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ORIGINAL PAPER

Finding acceleration parameter of a few spiral galaxies through MOND

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Abstract: In this study a few galaxies are considered as samples from the atlas of spiral galaxies for estimating the acceleration constant a_0 of Modified Newtonian dynamics (MOND). Considering the model of flat disk galaxy, the values of disk acceleration due to gravity (g_{disk}) are obtained which is equivalent to Newtonian gravity (g_N). This is further helpful in determining a_0 through modified Newtonian dynamics. The values obtained are 0.069–1.78 ($\times\ 10^{-13}\ \mathrm{km/s^2}$). Out of which 0.7–1.33 ($\times\ 10^{-13}\ \mathrm{km/s^2}$) are in a good agreement with the observed values. The MOND induced rotation curves of the galaxies are studied for the following interpolating functions:$\mu(x) = x(1+x^2)^{-\frac{1}{2}}$, $\mu(x) = \frac{x}{1+x}$ and$\mu(x) = 1 - e^{-x}$. This modification yields flat rotation curves. These curves are analyzed and compared with observational data. The simple interpolating function $\mu(x) = \frac{x}{1+x}$ is found to be a better fit to MOND. Also, the relation between acceleration constant and the surface mass density have been obtained and it is observed that acceleration constant increases with increase of surface mass density.

Keywords: Low mass spiral galaxies; Modified Newtonian dynamics (MOND); Flat rotation curves; Acceleration constant; Large scale structures

1. Introduction

It was noticed by Zwicky that the motion of galaxies in the Coma cluster was too rapid to be accounted for by its luminous matter [1]. Zwicky postulated that to explain the large velocities, additional mass was required than the known luminous matter. The additional required matter was called dark matter and has eluded discovery till date. Therefore, to explain the various dynamics of the galaxies and galactic systems, the hidden mass hypothesis was introduced in cosmological models. The missing mass problem was reviewed on the basis of mass-to-light ratios of spiral galaxies, elliptical galaxies and lenticular galaxies, binary galaxies, small groups of galaxies and clusters of galaxies. The studies concluded that the case for invisible mass in the universe was very strong [2]. It was clearly established that the average mass-to-light ratio was much larger than that of the visible part of the individual galaxies [3]. Further studies of galactic dynamics led to new assumptions to explain the observations such as the mass discrepancy. In1970's it became clear from the work of Rubin and Ford that the galactic rotational velocities were nearly constant at large distances from the galactic centers, which were contrary to the expected Keplerian decline [4]. Considering Newtonian dynamics, it was found necessary to postulate the existence of unseen dark matter to explain the flat rotation curves. In 1983, Milgrom proposed modification to the Newtonian mechanics, where it was suggested that the force law could be modified by inclusion of a parameter a_0 known as the acceleration constant [5]. This modified Newtonian dynamics (MOND) is an alternative paradigm which departs from standard Newtonian dynamics for low acceleration, without invoking the presence of dark matter [5, 6]. A non-relativistic theory for gravity has been built on the basis of MOND which accounts for the hidden mass problem [7]. MOND theory has been successful in explaining certain cases of galactic dynamics. One of the phenomenological laws of galactic dynamics underlying MOND is the baryonic Tully-Fisher relation $V^4 = GMa_0$ [5]. This relation is further investigated for a sample of galaxies with 21 cm rotation curves and obtained the fit with minimum scatter [8]. This is also applied to measure the baryonic mass including gas mass

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*Corresponding author, E-mail: sujata.14phd@cmr.edu.in along with stellar mass against constant rotational speed of galaxies which is in close conformity with the MOND predictions [9]. The analog of the universal rotation curve within the MOND framework and the baryonic Tully-Fisher relation predicted by universal rotation curves is in a good agreement [10]. In one of the studies rotation curves for four galaxies were considered at Cepheid-based distances out of which two rotation curves predicted by MOND paradigm are compatible with observed curves [11]. Based only on the distribution of visible matter, MOND theory predicts very well the observed dynamics such as rotation curves of dwarf to giant spirals, ellipticals, dwarf spheroidal and galaxy groups [5, 12]. The rotation curve was also determined by using the Tuorla-Heidelberg model for the mass distribution of the Milky Way galaxy which is in a good agreement with MOND [13]. The rotation curves predicted by MOND were also investigated through microlensing surveys and radio-frequency observations in the Milky Way galaxy and the results are in a good agreement with MOND [14]. The Newton's second law of motion is modified in the MOND formalism as, $m\mu\left(\frac{a}{a_0}\right)a = F$ where a_0 is indicative of the transition between Newtonian and deep-MOND regimes. The function $\mu\left(\frac{a}{a_0}\right)$ is known as the interpolating function [5, 15, 16]. The force acting here is only due to gravity and depends on the mass of the particle and the distribution of the mass which governs the force. The value of a_0 has been found to be $1.2 \times 10^{-13}\ \mathrm{km}/s^2$ obtained from galactic dynamics [17]. There, are two choices of interpolating functions that is simple and standard interpolating functions, $\mu(x) = \frac{x}{1+x}$ and $\mu(x) = x(1+x^2)^{-\frac{1}{2}}$ respectively. There is a strong relation found between observed radial acceleration and that due to the baryons by substituting MOND interpolating function for a sample of 153-disc galaxies with different morphologies, masses, sizes and gas fractions [18]. The radial acceleration relation is fitted to individual galaxies from the Spitzer Photometry & Accurate Rotation Curves (SPARC) database where the result is in favor of MOND model [19, 20]. Several other studies are in support of MOND [11, 13–15, 17, 21–24]. The conflict between the relativistic gravitation theory and MOND theory was resolved by considering generalized Tensor-Vector-Scalar (TeVeS) theory [25]. Relativistic MOND theory thus, resolves the problem of gravitational lensing and merging of galaxy clusters [25]. In the recent past the relativistic gravitational theory has been found to explain MOND phenomenology at small acceleration scales. It also demonstrates its agreement with the observed cosmic microwave background and matter power spectra on linear cosmological scale [26]. According to the assumption of Newtonian gravity the rotation velocities should decrease with the increase of distance from the galactic centers, which is known as Keplerian decline [5]. However, when the acceleration due to gravity is much smaller than a_0 Its rate of change increases with the square root of mass and decreases linearly with distance and the rotational velocity remains constant at large distances [5, 12, 15, 16]. The general laws of galactic dynamics predicted by MOND are well obeyed by the data with a_0 appearing with different independent roles. For example, the value of a_0 is equivalent to the square root of cosmological constant in natural units [5]. The value of $a_0 = 1.2 \times 10^{-13}\ \mathrm{km/s}^2$ is found to be of the same order as, $CH_0 = 5 \times 10^{-13}\ \mathrm{km/s}^2$ in cold dark matter cosmology where, H_0 is known as Hubble constant (ratio of velocity to distance) and C is the speed of light, where CH_0 has the dimension of acceleration parameter [5]. In the Ursa Major Cluster around thirty spiral galaxies are tested against MOND to produce the rotation curves which is in good agreement with the observed data [27]. Hence determination of acceleration constant a_0 is an important aspect. Hence, in this study, an attempt is made to determine the value of a_0 by using the model of flat disk galaxies in the context of MOND. A set of six flat rotating spiral galaxies are considered whose masses are less than $10^{11} M_\odot$ (mass of the sun) [28–33]. A motivating factor to consider low mass spirals is that the shape of the rotation curves is in excellent agreement with current data [34]. Hence, low mass spiral galaxies may have a low acceleration limit [5]. The value of a_0 is determined using the rotation velocities at large distances where the accelerations are small and are compared with observed data [35]. MOND aims not only to explain the rotation curves but also proposes an alternative theory of gravitation. Therefore, finding of acceleration constant in this study, analytically, by the model of flat disk galaxy, and then incorporating the same to obtain modified acceleration due to gravity g for different interpolating functions, is an interesting avenue. It may be noted that the only constraints imposed on the interpolating functions are Eqs. (4) and (5) of Sect. 2. Otherwise the choice of the function is arbitrary. The intended purpose of utilizing different interpolating functions is to obtain a better fit to MOND. Also, the dynamical central surface density is an important factor as it is purely the universal function of baryonic central surface density. And the relation between the two holds good in nonrelativistic MOND theories in the disc galaxy. It includes spiral and lenticular galaxies [36].

In Sect. 2 we present the basic formalism of MOND. In Sect. 3 we calculate the acceleration constant a_0 from flat disk models of galaxies. Section 4 presents results and discussion.

Table 1 Acceleration constant a_0 for different galaxies calculated

Galaxy Name	$\Sigma_0\,(10^6 M_\odot/kpc^2)$	R_D (kpc)	$a_0(10^{-13} km/s^2)$	g_{disk} (km/s^2)
NGC 300	194	1.35	1.33	232.959
NGC 2903	5914	2.50	1.17	2673.93
NGC 3198	837	3.31	0.763	585.554
NGC 2403	1081	1.65	0.924	1592.26
NGC 4395	1383	2.91	1.78	5273.49
UGC 10026	328	3.67	0.069	1475.34

Σ_0 is surface mass density, R_D is the disk scale length, g_{disk} is acceleration due to gravity in disk models of galaxies

2. The MOND formalism

In the standard Newtonian mechanics, the gravitational attractive force F between two masses M and m separated by a distance R is

$$F = G\frac{Mm}{R^2} \tag{1}$$

where G is the universal gravitational constant.

In MOND it was assumed that instead of the Newton's second law of motion,

$$F = ma \tag{2}$$

the following holds:

$$F = m\mu\left(\frac{a}{a_0}\right)a \tag{3}$$

where $\mu\left(\frac{a}{a_0}\right)$ is known as the interpolating function. The dimensionality of the original law is provided by a_0 in the denominator of the right-hand side of Eq. (3). The interpolating function imply the following conditions,

$$\mu\left(\frac{a}{a_0}\right) \approx 1 \quad \text{for } a \gg a_0 \tag{4}$$

$$\mu\left(\frac{a}{a_0}\right) \approx \frac{a}{a_0} \quad \text{for } a \ll a_0 \tag{5}$$

where, a_0 has the dimension of acceleration. The standard interpolating function in Eq. (5) has been substituted in this study to calculate $a_{0.}$ If the value of acceleration is larger than the acceleration constant, then the Newtonian second law is restored according to Eq. (4). In the limit of small acceleration, Eq. (1) and Eq. (3) can be combined. Hence, the equation becomes

$$\frac{a^2}{a_0} = \frac{GM}{R^2} \tag{6}$$

The rotational velocities of galaxies for circular orbits can be written as,

$$v^2 = aR \tag{7}$$

Substituting Eq. (7) in Eq. (6), the equation becomes,

$$v^4 = GMa_0 \tag{8}$$

Equation (8) is known as the famous baryonic Tully-Fisher relation in MOND theory [5, 12, 15, 16]. So, MOND will account for the force on galactic and extragalactic scales by modifying Newtonian gravity.

3. Acceleration constant from flat disk model of galaxies

The calculation of the acceleration due to gravity in the flat galaxies will be carried out without considering the assumptions of dark matter [5].

Denoting acceleration a as g, and then combining Eq. (1) and Eq. (2) the Newtonian acceleration becomes

$$g_N = \frac{GM}{R^2} \tag{9}$$

For flat disk galaxy, $g_N = g_{disk}$ [37, 38], can be written as

$$g_{disk} = 4\pi G\Sigma_0\left(\frac{R_D}{R}\right)(y)^2[I_0(y)K_0(y) - I_1(y)K_1(y)] \tag{10}$$

where $y = R/2R_D$, Σ_0 is the surface mass density at the galactic center, R is the distance from the galactic center, R_D is the disk scale length, I_0, I_1, are the modified Bessel functions of the first kind and K_0 and K_1 are the modified Bessel functions of the second kind. The actual acceleration g can be written in terms of g_N.

$$g = \frac{g_N}{\mu(x)} \tag{11}$$

Here, three interpolating functions, $\mu(x)$ are considered.

$$\begin{bmatrix} \mu(x) = x(1+x^2)^{-\frac{1}{2}} \\ \mu(x) = \frac{x}{1+x} \\ \mu(x) = 1 - e^{-x} \end{bmatrix} \tag{12}$$

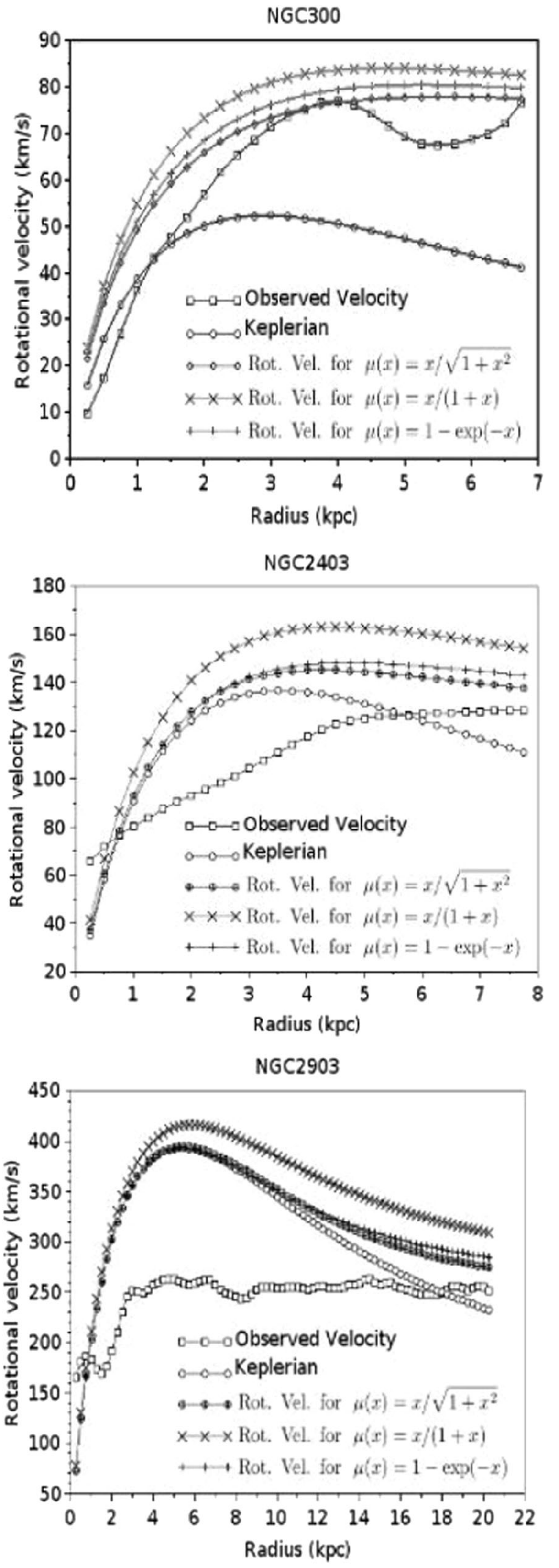

Fig. 1 Representation of curves of MOND (for three different interpolating functions), observed velocity and Keplerian decline

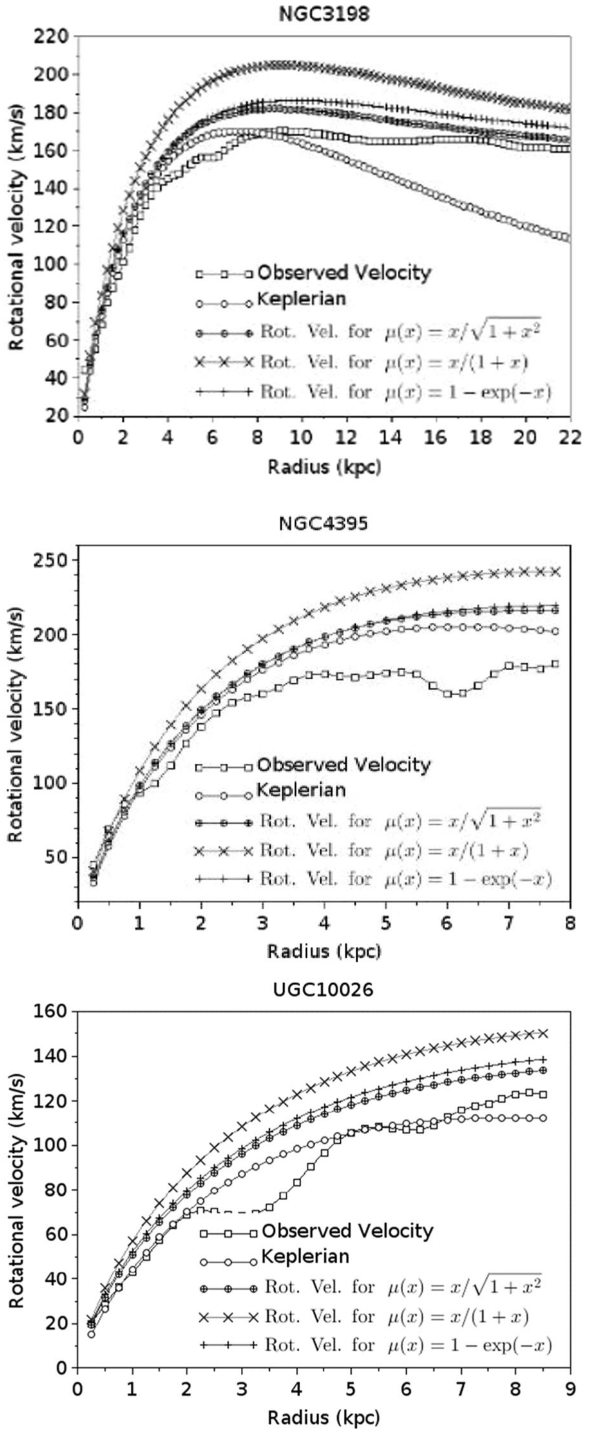

Fig. 1 continued

$$x = \frac{g}{a_0} \tag{13}$$

For $g \gg a_0$, $\mu(x) = 1$ and for $g \ll a_0$, $\mu(x) = x$. So, substituting Eq. (13) in Eq. (11) for $g \ll a_0$

Table 2 Comparison of calculated value of the acceleration constant with that of observed values

Sl. No	Name of spiral galaxy	Mass of spiral galaxy ($M_\odot$)	Distance of galaxies to which extent it is measured (kpc)	a_0 Calculated $10^{-13}\,\frac{km}{s^2}$	a_0 Observed $10^{-13}\,\frac{km}{s^2}$	Referenes
1	NGC 300	2.9×10^{10}	6.75	1.33	1.1–1.5	[21]
2	NGC 2903	4.9×10^{10}	21.25	1.17	1.2	[36, 37]
3	NGC 3198	8.5×10^{10}	22	0.763	1.1–1.5 Average is 0.7	[21]
4	NGC 2403	2.4×10^{4}	7.75	0.924	0.9	[22, 23]
5	NGC 4395	1×10^{4}	7.75	1.78	1.24	[41]
6	UGC 10026	1×10^{9}	8.5	0.069	1.2	[43]

Also it contains the mass and distance of galaxies to which extent these are measured

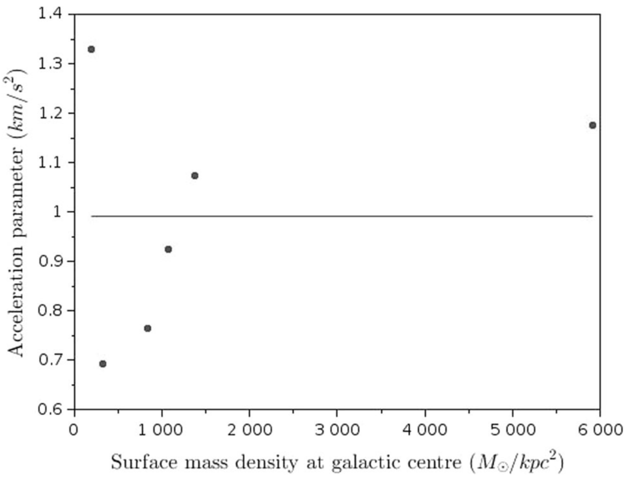

Fig. 2 Variation of the acceleration parameter a_0 with surface mass density Σ_0

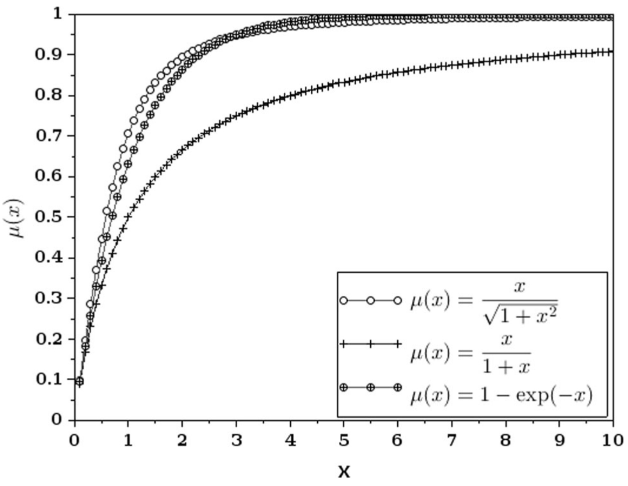

Fig. 3 Shapes of the interpolating Functions $\mu(x)$ where, $x = \frac{g}{a_0}$

$$g = \sqrt{g_N a_0} \tag{14}$$

According to the Newton's law Eq. (7) can be written as

$$g_N = \frac{v^2}{R} \tag{15}$$

Under MOND, Eq. (15) becomes,

$$g = \frac{v^2}{R} \tag{16}$$

The observation data of rotational velocity and radius for these galaxies are available in the literature [35]. Which are further analyzed to obtain rotational velocities under MOND.

Substituting Eq. (16) in Eq. (14) the equation becomes,

$$a_0 = \frac{v^4}{R^2 g_N} \tag{17}$$

The acceleration constant a_0 can be determined using Eq. (17). That MOND is significant over dark matter theory has been explained by some of the researchers [39, 40]. Substituting g_{disk} values of all the six spiral galaxies NGC 300, NGC 2403, NGC 2903, NGC 3198, NGC 4395 and UGC 10026 in Eq. (16), a_0 values have been calculated and presented in Table 1.

Here NGC refers to galaxies in the New General Catalog of nebulae and star clusters and UGC refers to the Uppsala General Catalog.

4. Results and discussion

The acceleration constant a_0 is estimated for the galaxies: NGC 300, NGC 2403, NGC 2903, NGC 3198, NGC 4395 and UGC 10026 and the results are presented in Table 1. The surface mass density Σ_0 at the galactic center for flat disks is assumed as those values at 0.25 kpc for these spiral galaxies [35]. The disc scale length R_D is also estimated from the data available therein. Figure 1 shows the curves of the observed velocity, along with the Keplerian decline due to the field of the flat disks alone, and the calculated rotational velocities for three different interpolating functions. The mass and the distances of galaxies, up to which the graphs have been plotted, are mentioned in Table 2.

For NGC 300, the MOND fitted curve becomes close to the observed rotational velocities near 4 kpc, and at about 7 kpc. For NGC 2403, the calculated values become close to the observed values at large radii from 4.5 to 7.75 kpc. However, the calculated values are much larger at low values of the distances upto 4.5 kpc for NGC 2903. For NGC 2903, the calculated values are much larger up to 14.5 kpc and close to the observed values from 14.5 to 20.5 kpc. For NGC 3198, the calculated values show much better agreement throughout the range of distances from about 0.25 to 22 kpc as compared to that of the other galaxies considered. For NGC 4395, the calculated and the observed velocities show good agreement at 7 kpc onwards. In case of UGC 10026, the calculated and observed velocities are in a good agreement at 8 kpc onwards. There is reasonable agreement between the observed rotational curves and the curves obtained by the MOND approach away from the galactic center. The observed rotation curves in Fig. 1, are not almost flat but show some variation. However, they tend to exhibit flatness at large radii [40].

In Fig. 1 three MOND fits are shown for three different interpolating functions. It is observed that there is some deviation between MOND fit curve and the observed curve at low radii for all the galaxies considered. Figure 1 also shows the Keplerian decline. If this is compared either with MOND fit or observed velocity then one can observe that the rotational velocity is decreasing under Newtonian gravity as one moves away from the galactic center. This leads to mass discrepancy. Hence, Newtonian law fails to explain the rotation curves or the dynamics of galaxies.

In Fig. 2, the variation between acceleration parameters at low acceleration limit with the surface mass density is shown. This shows the increase in a_0 along with the increase in Σ_0. The surface mass density based on the luminosity profile is approximately given by $\Sigma = \Sigma_0\, e^{-R/RD}$ [41]. If the mass density is low at the galactic center then the value of acceleration constant is also small. It appears that estimation of acceleration constant and hence a test of modified law of gravity is coupled with mass distribution of the galaxies. This is one one of the important observations of this study.

In Fig. 3, the variation between interpolating function and the ratio of true acceleration to the acceleration constant have been shown. The interpolating function basically links the Newtonian acceleration produced by the visible matter to the true gravitational acceleration. It is observed that the exponential interpolating function $\mu(x) = 1 - e^{-x}$ approaches Newtonian gravity quickly. Whereas, the simple interpolating function $\mu(x) = \frac{x}{1+x}$ approaches Newtonian gravity slowly. The simple interpolating function shows a better fit to MOND than the standard interpolating function $\mu(x) = x(1+x^2)^{-\frac{1}{2}}$. If compared with observed data, a simple interpolating function shows a better fit to MOND than the standard one.

This study shows that the a_0 values of four galaxies 0.7-1.33 ($\times 10^{-13}$ km/s^2) are nearer to the MOND acceleration constant when compared with the observed data as shown in Table 2. In the case of the galaxies NGC 4395 and UGC 10026, the values of acceleration constant do not show very good agreement with the MOND acceleration and the rotational curves are not exactly flat at low radii. The value of MOND acceleration is $a_0 = 1.2 \times 10^{-13}$ km/s^2. The best fit values of a_0 found from MOND mass models of 27 dwarf and low surface brightness galaxies are lying in the range of 1.1×10^{-13} km/s^2 to 1.5×10^{-13} km/s^2. Here the average value obtained for a_0 is 0.7×10^{-13} km/s^2 [21]. The value of a_0 was determined to be 1.21×10^{-13} km/s^2 from the mass modeling of a number of nearby galaxies with the standard interpolating function [42]. This value was further confirmed by using a sample of rotation curves of galaxies belonging to the Ursa Major galaxy group [27]. However, using an updated value of the distance to the Ursa Major group would bring the value of a_0 down to 0.9×10^{-13} km/s^2 [10]. The results of the acceleration constant for the NGC 3198 galaxy gives $a_0 = 1.2 \times 10^{-13}$ km/s^2 [37]. Considering nineteen low-rotating elliptical galaxies and applying Bayesian inference analysis, it was found that all the galaxies are consistent with a universal acceleration of value of 1.5×10^{-13} km/s^2 [43]. This value also agrees well with that reported for 153 rotationally supported galaxies from the Spitzer Photometry & Accurate Rotation Curves (SPARC) database [18]. Again this value is confirmed where the rotation curves are estimated from disk photometry data by using MOND's critical acceleration scale [44]. The comparison is presented in Table 2.

The calculated values of a_0 are shown in Table 1. According to the procedure followed in this study, the a_0 values for all the galaxies are nearer to the $a_0 = 1.22 \times 10^{-13}$ km/s^2 . The average value calculated is found to be 0.965×10^{-13} km/s^2. This is very close to $a_0 = 1.22 \times 10^{-13}$ km/s^2 calculated in MOND. In this study 79% of the sample leads to quality fit to MOND.

5. Conclusions

The small values of the observed rotational velocities while measuring experimentally, may be due to some undetected missing baryons while observing the mass distribution of stars of spirals of galaxies. Possibly this is due to the error in calculating mass to light ratio of stellar disk from observation. It is observed that the flatness of rotation curves are matching well at large radii [45].

The interpolating functions represent a smooth transition from Newtonian gravity to MOND.

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