Chiral and flavor oscillations in the interaction picture

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Abstract. We provide a concise review of how chiral and flavor oscillations can be described in quantum field theory using a finite-time interaction picture approach, where the mass and mixing terms in the Lagrangian can be treated as perturbations. We derive the oscillation formulas for both chiral and flavor transitions and demonstrate that, within the adopted approximations, they match the exact results obtained through non-perturbative methods. Finally, we point out the strong similarities and the differences between these two phenomena.

1. Introduction

The weak interaction is a strange beast. When parity was widely believed to be strictly conserved in all physical processes, Lee and Yang challenged this assumption, suggesting that weak interactions might violate it [1]. Shortly after, Wu and collaborators provided experimental confirmation, demonstrating that only left-handed neutrinos participate in weak processes [2]. More generally, charged current weak interactions involve only left-chiral fermions and right-chiral antifermions [3, 4, 5].

Furthermore, to address the solar neutrino puzzle [6], Pontecorvo and collaborators proposed that neutrino flavor states – those involved in weak interactions – could be expressed as linear superpositions of neutrino mass states, leading to the phenomenon of neutrino flavor oscillations [7, 8, 9, 10, 11]. This idea was later experimentally confirmed and is now well established [12, 13, 14, 15].

Another approach proposed to solve the solar neutrino puzzle was based on the observation that the chirality of massive particles is not conserved during their propagation. As a result, particles produced with definite chirality in weak interactions undergo chiral oscillations [16]. Although it was soon realized that this effect is too small to account for the missing solar neutrinos, the phenomenon itself is genuine and has been further investigated over the years

[17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. It has been shown that a massive fermion produced in a weak decay at time $t = t_i$ with a definite (e.g., left) chirality has a probability of being detected in another weak process at time $t = t_f$ with the same chirality, given by

$$P_{L\to L}(\mathbf{p}, \Delta t) \;=\; 1 - \frac{m^2}{\omega_{\mathbf{p}}^2}\sin^2\left(\omega_{\mathbf{p}}\,\Delta t\right)\,, \qquad \Delta t \equiv t_f - t_i\,, \tag{1}$$

where $\omega_{\mathbf{p}} = \sqrt{|\mathbf{p}|^2 + m^2}$ is the fermion energy. Although such expressions clearly shows that such effect is suppressed for very energetic particles ($\omega_{\mathbf{p}} \gg m$), chiral oscillations remain of phenomenological interest. Notably, it has been recently suggested that this effect could be observable in the cosmic neutrino background [28, 25] and in electronic transport in graphene layers, where chiral symmetry is explicitly broken by external potential barriers [29].

Within the phenomenological understanding of flavor oscillations, the problem arose regarding the correct theoretical description in quantum field theory (QFT) [30, 31, 32, 33, 34, 35, 36, 37]. In Ref. [32], an approach was proposed in which a *flavor Fock space* is explicitly constructed (see also Refs. [38, 39, 40, 41]). Within this approach, the flavor survival probability (e.g., for electron neutrinos) was derived in Ref. [42] and is given by

$$P_{e\to e}(\mathbf{p}, \Delta t) = 1 - \sin^2(2\theta)\left[|U_{\mathbf{p}}|^2 \sin^2\left(\frac{\Omega_{\mathbf{p}}^-}{2}\Delta t\right) + |V_{\mathbf{p}}|^2 \sin^2\left(\frac{\Omega_{\mathbf{p}}^+}{2}\Delta t\right)\right], \tag{2}$$

where $\Omega_{\mathbf{p}}^{\pm} \equiv \omega_{\mathbf{p},2} \pm \omega_{\mathbf{p},1}$, $\omega_{\mathbf{p},j} = \sqrt{|\mathbf{p}|^2 + m_j^2}$ (m_1 and m_2 are the neutrino masses) and

$$|U_{\mathbf{p}}| \;\equiv\; \left(\frac{\omega_{\mathbf{p},1} + m_1}{2\omega_{\mathbf{p},1}}\right)^{\frac{1}{2}} \left(\frac{\omega_{\mathbf{p},2} + m_2}{2\omega_{\mathbf{p},2}}\right)^{\frac{1}{2}} \left(1 + \frac{|\mathbf{p}|^2}{(\omega_{\mathbf{p},1} + m_1)(\omega_{\mathbf{p},2} + m_2)}\right).$$

$$|V_{\mathbf{p}}| \;=\; \frac{|\mathbf{p}|}{\sqrt{4\omega_{\mathbf{p},1}\omega_{\mathbf{p},1}}} \left(\sqrt{\frac{\omega_{\mathbf{p},2} + m_2}{\omega_{\mathbf{p},1} + m_1}} - \sqrt{\frac{\omega_{\mathbf{p},1} + m_1}{\omega_{\mathbf{p},2} + m_2}}\right). \tag{3}$$

Let us stress that the above formula also naturally arises in relativistic quantum mechanics, employing the Dirac equation [18]. A similar QFT approach has been also recently proposed for chiral oscillations [43], which permits to recover the formula (1) in a full fledged QFT language.

In this paper, we provide a brief review of the perturbative method to deal with flavor and chiral oscillations in QFT, originally introduced in Refs. [44] and [45], respectively. The key aspects of this approach are as follows:

(i) The mixing (or mass) term in the Lagrangian can be treated as a perturbation.
(ii) In the interaction picture, fields with definite flavor (chirality) can be expanded as free fields, ensuring that flavor (chiral) states are unambiguously defined in QFT.
(iii) The probability of flavor (chiral) oscillations can be computed using the matrix elements of the time evolution operator $U(t_i, t_f)$.

It is important to emphasize, as discussed in Ref. [46], that the time evolution operator plays a central role in this analysis. Taking the limit $t_i \to -\infty$, $t_f \to +\infty$, which transforms U into the S-matrix, would eliminate the oscillation phenomenon. This observation is profound and closely linked to the time-energy uncertainty relation [47, 48, 49, 50, 51]. Notably, the computation in the interaction picture, within the approximations used to truncate the perturbative expansion, yields results that nontrivially agree with Eqs. (1), (2).

The paper is organized as follows: In Section 2, we briefly review how chiral oscillations can be treated within our interaction picture scheme. Then, in Section 3, we apply the same methods to analyze two-flavor oscillations. Throughout the paper, we highlight the differences and similarities between these two phenomena. Finally, in Section 4, we present our conclusions.

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2. Chiral oscillations in the interaction picture

The Lagrangian for a massive Dirac field ψ can be expressed in terms of its chiral components as

$$\mathcal{L} \;=\; \sum_{\sigma=L,R} \overline{\psi}_\sigma i\partial\!\!\!/ \psi_\sigma - m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right) , \tag{4}$$

where $\psi_{L(R)} = P_{L(R)}\psi$, with $P_{L(R)} \equiv \frac{1\mp\gamma_5}{2}$ denoting the chiral projection operators.

The central idea is to interpret the mass term as an interaction between massless fields of definite chirality, enabling its treatment using standard perturbative methods in QFT. Our starting point is the Dyson series for the time evolution operator:

$$U(t_i,t_f) \;=\; \mathcal{T}\exp\left[i\int_{t_i}^{t_f} d^4x \,:\mathcal{L}_{int}(x):\right] \;=\; \mathcal{T}\exp\left[-i\int_{t_i}^{t_f} d^4x \,:\mathcal{H}_{int}(x):\right] , \tag{5}$$

where $\mathcal{L}_{int} \;=\; -m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right)$, and the interaction Hamiltonian density is given by $\mathcal{H}_{int}(x) = -\mathcal{L}_{int}(x)$. The operator $\mathcal{T}$ denotes time ordering.

In the interaction picture, the fields ψ_σ ($\sigma = L, R$) are expanded as free fields [5]:

$$\psi_\sigma(x) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}}\left(u_{\mathbf{k},\sigma}\,\alpha_{\mathbf{k},\sigma}\,e^{-ikx} + v_{\mathbf{k},\sigma}\,\beta^\dagger_{\mathbf{k},\sigma}\,e^{ikx}\right) . \tag{6}$$

Since these fields are massless, their helicity coincides with their chirality.

We define the perturbative vacuum in the usual way:

$$\alpha_{\mathbf{k},\sigma}|0\rangle = 0 = \beta_{\mathbf{k},\sigma}|0\rangle\,, \tag{7}$$

and its excitations as

$$|\psi_{\mathbf{p},\sigma}\rangle \;\equiv\; \alpha^\dagger_{\mathbf{p},\sigma}|0\rangle\,. \tag{8}$$

The canonical anticommutation relations are given by

$$\{\alpha_{\mathbf{k},\rho},\alpha^\dagger_{\mathbf{q},\sigma}\} = \delta_{\mathbf{kq}}\delta_{\rho\sigma}\,, \quad \{\beta_{\mathbf{k},\rho},\beta^\dagger_{\mathbf{q},\sigma}\} = \delta_{\mathbf{kq}}\delta_{\rho\sigma}\,. \tag{9}$$

Next, we explicitly compute the interaction Hamiltonian $H_{int} = \int d^3x\,\mathcal{H}_{int}(x)$ in the interaction picture:

$$H_{int}(t) \;=\; m\sum_{\mathbf{p}}\left[\beta_{-\mathbf{p},R}\alpha_{\mathbf{p},L}\,e^{-2i|\mathbf{p}|t} + \alpha^\dagger_{\mathbf{p},R}\beta^\dagger_{-\mathbf{p},L}\,e^{2i|\mathbf{p}|t} + h.c.\right] . \tag{10}$$

Our goal is to compute the survival probability, i.e., the probability for the process $|\psi_{\mathbf{p},L}\rangle \to |\psi_{\mathbf{p},L}\rangle$ over a time interval $\Delta t \equiv t_f - t_i$, $\mathcal{P}_{L\to L}(\mathbf{p},\Delta t)$. At zeroth order, we obtain $\mathcal{P}_{L\to L}(\mathbf{p},\Delta t) = 1$. The first nontrivial contribution appears at second order in m. The corresponding Feynman diagrams are shown in the upper part of Fig. 1.

Retaining only terms up to quadratic order in m, the survival amplitude can be expressed as

$$\mathcal{A}_{L\to L}(\mathbf{p};t_i,t_f) \;=\; 1 - \frac{1}{2}\mathcal{A}^{(2)}_{L\to L}(\mathbf{p};t_i,t_f)\,, \tag{11}$$

where $\mathcal{A}^{(2)}_{L\to L}(\mathbf{p};t_i,t_f)$ represents the second-order contribution, which is proportional to m^2. Squaring this expression and neglecting all terms with mass dependence beyond second order, we obtain

$$\mathcal{P}_{L\to L}(\mathbf{p};\Delta t) \;=\; 1 - \Re e\left(\mathcal{A}^{(2)}_{L\to L}(\mathbf{p};t_i,t_f)\right) . \tag{12}$$

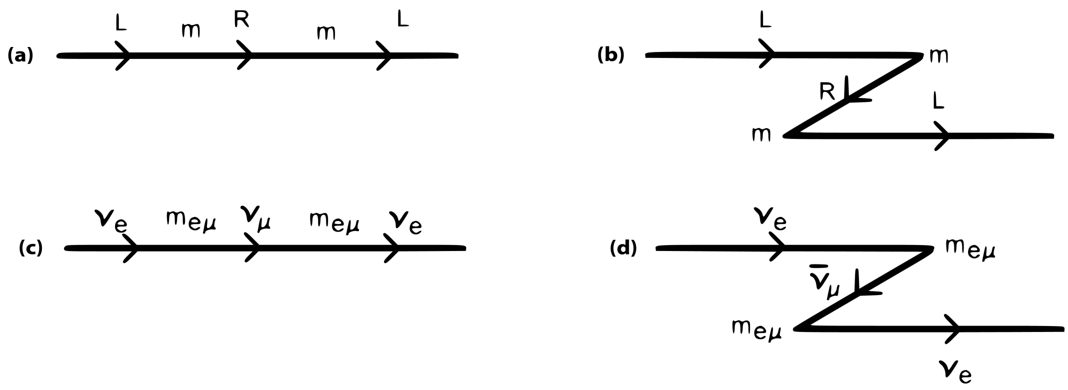

Figure 1. Second-order diagrams for the L (upper part) and e (lower part) survival probability. Time flows from left to right. The diagram (a) vanishes since it involves identical massless particles. The Z-type diagram (b) contributes nontrivially. The diagrams (c) and (d) both contribute to flavor oscillations.

The second-order amplitude is given explicitly by

$$\mathcal{A}^{(2)}_{L\to L}(\mathbf{p};t_i,t_f) \approx m^2 \int_{t_i}^{t_f} \mathrm{d}t_1 \int_{t_i}^{t_f} \mathrm{d}t_2\, e^{2i|\mathbf{p}|(t_1-t_2)} = \frac{m^2}{|\mathbf{p}|^2}\sin^2(|\mathbf{p}|(t_f-t_i))\,.$$

Thus, the survival probability takes the form

$$\mathcal{P}_{L\to L}(\mathbf{p};\Delta t) \;=\; 1-\frac{m^2}{|\mathbf{p}|^2}\sin^2\left(|\mathbf{p}|\,\Delta t\right)\,. \tag{13}$$

At leading order, this result coincides with Eq. (1). In fact, we have

$$\frac{m^2}{\omega_{\mathbf{p}}^2}\sin^2\left(\omega_{\mathbf{p}}\,\Delta t\right) \;=\; \frac{m^2}{|\mathbf{p}|^2}\sin^2\left(|\mathbf{p}|\,\Delta t\right)+O(m^4)\,. \tag{14}$$

This confirms that the quantum mechanical result is successfully recovered within the framework of finite-time QFT.

Although the above result is approximate, the agreement between Eq. (13) and Eq. (1) is highly non-trivial. To explore corrections beyond the leading order, we now consider the fourth-order contributions. One such term, proportional to m^4, arises from the square of the second-order amplitude:

$$\frac{1}{4}|\mathcal{A}^{(2)}_{L\to L}(\mathbf{p};t_i,t_f)|^2 \;=\; \frac{m^4}{4|\mathbf{p}|^4}\sin^4(|\mathbf{p}|\Delta t)\,. \tag{15}$$

However, this is not the only contribution at order m^4. The fourth-order amplitude itself provides an additional term:

$$\frac{1}{24}\mathcal{A}^{(4)}_{L\to L}(\mathbf{p};t_i,t_f) \;=\; \frac{m^4}{24|\mathbf{p}|^4}\sin^4(|\mathbf{p}|\Delta t)\,. \tag{16}$$

Thus, the survival probability at fourth order is given by

$$\mathcal{P}_{L\to L}(\mathbf{p};\Delta t) \;=\; 1-\frac{m^2}{|\mathbf{p}|^2}\sin^2\left(|\mathbf{p}|\,\Delta t\right)+\frac{m^4}{3|\mathbf{p}|^4}\sin^4(|\mathbf{p}|\Delta t)\,. \tag{17}$$

To verify consistency with the exact survival probability in Eq. (1), we expand the latter up to fourth order in m:

$$\mathcal{P}_{L\to L}(\mathbf{p};\Delta t) \;\approx\; 1-\frac{m^2\sin^2(|\mathbf{p}|\Delta t)}{|\mathbf{p}|^2}-\frac{m^4\sin(|\mathbf{p}|\Delta t)(|\mathbf{p}|\Delta t\cos(|\mathbf{p}|\Delta t)-\sin(|\mathbf{p}|\Delta t))}{|\mathbf{p}|^4}\,. \tag{18}$$

It is important to note that this expansion is only valid for short time intervals; for large Δt, the last term would diverge. Expanding further in Δt, we obtain:

$$\mathcal{P}_{L\to L}(\mathbf{p};\Delta t) \;\approx\; 1-m^2\Delta t^2+\frac{1}{3}m^2\Delta t^4\omega_{\mathbf{p}}^2+O\left(\Delta t^5\right)\,, \tag{19}$$

where $\omega_{\mathbf{p}}=\sqrt{|\mathbf{p}|^2+m^2}$ is the energy of the massive fermion. It is straightforward to see that this result matches the expansion of Eq. (17) in Δt, considering that $\sin\left(|\mathbf{p}|\,\Delta t\right)\approx|\mathbf{p}|\Delta t$.

3. Neutrino oscillations in the interaction picture

Following Ref. [44], we demonstrate that neutrino flavor oscillations can also be studied in the interaction picture, in a manner similar to chiral oscillations. However, in this case, the unperturbed modes are massive, with masses $m_e, m_\mu, \ldots$ (for simplicity, we will consider only two flavors), while the perturbation arises from the mixing term in the Lagrangian. The Hamiltonian density is therefore given by

$$\mathcal{H} \;=\; \mathcal{H}_0\,+\,\mathcal{H}_{int}\,, \tag{20}$$

where

$$\mathcal{H}_0 \;=\; \sum_{\sigma=e,\mu}\left[\overline{\nu}_\sigma\left(i\partial\!\!\!/-m_\sigma\right)\nu_\sigma\right]\,,\quad \mathcal{H}_{int} \;=\; m_{e\mu}\left(\overline{\nu}_e\nu_\mu+\overline{\nu}_\mu\nu_e\right)\,. \tag{21}$$

In the interaction picture, the neutrino fields ν_σ $(\sigma=e,\mu)$ can be expanded as

$$\nu_\sigma(x)=\frac{1}{\sqrt{V}}\sum_{\mathbf{k},r}\left[u^r_{\mathbf{k},\sigma}e^{-i\omega_{\mathbf{k},\sigma}t}\,\alpha^r_{\mathbf{k},\sigma}+v^r_{-\mathbf{k},\sigma}e^{i\omega_{\mathbf{k},\sigma}t}\,\beta^{r\dagger}_{-\mathbf{k},\sigma}\right]e^{i\mathbf{k}\cdot\mathbf{x}}\,, \tag{22}$$

with $\omega_{\mathbf{k},\sigma}=\sqrt{|\mathbf{k}|^2+m_\sigma^2}$. The perturbative vacuum is defined by the condition

$$\alpha^r_{\mathbf{k},\sigma}|0\rangle=0=\beta^r_{\mathbf{k},\sigma}|0\rangle\;, \tag{23}$$

while the ladder operators satisfy the standard anticommutation relations:

$$\{\alpha^r_{\mathbf{k},\rho},\alpha^{s\dagger}_{\mathbf{q},\sigma}\}=\delta_{\mathbf{kq}}\delta_{rs}\delta_{\rho\sigma}\,,\qquad \{\beta^r_{\mathbf{k},\rho},\beta^{s\dagger}_{\mathbf{q},\sigma}\}=\delta_{\mathbf{kq}}\delta_{rs}\delta_{\rho\sigma}\,. \tag{24}$$

Using these definitions, we can express the interaction Hamiltonian as

$$\begin{aligned} H_{int}(t) \;&=\; m_{e\mu}\sum_{s,s'=1,2}\sum_{\mathbf{p}}\Big[\beta^s_{\mathbf{p},\mu}\beta^{s\dagger}_{\mathbf{p},e}\delta_{ss'}W^*_{\mathbf{p}}(t)+\alpha^{r\dagger}_{\mathbf{p},\mu}\alpha^r_{\mathbf{p},e}\delta_{ss'}W_{\mathbf{p}}(t)\\ &+\;\beta^s_{-\mathbf{p},\mu}\alpha^{s'}_{e,\mathbf{p}}\left(Y^{ss'}_{\mathbf{p}}(t)\right)^*+\alpha^{s\dagger}_{\mathbf{p},\mu}\beta^{s'\dagger}_{-\mathbf{p},e}Y^{ss'}_{\mathbf{p}}(t)\,+\,e\leftrightarrow\mu\Big]\,, \end{aligned} \tag{25}$$

where we have introduced the functions

$$W_{\mathbf{p}}(t) \;=\; \overline{u}^s_{\mathbf{p},\mu}u^s_{\mathbf{p},e}e^{i\left(\omega_{\mathbf{k},\mu}-\omega_{\mathbf{k},e}\right)t} \;=\; W_{\mathbf{p}}\,e^{i(\omega_{\mathbf{p},\mu}-\omega_{\mathbf{p},e})t}, \tag{26}$$

$$Y^{ss'}_{\mathbf{p}}(t) \;=\; \overline{u}^s_{\mathbf{p},\mu}v^{s'}_{-\mathbf{p},e}e^{i\left(\omega_{\mathbf{k},\mu}+\omega_{\mathbf{k},e}\right)t} \;=\; Y^{ss'}_{\mathbf{p}}\,e^{i(\omega_{\mathbf{p},\mu}+\omega_{\mathbf{p},e})t}. \tag{27}$$

Their explicit forms are given by

$$W_{\mathbf{p}} = \sqrt{\frac{(\omega_{\mathbf{p},e}+m_e)\,(\omega_{\mathbf{p},\mu}+m_\mu)}{4\omega_{\mathbf{p},e}\omega_{\mathbf{p},\mu}}}\left(1-\frac{|\mathbf{p}|^2}{(\omega_{\mathbf{p},e}+m_e)(\omega_{\mathbf{p},\mu}+m_\mu)}\right), \tag{28}$$

$$Y^{22}_{\mathbf{p}} = -Y^{11}_{\mathbf{p}} = \frac{p_3}{\sqrt{4\omega_{\mathbf{p},e}\omega_{\mathbf{p},\mu}}}\left(\sqrt{\frac{\omega_{\mathbf{p},\mu}+m_\mu}{\omega_{\mathbf{p},e}+m_e}}+\sqrt{\frac{\omega_{\mathbf{p},e}+m_e}{\omega_{\mathbf{p},\mu}+m_\mu}}\right), \tag{29}$$

$$Y^{12}_{\mathbf{p}} = \left(Y^{21}_{\mathbf{p}}\right)^* = -\frac{p_1-ip_2}{\sqrt{4\omega_{\mathbf{p},e}\omega_{\mathbf{p},\mu}}}\left(\sqrt{\frac{\omega_{\mathbf{p},\mu}+m_\mu}{\omega_{\mathbf{p},e}+m_e}}+\sqrt{\frac{\omega_{\mathbf{p},e}+m_e}{\omega_{\mathbf{p},\mu}+m_\mu}}\right). \tag{30}$$

Let us now compute the survival probability $\mathcal{P}_{e\to e}(\mathbf{p},\Delta t)$, corresponding to the process $|\nu^r_{\mathbf{p},e}\rangle \to |\nu^r_{\mathbf{p},e}\rangle$. Keeping only terms up to the second order in $m_{e\mu}$, the amplitude can be expressed as

$$\mathcal{A}_{e\to e}(\mathbf{p};t_i,t_f) = 1-\frac{1}{2}\mathcal{A}^{(2)}_{e\to e}(\mathbf{p};t_i,t_f), \tag{31}$$

where $\mathcal{A}^{(2)}_{e\to e}(\mathbf{p};t_i,t_f)$ represents the second-order contribution, which is proportional to $m^2_{e\mu}$. The Feynman diagrams are depicted in the lower part of Fig.1. However, in this case, both diagrams (c) and (d) contribute non-trivially, with the vertex given by $m_{e\mu}$ instead of m. By squaring the amplitude we retain only terms of order $m^2_{e\mu}$ or lower, leading to

$$\mathcal{P}_{e\to e}(\mathbf{p};\Delta t) \approx 1-\mathrm{Re}\left(\mathcal{A}^{(2)}_{e\to e}(\mathbf{p};t_i,t_f)\right). \tag{32}$$

Explicitly, we obtain

$$\mathcal{P}_{e\to e}(\mathbf{p};\Delta t) = 1-4m^2_{e\mu}\left[\frac{W^2_{\mathbf{p}}}{(\omega_{\mathbf{p},e}-\omega_{\mathbf{p},\mu})^2}\sin^2\left(\frac{(\omega_{\mathbf{p},\mu}-\omega_{\mathbf{p},e})\Delta t}{2}\right)\right.$$
$$\left.+\frac{Y^2_{\mathbf{p}}}{(\omega_{\mathbf{p},e}+\omega_{\mathbf{p},\mu})^2}\sin^2\left(\frac{(\omega_{\mathbf{p},\mu}+\omega_{\mathbf{p},e})\Delta t}{2}\right)\right], \tag{33}$$

where

$$Y^2_{\mathbf{p}} = \sum_s \left(Y^{rs}_{\mathbf{p}}\right)^* Y^{rs}_{\mathbf{p}}, \tag{34}$$

and

$$Y_{\mathbf{p}} = \frac{|\mathbf{p}|}{\sqrt{4\omega_{\mathbf{p},e}\omega_{\mathbf{p},\mu}}}\left(\sqrt{\frac{\omega_{\mathbf{p},\mu}+m_\mu}{\omega_{\mathbf{p},e}+m_e}}+\sqrt{\frac{\omega_{\mathbf{p},e}+m_e}{\omega_{\mathbf{p},\mu}+m_\mu}}\right). \tag{35}$$

If we now introduce the following notation

$$|U_{\mathbf{p}}| = W_{\mathbf{p}}\,\frac{m_\mu-m_e}{\omega_{\mathbf{p},e}-\omega_{\mathbf{p},\mu}}$$
$$= \sqrt{\frac{(\omega_{\mathbf{p},e}+m_e)(\omega_{\mathbf{p},\mu}+m_\mu)}{4\omega_{\mathbf{p},e}\omega_{\mathbf{p},\mu}}}\left(1+\frac{|\mathbf{p}|^2}{(\omega_{\mathbf{p},e}+m_e)(\omega_{\mathbf{p},\mu}+m_\mu)}\right), \tag{36}$$

$$|V_{\mathbf{p}}| = Y_{\mathbf{p}}\,\frac{m_\mu-m_e}{\omega_{\mathbf{p},e}+\omega_{\mathbf{p},\mu}}$$
$$= \sqrt{\frac{(\omega_{\mathbf{p},e}+m_e)(\omega_{\mathbf{p},\mu}+m_\mu)}{4\omega_{\mathbf{p},e}\omega_{\mathbf{p},\mu}}}\left(\frac{|\mathbf{p}|}{\omega_{\mathbf{p},e}+m_e}-\frac{|\mathbf{p}|}{\omega_{\mathbf{p},\mu}+m_\mu}\right), \tag{37}$$

and $\sin 2\theta = 2m_{e\mu}/(m_\mu - m_e)$, then the survival probability can be rewritten as

$$\mathcal{P}_{e\to e}(\mathbf{p};\Delta t) = 1 - \sin^2 2\theta \left[|U_\mathbf{p}|^2 \sin^2\left(\frac{(\omega_{\mathbf{p},\mu} - \omega_{\mathbf{p},e})\Delta t}{2}\right) + |V_\mathbf{p}|^2 \sin^2\left(\frac{(\omega_{\mathbf{p},\mu} + \omega_{\mathbf{p},e})\Delta t}{2}\right)\right]. \tag{38}$$

Within our approximation, this result coincides with the oscillation probability (2). This is a remarkable result, as the method employed here differs significantly from the approach in Ref. [42].

It is also worth noting that we have followed the same computational steps for both chiral and flavor oscillations. Moreover, the flavor survival probability (33) reduces to the chiral survival probability (13) in the formal limit $m_e \to 0$, $m_\mu \to 0$, and $m_{e\mu} \to m$. In such limit, the term proportional to $|U_\mathbf{p}|$ disappears. Then, the only non-trivial contribution is given by the term proportional to $|V_\mathbf{p}|^2$, which describes fast oscillations.

4. Conclusions

Physical observables, by definition, are quantities that can be measured. Measuring a physical quantity requires interacting with the system, meaning that the nature of the interaction determines the relevant observables. In the case of weak interactions, the states with well-defined flavor and chirality are those that participate in the process, making them the physical states. For neutrinos, which interact almost exclusively via the weak force, flavor states can always be considered as the physical neutrino states. Therefore, it is crucial to establish a proper framework for describing states with definite flavor and chirality in QFT.

In this work, we have reviewed a perturbative approach to chiral and flavor oscillations, originally discussed in Refs. [44, 45], where chiral and flavor states are unambiguously defined within the interaction picture. Notably, this approach yields results that non-trivially agree with the non-perturbative findings reported in Refs. [42, 43]. Furthermore, the comparative analysis of chiral and flavor oscillations highlights their strong similarities, as the derivations we followed are nearly identical in both cases. However, in the case of flavor oscillations, diagram (c) in Fig. 1 provides a non-trivial contribution, while the corresponding diagram (a) does not give a contribution in the chiral oscillations case. This is the contribution which leads to the standard flavor oscillation formula [11]. In contrast, diagram (d), and the corresponding diagram (b) for chiral oscillations, describe rapid oscillations reminiscent of the *Zitterbewegung* phenomenon [24].

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