C_{\phi\widetilde{B}} \, , \qquad\qquad\qquad\qquad \beta_{\psi B} \supseteq -2ig'(Y_L + Y_R)\mathcal{Y}C_{\phi\widetilde{B}} \, , \qquad (100)$$

$$\beta_{\theta'} \supseteq -\frac{128\pi^2}{g'^2}\frac{m^2}{\Lambda^2}C_{\phi\widetilde{B}} \, .$$

5.2.3 ϕWB-operator

This operator is given by

$$Q_{\phi WB} = (\phi^\dagger\sigma^a\phi)W^a_{\mu\nu}B^{\mu\nu} = 2(\varphi t^a_L t^3_R\varphi)W^a_{\mu\nu}B^{\mu\nu} \tag{101}$$

The quadratic fluctuations are described by the matrices

$$c_{(a\alpha)(b\beta)} = -4g\epsilon^{abc}(\varphi t_L^c t_R^3 \varphi)B_{\alpha\beta}\,, \qquad\qquad a_{(a\alpha)\beta}^{\mu\nu} = -2(\varphi t_L^a t_R^3 \varphi)S^{\alpha\beta\mu\nu}\,,$$

$$c_{(a\alpha)\beta} = 2(D_\mu \varphi t_L^a t_R^3 D_\nu \varphi)S^{\alpha\beta\mu\nu} \qquad\qquad b_{(a\alpha)\beta}^\lambda = 2g^{\alpha\lambda}(\varphi t_L^a t_R^3 D_\beta \varphi)$$

$$+ 2(\varphi t_L^a t_R^3 D_\mu D_\nu \varphi)S^{\alpha\beta\mu\nu} \qquad\qquad\qquad - 2g^{\beta\lambda}(\varphi t_L^a t_R^3 D_\alpha \varphi)\,,$$

$$+ g\epsilon^{abc}W_{\alpha\beta}^b(\varphi t_L^c t_R^3 \varphi)\,, \tag{102}$$

$$c_{(a\alpha)i} = 4i(t_L^a t_R^3 \varphi)_i \partial^\mu B_{\mu\alpha} + 4i(t_L t_R^3 D^\mu \varphi)B_{\mu\alpha}\,, \qquad b_{(a\alpha)i}^\lambda = 4i(t_L^a t_R^3 \varphi)_i B^{\lambda\alpha}\,,$$

$$c_{\alpha i} = 4i(t_L^a t_R^3 \varphi)_i D^\mu W_{\mu\alpha}^a + 4i(t_L^a t_R^3 D^\mu \varphi)W_{\mu\alpha}^a\,, \qquad b_{\alpha i}^\lambda = 4i(t_L^a t_R^3 \varphi)W^{a\lambda\alpha}\,,$$

$$c_{ij} = -4(t_L^a t_R^3)_{ij}W_{\mu\nu}^a B^{\mu\nu}\,.$$

The symmetric tensor $S^{\alpha\beta\mu\nu}$ has been defined below eq. (75). We obtain the terms

$$\operatorname{tr} cM = -\left(2\lambda + \frac{5}{2}g^2 + 4g^2 C_2^{\mathrm{ad}} + \frac{3}{2}g'^2\right)(\phi^\dagger \sigma^a \phi)W_{\mu\nu}^a B^{\mu\nu}$$

$$- gg'(\phi^\dagger \phi)W_{\mu\nu}^a W^{a\mu\nu} - 3gg'(\phi^\dagger \phi)B_{\mu\nu}B^{\mu\nu} - 6\lambda gg'(\phi^\dagger \phi)^3$$

$$+ 24gg'(D_\mu \varphi t_L^a t_R^3 D^\mu \varphi)(\varphi t_L^a t_R^3 \varphi) + 6m^2 gg'(\phi^\dagger \phi)^2$$

$$- 3gg'(\phi^\dagger \phi)\bar{\psi}_L \sqrt{2}H\mathcal{Y}\psi_R + \mathrm{h.c.}\,,$$

$$-\frac{1}{4}\operatorname{tr} a_\lambda^\lambda M^2 = \left(3g^3 g' + 3gg'^3\right)(\phi^\dagger \phi)^3 - 24gg'(D_\mu \varphi t_L^a t_R^3 D^\mu \varphi)(\varphi t_L^a t_R^3 \varphi)\,,$$

$$-\frac{1}{12}\operatorname{tr} a_\lambda^\lambda[D_\mu,[D^\mu,M]] = 16gg'(\varphi t_L^a t_R^3 D_\mu D^\mu \varphi + D_\mu \varphi t_L^a t_R^3 D^\mu \varphi)(\varphi t_L^a t_R^3 \varphi)\,,$$

$$\frac{1}{3}\operatorname{tr} a^{\mu\nu}[D_\mu,[D_\nu,M]] = -16gg'(\varphi t_L^a t_R^3 D_\mu D^\mu \varphi + D_\mu \varphi t_L^a t_R^3 D^\mu \varphi)(\varphi t_L^a t_R^3 \varphi)\,,$$

$$\operatorname{tr} i\bar{\Gamma}\slashed{b}\Gamma = -2g'\bar{\psi}_L \sigma^{\mu\nu}W_{\mu\nu}^a \sigma^a \sqrt{2}H\mathcal{Y}(Y_L + Y_R)\tau^3 \psi_R$$

$$- 3g\bar{\psi}_L \sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}\tau^3 \psi_R + \mathrm{h.c.}\,,$$

$$-\frac{1}{6}\operatorname{tr} \bar{\Gamma}i\overleftrightarrow{\mathcal{D}}_a\Gamma = 12gg'(\phi^\dagger \phi)\bar{\psi}_L \sqrt{2}H\mathcal{Y}Y_R\tau^3 \psi_R + \mathrm{h.c.} \tag{103}$$

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Adding these terms leads to

$$32\pi^2\epsilon\mathcal{L}_{\phi WB}^{\text{div}} = \frac{C_{\phi WB}}{\Lambda^2}\Bigg(-\left(2\lambda+\frac{21}{2}g^2+\frac{3}{2}g'^2\right)(\phi^\dagger\sigma^a\phi)W_{\mu\nu}^a B^{\mu\nu}+6m^2gg'(\phi^\dagger\phi)^2$$

$$-\left(6\lambda gg'-3g^3g'-3gg'^3\right)(\phi^\dagger\phi)^3-gg'(\phi^\dagger\phi)W_{\mu\nu}^a W^{a\mu\nu}$$

$$-3gg'(\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu}+12gg'(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}Y_L\tau^3\psi_R \tag{104}$$

$$-2g'\bar\psi_L\sigma^{\mu\nu}W_{\mu\nu}^a\sigma^a\sqrt{2}H\mathcal{Y}(Y_L+Y_R)\tau^3\psi_R$$

$$-3g\bar\psi_L\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}\tau^3\psi_R+\text{h.c.}\Bigg)\,.$$

We find eight contributions to the beta functions:

$$\beta_{\phi WB}\supseteq\left(2\lambda+\frac{4}{3}g^2+\frac{19}{3}g'^2+2\gamma_\phi\right)C_{\phi WB}\,,\qquad \beta_{\psi\phi}\supseteq-12gg'Y_L\tau^3\mathcal{Y}C_{\phi WB}\,,$$

$$\beta_{\phi W}\supseteq gg'C_{\phi WB}\,,\qquad\qquad\qquad\qquad \beta_{\psi W}\supseteq 2g'(Y_L+Y_R)\tau^3\mathcal{Y}C_{\phi WB}\,,$$

$$\beta_{\phi B}\supseteq 3gg'C_{\phi WB}\,,\qquad\qquad\qquad\qquad \beta_{\psi B}\supseteq 3g\tau^3\mathcal{Y}C_{\phi WB}\,, \tag{105}$$

$$\beta_\phi\supseteq\left(6\lambda gg'-3g^3g'-3gg'^3\right)C_{\phi WB}\,,\qquad \beta_\lambda\supseteq 12gg'\frac{m^2}{\Lambda^2}C_{\phi WB}\,.$$

5.2.4 $\phi\widetilde{W}B$-operator

We are left with the last operator of class $X^2\phi^2$,

$$Q_{\phi\widetilde{W}B}=(\phi^\dagger\sigma^a\phi)\widetilde{W}_{\mu\nu}^a B^{\mu\nu}=2(\varphi t_L^a t_R^3\varphi)\widetilde{W}_{\mu\nu}^a B^{\mu\nu}\,. \tag{106}$$

The non-trivial elements of the fluctuation matrices are

$$c_{(a\alpha)(b\beta)}=-4g\epsilon^{abc}(\varphi t_L^c t_R^3\varphi)\widetilde{B}_{\alpha\beta}\,,\qquad b_{(a\alpha)\beta}^\lambda=-4\epsilon^{\alpha\beta\lambda\mu}(\varphi t_L^a t_R^3 D_\mu\varphi)\,,$$

$$c_{(a\alpha)\beta}=4ig(\varphi t_L^a t_R^3 t_L^b\varphi)\widetilde{W}_{\alpha\beta}^b\,,\qquad b_{(a\alpha)i}^\lambda=4i(t_L^a t_R^3\varphi)_i\widetilde{B}^{\lambda\alpha}\,,$$

$$c_{(a\alpha)i}=4i(t_L^a t_R^3 D^\mu\varphi)_i\widetilde{B}_{\mu\alpha}\,,\qquad b_{\alpha i}^\lambda=4i(t_L^a t_R^3\varphi)_i\widetilde{W}^{a\lambda\alpha}\,, \tag{107}$$

$$c_{\alpha i}=4i(t_L^a t_R^3 D^\mu\varphi)_i\widetilde{W}_{\mu\alpha}^a\,,$$

$$c_{ij}=-4(t_L^a t_R^3)_{ij}\widetilde{W}_{\mu\nu}^a B^{\mu\nu}\,.$$

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From these, we calculate the divergences

$$\text{tr } cM = -\left(2\lambda + \frac{5}{2}g^2 + 4g^2 C_2^{\text{ad}} + \frac{3}{2}g'^2\right)(\phi^\dagger \sigma^a \phi)\widetilde{W}^a_{\mu\nu}B^{\mu\nu}$$

$$- gg'(\phi^\dagger\phi)\widetilde{W}^a_{\mu\nu}W^{\mu\nu} - 3gg'(\phi^\dagger\phi)\widetilde{B}_{\mu\nu}B^{\mu\nu} \,,$$

$$\text{tr } i\bar{\Gamma}\slashed{\partial}\Gamma = 12igg'(\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}H\mathcal{Y}Y_L\tau^3\psi_R - 3ig\bar{\psi}_L\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}\tau^3\psi_R$$

$$- 2g'\bar{\psi}_L\sigma^{\mu\nu}W^a_{\mu\nu}\sigma^a\sqrt{2}H\mathcal{Y}(Y_L + Y_R)\tau^3\psi_R + \text{h.c.}$$

(108)

Adding these results, the divergent Lagrangian becomes

$$32\pi^2\epsilon\mathcal{L}^{\text{div}}_{\phi\widetilde{W}B} = \frac{C_{\phi\widetilde{W}B}}{\Lambda^2}\Bigg(-\left(2\lambda + \frac{21}{2}g^2 + \frac{3}{2}g'^2\right)(\phi^\dagger\sigma^a\phi)\widetilde{W}^a_{\mu\nu}B^{\mu\nu}$$

$$- gg'(\phi^\dagger\phi)\widetilde{W}^a_{\mu\nu}W^{a\mu\nu} + 12igg'(\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}H\mathcal{Y}Y_L\tau^3\psi_R$$

$$- 3gg'(\phi^\dagger\phi)\widetilde{B}_{\mu\nu}B^{\mu\nu} - 3ig\bar{\psi}_L\sigma^{\mu\nu}B_{\mu\nu}\sqrt{2}H\mathcal{Y}\tau^3\psi_R$$

$$- 2g'\bar{\psi}_L\sigma^{\mu\nu}W^a_{\mu\nu}\sigma^a\sqrt{2}H\mathcal{Y}(Y_L + Y_R)\tau^3\psi_R + \text{h.c.}\Bigg) \,.$$

(109)

The contributions to the RGEs are then

$$\beta_{\phi\widetilde{W}B} \supseteq \left(2\lambda + \frac{4}{3}g^2 + \frac{19}{3}g'^2 + 2\gamma_\phi\right)C_{\phi\widetilde{W}B} \,, \qquad \beta_{\psi\phi} \supseteq -12igg'Y_L\tau^3\mathcal{Y}C_{\phi\widetilde{W}B} \,,$$

$$\beta_{\phi\widetilde{W}} \supseteq gg'C_{\phi\widetilde{W}B} \,, \qquad\qquad\qquad\qquad \beta_{\psi W} \supseteq 2ig'(Y_L + Y_R)\tau^3\mathcal{Y}C_{\phi\widetilde{W}B} \,, \quad (110)$$

$$\beta_{\phi\widetilde{B}} \supseteq 3gg'C_{\phi\widetilde{W}B} \,, \qquad\qquad\qquad\qquad \beta_{\psi B} \supseteq 3ig\tau^3\mathcal{Y}C_{\phi\widetilde{W}B} \,.$$

5.3 Renormalization of the operator class ϕ^6 and $\phi^4 D^2$

This class collects the operators that consist only of scalar fields and derivatives, namely Q_ϕ, $Q_{\phi\square}$ and $Q_{\phi D}$. Using the relations in appendix A, these operators can be expressed in terms of the four real scalar fields φ_i. The basic building blocks in this representation are $(\varphi^2)^3$, $(\varphi D_\mu\varphi)^2$ and $(\varphi t_R^3 D_\mu\varphi)^2$.

5.3.1 ϕ^6-operator

We begin with the simplest case

$$Q_\phi = (\phi^\dagger\phi)^3 = \frac{1}{8}(\varphi_i\varphi_i)^3 \,.$$

(111)

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There is only one non-trivial matrix entry:

$$c_{ij} = -\frac{3}{4}(\varphi^2)^2\delta_{ij} - 3(\varphi^2)\varphi_i\varphi_j \; . \tag{112}$$

From the master formula we find

$$\text{tr } cM = -\left(54\lambda + \frac{9}{2}g^2 + \frac{3}{2}g'^2\right)(\phi^\dagger\phi)^3 + 24m^2(\phi^\dagger\phi)^2 \; . \tag{113}$$

The divergent Lagrangian is then given by

$$32\pi^2\epsilon\mathcal{L}_\phi^{\text{div}} = \frac{C_\phi}{\Lambda^2}\left(-\left(54\lambda + \frac{9}{2}g^2 + \frac{3}{2}g'^2\right)(\phi^\dagger\phi)^3 + 24m^2(\phi^\dagger\phi)^2\right) \; . \tag{114}$$

We obtain the two beta-function contributions

$$\beta_\phi \supseteq \left(54\lambda - \frac{27}{2}g^2 - \frac{9}{2}g'^2 + 6\gamma_\phi\right)C_\phi \; , \qquad \beta_\lambda \supseteq 48\frac{m^2}{\Lambda^2}C_\phi \; . \tag{115}$$

5.3.2 $\phi\Box$-operator

For the next operator, we have

$$Q_{\phi\Box} = (\phi^\dagger\phi)\Box(\phi^\dagger\phi) = -(\varphi D_\mu\varphi)^2 \; , \tag{116}$$

dropping a total derivative. The fluctuation matrices are

$$c_{ij} = 2(D_\mu\varphi)_i(D^\mu\varphi)_j - \Box(\varphi^2)\delta_{ij} \; ,$$

$$b_{ij}^\mu = (D^\mu\varphi)_i\varphi_j - \varphi_i(D^\mu\varphi)_j \; , \tag{117}$$

$$a_{ij}^{\mu\nu} = -2\varphi_i\varphi_j g^{\mu\nu} \equiv a_{ij}g^{\mu\nu} \; .$$

From these, we find the terms of the master formula to be

$$\text{tr } cM = -\left(12\lambda + \frac{3}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + 2g'^2(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi)$$

$$+ \left(4\lambda^2 + 2\lambda g^2\right)(\phi^\dagger\phi)^3 + 4m^2(\phi^\dagger\phi) - \left(8\lambda + 2g^2\right)m^2(\phi^\dagger\phi)^2$$

$$- 2m^2\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R + (2\lambda + g^2)(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c. },$$

$$-\frac{1}{3}\text{tr }[D^\mu, N_{\mu\nu}]b^\nu = -\left(\frac{1}{2}g^2 + \frac{1}{6}g'^2\right)(\phi^\dagger\phi)\Box(\phi^\dagger\phi) - \frac{2}{3}g'^2(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi)$$

$$- \frac{2}{3}\lambda g^2(\phi^\dagger\phi)^3 + \frac{2}{3}g^2m^2(\phi^\dagger\phi)^2 - \frac{1}{3}g^2(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R$$

$$- \frac{1}{3}g'^2(\phi^\dagger i\overset{\leftrightarrow}{D}_\mu\phi)\bar\psi\gamma^\mu(Y_L P_L + Y_R P_R)\psi$$

$$- \frac{1}{6}g^2(\phi^\dagger i\overset{\leftrightarrow}{D}{}^a_\mu\phi)\bar\psi_L\gamma^\mu\sigma^a\psi_L + \text{h.c. },\tag{118}$$

$$-\text{tr } aM^2 = -\left(6g^2 + 2g'^2\right)(\phi^\dagger\phi)\Box(\phi^\dagger\phi) - 8g'^2(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi)$$

$$+ \left(36\lambda^2 - 8\lambda g^2\right)(\phi^\dagger\phi)^3 - \left(24\lambda - 8g^2\right)m^2(\phi^\dagger\phi)^2$$

$$+ 4m^4(\phi^\dagger\phi) - 4g^2(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c. },$$

$$\text{tr } i\bar\Gamma\slashed{b}\Gamma = \frac{1}{2}(\phi^\dagger i\overset{\leftrightarrow}{D}{}^a_\mu\phi)\bar\psi_L\gamma^\mu\sigma^a\langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I\psi_L + 2(\phi^\dagger i\overset{\leftrightarrow}{D}_\mu\phi)\bar\psi_R\gamma^\mu\mathcal{Y}^\dagger\mathcal{Y}\tau^3\psi_R$$

$$+ (\phi^\dagger i\overset{\leftrightarrow}{D}_\mu\phi)\bar\psi_L\gamma^\mu\left(\langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I - 2\mathcal{Y}\mathcal{Y}^\dagger\right)\tau^3\psi_L$$

$$+ 4i\bar\psi_R\gamma^\mu\mathcal{Y}^\dagger\varphi(\tau^1 t^1_R + \tau^2 t^2_R)D_\mu\varphi\mathcal{Y}\psi_R ,$$

$$-\frac{1}{6}\text{tr } \bar\Gamma i\overset{\leftrightarrow}{\mathcal{D}}_a\Gamma = 6(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\mathcal{Y}^\dagger\mathcal{Y}\psi_R + \text{h.c.}$$

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The results sum up to the divergent Lagrangian

$$
\begin{aligned}
32\pi^2\epsilon\mathcal{L}^{\mathrm{div}}_{\phi\Box} = \frac{C_{\phi\Box}}{\Lambda^2}\Bigg(& -\left(12\lambda + 8g^2 + \frac{8}{3}g'^2\right)(\phi^\dagger\phi)\Box(\phi^\dagger\phi) - \frac{20}{3}g'^2(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi) \\
& + \left(40\lambda^2 - \frac{20}{3}\lambda g^2\right)(\phi^\dagger\phi)^3 - \left(32\lambda - \frac{20}{3}g^2\right)m^2(\phi^\dagger\phi)^2 \\
& + 8m^4(\phi^\dagger\phi) + (\phi^\dagger\phi)\bar{\psi}_L\sqrt{2}H\left(2\lambda\mathcal{Y} - \frac{10}{3}g^2\mathcal{Y} + 6\mathcal{Y}\mathcal{Y}^\dagger\mathcal{Y}\right)\psi_R \\
& - 2m^2\bar{\psi}_L\sqrt{2}H\mathcal{Y}\psi_R - (\phi^\dagger i\overset{\leftrightarrow}{D}_\mu\phi)\bar{\psi}_R\gamma^\mu\left(\frac{1}{3}g'^2Y_R - 2\mathcal{Y}^\dagger\mathcal{Y}\tau^3\right)\psi_R \quad (119) \\
& - (\phi^\dagger i\overset{\leftrightarrow}{D}_\mu\phi)\bar{\psi}_L\gamma^\mu\left(\frac{1}{3}g'^2Y_L + (2\mathcal{Y}\mathcal{Y}^\dagger - \langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I)\tau^3\right)\psi_L \\
& - (\phi^\dagger i\overset{\leftrightarrow}{D}{}^a_\mu\phi)\bar{\psi}_L\gamma^\mu\sigma^a\left(\frac{1}{6}g^2 - \frac{1}{2}\langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I\right)\psi_L \\
& + 4i\bar{\psi}_R\gamma^\mu\mathcal{Y}^\dagger\varphi(\tau^1t^1_R + \tau^2t^2_R)D_\mu\varphi\mathcal{Y}\psi_R + \mathrm{h.c.}\Bigg)\;.
\end{aligned}
$$

In total, the $\phi\Box$-operator contributes to the beta-functions with

$$\beta_{\phi\Box} \supseteq \left(12\lambda - 4g^2 - \frac{4}{3}g'^2 + 4\gamma_\phi\right)C_{\phi\Box}\,, \qquad \beta_{\psi\phi} \supseteq \left(-2\lambda\mathcal{Y} + \frac{10}{3}g^2\mathcal{Y} - 6\mathcal{Y}\mathcal{Y}^\dagger\mathcal{Y}\right)C_{\phi\Box}\,,$$

$$\beta_{\phi D} \supseteq \frac{20}{3}g'^2C_{\phi\Box}\,, \qquad\qquad\qquad \beta_{\phi\psi} \supseteq \left(\frac{1}{3}g'^2Y_R - 2\tau^3\mathcal{Y}^\dagger\mathcal{Y}\right)C_{\phi\Box}\,,$$

$$\beta_\phi \supseteq \left(-40\lambda^2 + \frac{20}{3}\lambda g^2\right)C_{\phi\Box}\,, \qquad\qquad \beta^{(1)}_{\phi\psi} \supseteq \left(\frac{1}{3}g'^2Y_L + (2\mathcal{Y}\mathcal{Y}^\dagger - \langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I)\tau^3\right)C_{\phi\Box}\,,$$

$$\beta_\lambda \supseteq \left(-64\lambda + \frac{40}{3}g^2\right)\frac{m^2}{\Lambda^2}C_{\phi\Box}\,, \qquad\qquad \beta^{(3)}_{\phi\psi} \supseteq \left(\frac{1}{6}g^2 - \frac{1}{2}\langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I\right)C_{\phi\Box}\,,$$

$$\beta_{m^2} \supseteq -8\frac{m^4}{\Lambda^2}C_{\phi\Box}\,, \qquad\qquad\qquad \beta_{\phi ud} \supseteq 2\mathcal{Y}^\dagger_u\mathcal{Y}_dC_{\phi\Box}\,,$$

$$\beta_\mathcal{Y} \supseteq -2\frac{m^2}{\Lambda^2}\mathcal{Y}C_{\phi\Box}\,.$$

$$(120)$$

5.3.3 ϕD-operator

The ϕD operator can be decomposed as

$$Q_{\phi D} = -\frac{1}{4}Q_{\phi\Box} + Q_{\phi R}\;. \tag{121}$$

where

$$Q_{\phi R} = -(\varphi t_R^3 D_\mu \varphi)^2 \ . \tag{122}$$

$Q_{\phi\square}$ has already been treated above. For $Q_{\phi R}$ we find the fluctuations

$$a_{ij}^{\mu\nu} = -2(t_R^3\varphi)_i(t_R^3\varphi)_j g^{\mu\nu} \equiv a_{ij} g^{\mu\nu} \ ,$$

$$b_{ij}^\lambda = 2(\varphi t_R^3 D^\lambda \varphi) t_{Rij}^3 + (t_R^3\varphi)_i(t_R^3 D^\lambda \varphi)_j - (t_R^3 D^\lambda \varphi)_i(t_R^3\varphi)_j \ ,$$

$$b_{(a\alpha)i}^\lambda = g(\varphi t_L^a t_R^3 \varphi)(t_R^3\varphi)_i g^{\lambda\alpha} \ ,$$

$$b_{\alpha i}^\lambda = \frac{g'}{4}\varphi^2(t_R^3\varphi)_i g^{\lambda\alpha} \ ,$$

$$c_{ij} = 6(t_R^3 D_\mu\varphi)_i(t_R^3 D^\mu\varphi)_j + 2(t_R^3\varphi)_i(t_R^3 D^2\varphi)_j + 2(t_R^3 D^2\varphi)_i(t_R^3\varphi)_j \ ,$$

$$c_{(a\alpha)(b\beta)} = 2g^2(\varphi t_L^a t_R^3\varphi)(\varphi t_L^b t_R^3\varphi)g_{\alpha\beta} \ ,$$

$$c_{\alpha\beta} = \frac{g'^2}{8}(\varphi^2)^2 g_{\alpha\beta} \ ,$$

$$c_{(a\alpha)\beta} = \frac{gg'}{2}\varphi^2(\varphi t_L^a t_R^3\varphi)g_{\alpha\beta} \ ,$$

$$c_{(a\alpha)i} = -4g(\varphi t_R^3 D_\alpha\varphi)(t_L^a t_R^3\varphi)_i - 3g(\varphi t_L^a t_R^3\varphi)(t_R^3 D_\alpha\varphi)_i - 2g(\varphi t_L^a t_R^3 D_\alpha\varphi)(t_R^3\varphi)_i \ ,$$

$$c_{\alpha i} = -g'(\varphi t_R^3 D_\alpha\varphi)\varphi_i - \frac{3}{4}g'\varphi^2(t_R^3 D_\alpha\varphi)_i - \frac{g'}{4}\partial_\alpha(\varphi^2)(t_R^3\varphi)_i \ .$$

$$\tag{123}$$

The divergences derived from these matrices are

$$\text{tr } cM = -\left(6\lambda + \frac{29}{2}g^2 + 4g'^2\right)(\phi^\dagger D_\mu \phi)^*(\phi^\dagger D^\mu \phi)$$

$$-\left(3\lambda + \frac{19}{8}g^2 + \frac{13}{8}g'^2\right)(\phi^\dagger \phi)\Box(\phi^\dagger \phi) - m^4(\phi^\dagger \phi)$$

$$-\left(\lambda^2 - \frac{1}{2}\lambda g^2 + 2\lambda g'^2 - g^4 - 2g^2 g'^2 - g'^4\right)(\phi^\dagger \phi)^3$$

$$+\left(2\lambda - \frac{1}{2}g^2 + 2g'^2\right)m^2(\phi^\dagger \phi)^2 + \frac{1}{2}m^2\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R$$

$$-\left(\frac{1}{2}\lambda - \frac{1}{4}g^2 + g'^2\right)(\phi^\dagger \phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c.}\ ,$$

$$-\frac{1}{3}\text{tr }[D^\mu, N_{\mu\nu}]b^\nu = -\frac{5}{6}g'^2(\phi^\dagger D_\mu \phi)^*(\phi^\dagger D^\mu \phi) - \left(\frac{1}{8}g^2 + \frac{5}{24}g'^2\right)(\phi^\dagger \phi)\Box(\phi^\dagger \phi)$$

$$+\frac{1}{6}g^2 m^2(\phi^\dagger \phi)^2 - \frac{1}{6}\lambda g^2(\phi^\dagger \phi)^3 - \frac{1}{12}g^2(\phi^\dagger \phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R$$

$$-\frac{5}{12}g'^2(\phi^\dagger i\overset{\leftrightarrow}{D}_\mu \phi)\bar\psi_R\gamma^\mu Y_R\psi_R - \frac{5}{12}g'^2(\phi^\dagger i\overset{\leftrightarrow}{D}_\mu \phi)\bar\psi_L\gamma^\mu Y_L\psi_L$$

$$-\frac{1}{24}g^2(\phi^\dagger i\overset{\leftrightarrow}{D^a_\mu} \phi)\bar\psi_L\gamma^\mu \sigma^a\psi_L + \text{h.c.}\ ,$$
(124)

$$-\text{tr } aM^2 = -2g^2(\phi^\dagger D_\mu \phi)^*(\phi^\dagger D^\mu \phi) - \left(-\frac{1}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger \phi)\Box(\phi^\dagger \phi)$$

$$-\left(\lambda^2 - \lambda g^2 + \lambda g'^2 + \frac{1}{4}g^4 + \frac{1}{2}g^2 g'^2 + \frac{1}{4}g'^4\right)(\phi^\dagger \phi)^3$$

$$+\left(2\lambda - g^2 + g'^2\right)m^2(\phi^\dagger \phi)^2 - m^4(\phi^\dagger \phi)$$

$$+g^2(\phi^\dagger \phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c.}\ ,$$

$$\text{tr } i\bar\Gamma \slashed{b}\Gamma = \frac{1}{8}(\phi^\dagger i\overset{\leftrightarrow}{D^a_\mu} \phi)\bar\psi_L\gamma^\mu \sigma^a\langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I\psi_L + \frac{5}{2}(\phi^\dagger i\overset{\leftrightarrow}{D}_\mu \phi)\bar\psi_R\gamma^\mu \mathcal{Y}^\dagger \mathcal{Y}\tau^3\psi_R$$

$$-\frac{5}{4}(\phi^\dagger i\overset{\leftrightarrow}{D}_\mu \phi)\bar\psi_L\gamma^\mu \left(2\mathcal{Y}\mathcal{Y}^\dagger - \langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I\right)\tau^3\psi_L$$

$$-i\bar\psi_R\gamma^\mu \mathcal{Y}^\dagger \varphi\left(\tau^1 t_R^1 + \tau^2 t_R^2\right)D_\mu\varphi\mathcal{Y}\psi_R$$

$$-\left(\frac{1}{2}g^2 + \frac{1}{2}g'^2\right)(\phi^\dagger \phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R + \text{h.c.}\ ,$$

$$-\frac{1}{6}\text{tr }\bar\Gamma i\overset{\leftrightarrow}{\mathcal{D}}_a\Gamma = \frac{1}{2}(\phi^\dagger \phi)\bar\psi_L\sqrt{2}H\mathcal{Y}\mathcal{Y}^\dagger \mathcal{Y}\psi_R + \text{h.c.}$$

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We reduce contractions with the Levi-Civita tensor using

$$\epsilon_{ijkl}(D_\mu\varphi)_i(t_R^3 D^\mu\varphi)_j(t_R^3\varphi)_k\varphi_l = (\varphi t_R^3 D_\mu\varphi)^2 - \frac{1}{4}(\varphi D_\mu\varphi)^2 + \frac{1}{4}\varphi^2(D_\mu\varphi)^2 \ . \tag{125}$$

Combining the results in (119) and (124) according to (121), we obtain

$$
\begin{aligned}
32\pi^2\epsilon\mathcal{L}_{\phi D}^{\rm div} = \frac{C_{\phi D}}{\Lambda^2}\Bigg(&-\left(6\lambda + \frac{33}{2}g^2 + \frac{19}{6}g'^2\right)(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi) + m^2\bar\psi_L\sqrt{2}H\mathcal{Y}\psi_R \\
&-\frac{5}{3}g'^2(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \left(12\lambda - 3g^2 + 3g'^2\right)m^2(\phi^\dagger\phi)^2 \\
&-\left(12\lambda^2 - 3\lambda g^2 + 3\lambda g'^2 - \frac{3}{4}g^4 - \frac{3}{2}g^2g'^2 - \frac{3}{4}g'^4\right)(\phi^\dagger\phi)^3 \\
&-(\phi^\dagger\phi)\bar\psi_L\sqrt{2}H\left(\lambda\mathcal{Y} - \frac{3}{2}g^2\mathcal{Y} + \frac{3}{2}g'^2\mathcal{Y} + \mathcal{Y}\mathcal{Y}^\dagger\mathcal{Y}\right)\psi_R \\
&-(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)\bar\psi_R\gamma^\mu\left(\frac{1}{3}g'^2 Y_R - 2\tau^3\mathcal{Y}^\dagger\mathcal{Y}\right)\psi_R - 4m^4(\phi^\dagger\phi) \\
&-(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)\bar\psi_L\gamma^\mu\left(\frac{1}{3}g'^2 Y_L + (2\mathcal{Y}\mathcal{Y}^\dagger - \langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I)\tau^3\right)\psi_L \\
&-2i\bar\psi_R\gamma^\mu\mathcal{Y}^\dagger\varphi(\tau^1 t_R^1 + \tau^2 t_R^2)D_\mu\varphi\mathcal{Y}\psi_R + \text{h.c.}\Bigg) \ .
\end{aligned}
\tag{126}
$$

We find the following set of beta-function entries:

$$\beta_{\phi D} \supseteq \left(6\lambda + \frac{9}{2}g^2 - \frac{5}{6}g'^2 + 4\gamma_\phi\right)C_{\phi D} \ , \qquad\qquad \beta_{\phi\Box} \supseteq \frac{5}{3}g'^2 C_{\phi D} \ ,$$

$$\beta_\phi \supseteq \left(12\lambda^2 - 3\lambda(g^2 - g'^2) - \frac{3}{4}(g^2 + g'^2)^2\right)C_{\phi D} \ , \qquad \beta_{m^2} \supseteq 4\frac{m^4}{\Lambda^2}C_{\phi D} \ ,$$

$$\beta_\lambda \supseteq \left(24\lambda - 6(g^2 - g'^2)\right)\frac{m^2}{\Lambda^2}C_{\phi D} \ , \qquad\qquad \beta_{\mathcal{Y}} \supseteq \frac{m^2}{\Lambda^2}\mathcal{Y}C_{\phi D} \ ,$$

$$\beta_{\psi\phi} \supseteq \left(\lambda\mathcal{Y} - \frac{3}{2}(g^2 - g'^2)\mathcal{Y} + \mathcal{Y}\mathcal{Y}^\dagger\mathcal{Y}\right)C_{\phi D} \ , \qquad \beta_{\phi ud} \supseteq -\mathcal{Y}_u^\dagger\mathcal{Y}_d C_{\phi D} \ ,$$

$$\beta_{\phi\psi} \supseteq \left(\frac{1}{3}g'^2 Y_R - 2\tau^3\mathcal{Y}^\dagger\mathcal{Y}\right)C_{\phi D} \ ,$$

$$\beta_{\phi\psi}^{(1)} \supseteq \left(\frac{1}{3}g'^2 Y_L + (2\mathcal{Y}\mathcal{Y}^\dagger - \langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I)\tau^3\right)C_{\phi D} \ .$$

$$\tag{127}$$

This completes our calculation for the bosonic operators.

6   Summary of results

We summarize our results by listing all the contributions to the renormalization-group beta functions that arise from the pure bosonic operators of dimension 6 in SMEFT. All results are in agreement with [12–14] (see also the compilation in [18]).

Adding the individual contributions derived in section 5 we obtain:

$$\beta_G \supseteq 15 g_s^2 C_G \,, \qquad \beta_{\widetilde{G}} \supseteq 15 g_s^2 C_{\widetilde{G}} \,, \qquad \beta_W \supseteq \frac{29}{2} g^2 C_W \,, \qquad \beta_{\widetilde{W}} \supseteq \frac{29}{2} g^2 C_{\widetilde{W}} \qquad (128)$$

$$\beta_{\phi G} \supseteq \left(6\lambda - 14 g_s^2 - \frac{9}{2} g^2 - \frac{3}{2} g'^2 + 2\gamma_\phi \right) C_{\phi G} \qquad (129)$$

$$\beta_{\phi \widetilde{G}} \supseteq \left(6\lambda - 14 g_s^2 - \frac{9}{2} g^2 - \frac{3}{2} g'^2 + 2\gamma_\phi \right) C_{\phi \widetilde{G}} \qquad (130)$$

$$\beta_{\phi B} \supseteq \left(6\lambda - \frac{9}{2} g^2 + \frac{85}{6} g'^2 + 2\gamma_\phi \right) C_{\phi B} + 3 g g' C_{\phi W B} \qquad (131)$$

$$\beta_{\phi \widetilde{B}} \supseteq \left(6\lambda - \frac{9}{2} g^2 + \frac{85}{6} g'^2 + 2\gamma_\phi \right) C_{\phi \widetilde{B}} + 3 g g' C_{\phi \widetilde{W} B} \,, \qquad (132)$$

$$\beta_{\phi W} \supseteq -15 g^3 C_W + \left(6\lambda - \frac{53}{6} g^2 - \frac{3}{2} g'^2 + 2\gamma_\phi \right) C_{\phi W} + g g' C_{\phi W B} \qquad (133)$$

$$\beta_{\phi \widetilde{W}} \supseteq -15 g^3 C_{\widetilde{W}} + \left(6\lambda - \frac{53}{6} g^2 - \frac{3}{2} g'^2 + 2\gamma_\phi \right) C_{\phi \widetilde{W}} + g g' C_{\phi \widetilde{W} B} \qquad (134)$$

$$\beta_{\phi W B} \supseteq 3 g^2 g' C_W + 2 g g' C_{\phi W} + 2 g g' C_{\phi B} + \left(2\lambda + \frac{4}{3} g^2 + \frac{19}{3} g'^2 + 2\gamma_\phi \right) C_{\phi W B} \qquad (135)$$

$$\beta_{\phi \widetilde{W} B} \supseteq 3 g^2 g' C_{\widetilde{W}} + 2 g g' C_{\phi \widetilde{W}} + 2 g g' C_{\phi \widetilde{B}} + \left(2\lambda + \frac{4}{3} g^2 + \frac{19}{3} g'^2 + 2\gamma_\phi \right) C_{\phi \widetilde{W} B} \qquad (136)$$

$$\beta_\phi \supseteq \left(18\lambda g^2 - 9 g^4 - 3 g^2 g'^2 \right) C_{\phi W} + \left(6\lambda g'^2 - 3 g^2 g'^2 - 3 g'^4 \right) C_{\phi B}$$

$$+ \left(6\lambda gg' - 3g^3g' - 3gg'^3\right) C_{\phi WB} + \left(54\lambda - \frac{27}{2}g^2 - \frac{9}{2}g'^2 + 6\gamma_\phi\right) C_\phi$$

$$+ \left(-40\lambda^2 + \frac{20}{3}\lambda g^2\right) C_{\phi\Box} + \left(12\lambda^2 - 3\lambda(g^2 - g'^2) - \frac{3}{4}(g^2 + g'^2)^2\right) C_{\phi D} \tag{137}$$

$$\beta_{\phi\Box} \supseteq \left(12\lambda - 4g^2 - \frac{4}{3}g'^2 + 4\gamma_\phi\right) C_{\phi\Box} + \frac{5}{3}g'^2 C_{\phi D} \tag{138}$$

$$\beta_{\phi D} \supseteq \frac{20}{3}g'^2 C_{\phi\Box} + \left(6\lambda + \frac{9}{2}g^2 - \frac{5}{6}g'^2 + 4\gamma_\phi\right) C_{\phi D} \tag{139}$$

$$\beta_{qG} \supseteq 9g_s^2 C_G \mathcal{Y}_q + 9ig_s^2 C_{\widetilde{G}} \mathcal{Y}_q - 4g_s \mathcal{Y}_q C_{\phi G} - 4ig_s \mathcal{Y}_q C_{\phi\widetilde{G}} \tag{140}$$

$$\beta_{\psi W} \supseteq -g\mathcal{Y}C_{\phi W} - ig\mathcal{Y}C_{\phi\widetilde{W}} + 2g'(Y_L + Y_R)\tau^3 \mathcal{Y}C_{\phi WB} + 2ig'(Y_L + Y_R)\tau^3 \mathcal{Y}C_{\phi\widetilde{W}B} \tag{141}$$

$$\beta_{\psi B} \supseteq -2g'(Y_L + Y_R)\mathcal{Y}C_{\phi B} - 2ig'(Y_L + Y_R)\mathcal{Y}C_{\phi\widetilde{B}} + 3g\tau^3 \mathcal{Y}C_{\phi WB} + 3ig\tau^3 \mathcal{Y}C_{\phi\widetilde{W}B} \tag{142}$$

$$\beta_{q\phi} \supseteq 32g_s^2 \mathcal{Y}_q C_{\phi G} + 32ig_s^2 \mathcal{Y}_q C_{\phi\widetilde{G}} \tag{143}$$

$$\beta_{\psi\phi} \supseteq 9g^2 \mathcal{Y}C_{\phi W} + 12g'^2(Y_L^2 + Y_R^2)\mathcal{Y}C_{\phi B} + 9ig^2 \mathcal{Y}C_{\phi\widetilde{W}}$$
$$+ 12ig'^2(Y_L^2 + Y_R^2)\mathcal{Y}C_{\phi\widetilde{B}} - 12gg'Y_L\tau^3 \mathcal{Y}C_{\phi WB} - 12igg'Y_L\tau^3 \mathcal{Y}C_{\phi\widetilde{W}B}$$
$$+ \left(-2\lambda\mathcal{Y} + \frac{10}{3}g^2\mathcal{Y} - 6\mathcal{Y}\mathcal{Y}^\dagger\mathcal{Y}\right) C_{\phi\Box} + \left(\lambda\mathcal{Y} - \frac{3}{2}(g^2 - g'^2)\mathcal{Y} + \mathcal{Y}\mathcal{Y}^\dagger\mathcal{Y}\right) C_{\phi D} \tag{144}$$

$$\beta_{\phi\psi} \supseteq \left(\frac{1}{3}g'^2 Y_R - 2\tau^3 \mathcal{Y}^\dagger\mathcal{Y}\right)(C_{\phi\Box} + C_{\phi D}) , \qquad \beta_{\phi ud} \supseteq \mathcal{Y}_u^\dagger\mathcal{Y}_d (2C_{\phi\Box} - C_{\phi D}) \tag{145}$$

$$\beta_{\phi\psi}^{(1)} \supseteq \left(\frac{1}{3}g'^2 Y_L + \left(2\mathcal{Y}\mathcal{Y}^\dagger - \langle\mathcal{Y}\mathcal{Y}^\dagger\rangle_I\right)\tau^3\right)(C_{\phi\Box} + C_{\phi D}) \tag{146}$$

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$$\beta_{\phi\psi}^{(3)} \supseteq \left(\frac{1}{6}g^2 - \frac{1}{2}\langle \mathcal{Y}\mathcal{Y}^\dagger \rangle_I \right) C_{\phi\Box} \tag{147}$$

Here $\langle \ldots \rangle_I$ denotes a trace over isospin indices.

Finally, we collect the contributions from the bosonic dimension-6 operators to the beta functions of couplings in the SM at dimension 4:

$$\beta_{g_s} \supseteq -8g_s \frac{m^2}{\Lambda^2} C_{\phi G} \,, \qquad \beta_g \supseteq -8g \frac{m^2}{\Lambda^2} C_{\phi W} \,, \qquad \beta_{g'} \supseteq -8g' \frac{m^2}{\Lambda^2} C_{\phi B} \tag{148}$$

$$\beta_\lambda \supseteq 36g^2 \frac{m^2}{\Lambda^2} C_{\phi W} + 12g'^2 \frac{m^2}{\Lambda^2} C_{\phi B} + 12gg' \frac{m^2}{\Lambda^2} C_{\phi WB} + 48 \frac{m^2}{\Lambda^2} C_\phi$$
$$+ \left(-64\lambda + \frac{40}{3}g^2 \right) \frac{m^2}{\Lambda^2} C_{\phi\Box} + \left(24\lambda - 6(g^2 - g'^2) \right) \frac{m^2}{\Lambda^2} C_{\phi D} \tag{149}$$

$$\beta_{m^2} \supseteq 4\frac{m^4}{\Lambda^2} \left(-2C_{\phi\Box} + C_{\phi D} \right) \,, \qquad \beta_{\mathcal{Y}} \supseteq \frac{m^2}{\Lambda^2} \mathcal{Y} \left(-2C_{\phi\Box} + C_{\phi D} \right) \tag{150}$$

$$\beta_{\theta_s} \supseteq -\frac{128\pi^2}{g_s^2} \frac{m^2}{\Lambda^2} C_{\phi\widetilde{G}} \,, \qquad \beta_\theta \supseteq -\frac{128\pi^2}{g^2} \frac{m^2}{\Lambda^2} C_{\phi\widetilde{W}} \,, \qquad \beta_{\theta'} \supseteq -\frac{128\pi^2}{g'^2} \frac{m^2}{\Lambda^2} C_{\phi\widetilde{B}} \tag{151}$$

7 Conclusions

We have shown how functional methods provide an efficient way to compute UV divergences to one loop in SMEFT. Using the background-field method and a super-heat-kernel expansion, we derived a master formula for the one-loop divergences of EFTs that generalizes a known formula, originally due to 't Hooft [8]. The generalization allows for the addition of a non-standard term of the form $a^{\mu\nu} D_\mu D_\nu$ to the fluctuation operator $\Delta = D^\mu D_\mu + Y$, treated to first order in the field-dependent quantity $a^{\mu\nu}$.

As an application of this master formula we computed the complete one-loop divergences from insertions of the purely bosonic dimension-6 operators in the Warsaw basis [11] of SMEFT. We derived the corresponding RGEs, describing the RG mixing of the bosonic dimension-6 operators into any SMEFT operator of dimension 4 or 6. Our analysis serves as an independent confirmation of results previously obtained in the literature [12–14].

We have also discussed how the RG beta-functions (anomalous dimensions) for operator coefficients in SMEFT are related to the one-loop divergences, demonstrating that this relation is governed by chiral dimensions. In future work, we plan to return to the renormalization of the remaining dimension-6 operators in SMEFT using functional methods.

Acknowledgements

A   Details on the Higgs field representation

We express the Higgs field degrees of freedom as ($j \in \{0, 1, 2, 3\}$)

$$H \equiv \frac{1}{\sqrt{2}} \left(\widetilde{\phi}, \phi \right) = i\tau^j \varphi_j , \tag{152}$$

with τ^a ($a \in \{1, 2, 3\}$) the generators of SU(2) and $\tau^0 = -\frac{i}{2}\mathbf{1}$. Under an electroweak gauge transformation $H \to g_L H g_R^\dagger$, with $g_L \in \mathrm{SU}(2)_L$ and g_R belonging to the $\mathrm{U}(1)_Y$ subgroup of $\mathrm{SU}(2)_R$. Since $\mathrm{SU}(2) \otimes \mathrm{SU}(2)$ is the universal covering group of $\mathrm{SO}(4)$, we can express the transformation properties of φ_i in terms of SO(4) generators. The covariant derivative acting on the fields φ_i is given by

$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + ig W_\mu^a t_{Lij}^a \varphi_j + ig' B_\mu t_{Rij}^3 \varphi_j , \tag{153}$$

with the SO(4) generators

$$
\begin{aligned}
t_{Lij}^a &\equiv +2 \operatorname{tr} (\tau^i)^\dagger \tau^a \tau^j = -\frac{i}{2} \left(\epsilon^{aij} + \delta^{ai}\delta^{0j} - \delta^{0i}\delta^{aj} \right) , \\
t_{Rij}^a &\equiv -2 \operatorname{tr} (\tau^i)^\dagger \tau^j \tau^a = -\frac{i}{2} \left(\epsilon^{aij} - \delta^{ai}\delta^{0j} + \delta^{0i}\delta^{aj} \right) .
\end{aligned}
\tag{154}
$$

Here $a, b \in \{1, 2, 3\}$ and $i, j, k, l \in \{0, 1, 2, 3\}$. The antisymmetric tensor ϵ^{aij} is defined such that $\epsilon^{aij} = 0$ if $i = 0$ or $j = 0$. In matrix form we can write

$$
t_L^1 = -\frac{i}{2} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} , \quad
t_L^2 = -\frac{i}{2} \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} , \quad
t_L^3 = -\frac{i}{2} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} ,
$$

$$
t_R^1 = -\frac{i}{2} \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} , \quad
t_R^2 = -\frac{i}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} , \quad
t_R^3 = -\frac{i}{2} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} .
\tag{155}
$$

These matrices fulfill the SU(2) algebra

$$\left[t_L^a, t_L^b\right] = i\epsilon^{abc}t_L^c \,, \qquad\qquad \left[t_R^a, t_R^b\right] = i\epsilon^{abc}t_R^c \,,$$

$$\left\{t_L^a, t_L^b\right\} = \frac{1}{2}\delta^{ab} \,, \qquad\qquad \left\{t_R^a, t_R^b\right\} = \frac{1}{2}\delta^{ab} \,, \tag{156}$$

$$\mathrm{tr}\, t_L^a t_L^b = \delta^{ab} \,, \qquad\qquad \mathrm{tr}\, t_R^a t_R^b = \delta^{ab} \,,$$

with $\left[t_L^a, t_R^b\right] = 0$ and $\mathrm{tr}\, t_L^a t_R^b = 0$. Some useful identities are

$$t_{Lik}^a t_{Lkj}^b = \frac{1}{4}\delta^{ab}\delta_{ij} + \frac{i}{2}\epsilon^{abc}t_{Lij}^c \,, \qquad t_{Lij}^a t_{Lkl}^a = \frac{1}{4}\left(\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl} + \epsilon_{ijkl}\right) \,,$$

$$t_{Rik}^a t_{Rkj}^b = \frac{1}{4}\delta^{ab}\delta_{ij} + \frac{i}{2}\epsilon^{abc}t_{Rij}^c \,, \qquad t_{Rij}^a t_{Rkl}^a = \frac{1}{4}\left(\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl} - \epsilon_{ijkl}\right) \,. \tag{157}$$

Here ϵ_{ijkl} denotes the totally anti-symmetric 4-dimensional tensor. Using the real representation for the Higgs field, the SM equations of motion read

$$(D_\mu G^{\mu\nu})^A = g_s\bar{q}\gamma^\nu T^A q \,,$$

$$(D_\mu W^{\mu\nu})^a = igt_{Lij}^a\varphi_i(D^\nu\varphi)_j + g\bar{\psi}\gamma^\nu\tau^a P_L\psi \,,$$

$$\partial_\mu B^{\mu\nu} = ig't_{Rij}^3\varphi_i(D^\nu\varphi)_j + g'\bar{\psi}\gamma^\nu(Y_L P_L + Y_R P_R)\psi \,, \tag{158}$$

$$(D_\mu D^\mu\varphi)_i = m^2\varphi_i - \frac{\lambda}{2}(\varphi_j\varphi_j)\varphi_i - i\bar{\psi}\sqrt{2}\left(\tau^i\mathcal{Y}P_R - \mathcal{Y}^\dagger(\tau^i)^\dagger P_L\right)\psi \,,$$

$$i\slashed{D}\psi = \sqrt{2}(H\mathcal{Y}P_R + \mathcal{Y}^\dagger H^\dagger P_L)\psi \,.$$

B SM fluctuation operator

The SM fluctuation operator can be cast in the form of (5) by choosing the Feynman gauge. In the electroweak sector we use a gauge fixing term that cancels the mixing between the gauge fields and the would-be Goldstone bosons in the SM [29]. For QCD we take the usual Yang-Mills gauge fixing [4]. The gauge-fixing Lagrangian reads[6]

$$\mathcal{L}_{\mathrm{g.f.}} = -\frac{1}{2}f^A f^A - \frac{1}{2}f^a f^a - \frac{1}{2}f^2 \,. \tag{159}$$

with

$$f^A = D_\mu\alpha^{A\mu} \,, \qquad f^a = D_\mu\omega^{a\mu} - igt_{Lij}^a\xi_i\varphi_j \,, \qquad f = \partial_\mu\beta^\mu - ig't_{Rij}^3\xi_i\varphi_j \,. \tag{160}$$

---

[6]A general discussion of gauge fixing in SMEFT can be found in [30]

The covariant derivatives act on α_μ and ω_μ in the adjoint representation. The Faddeev–Popov Lagrangian is quadratic in the ghost fields, which do not mix with the other degrees of freedom, so its divergences can be calculated separately:

$$32\pi^2\epsilon\mathcal{L}^1_{\text{div, ghost}} = \frac{1}{6}g_s^2 C_3^{\text{ad}}G^A_{\mu\nu}G^{A\mu\nu} + \frac{1}{6}g^2 C_2^{\text{ad}}W^a_{\mu\nu}W^{a\mu\nu} - \frac{\lambda}{2}(\phi^\dagger\phi)^2\left(\frac{3g^4 + 2g^2g'^2 + g'^4}{2\lambda}\right) .$$
(161)

Here $C_N^{\text{ad}} = N$ is the Dynkin index of SU(N) in the adjoint representation. The bosonic building blocks N^μ and M in (6) are given by

$$N^\mu_{IJ} = \begin{pmatrix} g_s f^{ABC}G^{C\mu}g_{\alpha\beta} & & & \\ & g\epsilon^{abc}W^{c\mu}g_{\lambda\rho} & & \\ & & 0 & \\ & & & igW^{d\mu}t^d_{Lij} + ig'B^\mu t^3_{Rij} \end{pmatrix} ,$$

$$M_{IJ} = \begin{pmatrix} 2g_s f^{ABC}G^C_{\alpha\beta} & 0 & 0 & 0 \\ 0 & 2g\epsilon^{abc}W^c_{\lambda\rho} + \dfrac{g^2}{4}(\varphi_k\varphi_k)\delta^{ab}g_{\lambda\rho} & gg'(\varphi t^a_L t^3_R\varphi)g_{\lambda\kappa} & -2g(t^a_L D_\lambda\varphi)_j \\ 0 & gg'(\varphi t^b_L t^3_R\varphi)g_{\rho\sigma} & \dfrac{g'^2}{4}(\varphi_k\varphi_k)g_{\sigma\kappa} & -2g'(t^3_R D_\sigma\varphi)_j \\ 0 & -2g(t^b_L D_\rho\varphi)_i & -2g'(t^3_R D_\kappa\varphi)_i & M_{ij} \end{pmatrix} ,$$
(162)

with the field indices $I = (A\alpha, a\lambda, \sigma, i)$ and $J = (B\beta, b\rho, \kappa, j)$. Here we have defined

$$M_{ij} \equiv \left(\left(\frac{\lambda}{2} + \frac{g^2}{4}\right)(\varphi_k\varphi_k) - m^2\right)\delta_{ij} + \left(\lambda - \frac{g^2}{4}\right)\varphi_i\varphi_j - g'^2(t^3_R\varphi)_i(t^3_R\varphi)_j .$$
(163)

The fermion-boson mixing terms in (4) are given by

$$\Gamma^T = (-i)\begin{pmatrix} g_s T^B\gamma^\beta\psi \\ g\tau^b\gamma^\rho P_L\psi \\ g'\gamma^\kappa(Y_L P_L + Y_R P_R)\psi \\ \sqrt{2}(\tau^j\mathcal{Y}P_R - \mathcal{Y}^\dagger(\tau^j)^\dagger P_L)\psi \end{pmatrix} , \quad \bar\Gamma = (-i)\begin{pmatrix} g_s\bar\psi T^A\gamma^\alpha \\ g\bar\psi\tau^a\gamma^\lambda P_L \\ g'\bar\psi\gamma^\sigma(Y_L P_L + Y_R P_R) \\ \sqrt{2}\bar\psi(\tau^i\mathcal{Y}P_R - \mathcal{Y}^\dagger(\tau^i)^\dagger P_L) \end{pmatrix} ,$$
(164)

Note that in our conventions $\bar\Gamma_I = S_{IJ}\Gamma^\dagger_J\gamma_0$ with $S = \text{diag}(-1,-1,-1,1)$ and I, J labels for the bosonic variables. Finally, the pure fermionic terms in (7) read

$$r = \sqrt{2}H\mathcal{Y} , \quad R_\mu = g_s G_\mu + g'B_\mu Y_R ,$$

$$l = \sqrt{2}\mathcal{Y}^\dagger H^\dagger , \quad L_\mu = g_s G_\mu + gW_\mu + g'B_\mu Y_L .$$
(165)

$Y_{L,R}$ are the hypercharge matrices

$$Y_L = \text{diag}\left(\frac{1}{6}, \frac{1}{6}, -\frac{1}{2}, -\frac{1}{2}\right) \qquad \text{and} \qquad Y_R = \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, 0, -1\right) . \tag{166}$$

With these building blocks, and using the master formula in (9), we may verify the one-loop renormalization group equations of the SM. First, including the ghost contributions, we find for the one-loop divergences

$$32\pi^2 \epsilon \mathcal{L}_{\text{div}}^{\text{SM}} =$$

$$-\frac{1}{2}\langle G^{\mu\nu}G_{\mu\nu}\rangle\left(-\frac{22N_c - 4N_f}{3}\right)g_s^2 + 2C_F g_s^2 \bar{q} i \slashed{D} q - 8C_F g_s^2\left(\bar{q}\sqrt{2}H\mathcal{Y}P_R q + \text{h.c.}\right)$$

$$-\frac{1}{2}\langle W^{\mu\nu}W_{\mu\nu}\rangle\left(-\frac{44}{3} + \frac{2}{3}(N_c + 1)f + \frac{1}{3}\right)g^2 - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}\left(\left(\frac{22N_c}{27} + 2\right)f + \frac{1}{3}\right)g'^2$$

$$+D^\mu \phi^\dagger D_\mu \phi\left(-6g^2 - 2g'^2 + 2\langle \mathcal{Y}^\dagger \mathcal{Y}\rangle\right) + m^2 \phi^\dagger \phi\left(-\frac{3}{2}g^2 - \frac{1}{2}g'^2 - 6\lambda\right)$$

$$-\frac{\lambda}{2}(\phi^\dagger\phi)^2\left(-3g^2 - g'^2 - 12\lambda - \frac{3}{4\lambda}(3g^4 + 2g^2 g'^2 + g'^4) + \frac{4}{\lambda}\langle(\mathcal{Y}^\dagger\mathcal{Y})^2\rangle\right)$$

$$+\bar{\psi}_L\left(\frac{3}{2}g^2 + 2g'^2 Y_L^2\right)i\slashed{D}\psi_L + \bar{\psi}_R 2g'^2 Y_R^2 i \slashed{D}\psi_R - 8g'^2\left(\bar{\psi}_L\sqrt{2}H Y_L \mathcal{Y}Y_R\psi_R + \text{h.c.}\right)$$

$$+\bar{\psi}_L\langle \mathcal{Y}\mathcal{Y}^\dagger\rangle_I i\slashed{D}\psi_L + 2\bar{\psi}_R\mathcal{Y}^\dagger\mathcal{Y}i\slashed{D}\psi_R - 2\left(\bar{\psi}_L\sqrt{2}H(\langle \mathcal{Y}\mathcal{Y}^\dagger\rangle_I - \mathcal{Y}\mathcal{Y}^\dagger)\mathcal{Y}\psi_R + \text{h.c.}\right) \tag{167}$$

Here $\langle \ldots \rangle_I$ represents a trace over isospin indices only. $N_c = 3$, $f = 3$, and $N_f = 6$ denote the number of colours, fermion generations, and quark flavours, respectively, and $C_F = (N_c^2 - 1)/2N_c$. The quark fields are written as $q = (u, d, 0, 0)^T$.

From the divergences in (167) we obtain the beta functions of the SM:

$$\beta_{g_s} = -\frac{11N_c - 2N_f}{3}g_s^3 = -7g_s^3 \tag{168}$$

$$\beta_g = -\left(\frac{22}{3} - \frac{N_c + 1}{3}f - \frac{1}{6}\right)g^3 = -\frac{19}{6}g^3 \tag{169}$$

$$\beta_{g'} = \left(\left(\frac{11N_c}{27} + 1\right)f + \frac{1}{6}\right)g'^3 = \frac{41}{6}g'^3 \tag{170}$$

$$\beta_\lambda = -3(3g^2 + g'^2)\lambda + 12\lambda^2 + \frac{3}{4}(3g^4 + 2g^2 g'^2 + g'^4) + 4\lambda\langle \mathcal{Y}^\dagger\mathcal{Y}\rangle - 4\langle(\mathcal{Y}^\dagger\mathcal{Y})^2\rangle \tag{171}$$

$$\beta_{m^2} = m^2\left(-\frac{9}{2}g^2 - \frac{3}{2}g'^2 + 6\lambda + 2\langle \mathcal{Y}^\dagger\mathcal{Y}\rangle\right) \tag{172}$$

$$\beta_{\mathcal{Y}} = \frac{3}{2}\left(2\mathcal{Y}\mathcal{Y}^\dagger - \langle \mathcal{Y}\mathcal{Y}^\dagger\rangle_I\right)\mathcal{Y}$$

$$-\left(\frac{9}{4}g^2 + \left(\frac{3}{4} + 6Y_L Y_R\right)g'^2 - \langle \mathcal{Y}^\dagger \mathcal{Y} \rangle + 6C_F g_s^2 P_q\right)\mathcal{Y} \qquad (173)$$

with $3/4 + 6Y_L Y_R = \mathrm{diag}(17/12, 5/12, 3/4, 15/4)$, $P_q = \mathrm{diag}(1, 1, 0, 0)$, in agreement with the results compiled in [18] (see also [7] for further details).

References

[1] I. Jack and H. Osborn, Nucl. Phys. B **249** (1985) 472.

[2] C. Lee and C. Rim, Nucl. Phys. B **255** (1985) 439.

[3] H. Neufeld, J. Gasser and G. Ecker, Phys. Lett. B **438** (1998) 106 [hep-ph/9806436].

[4] L. F. Abbott, Acta Phys. Polon. B **13** (1982) 33.

[5] H. Neufeld, Eur. Phys. J. C **7** (1999) 355 [hep-ph/9807425].

[6] M. Knecht, H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. C **12** (2000) 469 [hep-ph/9909284].

[7] G. Buchalla, O. Catà, A. Celis, M. Knecht and C. Krause, Nucl. Phys. B **928** (2018) 93 [arXiv:1710.06412 [hep-ph]].

[8] G. 't Hooft, Nucl. Phys. B **62** (1973) 444.

[9] A. Agadjanov, D. Agadjanov, A. Khelashvili and A. Rusetsky, Eur. Phys. J. A **49** (2013) 120 [arXiv:1307.1451 [hep-ph]].

[10] W. Buchmüller and D. Wyler, Nucl. Phys. B **268**, 621 (1986).

[11] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010**, 085 (2010) [arXiv:1008.4884 [hep-ph]].

[12] E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1310** (2013) 087 [arXiv:1308.2627 [hep-ph]].

[13] E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1401** (2014) 035 [arXiv:1310.4838 [hep-ph]].

[14] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1404** (2014) 159 [arXiv:1312.2014 [hep-ph]].

[15] R. Alonso, H. M. Chang, E. E. Jenkins, A. V. Manohar and B. Shotwell, Phys. Lett. B **734** (2014) 302 [arXiv:1405.0486 [hep-ph]].

[16] J.-N. Toelstede, Master thesis Ludwig-Maximilians-Universität München (2018).

[17] C. Grojean, E. E. Jenkins, A. V. Manohar and M. Trott, JHEP **1304** (2013) 016 [arXiv:1301.2588 [hep-ph]]; J. Elias-Miro, J. R. Espinosa, E. Masso and A. Pomarol, JHEP **1311** (2013) 066 [arXiv:1308.1879 [hep-ph]]; JHEP **1308** (2013) 033 [arXiv:1302.5661 [hep-ph]]; G. M. Pruna and A. Signer, JHEP **1410** (2014) 014 [arXiv:1408.3565 [hep-ph]]; V. Cirigliano, W. Dekens, J. de Vries and E. Mereghetti, Phys. Rev. D **94** (2016) no.3, 034031 [arXiv:1605.04311 [hep-ph]]; F. Feruglio, P. Paradisi and A. Pattori, Phys. Rev. Lett. **118** (2017) no.1, 011801 [arXiv:1606.00524 [hep-ph]]; C. Bobeth, A. J. Buras, A. Celis and M. Jung, JHEP **1704** (2017) 079 [arXiv:1609.04783 [hep-ph]]; A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, Phys. Rev. D **96** (2017) no.3, 035026 [arXiv:1704.05672 [hep-ph]]; C. Bobeth, A. J. Buras, A. Celis and M. Jung, JHEP **1707** (2017) 124 [arXiv:1703.04753 [hep-ph]]; C. Bobeth and A. J. Buras, JHEP **1802** (2018) 101 [arXiv:1712.01295 [hep-ph]]; A. J. Buras and M. Jung, JHEP **1806** (2018) 067 [arXiv:1804.05852 [hep-ph]]; J. Aebischer, C. Bobeth, A. J. Buras and D. M. Straub, arXiv:1808.00466 [hep-ph]; J. E. Camargo-Molina, A. Celis and D. A. Faroughy, arXiv:1805.04917 [hep-ph]; J. Aebischer, J. Kumar and D. M. Straub, arXiv:1804.05033 [hep-ph]; M. Chala, C. Krause and G. Nardini, JHEP **1807** (2018) 062 [arXiv:1802.02168 [hep-ph]].

[18] A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, Eur. Phys. J. C **77** (2017) no.6, 405 [arXiv:1704.04504 [hep-ph]].

[19] P. Van Nieuwenhuizen, Phys. Rept. **68** (1981) 189.

[20] J. F. Donoghue, E. Golowich and B. R. Holstein, *Dynamics of the standard model*, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **35** (2014).

[21] R. D. Ball, Phys. Rept. **182** (1989) 1.

[22] http://einstein.ucsd.edu/smeft/

[23] E. E. Jenkins, A. V. Manohar and M. Trott, Phys. Lett. B **726** (2013) 697 [arXiv:1309.0819 [hep-ph]].

[24] G. Buchalla, O. Catà and C. Krause, Phys. Lett. B **731** (2014) 80 [arXiv:1312.5624 [hep-ph]].

[25] R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. **64** (1991) 345.

[26] V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. **207** (2016) 432 [arXiv:1601.01167 [hep-ph]].

[27] Wolfram Research, Inc., Mathematica, Version 11.1 (2017).

[28] V. I. Borodulin, R. N. Rogalyov and S. R. Slabospitskii, arXiv:1702.08246 [hep-ph].

[29] A. Denner, G. Weiglein and S. Dittmaier, Nucl. Phys. B **440** (1995) 95 [hep-ph/9410338].

[30] A. Helset, M. Paraskevas and M. Trott, Phys. Rev. Lett. **120** (2018) no.25, 251801 [arXiv:1803.08001 [hep-ph]].