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# DETERMINATION OF $(j, \ell)$ FOR $\Delta(2200)$ FROM A DUAL ANALYSIS OF BACKWARD $\pi^+p$ SCATTERING*

Ernest Ma†
Physics Department, University of California
Santa Barbara, California 93106

and

Gordon L. Shaw††
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

## ABSTRACT

A negative parity $\Delta$ resonance with mass roughly equal to 2200 MeV was recently discovered from careful measurements by Baker *et al.* of $\pi^+p$ $180^o$ elastic scattering. Combining the $180^o$ data with angular distribution measurements near 2200 MeV, we investigate possible $(j, \ell)$ assignments of the $\Delta(2200)$. We find 1) the dominant contribution is $D_{35}$, ruling against a (56, L odd) quark model multiplet, and 2) the dual interference model gives a good fit to the data whereas a pure resonance fit does not.

There is considerable theoretical interest in knowing which (SU(6), L) multiplets are actually occupied by the higher mass baryon states. There has been recent speculation[1] that the three quarks combine to give only 56 representations with even parity and 70 representations with odd parity (and no 20's). Dynamical diquark-quark models which give these features have been formulated.[2] On the other hand, duality arguments seem to require 56's and 70's for all L ($\geq 2$).[3,4]

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†Supported in part by the National Science Foundation.
††Permanent address: Physics Department, University of California, Irvine, California 92664.

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A negative-parity $\Delta$ resonance with a mass roughly equal to 2200 MeV was recently discovered from careful measurements of $\pi^+p$ elastic scattering differential cross sections in the backward direction,[5] in accordance with the prediction[6] of a phenomenological interference model based on duality that was proposed some time ago.[7] It is therefore of great interest now, whether from the viewpoint of the quark model or from that of Regge recurrences, to determine the $(j, \ell)$ assignment of this new resonance. We cannot, of course, obtain this from the backward data alone; we need to have, in addition, a compatible set of angular distribution measurements near the backward direction, as well as a reliable method of amplitude analysis.

In view of its success in predicting the $\Delta(2200)$ in the one case,[6] and its many other fruitful applications elsewhere,[7,8] we base our analysis on the dual interference model,[7] and combine the $180^{\circ}$ data,[5] where $p_{lab}$ ranges from 2 to 6 GeV/c, with angular distribution data due to another experiment,[9] at three energies close to that of the new resonance, i.e., $p_{lab}$ = 2.18, 2.28, and 2.38 GeV/c. Our chief purpose is to investigate the possible $(j, \ell)$ assignments of the $\Delta(2200)$ and its neighboring resonances. It has been noted that the leading ($\underline{56}$, $L=3^-$) state would be a $G_{39}$, that it should decay readily into the elastic $\pi N$ channel, and that perhaps it is responsible for the deep dip in the backward data at 2200 MeV.[3]

As a result of our analysis we find:

1) There is little evidence for the existence of a $G_{39}$ which has been discussed as a possibility;[3,10] the dominant contribution is $D_{35}$, which can be considered as the second member of a Regge $\Delta_\beta$ family, of which the $S_{31}(1650)$ is the first. 2) Furthermore, in the context of our model, we find two distinct groups of resonances: one of negative parity at about 2200 MeV and the other of positive parity at about 2100 MeV. 3) Whereas, the analysis of Rey *et al.*[11] of the $180^{\circ}$

data alone could not distinguish between a pure resonance fit or one using our dual interference model, we find that only the inteference model gives a good fit to all the data.[5,9]

The concept of duality in strong interaction processes as defined in terms of Finite Energy Sum Rules for the imaginary part of the scattering amplitude has been extended in a model[6,7] suggested by a general feature of the Veneziano representation to include a statement for the real part as well. In this dual interference model, the sum of direct-channel resonances is identified with the signatured part of the Regge exchange amplitude, leaving the purely real nonsignatured part as an interfering background. At intermediate energies therefore, one simply writes the full amplitude as a sum of the contribution of the nonsignatured Regge part extrapolated from a solution at high energies and that of the direct-channel resonances whose defining parameters are determined at low energies. A smooth transition is thus provided for in this description of the scattering amplitude, from being a sum of resonances plus background to that of Regge poles. Indeed, several successful applications of this model have already been registered.[7,8]

We use our previous Regge fit to the high energy $\pi^{\pm}p$ data in terms of the exchange of the $\Delta_\delta$ and $N_\alpha$ trajectories. As usual we have

$$f(\sqrt{s},u) = f_1(\sqrt{s},u) - \cos\theta \; f_1(-\sqrt{s},u) \quad ,$$

and

$$g(\sqrt{s},u) = \sin\theta \, f_1(-\sqrt{s},u) \quad ,$$

so that the expressions for the differential cross section and the polarization of the outgoing nucleon are, respectively

$$d\sigma/d\Omega = |f|^2 + |g|^2 \quad , \qquad P = 2\,\mathrm{Im}(fg^*)/(d\sigma/d\Omega) \quad .$$

Furthermore,

$$f_1(\sqrt{s},u) = \frac{E_s+m}{2\sqrt{s}}\left[(\sqrt{u}-\sqrt{s}+2m)\,\frac{f_1(\sqrt{u},s)}{E_u+m} + (\sqrt{u}+\sqrt{s}-2m)\,\frac{f_1(-\sqrt{u},s)}{E_u-m}\right] .$$

For the $\Delta_\delta$, $N_\alpha$ respectively,

$$\frac{f_1(\sqrt{u},s)}{E_u+m} = \frac{\gamma_\Delta(\sqrt{u})}{\sqrt{u}}\,\frac{1-e^{-i\pi\alpha_\Delta-\frac{1}{2}}}{\Gamma\left(\alpha_\Delta+\frac{1}{2}\right)\cos\pi\alpha_\Delta}\left(\frac{s}{s_\Delta}\right)^{\alpha_\Delta-\frac{1}{2}} ,$$

and

$$\frac{f_1(-\sqrt{u},s)}{E_u-m} = -\frac{\gamma_N(\sqrt{u})}{\sqrt{u}}\,\frac{1+e^{-i\pi\alpha_N-\frac{1}{2}}}{\Gamma\left(\alpha_N+\frac{1}{2}\right)\cos\pi\alpha_N}\left(\frac{s}{s_N}\right)^{\alpha_N-\frac{1}{2}} ,$$

where the Regge trajectories $\alpha_\Delta$, $\alpha_N$ are functions of $\sqrt{u}$; $s_\Delta$, $s_N$ constants having the dimension of s; and $\gamma_\Delta$, $\gamma_N$ reduced residue functions.[12]

The parameters for these quantities were obtained by fitting $\pi^-p$ and $\pi^+p$ backward elastic scattering out to u=-1.1 GeV$^2$ for $p_{lab}$ from 6 to $\sim$ 17 GeV/c. These values are given in Ref. 6.

In Fig. 1, we plot the $\pi^+p$ 180$^o$ elastic cross section versus $p_{lab}$ as predicted by our fit and compare it to the recent data[5] for $p_{lab} \lesssim$ 6 GeV. The agreement is quite remarkable down to $p_{lab} \sim$ 4 GeV. We know therefore that the two sets of data are compatible and that our high-energy fit is reliable down to much lower energies.[13]

To get a good fit to the data for $p_{lab} \lesssim$ 4 GeV however, we must include resonances. To avoid double counting and be consistent with duality, we keep only the nonsignatured part of the Regge term, and add to it a sum of direct channel $\pi^+p$ resonances. As for the form of the resonance amplitude, we have used a constant-width, constant-elasticity Breit-Wigner parametrization with the additional requirement of MacDowell symmetry.

Keeping fixed the Regge parameters that we have obtained from the high-energy fits and those of the resonance parameters coming from low-energy data,[14] we now determine the parameters of the resonances situated at intermediate energies.

Even a casual glance at the recent $\pi^+p$ $180^o$ data[5] will discern the remarkable dip in the cross section at $p_{lab}$=2.1 GeV/c ($d\sigma/d\Omega$ = 18.3 $\mu$b/sr) before it rises to a peak at 2.6 GeV/c (312.6 $\mu$b/sr). As has been emphasized,[5,11] this is strong evidence for the existence of a negative-parity resonance at about 2200 MeV. To isolate the $(j,\ell)$ assignment of the new resonance, we must examine angular distribution data in this energy range. The fact that there is such a rapid change in the cross sections will then help to distinguish among many possibilities.

Looking over the various sets of available data for this purpose, we single out the only experiment[9] designed specifically to take measurements at backward angles, and where the angular distribution has been given in considerable detail. In particular, we use the data (59 points) at $p_{lab}$ = 2.18, 2.28, and 2.38 GeV/c. The reason for not using angular distribution data at higher energies is clear: the fact that there are probably many overlapping resonances resulting in a smooth cross section prevents us from a clear determination; whereas for $p_{lab} \lesssim$ 2.5 GeV/c, there is hope that the few dominant resonances are good enough for our procedure to be meaningful. We also check that the slope parameters used in obtaining the $180^o$ data[5] are indeed compatible with those values of the cross section in the angular distribution data[9] nearest $180^o$. (Other experiments designed for detection at all angles, but emphasizing the forward direction, appear not to have results compatible with the $180^o$ data.)

In fitting the combined data, the following contributions are held fixed: (1) the Regge background term as given by the nonsignatured part of the high-energy fit, and (2) those 7 resonances below 2 GeV as given by the Particle Data Group[14] (1972 averages). The 2200 MeV region is treated as being comprised of two groups of resonances, each of common mass and width[15]: one of positive parity containing $P_{33}$ and $P_{31}$, and one of negative parity containing $D_{35}$, $G_{39}$, $G_{37}$, $S_{31}$ and $D_{33}$. All but one of these resonances have been considered to be phenomenological possibilities (see 1974 Particle Data Group[14]), while the exception, $G_{37}$, is certainly possible on the basis of the quark model. The three prominent $\Delta_\delta$ resonances of higher mass plus the one resonance ($D_{35}$) at around 2 GeV are also varied.

Without assuming which one of the partial waves within a particular group is dominant, we search for the best fit to the data. Our results are given in Table I and Fig. 1-2. The $\chi^2$ of this fit is 119 for 97 points. (The error bars for 5 points in the angular distribution data which have $\cos\theta < -.95$ have been doubled so that they overlap with the backward data projections.[5]) The evidence is therefore quite good for a large $D_{35}$ contribution to the negative-parity group while the other partial waves together make up only about a third of the total elasticity; as well as for a clear separation of two groups of resonances in the 2200 MeV region; the one of positive parity having comparable amounts of $P_{33}$ and $P_{31}$.

We note that there is a difficulty in using polarization data in the 2200 MeV region since we expect rapid variations in polarization <u>both</u> as a function of $p_{lab}$ and $\cos\theta$. This is illustrated in Table II. Thus it is important for any experimental determination of the polarization to quote the spread in energy and angle.

To check the overall correctness of our results, we have tried many other different combinations of partial waves. For example, we have tried eliminating all but the $D_{35}$ and one other resonance ($G_{39}$, etc.) but the resulting fit still favors a dominant amount of $D_{35}$. Excellent fits with no $G_{39}$ are found. We have also tried using a pure resonance fit but we were unable to obtain a good fit at all with any combination of partial waves. This fact points directly to the validity of the of the dual interference model as a useful method of amplitude analysis in the intermediate energy range.

Most of our knowledge of $\pi N$ resonances comes from detailed phase shift analysis. However it is clear[16] that these analyses are enormously difficult and cannot go much above $p_{lab} \sim 2$ GeV/c. New approaches are needed to extract the resonant parameters for $p_{lab}$ up to 3.5 GeV/c which are needed to further test theoretical models such as i) the $SU(6) \times O(3)$ quark model for the baryon mass spectrum ii) $SU(6)_W$ schemes for the partial decay widths, iii) linearly rising direct channel Regge trajectories, iv) local duality, etc.

We suggest more extensive use of our successful dual resonance—Regge interference formalism to directly analyze $\pi N$ data. We need only restrict ourselves to the nondiffractive data, i.e., not consider forward $\pi^{\pm}p$ elastic scattering in applying the model. The region around $90^{\circ}$ could be treated by having a background term of the quark rearrangement form recently developed by Gunion, Brodsky and Blankenbecler[17] interfering with the direct channel resonances.

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11. C. A. Rey, A. J. Lennox, J. A. Poirier, and K. P. Pretzl, Phys. Rev. Letters 32, 908 (1974); also Phys. Rev. Letters 33, 250 (1974) where corrected values for their dual model fit are given.

12. For phenomenological simplicity, we have preserved in the above, explicit MacDowell symmetry of the amplitudes. Now, because of the simple functional forms chosen for the trajectories, the $\Delta_\delta$ and the $N_\alpha$ are accompanied respectively by a $\Delta_\gamma$ and an $N_\beta$; although theoretically, the opposite-parity partner of a given Regge trajectory can hide behind a fixed cut in the J-plane, as in the case of the Reggeon calculus.

13. To include $\pi$N charge exchange data, it would be necessary to add the $N_\gamma$ trajectory.

14. Particle Data Group, Phys. Letters 39B, 1 (1972); Phys. Letters 50B, 1 (1974).

15. These are both theoretical arguments and experimental evidence that $\Delta$ states in a given (SU(6), L) multiplet are degenerate in mass. See, Dalitz, Ref. 4.

16. See, e.g., Panel on New Directions and Methods, Baryon Resonances-73, (Purdue University, West Lafayette, Indiana, 1973), p. 1.

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TABLE I

Resonance parameters for the dual model fit presented in Figs. 1 and 2. The parameters for the seven resonances below 2000 MeV are taken from the 1972 Particle Data Group averages and are not varied. The masses, widths, and elasticities of the heavier resonances are varied in the fit. The values of the last three resonances should be considered as effective quantities (since there are undoubtedly many other resonances in this energy range). This set of parameters is a typical representation of the good fits (see text).

| Resonance (mass in MeV) | Width (MeV) | Elasticity (%) |
|---|---|---|
| $P_{33}(1236)$ | 120 | 99.4 |
| $S_{31}(1637)$ | 164 | 29 |
| $D_{33}(1685)$ | 245 | 15 |
| $P_{33}(1710)$ | 324 | 10 |
| $F_{35}(1877)$ | 259 | 17 |
| $P_{31}(1889)$ | 305 | 25 |
| $F_{37}(1947)$ | 199 | 44 |
| $D_{35}(2013)$ | 445 | 12.0 |
| $P_{33}(2107)$ | 231 | 19.2 |
| $P_{31}$ | | 14.2 |
| $D_{35}(2207)$ | 332 | 13.6 |
| $G_{39}$ | | 1.4 |
| $G_{37}$ | | 1.6 |
| $S_{31}$ | | 2.0 |
| $D_{33}$ | | 2.7 |
| $H_{3,11}(2479)$ | 455 | 10.4 |
| $J_{3,15}(2954)$ | 298 | 1.9 |
| $L_{3,19}(3440$ | 660 | 3 |

TABLE II

Polarization as a function of $p_{lab}$ and cos $\theta$ (corresponding to the dual model fit presented in Figs. 1 and 2) illustrating the rapid variation in both variables in the 2200 MeV region.

| $p_{lab}$ (GeV/c) | cos $\theta$ = -0.98 | -0.97 | -0.96 | -0.95 | -0.94 | -0.93 |
|---|---|---|---|---|---|---|
| 2.08 | -0.80 | -0.73 | -0.58 | -0.41 | -0.25 | -0.10 |
| 2.09 | -0.61 | -0.57 | -0.43 | -0.28 | -0.13 | -0.01 |
| 2.10 | -0.39 | -0.36 | -0.26 | -0.13 | 0.00 | 0.13 |
| 2.11 | -0.18 | -0.15 | -0.07 | 0.04 | 0.15 | 0.26 |
| 2.12 | 0.01 | 0.05 | 0.12 | 0.20 | 0.29 | 0.38 |
| 2.13 | 0.16 | 0.22 | 0.28 | 0.35 | 0.42 | 0.49 |
| 2.14 | 0.26 | 0.34 | 0.41 | 0.48 | 0.54 | 0.60 |

FIGURE CAPTIONS

1. Plots of $d\sigma/d\Omega$ at $\cos\theta = -1$ versus $p_{lab}$. The data points are those of Ref. 5. The dashed curve is the prediction of our previous[6] Regge fit to the high energy data ($\gtrsim$ 6 GeV/c). The parameters are given in Ref. 6. Note that the dashed Regge curve is a good average of $180^{\circ}$ points down to the lowest energy. The nonsignatured parts of these Regge amplitudes are taken to be the background term for the present dual model fits and are not varied. The solid curve is the present dual model fit: the resonance parameters are given in Table I.

2. Plots of $d\sigma/d\Omega$ versus $\cos\theta$ for $p_{lab}$ = 2.18, 2.28, and 2.38 GeV/c. The data points are those of Ref. 9. The solid curve is the same fit as in Fig. 1.

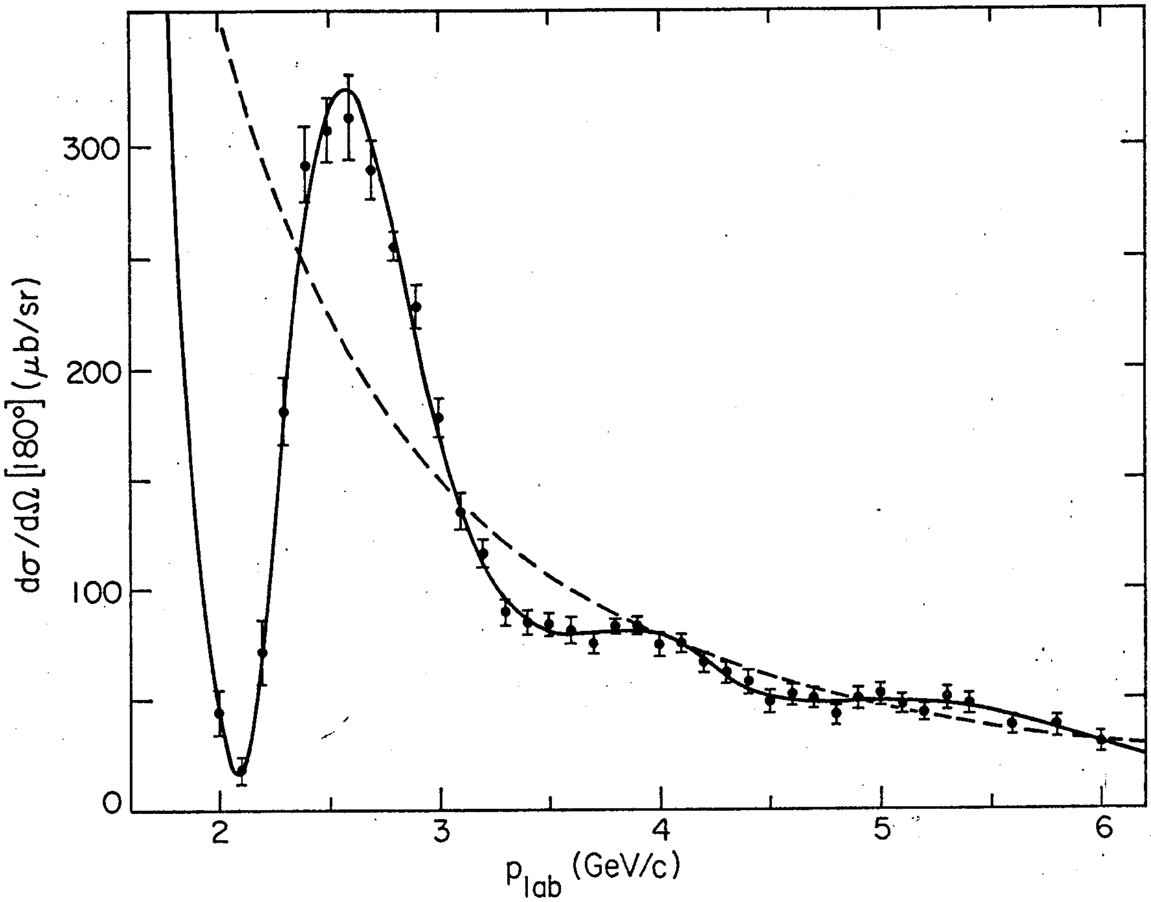


Fig. 1

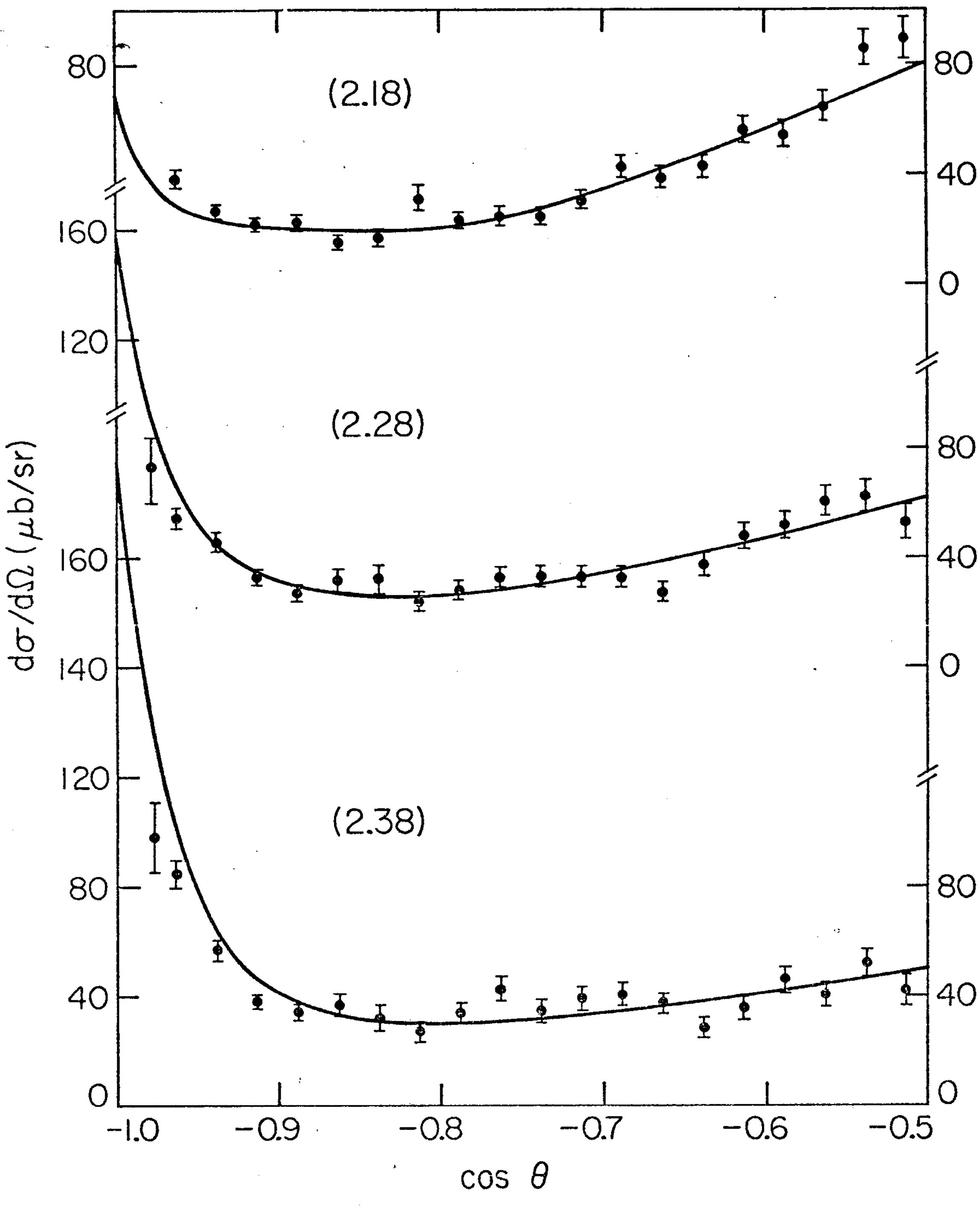
(2.18)
(2.28)
(2.38)
dσ/dΩ (μb/sr)
cos θ
80
160
120
160
140
120
80
40
0
80
40
0
80
40
0
80
40
0
-1.0
-0.9
-0.8
-0.7
-0.6
-0.5

Fig. 2