Quantum Vacuum Gravitation Matter-Antimatter Antigravity

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Abstract. Without stating any assumptions or making postulates we show that the electromagnetic quantum vacuum plays a primary role in quantum electrodynamics, particle physics, gravitation and cosmology. Photons are local oscillations of the electromagnetic quantum vacuum field guided by a non-local vector potential wave function. The electron-positron elementary charge emerges naturally from the vacuum field and is related to the photon vector potential. We establish the masse-charge equivalence relation showing that the masses of all particles (leptons, mesons, baryons) and antiparticles have electromagnetic origin. In addition, we deduce that the gravitational constant G is an intrinsic property of the electromagnetic quantum vacuum putting in evidence the electromagnetic nature of gravity. We show that Newton's gravitational law is equivalent to Coulomb's electrostatic law. Furthermore, we draw that G is the same for matter and antimatter but gravitational forces could be repulsive between particles and antiparticles because their masses bear naturally opposite signs. The electromagnetic quantum vacuum field may be the natural link between particle physics, quantum electrodynamics, gravitation and cosmology constituting a basic step towards a unified field theory.

Keywords: Electromagnetic field ground state; quantum vacuum; elementary charges; elementary particles; mass-charge relation; photons; electromagnetic gravitation; matter-antimatter antigravity; unified field theory

Introduction

Despite the theoretical and experimental advances achieved in last years, the real nature of gravitation is still unknown. After Newton's theory and the elaboration of general relativity a large number of well-validated astrophysical observations entailed the necessity of introducing new physical concepts like modified gravity [1-3] and dark matter [4,5] for their explanation. However, recent experiments [6] have shown, with a high degree of statistical accuracy, that the modified gravity models should not be valid. On the other hand, the nature of dark matter remains totally unknown [7] yielding an intriguing puzzle for understanding the observed galactic gravitational anomalies.

Understanding gravitation's nature is certainly among the greatest challenges of modern physics. In what follows, we show that the mass effect as well as the gravitational constant emerge from the electromagnetic quantum vacuum field and consequently have purely electromagnetic origin. Through this lens, the equivalence between Newton's gravitational law and Coulomb's electrostatic law is straightforward. Furthermore, the gravitational repelling between matter and antimatter appears naturally.

The Electromagnetic Quantum Vacuum

Quantum field theory fails to evaluate the vacuum energy density by a factor of 120 orders of magnitude. In the case of the electromagnetic field the zero-point energy corresponding to the fundamental eigenstate of the harmonic oscillator Hamiltonian writes with the well-known expression $\sum_{k,\lambda}\hbar\omega_k/2$, where $\hbar$ is Planck's reduced constant and the summation runs over all the modes k with angular frequency ω_k while λ takes only two values corresponding to circular left or right polarization. Thus, at any point in free space the total summation yields an infinite vacuum energy density. However, when the summation is upper limited to Planck's energy $(\sim 10^{19} GeV)$ one gets the "astronomic" value $\sim 10^{110}\, J\, m^{-3}$ while the astrophysical observations give roughly $\sim 10^{-10}\, J\, m^{-3}$ [8]. This unphysical theoretical result, the worst ever in the history of science, is mainly due to the mathematical ambiguity during the field quantization procedure according to the harmonic oscillator model consisting of replacing the commuting classical canonical variables of momentum and position by the corresponding non-commuting quantum mechanics Hermitian operators [9-12]. Electromagnetic waves are not composed of harmonic oscillators and no experiment has ever demonstrated that a single photon state is a harmonic oscillator. It has to be emphasized that, in contrast with material oscillators, the zero-point energy in quantum field theory resulting from the harmonic oscillator Hamiltonian does not correspond to a physical state and this is what indeed the recent astrophysical observations confirm.

As frequently pointed out [9], we recall that the interpretations of the spontaneous emission and the Lamb shift in quantum electrodynamics (QED) are not due to the zero-point energy term $\sum_{k,\lambda}\hbar\omega_k/2$, but to the commutation property $\left[a_{k\lambda}, a_{k\lambda}^{+}\right]=1$ of the k-mode and λ-polarization photon creation $a_{k\lambda}^{+}$ and annihilation $a_{k\lambda}$ non-Hermitian operators respectively. As about the well-known Casimir effect, it can be fully explained using the source fields or Lorentz's forces [13-15] without invoking at all the zero-point energy. It has to be underlined again that the zero-point energy term $\sum_{k,\lambda}\hbar\omega_k/2$, being a constant, commutes with all quantum mechanics Hermitian operators corresponding to physical observables and consequently should have absolutely no influence to any physical process.

On the other hand, we recall [10-12,17-19] that the quantized vector potential of the electromagnetic field for a cavity free k-mode photon with angular frequency ω_k and polarization λ (circular left or right) writes in the classical and quantum representations respectively

$$\vec{\alpha}_{k\lambda}(\vec{r},t)=\xi\omega_k\left[\hat{\varepsilon}_{k\lambda}\, e^{i\left(\vec{k}\cdot\vec{r}-\omega_k t+\theta\right)}+\hat{\varepsilon}_{k\lambda}^{*}\, e^{-i\left(\vec{k}\cdot\vec{r}-\omega_k t+\theta\right)}\right]=\omega_k\,\vec{\Xi}_{k\lambda}(\vec{r},t) \tag{1a}$$

$$\tilde{\alpha}_{k\lambda}(\vec{r},t)=\xi\omega_k\left[\hat{\varepsilon}_{k\lambda}\, a_{k\lambda} e^{i\left(\vec{k}\cdot\vec{r}-\omega_k t+\theta\right)}+\hat{\varepsilon}_{k\lambda}^{*}\, a_{k\lambda}^{+} e^{-i\left(\vec{k}\cdot\vec{r}-\omega_k t+\theta\right)}\right]=\omega_k\,\tilde{\vec{\Xi}}_{k\lambda}(\vec{r},t) \tag{1b}$$

where $\hat{\varepsilon}_{k\lambda}$ is the polarization complex unit vector, $\vec{k}$ the wave vector with amplitude $\left|\vec{k}\right|=2\pi/\lambda_k$, λ_k being here the wavelength of the mode k and θ a phase parameter.

The vector potential amplitude quantization constant ξ is obtained by normalizing the electromagnetic field energy over a period to the single photon energy $\hbar\omega_k$ and is given by

$$\xi = \frac{\hbar}{4\pi ec} = \frac{1}{(4\pi)^2}\frac{e\mu_0}{\alpha} = \pm 1.747\,10^{-25}\ V\,m^{-1}s^2 \tag{2}$$

where e is the electron or positron charge, c the speed of light in vacuum, $\alpha \simeq 1/137$ is the fine structure constant and μ_0 the vacuum magnetic permeability.

Obviously, for $\omega_k = 0$ the resulting electromagnetic field ground state does not correspond to a perfectly empty space because the fundamental function $\vec{\Xi}(\vec{r},t)$ in the vector potential expression still subsists and becomes in classical and quantum representations

$$\vec{\Xi}_{0\lambda} = \xi\left[\hat{\varepsilon}_\lambda\, e^{i\theta} + cc\right] \tag{3a}$$

$$\tilde{\vec{\Xi}}_{0\lambda} = \xi\left[\hat{\varepsilon}_\lambda\, a_{k\lambda} e^{i\theta} + cc\right] \tag{3b}$$

where *cc* is the complex conjugate

Hence, in total absence of energy, of vector potential as well as of electric and magnetic fields, $\Xi_{0\lambda}$ is the electromagnetic field ground state, the electromagnetic quantum vacuum corresponding to light at zero frequency, "Dark Light". It is a real cosmic field $(\lambda_k \to \infty)$ permeating all of space and having electric potential nature according to the physical dimensions of the vector potential quantization amplitude ξ. We have shown previously that the electromagnetic field ground state complements naturally the normal ordering Hamiltonian in QED and describes a real physical vacuum state in both classical and quantum electromagnetic theories [10-12]. In addition, it explains directly the vacuum effects such as the spontaneous emission and the Lamb shift as well as the Fulling-Davies-Unruh temperature [16].

The phase parameter θ may take any value and consequently the electromagnetic quantum vacuum is composed of all possible states $\vec{\Xi}_{k\lambda}(\vec{r},t)$ corresponding to all modes and polarizations

$$\vec{\Xi}_{k\lambda}(\vec{r},t) = \xi\left[\hat{\varepsilon}_{k\lambda}\, e^{i\left(\vec{k}\cdot\vec{r}-\omega_k t+\varphi\right)} + cc\right] \tag{4a}$$

$$\tilde{\vec{\Xi}}_{k\lambda}(\vec{r},t) = \xi\left[\hat{\varepsilon}_{k\lambda}\, a_{k\lambda} e^{i\left(\vec{k}\cdot\vec{r}-\omega_k t+\varphi\right)} + cc\right] \tag{4b}$$

An angular frequency operator $\tilde{\Omega}$ can be readily defined [16,20]

$$\tilde{\Omega} = -i\,c\vec{\nabla} \tag{5}$$

We get the equation governing the fundamental function of the vector potential $\vec{\Xi}_{k\lambda}(\vec{r},t)$ in vacuum

$$i\frac{\partial}{\partial t}\begin{bmatrix}\vec{\Xi}_{k\lambda}(\vec{r},t)\\ \tilde{\Xi}_{k\lambda}(\vec{r},t)\end{bmatrix} = \tilde{\Omega}\begin{bmatrix}\vec{\Xi}_{k\lambda}(\vec{r},t)\\ \tilde{\Xi}_{k\lambda}(\vec{r},t)\end{bmatrix} = \begin{bmatrix}\vec{\alpha}_{k\lambda}(\vec{r},t)\\ \tilde{\alpha}_{k\lambda}(\vec{r},t)\end{bmatrix} \tag{6}$$

The last relation characterize the duality of the electromagnetic quantum vacuum which has both classical and quantum properties resulting to the simultaneous wave-particle intrinsic property of single photons. According to equation (6), photons are generated by the action of the angular frequency operator $\tilde{\Omega}$ upon the electromagnetic vacuum state $\Xi_{k\lambda}(\vec{r},t)$ creating a real vector potential of a *k*-mode and λ-polarization photon and consequently a photon. Thus, photons are local oscillations of the electromagnetic quantum vacuum extending over a wavelength, with circular polarization (left or right), guided by a non-local vector potential wave function.

Due to the electromagnetic vacuum fluctuations, space is permanently full of transient photons of all frequencies underlying the cosmic radiation background. Following the above it can be demonstrated

that the electromagnetic quantum vacuum field fluctuations provide a satisfactory explanation to the vacuum energy scale riddle [20].
From Heisenberg's uncertainty relation it is evident that the lower the frequency f the longer the transient photons lifetime, which could also explain the origin of the $1/f^{n}$ noise observed in all astrophysical measurements. Under these conditions, free space is not Lorentz invariant and an observer with uniform velocity, in absence of any other reference frame, would be able to detect his motion from the Doppler shift of the electromagnetic vacuum fluctuations spectrum.

The electromagnetic nature of particles-antiparticles masses and of the gravitational constant G. Masse-charge equivalence, Newton's and Coulomb's laws equivalence.
The electron-positron elementary charge *e*, a fundamental physical constant, is obtained exactly from the electromagnetic vacuum quantized amplitude ξ [11,12,21]

$$e=\left(4\pi\right)^{2}\alpha\frac{\xi}{\mu_{0}} \tag{7}$$

Note that the last relation is neither a postulate nor a definition but derives naturally during the normalization of the vector potential quantization amplitude to a single photon state. In fact, all the QED fundamental constants $c=(\varepsilon_{0}\mu_{0})^{-1/2}$, $e=(4\pi)^{2}\alpha\xi/\mu_{0}$ and $\hbar=\alpha\xi^{2}(4\pi)^{3}\varepsilon_{0}^{-1/2}\mu_{0}^{-3/2}$ are expressed uniquely through the vacuum constants ε_{0}, μ_{0} and ξ entailing that they derive from the electromagnetic quantum vacuum field. Evidently, the photon vector potential amplitude $a_{0k}=\xi\omega_{k}$ and the elementary charge e are physically related to the electromagnetic vacuum through the vacuum constant ξ demonstrating that photons and leptons/antileptons are states of the vacuum field, putting the basis for a physical comprehension of their mutual transformation mechanism.
Now, from the historical experimental evidence we recall that Planck's constant $\hbar$ is intrinsically related to the energy quantization of the electromagnetic field at a single photon level. Despite of this characteristic physical origin Planck's constant is used in quantum physics for the description of all particles. Consequently, we may intuitively guess that the electromagnetic nature should be an inherent property of any particle. Indeed, the rest mass of the electron-positron is expressed directly through the elementary charge *e* and its magnetic moment [16, 22]

$$m_{e^{-},e^{+}}=2\pi c\,e^{2}\frac{\xi}{\mu_{B}} \tag{8}$$

with $\mu_{B}=9.274\,10^{-24}\ J\,T^{-1}$ is the Bohr magneton.
It is worthy to mention that the notion of mass introduced in the expression of Bohr's magneton in the literature is a classical concept associated to a quantum process for the description of the magnetic dipole moment $\vec{M}$ of a wave-particle with wave-vector $\vec{k}$ in a circular standing state of radius *r*. In a pure quantum description Bohr's magneton is simply the proportionality constant characterizing the magnetic dipole moment $\vec{M}=\mu_{B}(\vec{k}\times\vec{r})$.
With the same token the proton/antiproton mass writes

$$m_{P^{+},P^{-}}=2\pi c\,e^{2}\frac{\xi}{\mu_{P}} \tag{9}$$

where $\mu_{P}=5.050\,8\,10^{-27}\ J\,T^{-1}$ is the proton magneton.
Using the relation (2), the mass m_{i} of any elementary particle-antiparticle writes in a general way

$$m_{i}=2\pi c\,e^{2}\frac{\xi}{\mu_{i}}=\left(\frac{c\mu_{0}}{8\pi\alpha}\right)\frac{e^{3}}{\mu_{i}} \tag{10}$$

where e is the electron charge for particles or the positron charge for antiparticles and μ_i is the magneton of the particle *i*.

Obviously, the masses of particles and antiparticles are purely of electromagnetic nature. The magneton for the electron-positron is μ_B while that of any other particle-antiparticle can be expressed approximately through Bohr's magneton and the fine structure constant

$$\mu_{e^-,e^+} = \mu_B \quad ; \quad \mu_i \simeq \left(\frac{2\alpha}{n_i}\right)\mu_B \tag{11}$$

where n_i is simply a positive integer [16, 23].

Thus, the mass of any particle *i*, lepton, meson or baryon, as well as of any antiparticle, is expressed with a precision of roughly 1% through the elementary charge *e*

$$m_{e^-,e^+} = \mu_e e^3 \quad ; \quad m_i \simeq \left(\frac{n_i}{2\alpha}\right)\mu_e e^3 \tag{12}$$

The numerical value of the proportionality constant is $\mu_e = c\mu_0 / 8\pi\alpha\mu_B = 2.215\,10^{26}\, kg\cdot C^{-3}$.

e.g. muon mass is obtained for n_i = 3, pion for n_i = 4, kaon for n_i =14, rho for n_i = 22, nucleon for n_i = 27, lambda for n_i = 32, sigma for n_i = 34, tau for n_i = 51…etc, while Higgs boson mass is obtained for n_i = 3,574.

Finally, it can be easily demonstrated [16] that any particle moving in the electromagnetic quantum vacuum with an acceleration $\vec{\gamma}$ will experience the Fulling-Davies-Unruh temperature

$$T_H = \frac{\hbar}{2\pi c k_B}\left|\vec{\gamma}\right| \tag{13}$$

where k_B is Boltzmann's constant.

Following the last equations, we draw that the particles and antiparticles masses are quantum states of the vacuum field originating from charges and their magnetic moments. In fact, the presence in equation (12) of an integer n_i characterizing the particles-antiparticles masses implies that the electromagnetic vacuum must have a complex structure of quantum states yet to discover, which might be related to string theory. Equations (10) and (12) express simply the mass-charge equivalence. Apparently, strong and weak forces seem to be particular manifestations of the electromagnetic vacuum field while quarks and antiquarks should be also states of the same vacuum field. Through this lens, all neutral elementary particles-antiparticles must be composed of positive and negative charges and consequently, gravity should be an electromagnetic effect.

Indeed, considering Planck's length l_P = 1.616 10^{-35} *m*, which is the shorter possible wavelength for a single photon beyond which the electromagnetic energy density transforms to a black hole, the gravitational constant *G* is obtained exactly through the elementary charge *e* and the electromagnetic vacuum constants ξ, ε_0 and μ_0 revealing its electromagnetic nature

$$G = \frac{l_p^2}{4\pi\,\varepsilon_0\mu_0 e\xi} \tag{14}$$

Planck's length is a physical constant characterizing the granularity of the electromagnetic quantum vacuum field.

The origin of gravitation appears to lay on the modification of the electromagnetic quantum vacuum field fluctuations due to the presence of localized charge (mass) densities [22]. By the same token, light rays should follow the paths in the electromagnetic vacuum imposed by the charge densities in space modifying locally the vacuum fluctuations and the refractive index.

The masse-charge equivalence and the electromagnetic nature of the gravitation constant imply the equivalence of Newton's gravitation law to Coulomb's electrostatic law.

In fact, the well-known Newton's gravitational potential between two particles *i* and *j* with respective masses m_i and m_j at a distance r_{ij} writes

$$U_{Newton} = G\frac{m_i m_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0}\frac{e_i e_j}{r_{ij}}\eta_{ij} = U_{Coulomb} \tag{15}$$

where we have used the relations (2), (10) and (14).

Note that e_i and e_j denote the electron charge for particles or the positron one for antiparticles while η_{ij} is a dimensionless parameter depending on the magnetic moments of the interacting particles

$$\eta_{ij} = \frac{\pi\hbar c l_P^2}{\mu_0\mu_i\mu_j} \tag{16}$$

Note also that, when considering the algebraic sign of charges, and not their absolute values as usually, a minus sign has to be considered in Coulomb's law for the resulting positive electrostatic forces to characterize attraction (as in gravitation) and the negative ones repulsion (as in anti-gravitation).

We have used μ_B and l_p as intermediate constants in order to obtain physically meaningful relations expressing the mass-charge equivalence, the electromagnetic nature of the gravitational constant *G* and the equivalence between Newton's gravitational law and Coulomb's electrostatic law.

We recall that Bohr's magneton μ_B is generally considered as a positive quantity so as the Larmor angular frequencies $\omega_L = \mu_B\left|\vec{B}\right|/\hbar$ in a magnetic field $\vec{B}$ to be also positive and the corresponding absorbed and emitted photons to have positive energies $\hbar\omega_L$. Under this condition, considering that the product of the magnetons $\mu_i\mu_j$ of the interacting particles is a positive quantity we can draw a quite interesting feature concerning matter, antimatter and gravity. Obviously, since ξ and e have the same algebraic sign according to (2), the gravitational constant expressed by (14) is positive for both matter and antimatter. Hence, gravitational forces are attractive between bodies of ordinary matter as well as between bodies of antimatter, but they should be repulsive between matter and antimatter according to (15) since their masses are expressed through the electron and positron charge respectively and consequently bear opposite signs following the relation (10).

In fact, a particle and an antiparticle of opposite charge are attracted by Coulomb forces overcoming naturally the weak gravitational repulsion and annihilate mutually giving generally birth to photons. Conversely, matter and antimatter neutral structures must be repelled due to repulsive gravitational forces. This is in agreement with previous studies that have shown that CPT symmetry (Charge conjugation, Parity and Time reversal) and General Relativity cannot be compatible unless matter and antimatter are mutually repelled [24]. We expect the ALPHA-g, AEgIS and GBAR experiments at CERN to give a definite answer to that issue.

Note that, from equation (7) for the electron, ξ is negative and consequently the ordinary masses in equations (10) and (12) appear as negative and those of antimatter, obtained from the positron charge, as positive. This is because historically, a negative charge was conventionally attributed to the electron and a positive one to the positron.

Finally, since ξ has positive and negative values, the vector potential of photons emitted by matter have naturally opposite signs with respect to those emitted by antimatter. This might constitute an experimental criterion for exploring the presence of antimatter structures in the universe. In fact, recent works have shown that antihydrogen atoms have the same properties with those of ordinary hydrogen atoms and particularly the same energy levels [25,26]. We may reasonably assume that antimatter stars and probably antimatter galaxies should have the same birth, life and death as the ordinary matter ones, as well as similar radiation properties yielding a particular difficulty for their detection [27].

An experimental device capable of distinguishing the opposite signs of the polarized photons vector potential might be helpful to explore the antimatter distributions in the universe.

Conclusion

We have visited the basic physical features that derive naturally from the electromagnetic quantum vacuum, the cosmic Dark Light. Photons (electromagnetic waves) are oscillations of the vacuum field. Leptons/antileptons elementary charges are states of the same vacuum field and are directly related to the photon vector potential which puts the basis for understanding the mechanism of their mutual transformations. The masses of all particles and antiparticles originate from charges and their magnetic moments witnessing a complex structure for the electromagnetic vacuum involving quantum levels that might be related to string theory. The mass-charge equivalence relation results readily expressing that particle-antiparticle masses are proportional to the cubic power of the electron-positron charge respectively. Newton's gravitational potential for elementary masses is equivalent to Coulomb's electrostatic potential for elementary charges. Consequently, all elementary entities derive from the electromagnetic field ground state, in simple words, everything in universe emerge from the cosmic Dark Light.

The gravitational constant G is also an electromagnetic vacuum property and is the same for matter and antimatter. Conversely, the masses of particles and antiparticles bear naturally opposite signs entailing a mutual gravitational repulsion, on the condition that their magnetic moments have the same sign. Matter-antimatter antigravity may play an important role in the observed cosmic acceleration and has to be investigated experimentally.

The electromagnetic quantum vacuum is a cosmic field permeating everything in universe and whose fluctuations, according to Heisenberg's uncertainty principle, last longer the lower the frequency. This could also explain the origin of the $1/f^{n}$ noise observed in astrophysics.

Finally, it is of high importance to mention that we have made no hypothesis and advanced no postulates in order to obtain the above results which derive naturally from Maxwell's theory once the vector potential amplitude is normalized at a single photon level. Consequently, the electromagnetic quantum vacuum field may constitute the physical basis for the development of a coherent unified field theory. Furthermore, the extreme simplicity of the established formalism relating the electromagnetic vacuum to electromagnetism, particle physics and gravitation reveals that there is a real physical background behind the equations opening new perspectives for innovative studies on gravity-antigravity.

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