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Thermal fluctuation effects on shear viscosity to entropy ratio in five-dimensional Kerr–Newman black holes

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Abstract We investigate how thermal fluctuations affect the properties of five-dimensional Kerr–Newman black holes, focusing particularly on the shear viscosity to entropy ratio. Our analysis incorporates logarithmic corrections to the Bekenstein–Hawking entropy and examines their impact on black hole thermodynamics. We explore three approaches to studying the shear viscosity-entropy ratio in the presence of thermal fluctuations: considering independent shear viscosity, thermally corrected shear viscosity, and an independent ratio assumption. Notably, we find that the lower bound of $\eta/S \geq 1/4\pi$ remains valid even with thermal fluctuations, though the specific behavior depends on the black hole mass and correction parameter. Our results suggest that thermal fluctuations generally decrease the ratio for massive black holes while maintaining the universal lower bound. This work extends our understanding of quantum corrections to black hole transport properties.

1 Introduction

Black hole physics has long served as a fertile ground for exploring the interplay between general relativity, thermodynamics, and quantum mechanics. Since the seminal discovery of the Bekenstein–Hawking entropy [1,2], black hole thermodynamics has provided profound insight into the quantum nature of gravity [3–14]. In recent years, the study of higher-dimensional black holes has gained prominence due to their relevance in string theory, supergravity, and the AdS/CFT correspondence [15–23] Among these, five-dimensional black holes, such as the Kerr–Newman solutions, stand out due to their rich structure, incorporating electric charge, angular momentum, and additional spatial dimensions.

The five-dimensional Kerr–Newman black hole arises as a solution to the Einstein–Maxwell–Chern–Simons theory, which naturally emerges in higher-dimensional supergravity models [24]. The inclusion of Chern–Simons terms not only ensures gauge invariance but also leads to intriguing modifications in the thermodynamic and transport properties of black holes. These properties make five-dimensional Kerr–Newman black holes a versatile platform for studying quantum corrections and transport phenomena [25–33].

One of the remarkable developments in black hole physics is the realization that black holes behave as thermodynamic systems. Their entropy and temperature satisfy relations analogous to those of conventional thermodynamic systems. Furthermore, when black holes are embedded in an anti-de Sitter (AdS) background, the AdS/CFT correspondence provides a holographic interpretation of their transport properties in terms of strongly coupled quantum field theories. A key quantity in this context is the shear viscosity to entropy ratio, η/S, which has been conjectured to satisfy the universal Kovtun–Son–Starinets (KSS) bound, $\eta/S \geq 1/4\pi$ [34]. However, thermal fluctuations and quantum effects introduce corrections to this ratio, prompting investigations of the strength of the KSS bound under such modifications [35]. It is also worth noting that recent studies have demonstrated that thermal fluctuations can induce logarithmic corrections to black hole entropy, significantly impacting transport properties such as the shear viscosity to entropy ratio [36,37]. For instance, in the context of the STU black hole [38], these corrections were shown to potentially violate the KSS bound, highlighting the sensitivity of black hole thermodynamics to quantum effects.

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These findings underline the importance of analyzing similar corrections in higher-dimensional black holes to better understand their implications within the AdS/CFT framework.

Thermal fluctuations are expected to play a significant role in black hole thermodynamics, particularly for black holes of smaller mass or near extremality [39]. These fluctuations induce logarithmic corrections to the classical Bekenstein-Hawking entropy and may alter the black hole's thermodynamic stability and transport coefficients. In this work, we explore the impact of thermal fluctuations on the shear viscosity to entropy ratio in five-dimensional Kerr–Newman black holes. By incorporating quantum corrections to the entropy and considering various approaches to the viscosity correction, we aim to assess the validity of the KSS bound in the presence of thermal fluctuations.

The manuscript is organized as follows: Sect. 2 presents an overview of the five-dimensional Kerr–Newman black hole solution, its thermodynamic properties, and the formulation of the corrected entropy. The problem of quantum corrected thermodynamics of the five-dimensional Kerr–Newman black hole is studied in Sect. 3. Section 4 examines quantum corrections to the shear viscosity-entropy ratio through three distinct approaches: independent shear viscosity, thermally corrected viscosity, and invariant ratio assumption. Finally, in Sect. 6, we summarize our findings and discuss potential avenues for future research in the context of black hole transport properties and quantum gravity.

2 The five-dimensional Kerr–Newman black holes

The five-dimensional Kerr–Newman black hole represents a crucial extension of black hole physics beyond four dimensions, offering insights into both string theory and the AdS/CFT correspondence. Unlike its four-dimensional counterpart, this solution exhibits a richer structure due to the additional spatial dimension and the presence of two independent rotation parameters. The solution we study emerges from the five-dimensional Einstein–Maxwell theory with a Chern–Simons term, which naturally arises in various supergravity theories. The complete metric structure of this spacetime incorporates both electric charge and angular momentum, making it an ideal testing ground for examining how quantum corrections might modify classical black hole properties. Of particular interest is the behavior of the horizon geometry, which plays a crucial role in determining both thermodynamic and transport properties. We consider a charged and rotating black hole solution of five-dimensional gravity minimal coupled to a gauge gravity. The relevant action is given by the Yang–Mills–Chern–Simons Lagrangian [24],

$$S_5 = \frac{1}{4\pi^2}\int d^5x\left(\sqrt{-g}\left(R-\frac{3}{4}F^2\right)+\frac{1}{4}\epsilon^{abcde}A_aF_{bc}F_{de}\right). \tag{1}$$

The action S_5 represents a five-dimensional Einstein–Maxwell theory augmented with a Chern–Simons term. The first term ($\sqrt{-g}\left(R-\frac{3}{4}F^2\right)$) captures the standard Einstein–Maxwell dynamics, where R is the Ricci scalar and F^2 represents the tensor of the strength of the electromagnetic field. The second term $\epsilon^{abcde}A_aF_{bc}F_{de}$ is the Chern–Simons term, which distinguishes this theory from its four-dimensional counterpart. This term is crucial for maintaining gauge invariance in five dimensions and naturally emerges in various supergravity theories.

This action yields field equations that admit the Kerr–Newman black hole solution we study.

We are interested in the following black hole solution of the above action [24],

$$\begin{aligned} ds_5^2 = &-\frac{(a^2+r^2)(a^2+r^2-M)}{\Sigma^2}dt^2+\Sigma\Big(\frac{r^2dr^2}{f^2-Mr^2}+\frac{d\theta^2}{4}\Big)\\ &-\frac{MF}{\Sigma^2}(d\psi+\cos\theta d\phi)dt\\ &+\frac{\Sigma}{4}(d\psi^2+d\phi^2+2\cos\theta d\psi d\phi)\\ &+\frac{a^2MB}{4\Sigma^2}(d\psi+\cos\theta d\phi)^2, \end{aligned} \tag{2}$$

$$A = \frac{M\sinh 2\delta}{2\Sigma}\left(dt-\frac{1}{2}ae^{\delta}(d\psi+\cos\theta d\phi)\right) \tag{3}$$

where

$$\begin{aligned} B &= a^2+r^2-2Ms^3c^3-Ms^4(2s^2+3),\\ F &= a(r+2+a^2)(c^3+s^3)-aMs^3,\\ \Sigma &= r^2+a^2+Ms^2,\\ f &= r^2+a^2,\quad s\equiv\sinh\delta,\quad c\equiv\cosh\delta \end{aligned}$$

These black holes contains three physical parameters i.e. mass, electric charge and angular momentum respectively given by,

$$M_0 = \frac{3M}{2}\cosh 2\delta,\quad Q = Msc,\quad J = aM(c^3+s^3) \tag{4}$$

The inner (-) and outer (+) horizons of this black hole are given by,

$$r_\pm^2 = \frac{1}{2}\left[(M-2a^2)\pm\sqrt{M(M-4a^2)}\right]. \tag{5}$$

The Hawking temperature of the event horizon is given by,

$$T = \frac{1}{\pi\sqrt{M}}\frac{\sqrt{1-\frac{4a^2}{M}}}{c^3-s^3+(c^3+s^3)\sqrt{1-\frac{4a^2}{M}}}, \tag{6}$$

while the Bekenstein-Hawking entropy in Planck units is given by,

$$S_H = \pi\sqrt{2}M \sqrt{(c^6+s^6)M - 2(c^3+s^3)^2a^2 + (c^4+c^2s^2+s^4)\sqrt{M(M-4a^2)}}. \quad (7)$$

The physical parameters M_0, Q, and J characterizing this black hole solution have a rich interplay through the relations given in Eq. (4). The presence of both charge and angular momentum leads to an interesting horizon structure, as expressed in Eq. (5) for the inner and outer horizons. The corresponding thermodynamic quantities – temperature and entropy – given by Eqs. (6) and (7) respectively, form the foundation for studying quantum corrections in subsequent sections. These expressions reduce to the expected limits for Kerr and Reissner–Nordström black holes in five dimensions when appropriate parameters vanish. The existence of both charge and rotation parameters makes this solution particularly suitable for examining how thermal fluctuations might modify the classical picture of black hole thermodynamics and transport properties.

3 Corrected thermodynamics

The quantum nature of gravity suggests that black hole thermodynamics should incorporate thermal fluctuations around equilibrium. These fluctuations introduce corrections to the classical Bekenstein–Hawking entropy, with the leading correction typically taking a logarithmic form. For the five-dimensional Kerr–Newman black hole under consideration, the corrected entropy incorporates these quantum effects through,

$$S = S_H - \frac{1}{2}\ln S_c'', \quad (8)$$

where S_c'' represents the second derivative of the classical entropy with respect to the inverse temperature β, evaluated at the equilibrium temperature β_0,

$$S_c'' = \frac{\partial^2 S_H}{\partial \beta_k^2}|_{\beta_k=\beta_0}.$$

This correction term arises from a saddle-point approximation of the partition function and captures the leading quantum effects on black hole thermodynamics. This logarithmic correction can alternatively be expressed in terms of the horizon temperature,

$$S = S_H - \frac{\alpha}{2}\ln(S_H T^2), \quad (9)$$

where α is a parameter that controls the strength of the thermal fluctuations [35]. This formulation proves particularly useful for analyzing how quantum effects modify the thermodynamic stability and transport properties of the black hole.

We can see the behavior of the logarithmic corrected entropy graphically by Fig. 1 where we used Planck units ($\hbar = k_B = \ell_p = 1$). We can see that the effect of the logarithmic correction is increasing of the entropy.

Figure 1 reveals the fundamental relationship between thermal fluctuations and black hole entropy in five-dimensional Kerr-Newman spacetime. The plots compare the classical entropy ($\alpha = 0$) with quantum-corrected entropy ($\alpha = 1$) across different mass regimes and rotation states. In both rotating ($a = 0.5$) and non-rotating ($a = 0$) configurations, the entropy demonstrates a consistent monotonic increase with mass, reflecting the deep connection between a black hole's mass and its information content.

The quantum-corrected entropy consistently exceeds the classical values, with the divergence becoming particularly pronounced at smaller mass scales ($M < 0.5$). This enhancement indicates that thermal fluctuations introduce additional microscopic degrees of freedom that contribute to the black hole's total entropy.

The presence of rotation ($a = 0.5$) manifests itself in a steeper entropy growth curve compared to the non-rotating case, suggesting that angular momentum provides additional channels for storing information in the black hole system. This behavior aligns with our theoretical understanding that both quantum effects and rotation contribute to the rich thermodynamic structure of higher-dimensional black holes. The enhanced effect of quantum corrections at smaller masses also supports the broader theoretical framework in which quantum gravity effects become increasingly relevant at smaller scales.

Having established the form of the quantum-corrected entropy, we now examine its implications for the black hole's thermodynamic stability. A key thermodynamic quantity that reveals stability characteristics is the specific heat, which measures the system's response to temperature changes. We calculate this through the standard thermodynamic relation,

$$C = T\frac{dS}{dT}. \quad (10)$$

This expression, incorporating our quantum corrections to the entropy, allows us to analyze how thermal fluctuations modify the black hole's thermal stability. The derivative captures both the classical contribution from the Bekenstein-Hawking entropy and the quantum corrections from thermal fluctuations, providing insight into the interplay between these effects.

In Fig. 2 we can see the behavior of the specific heat in terms of the mass parameter. We find that logarithmic correction does not have many important effects in the specific heat when the parameters c and s are large. In that case, for $c \geq s$ we can see a phase transition while in the case of $c < s$, the black hole is in unstable phase without any phase transition and critical points (see Fig. 2a). On the other hand, for the

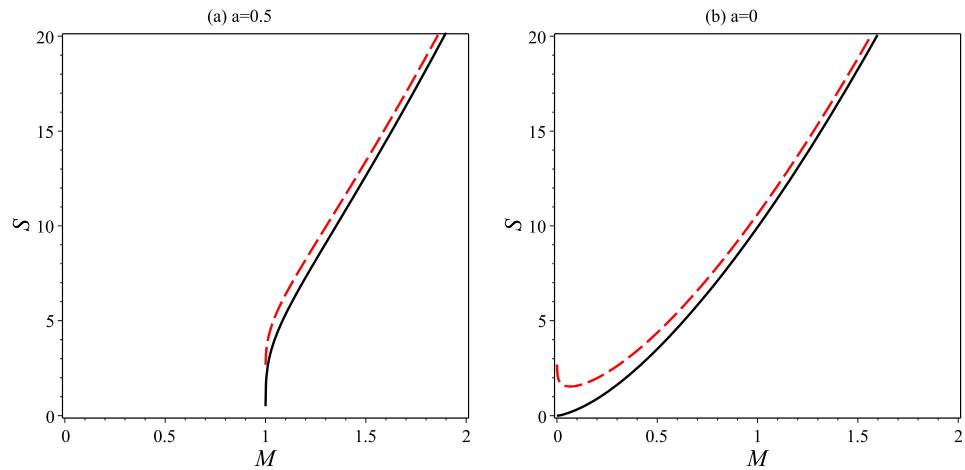

Fig. 1 Entropy S $[k_B]$ in terms of M $[\ell_p]$ with $c = s = 1$. (**a**) $a = 0.5$ and (**b**) $a = 0$; $\alpha = 0$ (black solid) and $\alpha = 1$ (red dashed)

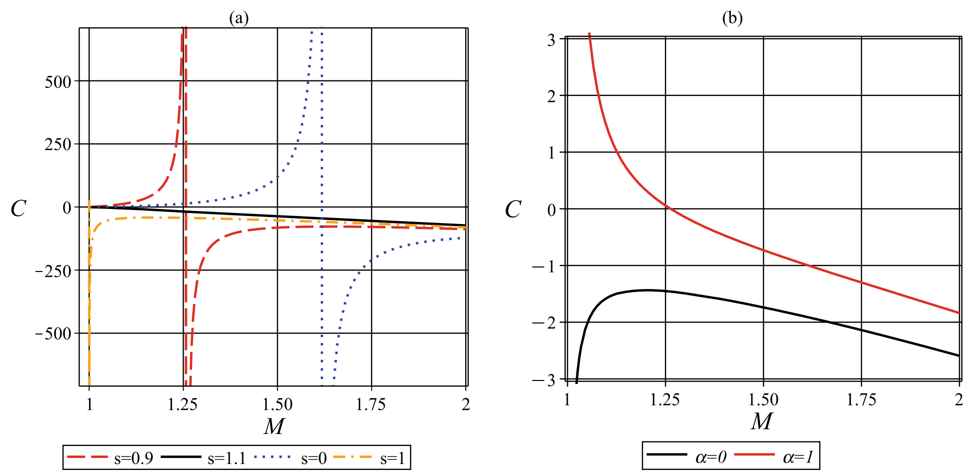

Fig. 2 Specific heat C $[k_B]$ in terms of M $[\ell_p]$: (**a**) for $c = \alpha = 1$, and $a = 0.5$, with variation of s; (**b**) for $c = s = 0.2$, and $a = 0.5$ with $\alpha = 0$ (black solid) and $\alpha = 1$ (red solid)

small values of c and s parameters we show that logarithmic correction is important to have a stable black hole. In Fig. 2b we can see that the black hole is completely unstable in the absence of logarithmic correction ($\alpha = 0$), however, the effect of the thermal fluctuations are the presence of some stable regions for $M \leq M_c$ ($M_c \approx 1.25$ in Fig. 2b). For the non-rotating case ($a = 0$) we have always an unstable black hole.

4 Shear viscosity to entropy ratio

The shear viscosity to entropy ratio ($\frac{\eta}{S}$) serves as a crucial probe of how quantum effects modify the transport properties of black holes. This ratio has garnered significant attention due to its connection to the AdS/CFT correspondence and the proposed universal bound,

$$\frac{\eta}{S} \geq \frac{1}{4\pi}, \tag{11}$$

known as the KSS bound. In the context of five-dimensional Kerr–Newman black holes, thermal fluctuations introduce corrections to both the entropy and potentially the shear viscosity, making it essential to examine how these quantum effects influence this fundamental ratio.

For strongly coupled quantum field theories, this ratio approaches the KSS bound, making it a valuable tool for understanding quantum corrections to classical black hole physics. Our analysis explores three distinct approaches to incorporating thermal fluctuations: treating shear viscosity as independent of quantum corrections, considering thermally corrected shear viscosity, and assuming the ratio itself remains unchanged. Each approach provides unique insights into how quantum effects modify transport properties while potentially preserving or violating the KSS bound.

4.1 Independent shear viscosity

We can assume that the shear viscosity is independent of thermal fluctuations. This means that logarithmic correction does not contribute to shear viscosity. In that case, we use uncorrected entropy given by the Eq. (7) and temperature given by the Eq. (6) together diffusion constant [40] to obtain the shear viscosity to entropy ratio as,

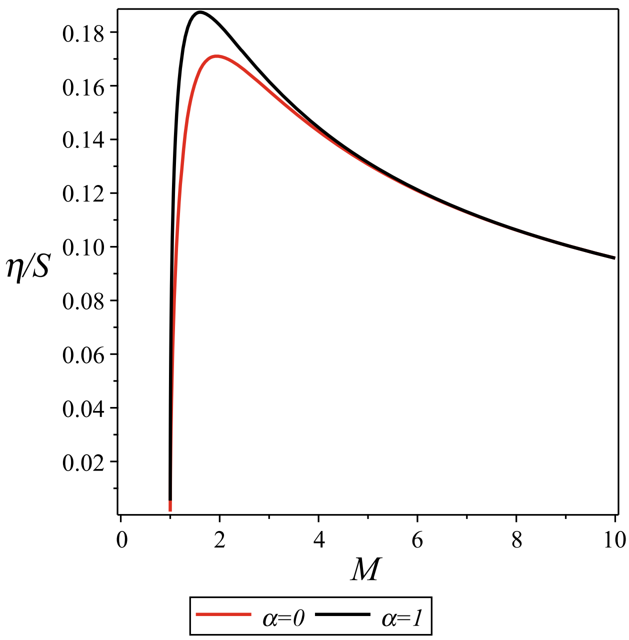

Fig. 3 Effect of the logarithmic correction on the shear viscosity to entropy ratio $\frac{\eta}{S}$ $[\frac{\hbar}{k_B}]$ versus M $[\ell_p]$ for $c = 0.8$, $s = 0.2$, $D = 1$ and $a = 0.5$; $\alpha = 0$ (black solid) and $\alpha = 1$ (red solid)

$$\frac{\eta}{S} = \frac{1}{\pi\sqrt{M}}\frac{S_H D}{S_H - \frac{\alpha}{2}\ln(S_H T^2)} \times \frac{\sqrt{1-\frac{4a^2}{M}}}{c^3 - s^3 + (c^3+s^3)\sqrt{1-\frac{4a^2}{M}}}, \tag{12}$$

where S_H is given by the the Eq. (7). We find that the effect of thermal fluctuation is decreasing of ratio, while the lower bound is kept for the massive black hole. A typical behavior can be seen in Fig. 3. It is illustrated that for large M the ratio yields $\frac{1}{4\pi}$ for both the cases of $\alpha = 0$ and $\alpha = 1$.

Figure 3 shows how thermal fluctuations affect the shear viscosity to entropy ratio ($\frac{\eta}{S}$) in five-dimensional Kerr–Newman black holes, with parameters set to $c = 0.8$, $s = 0.2$, $D = 1$, and the rotation parameter $a = 0.5$. The plot compares the classical case ($\alpha = 0$, solid black line) with the quantum corrected case ($\alpha = 1$, red dashed line). The ratio exhibits a nonmonotonic behavior with respect to the black hole mass M, initially increasing sharply to reach a peak around $M \approx 2$, followed by a gradual decrease for larger masses. In particular, thermal fluctuations reduce the maximum value of $\frac{\eta}{S}$ but preserve the overall qualitative behavior. For large masses ($M > 8$), the classical and quantum-corrected ratios converge toward the universal lower bound of $1/4\pi$, suggesting the robustness of this bound even in the presence of quantum effects. The suppression of $\frac{\eta}{S}$ by thermal fluctuations aligns with the expectation that quantum effects generally enhance dissipative processes in dual field theory, while maintaining the fundamental constraints imposed by unitarity.

4.2 Corrected shear viscosity due to thermal fluctuations

Thermal fluctuations can modify both the entropy and transport coefficients of black holes. While the corrections to entropy are well established through logarithmic terms, the quantum modifications to shear viscosity require careful consideration. We propose that the shear viscosity, like other thermodynamic quantities, receives corrections from thermal fluctuations. The relationship between shear viscosity η and the diffusion constant D can be modified in the presence of quantum effects according to,

$$\frac{\eta}{D} = S_H T + \mathcal{O}(\alpha). \tag{13}$$

It is easy to find that the shear viscosity-entropy ratio may be an increasing or a decreasing function of α. Also, this ratio may yield a constant for a suitable value of $\mathcal{O}(\alpha)$. Therefore, we can conclude that the lower bound may hold. However, for the appropriate value of correction terms, the lower bound may violated and the shear viscosity to entropy ratio vanishes ($S_H T \approx \mathcal{O}(\alpha)$). Among the various methods for calculating correction terms, the Kubo formula [41], which relates the shear viscosity to the correlation function of the stress–energy tensor with zero spatial momentum, is famous.

There is also another way to obtain the corrected shear viscosity due to thermal fluctuations, which is explained in the next subsection.

4.3 Independent ratio

An alternative approach to understanding how thermal fluctuations affect the shear viscosity-to-entropy ratio is to postulate that the ratio itself remains invariant under quantum corrections. This assumption stems from the fundamental nature of the KSS bound and its possible deeper connection to quantum gravity. Under this hypothesis, we maintain,

$$\frac{\eta}{S} = \frac{1}{4\pi}. \tag{14}$$

This approach allows us to derive the quantum corrections to shear viscosity by requiring the ratio to remain constant even as the entropy receives logarithmic corrections. The resulting expression for the shear viscosity must then compensate for the changes in entropy to preserve the universal bound. This provides a unique perspective on how thermal fluctuations might modify transport properties while maintaining fundamental quantum gravity constraints.

By combining Eq. (7) with (9) and utilizing the invariance of the $\frac{\eta}{S}$ ratio under quantum corrections, we derive a comprehensive expression for the shear viscosity that incorporates both classical and thermal fluctuation effects,

$$\eta = \frac{\sqrt{2}}{4} M \times \sqrt{(c^6+s^6)M - 2(c^3+s^3)^2 a^2 + (c^4+c^2s^2+s^4)\sqrt{M(M-4a^2)}}$$
$$- \frac{\alpha}{8\pi} \ln \times \frac{\sqrt{2}\sqrt{\left(c^6+s^6\right) M - 2\left(c^3+s^3\right)^2 a^2 + \left(c^4+c^2s^2+s^4\right)\sqrt{M\left(M-4a^2\right)}}\left(M-4a^2\right)}{\pi M \left(c^3 - s^3 + \left(c^3+s^3\right)\sqrt{1-4\frac{a^2}{M}}\right)^2}. \quad (15)$$

The analysis under the independent ratio assumption leads to an expression for shear viscosity that maintains the universal bound while incorporating quantum corrections. As demonstrated by Eq. (15), this approach yields a complex relationship between shear viscosity, black hole mass, rotation parameter, and thermal fluctuation strength. Notably, this formulation ensures that the KSS bound remains valid for all values of the correction parameter α, providing a consistent framework for understanding quantum effects on transport properties. The persistence of the lower bound across all parameter ranges suggests its fundamental nature in quantum gravity, even when thermal fluctuations significantly modify both the entropy and shear viscosity individually. This robustness adds weight to conjectures about the universal character of the η/S ratio in strongly coupled quantum systems, including those described by higher-dimensional black hole geometries.

5 Holographic interpretation of results

In the dual QFT, the logarithmic corrections to the entropy, and consequently to the viscosity, likely correspond to higher-order quantum effects in the thermal plasma. These corrections are expected to manifest themselves as deviations in the scaling behavior of the viscosity and other transport coefficients. Moreover, the intricate dependence of η (15) on the black hole's rotation and charge parameters suggests that the dual plasma's properties are influenced by additional conserved charges or angular momentum.

The correction terms in Eq. (15) also provide a framework to understand the impact of thermal fluctuations on the stability and hydrodynamic behavior of the dual system. The logarithmic dependence on entropy and temperature implies that quantum corrections introduce new scales into the system, which may influence its critical behavior near phase transitions or in non-equilibrium states.

Equation (15) is in fact logarithmic corrected shear viscosity due to thermal fluctuation. The holographic correspondence provides a powerful framework for understanding universal transport properties in strongly coupled quantum systems. Through this duality, the transport coefficients of the boundary field theory are intimately connected to the dynamics of bulk gravitational fields near the black hole horizon. The shear viscosity to entropy ratio emerges as a particularly significant quantity, as its universal value of $1/4\pi$ in holographic theories suggests a fundamental bound on quantum dissipation. In this context, thermal fluctuations in five-dimensional Kerr–Newman black holes not only modify the transport coefficients but also reveal deeper connections between gravitational dynamics and quantum transport phenomena. The relationship between different transport coefficients, electrical conductivity, bulk viscosity, and thermal conductivity, can be understood through the holographic dictionary, where the horizon properties encode the boundary transport behavior. This connection becomes especially relevant when considering quantum corrections, as thermal fluctuations in the bulk geometry manifest themselves as modifications to transport coefficients in the dual-field theory, while preserving certain universal relations that appear to transcend specific models.

In quantum critical systems, several transport coefficients exhibit universal relations with the shear viscosity (η).

5.1 Electrical conductivity

The electrical conductivity (σ) near quantum critical points follows a universal relation with shear viscosity [42],

$$\sigma = B\frac{e^2\eta}{T^2}, \quad (16)$$

where e denotes the electron charge, and B is a constant related to a suitable measure of the number of degrees of freedom in the system. This relation emerges from the quantum critical nature of the system and has been verified in quark-gluon plasma experiments at RHIC, and cold atom systems near unitarity.

By using the shear viscosity (15) in the equation (16) one can obtain the electrical conductivity in terms of M.

In Fig. 4, the electrical conductivity of the five-dimensional Kerr–Newman black hole in unit of electrical charge exhibits a rich and complex behavior as a function of mass M, with parameters set to $c = 0.8$, $s = 0.2$, $B = 0.005$ and rotation parameter $a = 0.5$. The plot compares two cases: the classical behavior without thermal fluctuations ($\alpha = 0$, shown by the black solid line) and the quantum-corrected case includ-

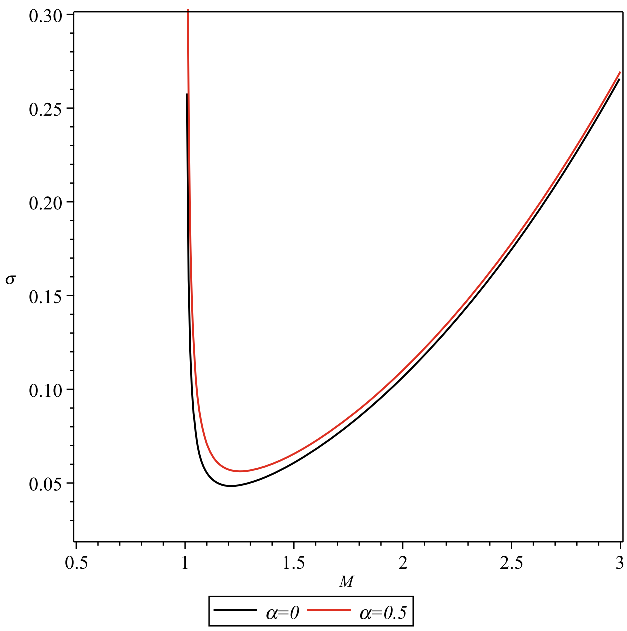

Fig. 4 The electrical conductivity σ $[\frac{e^2\hbar}{]}$ in terms of M $[\ell_p]$ for $c = 0.8$, $s = 0.2$, $B = 0.005$, $e = 1$ and $a = 0.5$; $\alpha = 0$ (black solid) and $\alpha = 1$ (red solid)

ing thermal fluctuations ($\alpha = 1$, depicted by the red dashed line). The conductivity shows distinctive features across different mass regimes. For small masses ($M < 1$), both curves show a sharp decrease, leading to a pronounced minimum around $M \approx 1$. This minimum suggests a critical point where the transport properties of the black hole undergo a significant transition. Beyond this point, the conductivity increases monotonically with mass, with both classical and quantum-corrected cases showing similar qualitative behavior. Thermal fluctuations introduce subtle but noticeable modifications to conductivity, particularly evident in the intermediate mass range, where the quantum-corrected curve ($\alpha = 1$) shows slightly higher values compared to the classical case. This enhancement aligns with the theoretical expectation that quantum effects generally modify transport coefficients while preserving their fundamental scaling relations. For larger masses ($M > 2$), both curves converge, indicating that thermal fluctuation effects become less prominent in the high-mass regime, consistent with the general principle that quantum corrections are most significant for smaller black holes. This behavior provides important insights into how quantum effects modify the transport properties of black holes while maintaining the universal relationships predicted by holographic correspondence. The general behavior of the electrical conductivity obtained in this model is in agreement with previous results obtained from the other holographic models [43].

Our results at the intermediate mass shows,

$$\frac{\sigma}{T} \approx 0.4, \tag{17}$$

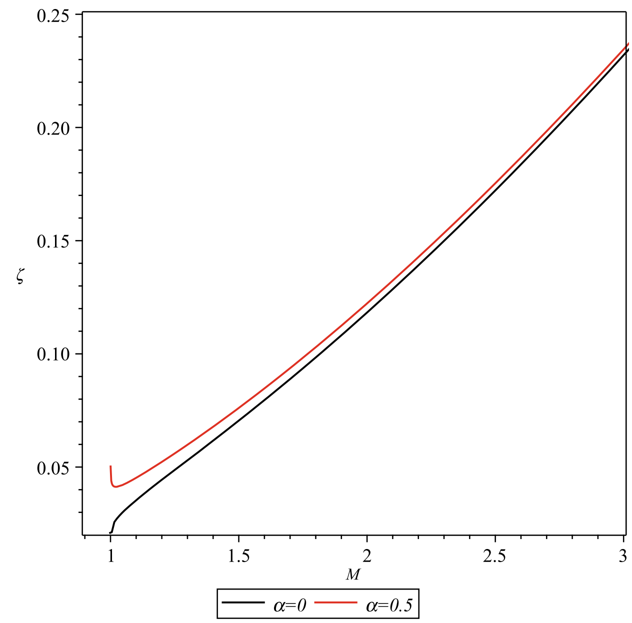

Fig. 5 The bulk viscosity ζ $[\hbar]$ in terms of M $[\ell_p]$ for $c = 0.8$, $s = 0.2$, $c_s = 0.5$, and $a = 0.5$; $\alpha = 0$ (black solid) and $\alpha = 1$ (red solid)

which is in agreement with the claim of the Ref. [44].

5.2 Bulk viscosity

The bulk viscosity (ζ) satisfies Buchel's bound in terms of shear viscosity,

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right), \tag{18}$$

where c_s is the speed of sound in the medium. Assuming the lower bound we can obtain the following relation for the bulk viscosity in terms of the shear viscosity,

$$\zeta = 2\left(\frac{1}{3} - c_s^2\right)\eta. \tag{19}$$

By using the shear viscosity (15) in the equation (19) one can obtain the bulk viscosity in terms of M.

In Fig. 5, we analyze the bulk viscosity behavior of the five-dimensional Kerr–Newman black hole, plotted against mass M with parameters $c = 0.8$, $s = 0.2$, $c_s = 0.5$, and rotation parameter $a = 0.5$. The plot compares the classical ($\alpha = 0$, black solid line) and quantum corrected ($\alpha = 1$, red dashed line) cases. The bulk viscosity exhibits interesting behavior across different mass regimes. For small masses ($M \approx 1$), both curves show a characteristic minimum, suggesting a critical point where the transport properties undergo a significant transition. Beyond this point, the bulk viscosity increases monotonically with mass, with both classical and quantum-corrected cases showing similar qualitative behavior. The thermal fluctuations introduce modifications to the bulk viscosity, particularly evident in the intermediate mass range, where the quantum-corrected curve shows slightly higher values compared to the classical case. This enhancement indicates that quantum effects tend to increase the number of dissipative processes in the bulk. For larger masses, both curves demonstrate similar growth patterns, suggesting that thermal fluctuation effects become less prominent in the high-mass regime. This behavior is consistent with the theoretical expectation that quantum corrections are most significant for smaller black holes. The choice of sound speed $c_s = 0.5$ ensures that the bulk viscosity remains positive and satisfies causality constraints, leading to physically meaningful results that align with holographic predictions and the general framework of quantum field theory in curved spacetime.

5.3 Thermal conductivity

The thermal conductivity (κ) follows a Wiedemann-Franz law,

$$\frac{\kappa}{\sigma} = LT, \tag{20}$$

where L is a Lorenz number that is given by,

$$L = \frac{\pi^2 k_B^2}{3e^2}, \tag{21}$$

where k_B is Boltzmann constant. So, we can calculate the thermal conductivity using the electrical conductivity given by equation (16).

Figure 6 reveals a fascinating aspect of thermal conductivity (κ) in five-dimensional Kerr–Newman black holes, displaying behavior that is markedly different from other transport coefficients. The plot shows κ versus mass M with parameters $c = 0.8, s = 0.2, e = 1, B = 0.005$, and $a = 0.5$, comparing classical ($\alpha = 0$) and quantum corrected ($\alpha = 1$) cases.

The most striking feature is the scale of the thermal conductivity values, shown in units of 10^{-9}, which is several orders of magnitude smaller than both the electrical conductivity and the bulk viscosity. This dramatic suppression of thermal transport relative to other transport phenomena suggests a hierarchical structure in the dissipative processes of holographic systems.

Thermal conductivity exhibits a sharp peak near $M \approx 1$, followed by a rapid decrease to a minimum, before gradually increasing again for larger masses. This nonmonotonic behavior differs significantly from the more straightforward patterns seen in electrical conductivity and bulk viscosity. The quantum corrections (red dashed line) modify this behavior most prominently near the peak region, where thermal fluctuations appear to smooth out the classical singularity-like feature.

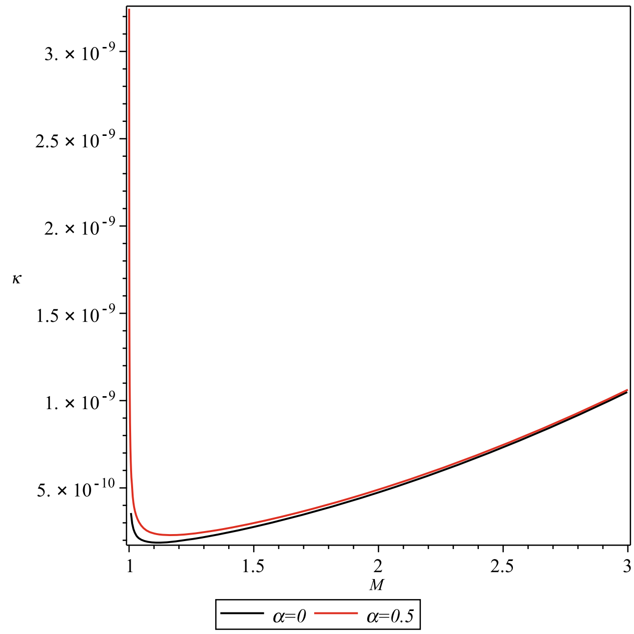

Fig. 6 The thermal conductivity κ $[\frac{k_B^2}{\hbar}]$ in terms of M $[\ell_p]$ for $c = 0.8$, $s = 0.2$, $e = 1$, $B = 0.005$ and $a = 0.5$; $\alpha = 0$ (black solid) and $\alpha = 1$ (red dashed)

Perhaps most intriguingly, the quantum-corrected and classical curves nearly coincide for large masses, but with a subtle separation that persists even in this regime. This behavior aligns with the Wiedemann–Franz law [45] and suggests that thermal fluctuations maintain a small but finite effect on thermal transport even in the classical limit, a feature not as apparent in other transport coefficients.

6 Conclusion

In this paper, the effects of thermal fluctuations on the properties of five-dimensional Kerr–Newman black holes were analyzed, with a particular focus on the shear viscosity to entropy ratio. The study incorporated quantum corrections to the classical Bekenstein–Hawking entropy through logarithmic terms, which captured the leading effects of thermal fluctuations. These corrections were shown to significantly modify the thermodynamic and transport properties of the black hole, especially for smaller mass black holes or near-extremal configurations.

The corrected entropy was found to increase due to the presence of thermal fluctuations, enhancing the black hole's information content and microscopic degrees of freedom. The specific heat analysis revealed that the thermal corrections altered the stability characteristics of the black hole. While massive black holes exhibited stable behavior, smaller black holes displayed unstable phases with critical points emerging in certain parameter regimes.

The shear viscosity to entropy ratio was examined under three different scenarios: assuming the shear viscosity to be independent of quantum corrections, incorporating corrections directly into the viscosity, and maintaining the ratio itself as invariant. Across all approaches, the study confirmed the robustness of the KSS bound, $\eta/S \geq 1/4\pi$, even in the presence of quantum effects. The findings demonstrated that while thermal fluctuations tended to decrease the ratio, they did not violate the universal bound, which underscores its fundamental significance in quantum gravity and holographic duality. This study's findings enhance our comprehension of the impact of quantum corrections on the transport properties and thermodynamic behavior of black holes. The persistence of the KSS bound across all examined scenarios highlights its potential universality, even under significant thermal fluctuations. We believe that these findings support the broader theoretical framework that relates black hole physics to strongly coupled quantum systems through the AdS/CFT correspondence.

In the near future, we plan to analyze extensions to higher-dimensional rotating black holes with more complex charge configurations or cosmological backgrounds. We aim to investigate non-logarithmic quantum corrections [46,47] and their impact on transport coefficients, such as shear viscosity [48], bulk viscosity and electrical conductivity. Employing holographic techniques to derive more rigorous formulations of quantum corrections to transport properties will also be a focus of our research. Furthermore, we intend to examine the implications of these results within the framework of the fluid-gravity correspondence, with the goal of illuminating the connections between black hole dynamics and relativistic hydrodynamics. These planned investigations are expected to provide deeper insights into the interplay between quantum gravity and black hole transport phenomena.

The corrected expression for shear viscosity serves as a starting point for exploring more complex scenarios within the AdS/CFT framework. Future work could extend this analysis to higher-dimensional rotating black holes with more intricate charge configurations, such as those in gauged supergravity theories.

Furthermore, holographic techniques could be employed to derive more precise quantum corrections to transport properties, using tools such as the Kubo formula [49,50]. These efforts would deepen the understanding of the interplay between quantum corrections in black hole thermodynamics and their dual field-theoretic descriptions, offering new insights into the nature of strongly coupled systems and quantum gravity.

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