NON-ABELIAN COLOR FIELDS FROM RELATIVISTIC COLOR CHARGE CONFIGURATIONS IN THE CLASSICAL LIMIT

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We study the dynamics of color fields generated by simple configurations of relativistic particles with Abelian and non-Abelian ($SU(2)$) charges in the classical limit. We find that chromodynamic (non-Abelian) systems generally show Coulomb-like features by analogy with electrodynamics. A peculiar feature in the non-Abelian case is the additional strength of the chromoelectric and chromomagnetic fields caused by the contribution of changing the color charge. The presence of this non-Abelian additional term in the chromoelectric and chromomagnetic fields creates a "color charge glow". This situation is especially relevant to the very initial phase of ultra-relativistic heavy-ion collisions, where the initial partonic state is governed by high (non-equilibrium) parton densities and strong local color fluctuations.

Our present study is motivated by recent investigations of peripheral ultra-relativistic heavy-ion collisions at the RHIC and the LHC, for which a large electric charge eZ of the colliding nuclei leads to the generation of intense electric and magnetic fields during the passage time of the charged 'spectators', as discussed in Ref. [1]. It is speculated that a very strong electromagnetic field - of short duration essentially in the pre-equilibrium phase - might have an important impact on particle production [2,3]. Fortunately, a large theoretical (phenomenological) activity combined with enthusiasm in studying the chiral magnetic effect (CME) and, in general, strongly interacting matter in magnetic fields in recent years [4] is balanced by a careful analysis of the corresponding RHIC and LHC experimental data [5] on the charge separation dependence of azimuthal correlations and, in particular, correlations with respect to the reaction plane. Actually, the measurements of the ALICE Collaboration [6] of the pion angular correlations are consistent with the qualitative expectations from the chiral magnetic effect. However, these results are also consistent with local charge conservation in combination with a large elliptic flow v_2. Such a classical field dynamics calls for a transport formulation and the development of an extended transport code for ultra-relativistic collision processes, including the color dynamics of the gluon fields.

In order to proceed resolving this task we develop an practical approximation for configurations of two SU(2)-color group charges which are moving along a straight line towards each other. The classical character of the non-Abelian field means, as usual, that the field operators are replaced by their average values for the quantum state, and the off-diagonal elements are neglected. Quarks are treated as classical point-like massive particles possessing a color charge. Recalling briefly the results from classical electrodynamics we have the field of a point-like charge propagating along the trajectory $\mathbf{r(t)}$ is described by the (retarded) Liénard-Wiechert potential at the observation point $\mathbf{r_0}$ [7]

$$\varphi = \frac{1}{4\pi} \left[\frac{e}{R - \mathbf{vR}} \right]_{t'}, \quad \mathbf{A} = \frac{1}{4\pi} \left[\frac{\mathbf{e\,v}}{\mathbf{R - vR}} \right]_{\mathbf{t'}}. \tag{1}$$

Here we stick to the standard system of units with speed of light $c = 1$, with dimensionless electrodynamic e and non-Abelian g charges, i.e., have the proper factors of $\hbar c$ to keep the proper dimensions of magnitudes. Then characteristic distances in the problem are of an order of 1 fm and the potentials and field strengths are measured in units of m_π and m_π^2, respectively, where m_π is the π-meson rest mass. In Eq. (1), φ is the 0'th component of the potential and $\mathbf{A}$ is its vector component, $\mathbf{v}$ is the particle velocity at some retarded time t' which is determined by the distance between the observation point and the particle $R = |\mathbf{R}|$ and $\mathbf{R} = \mathbf{r_0} - \mathbf{r}(t')$ is the radius-vector from the charge position to the observation point $\mathbf{r_0}$. The retarded and laboratory time t

are related by $R = t - t'$. The electric and magnetic fields then are given by

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi = \frac{1}{4\pi} \left[\frac{\mathbf{e}}{\mathbf{R^2}} \frac{(1 - \mathbf{v^2})(\mathbf{n} - \mathbf{v})}{(1 - \mathbf{vn})^3} + \frac{\mathbf{e}}{\mathbf{R}} \frac{\mathbf{n} \times (\mathbf{n} - \mathbf{v}) \times \dot{\mathbf{v}}}{(1 - \mathbf{vn})^3} \right]_{\mathbf{t'}} ,$$

$$\mathbf{H} = \nabla \times \mathbf{A} = \frac{1}{4\pi} \left[-\frac{\mathbf{e}}{\mathbf{R^2}} \frac{\mathbf{n} \times \mathbf{v}}{(1 - \mathbf{vn})^3}(1 - \mathbf{v^2} + \dot{\mathbf{v}}\mathbf{R}) - \frac{\mathbf{e}}{\mathbf{R}} \mathbf{n} \times \dot{\mathbf{v}} \right]_{\mathbf{t'}} = \mathbf{n} \times \mathbf{E} .$$

(2)

In SU(2)-color group QCD Lagrangian has the form $\mathcal{L} = -\widetilde{G}^{\mu\nu}\widetilde{G}_{\mu\nu}/4 - \widetilde{j}^\mu \widetilde{A}_\mu$, where the color vector $\widetilde{A}_\mu = (A_\mu^1, A_\mu^2, A_\mu^3)$ represents a triplet of the Yang-Mills fields of different colors ("isospin"), $\widetilde{j}^\mu$ is the current density of external color sources and $\widetilde{G}_{\mu\nu} = \partial_\mu \widetilde{A}_\nu - \partial_\nu \widetilde{A}_\mu + g\widetilde{A}_\mu \times \widetilde{A}_\nu$ is the gluon field tensor with the covariant derivative acting as $\widetilde{D^\mu} f = \partial^\mu \widetilde{f} + g\widetilde{A}^\mu \times \widetilde{f}$. The product sign $\times$ corresponds to the vector product in the color space. The classical equations of motion, as known, then read $\widetilde{D^\mu G_{\mu\nu}} = \widetilde{j}_\nu$. The most significant difference between these equations and the electrodynamic equations is the compatibility conditions of the system $\widetilde{D^\mu j}_\mu = 0$, that generally implies that the color vector charge is not conserved in a sense similar to electrodynamics. In the non-Abelian case we cannot benefit from an analysis of the general Yang-Mills solutions but one can construct approximate solutions with properties similar to the solutions of electrodynamics. Here we focus on the analysis of some simple examples important for practical applications treating the relativistic color objects propagating along a straight line. As an approximate solution, we consider a superposition of the Liénard-Wiechert potentials in which the vector of the particle color charge can change in time and should be taken at the retarded time. In this context the compatibility condition for a point-like charge becomes $\dot{\widetilde{C}} = g \left[\widetilde{\varphi}(t, \mathbf{r}) - \mathbf{v} \, \widetilde{\mathbf{A}}(t, \mathbf{r}) \right] \times \widetilde{C}$. From these equations it is easy to see that the modulus of the color charge vector remains constant. In the following we take the color charge as a unit vector and specify the magnitude of the charge to be given by the coupling constant g ($\alpha_g = g^2/(4\pi) = 0.3$), thus taking the coupling strength out as an independent factor. The consideration of the non-Abelian charge as a function of time leads to the generation of additional terms in the expressions for the chromoelectric and chromomagnetic fields of color point-like charges

$$\widetilde{\mathbf{E}} = \frac{1}{4\pi} \left[\frac{\widetilde{C}}{R^2} \frac{(1 - \mathbf{v^2})(\mathbf{n} - \mathbf{v})}{(1 - \mathbf{vn})^3} + \frac{\widetilde{C}}{R} \frac{\mathbf{n} \times (\mathbf{n} - \mathbf{v}) \times \dot{\mathbf{v}}}{(1 - \mathbf{vn})^3} + \frac{\widetilde{D}}{R} \frac{\mathbf{n} - \mathbf{v}}{(1 - \mathbf{vn})^2} \right]_{t'} ,$$

$$\widetilde{\mathbf{H}} = \frac{1}{4\pi} \left[-\frac{\widetilde{C}}{R^2} \frac{\mathbf{n} \times \mathbf{v}}{(1 - \mathbf{vn})^3}(1 - \mathbf{v^2} + \dot{\mathbf{v}}\mathbf{R}) - \frac{\widetilde{\mathbf{C}}}{\mathbf{R}} \mathbf{n} \times \dot{\mathbf{v}} - \frac{\widetilde{\mathbf{D}}}{\mathbf{R}} \frac{\mathbf{n} \times \mathbf{v}}{(1 - \mathbf{vn})^2} \right]_{t'} = \mathbf{n} \times \widetilde{\mathbf{E}} .$$

(3)

These additional terms formally look like radiation terms. It can be demonstrated that the solutions of the Yang–Mills equations are automatically correct with accuracy of order g, in reality g^3, see [8, 9], if the compatibility conditions are satisfied up to this order. Now let us consider the field of two color charges $\widetilde{P}, \widetilde{Q}$ moving along the z-axis towards each other with velocities v and $-w$, respectively. If their encounter is supposed to take place at time $t = 0$, their coordinates in the laboratory system are given as $z_1 = vt, z_2 = -wt$. Obviously, when the particles are far from each other their interaction can be assumed weak and their charges remain constant with high accuracy. Adapting the typical scale of non-Abelian (strong) interactions of about 1 fm we assume that the interaction is switched on just at this distance (scale) denoted as D. Obviously, this generates the characteristic time scale in the problem. Then the first "milestone" appears when the particles enter the interaction area, i.e. $T = -D/(v + w)$. The second time instant of importance is t_1' when a signal of the appearance of charge $\widetilde{Q}_T$ at the distance of 1 fm reaches the first particle (with the color charge $\widetilde{P}$). Similarly for the second particle, the time when the charge $\widetilde{P}_T$ comes at the same distance is denoted by t_2', where $\widetilde{P}_T, \widetilde{Q}_T$ are the color charges before entering the zone of interaction, i.e. before the time T, $t_1' = \frac{1-w}{1+v} T$, $t_2' = \frac{1-v}{1+w} T$. These expressions are easily extracted from the scheme in Fig. 1 by writing out the relations for the arrival of the "light" signals and charges at the points of interest. In the figure, these events take the form of the corresponding triangles. Just at these times the color charges start to rotate with respect to their constant color vector that is the vector peculiar to the partner charge before it entered the interaction area. This regime is going on up to the time $t'' = \frac{1-w}{1+v} \frac{1-v}{1+w} T$, which is the same for both particles. It is also the time necessary for a signal to reach the partner providing information on the beginning of rotation starting from its asymptotic charge value $\widetilde{P}_T, (\widetilde{Q}_T)$.

The velocity of a relativistic particle is determined by the relation $v = \left(1 - \frac{m^2}{\mathcal{E}^2} \right)^{1/2} \simeq 1 - \frac{m^2}{2\,\mathcal{E}^2}$, where m is the particle mass and $\mathcal{E}$ its energy. It allows us to estimate the order of magnitude for the characteristic time in the problem as $t' \sim \frac{m^2}{\mathcal{E}^2} T$, $t'' \sim \frac{m^4}{\mathcal{E}^4} T$. The collision energies of present heavy-ions facilities (RHIC and LHC) allow us to estimate the corresponding factors as $\mathcal{E}/m \sim 10$–10^2 and more. Another interesting scale appear in dipole configuration as distance δ between the dipole charges and we assume it to be of the order of the interquark distance in the nucleon; i.e. around $\delta = 1$ fm. Then in the laboratory system the dipole size (due
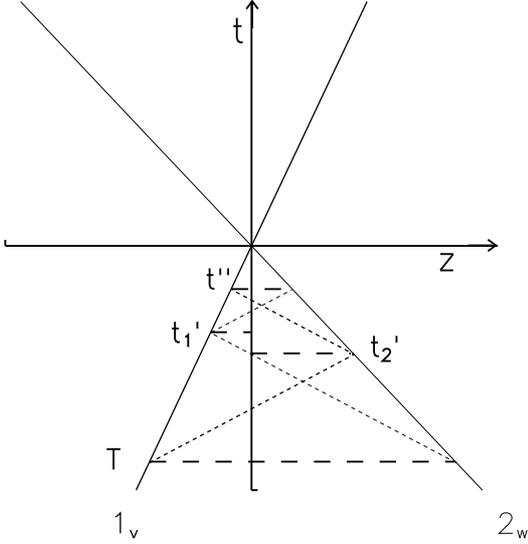
Figure 1. The time scheme for the meeting of two color particles. The solid lines are the particle trajectories, the dashed lines show projections on the time axis, the dotted lines are the light signals. Notations for the parameters T, $D = 1 \sim$ fm, t'_1, t'' are given in the text.
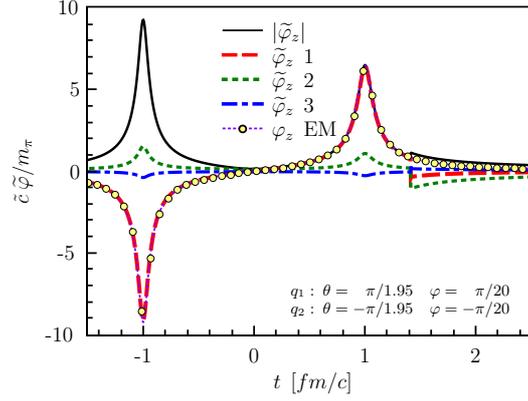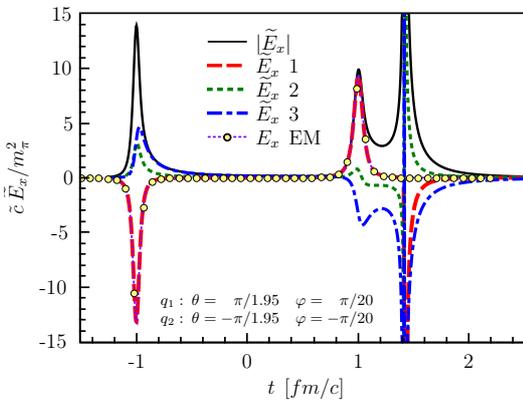

Figure 2. Time evolution of three color components for the scalar $\widetilde{\phi}$ for two oppositely directed charges in color space as a function of time is given by the dashed lines. The solid line shows the absolute value of the isovector potential $|\widetilde{\phi}|$. The electrodynamic potential with charges $\pm e$ corresponding to the coupling constant $\alpha_e = 0.3$ is displayed by open circles.

to Lorentz contraction) will be $\Delta = \delta \, (1 - v^2)^{1/2} \sim \delta \, \frac{m}{\mathcal{E}}$. Let us denote the time corresponding to this scale as $t_3 \sim m/\mathcal{E}$. Thus, for the relativistic problem of interest we obtain the following time hierarchy of interaction stages $t'' \ll t' \ll t_3 \ll T$. Due to the chosen geometry of the problem an approximate solution of the Yang-Mills equations for two color charges can be represented as the following superposition: $\widetilde{\varphi} = [\varphi_1 \widetilde{P}]_{t'} + [\varphi_2 \widetilde{Q}]_{t'}$, $\widetilde{A}_z = v \, [\varphi_1 \widetilde{P}]_{t'} - w \, [\varphi_2 \widetilde{Q}]_{t'}$. The scalar potentials can be specified as $\varphi_1 = \varphi_2 = \frac{1}{4\pi} \, \frac{g}{z_2 - z_1}$. Also, taking into account the form of the vector potential, we arrive at the following expression for the compatibility conditions:

$$\dot{\widetilde{P}} = \alpha_g \, \frac{1 + v \, w}{|z_1 - z_2|} \, \widetilde{Q}(t - t^{**}_{12}) \times \widetilde{P} \, , \quad \dot{\widetilde{Q}} = \alpha_g \, \frac{1 + v \, w}{|z_1 - z_2|} \, \widetilde{P}(t - t^{*}_{21}) \times \widetilde{Q} \, . \tag{4}$$

Details of derivation can be found in [10] As an example, let us consider the field created by two relativistic particles moving with velocities: $v = 1 - 2 \cdot 10^{-2}$ and $|w| = 1 - 1 \cdot 10^{-2}$. The energy-mass ratio for the first particle is $\mathcal{E}/m \simeq 16$, and for the second particle is $\mathcal{E}/m \simeq 22$. Let us take the coordinates of the observation point $\mathbf{r}_0$ as $x = 2$, $y = 0$, $z = 1$ (fm). For comparison, we consider also the field created by particles with electric charge $\pm e$ of the same interaction strength as the color charges ($e^2/(4\pi) = g^2/(4\pi) = 0.3$) moving with the same velocities. The potentials for the scalar $\widetilde{\varphi}$ field of color particles is presented in Fig. 2 where the initial angles

Figure 3. Three color components of the chromoelectric $\widetilde{E}$ field strength for two moving color charges of the opposite signs as a function of time (see the text).
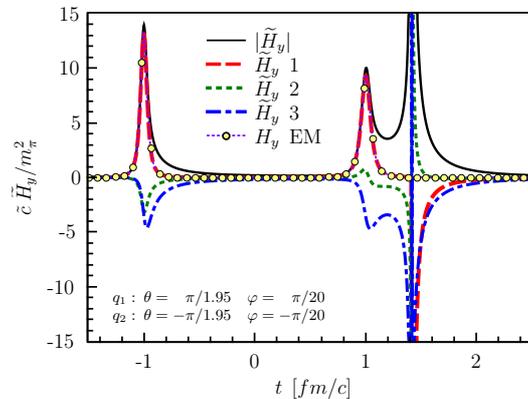
Figure 4. Three color components of the chromomagnetic $\widetilde{H}$ field strength for the same problem as in Fig. 3.

in the color space are determined as $\theta = \pi/1.95$, $\phi = \pi/20$ for the first particle and $\theta = -\pi/1.95$, $\phi = -\pi/20$ for the second one. In this case the color charge of the first particle, for example, will be $\widetilde{P} = (P_1, P_2, P_3)$, with $P_1 = \sin\theta\cos\phi$, $P_2 = \sin\theta\sin\phi$, $P_3 = \cos\theta$. This configuration of color charges $\widetilde{P}$, $\widetilde{Q}$ corresponds to almost oppositely directed charges at the initial stage. The dashed lines in this figure show the three color components of the scalar potential. The open circles are plotted for the potentials corresponding to the enhanced electrodynamic coupling. The modulus of the scalar $|\widetilde{\varphi}|$ is displayed by the solid lines in Fig. 2. According to the choice of the geometry, we see that the first maximum corresponds to the passage of the first particle at the closest distance to the observation point and then the second maximum is the corresponding passage of the second charge. Both bumps are located symmetrically with respect to the point $t = 0$. For the selected configuration of color charges, one of the components of the potentials, shown by the dashed lines, dominates and almost coincides with the appropriate modulus of vectors in the color space (solid line). The meeting point is located at a distance of $R = (x^2 + z^2)^{1/2} = \sqrt{5} \sim 2.24$ from the observation point. It is seen that there is a noticeable difference between the scalar potential as compared to the electrodynamic case (open circles) at the appropriate time when a signal on a meeting of the particles has arrived. The time dependence of the strength components of the chromoelectric $\widetilde{E}_x$ and chromomagnetic $\widetilde{H}_y$ fields is shown in Fig. 3, 4 by the three dashed lines; the solid lines correspond to the modulus of the chromofields $|\widetilde{E}_x|$ and $|\widetilde{H}_y|$; the open circles show the electrodynamic vector field corresponding to charges $\pm e$ and coupling constant $\alpha_e = 0.3$. We have cut the singular peaks at some threshold and thus the lines look somewhat irregular. The E_x and H_y components are dominating. In both cases two maxima (minima) caused by passing the color charges in the vicinity of the observation point are clearly visible. Note that here the color field strength is plotted in dimensionless units where $\widetilde{C}$ is the color charge of the appropriate field component and m_π is the pion mass. The charge velocities considered roughly correspond to the RHIC energy where the maximal electromagnetic field eH_y/m_π^2 reaches a few units [1]. This value is essentially smaller than those in the color charge case (see Fig. 3). It is seen that for the color charge configuration considered the chromoelectric and chromomagnetic field are quite similar to the field in the case of enhanced electrodynamics. Some difference in the hight of the first two maxima are caused by different velocities of color charges. A significant difference at the third maximum is due to the arrival of a signal from the meeting point of particles to the observation point, where there is a noticeable additional contribution of chromoelectric and chromomagnetic fields associated with the temporal change of the particle color charges. This "color glow" effect is not an artifact of the approximation made but results from the pure non-abelian term proportional to $\widetilde{D}$. The longitudinal component is strongly suppressed, as it should be due to relativistic effects, but the signal from the meeting point of the particles leads to almost equal contributions. In a similar manner the collision of a color charge and color dipole as well as of two color dipoles were considered. Such configurations are just the elementary basic examples in ultra-relativistic heavy-ion collisions in the existing phenomenological models. More results can be found in [10].

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