

# TRANSVERSE PHASE SPACE TOMOGRAPHY USING MACHINE LEARNING AT THE CLARA ACCELERATOR TEST FACILITY

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## Abstract

Phase space tomography is a powerful technique for characterising beams in particle accelerators, and has found widespread use at many facilities. However, conventional tomography techniques require significant computational resources, particularly when reconstructing the charge distribution for two or more degrees of freedom. Here, we describe a novel technique that employs machine learning and image compression for transverse phase space tomography in two degrees of freedom. The use of machine learning allows the beam distribution in 4D phase space to be reconstructed more quickly than by conventional tomography techniques, while the application of image compression can dramatically reduce the size of the data sets involved in the analysis. The new method has been deployed on the CLARA accelerator at Daresbury Laboratory to characterise electron bunches with moderate energy (35 MeV) and charges up to 100 pC. We compare the machine learning technique against a conventional tomography algorithm (algebraic reconstruction) applied to the same data set, and show that the results are at least as good in terms of predicting the observed beam profiles for a range of quadrupole strengths.

## INTRODUCTION

Phase space tomography [1, 2] is a powerful technique for characterising a beam's charge distribution in phase space in one or more degrees of freedom. Tomography in two transverse degrees of freedom provides a detailed understanding of the beam substructure, and also allows for characterization of the betatron coupling. However, applying the technique for multiple degrees of freedom generally requires significant computational resources. Storage of a 4D phase space distribution with  $N$  data points along each axis requires a data structure with  $N^4$  values, and the memory resources required to manipulate the input data can be much larger.

High-dimensional tomography methods may be of particular use for characterizing and operating advanced accelerators, such as high-brightness Free Electron Laser (FEL) drivers and injectors for machines using novel acceleration methods. Recent simulation work [3, 4] has demonstrated a technique for 5D tomography, revealing the transverse phase space as a function of longitudinal position. Techniques leading to a reduction in the computational resources required for high-dimensional tomography are therefore of widespread interest.

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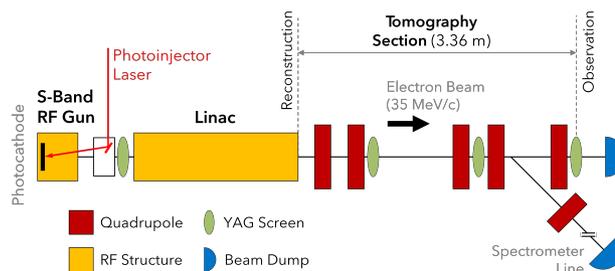


Figure 1: Schematic layout of the CLARA front end at the time of this study. Distances between elements are approximately to scale. For clarity not all elements are shown.

In principle, images can be stored in a compressed form (for example, as discrete cosine transforms) to reduce the size of the data structures involved in tomography, while retaining sufficient information to reconstruct the phase space to a good resolution. However, conventional tomography algorithms are formulated on the basis that the input data are direct projections of the initial phase space (e.g. beam images obtained for a range of betatron phase advances). Therefore, it is not obvious how compressed data can be used in the context of an established tomography algorithm.

Machine learning (ML) techniques such as artificial neural networks (ANNs) offer an alternative to conventional algorithms, and can be used for tomographic analysis of data stored in a compressed form. Similar methods are already used extensively for image analysis and tomography, particularly in medical contexts [5].

These proceedings describe experimental studies aimed at demonstrating transverse phase space tomography using machine learning. The implementation of the technique is explicitly designed to work with beam images stored in a compressed format. The experiments described here were performed on the CLARA front end [6, 7] at STFC Daresbury Laboratory in 2022. Earlier tomography studies on CLARA are reported in [8].

## EXPERIMENTAL METHOD

### Data Collection

Figure 1 shows a schematic of CLARA, as operated at the time of these studies. The accelerator consists of an S-band RF photoinjector and short linac, providing electron bunches (up to 250 pC) at a maximum momentum of 35 MeV/c and 10 Hz repetition rate, with transverse emittance below  $1 \mu\text{m}$ . As shown in Fig. 1, the accelerating structures are followed by a transport and diagnostics section

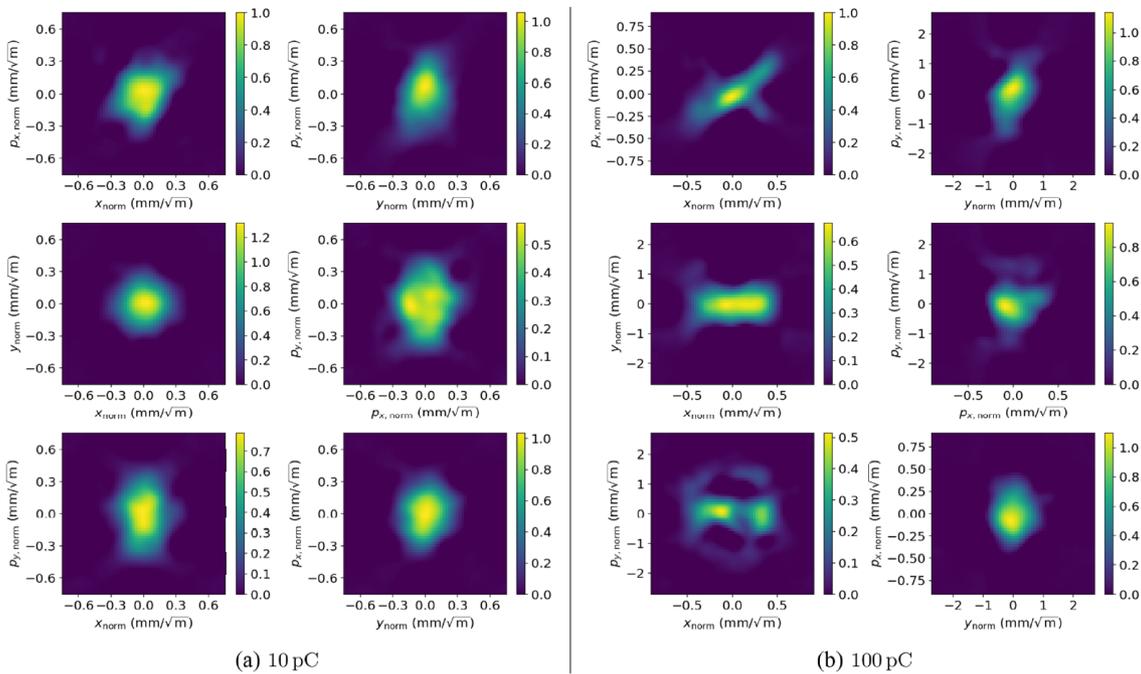


Figure 2: Projections of the 4D charge distribution at the reconstruction point, shown in normalised phase space for bunch charges of (a) 10 pC and (b) 100 pC.

that includes five quadrupoles. The tomography technique in [8] was applied to CLARA, operated at 35 MeV/c.

Measurements were taken with the aim of reconstructing the phase space at the linac exit (the reconstruction point; see Fig. 1). Beam images were collected using a YAG screen at the end of the diagnostics section (the observation point) for a range of optics configurations. Before beam time, a design model of the machine [9] was used to find 32 sets of quadrupole settings, producing a range of phase advances between the reconstruction and observation points. Settings were chosen to keep the Twiss beta functions approximately constant, avoiding beams with very small sizes or large aspect ratios. Measurements were ordered to minimise the need for magnet degaussing between steps.

Offline analysis of the experimental data indicated a discrepancy between the design model of the accelerator and the operational settings during beam time. Throughout this work, a “calibrated” model of the beam optics is used, initialised with the measured Twiss functions at the reconstruction point. This improves consistency with the experimental data, but reduces the range of betatron phase advances.

### Image Processing

In this study, tomography analyses are exclusively performed in normalised phase space; this improves the accuracy of reconstructions [10] and simplifies the analysis. In the horizontal plane, the normalised coordinates are defined as  $x_{\text{norm}} = x/\sqrt{\beta_x}$  and  $p_{x,\text{norm}} = p_x\sqrt{\beta_x} + \alpha_x/\sqrt{\beta_x}$ , where  $(x, p_x)$  are the standard phase space coordinates, and  $\alpha_x$  and  $\beta_x$  are the local values of the Twiss optics functions. When transformed into normalised coordinates, the linear

transport matrices are simply rotation matrices about angles corresponding to the betatron phase advances.

Before analysis, a background frame (taken with the photoinjector laser turned off) was subtracted from each beam image to remove contributions from dark current. Images were then cropped to maximize the area occupied by the beam, scaled from camera pixels to physical units, and transformed into normalised coordinates  $(x_{\text{norm}}, y_{\text{norm}})$ . For each step of the scan, the Twiss optics functions at the observation point were determined using the calibrated optics model.

In this implementation of the tomography technique, beam images are represented by their 2D discrete cosine transforms (DCTs) [11]. Compression is achieved by truncating the (type II) DCT expansion to a finite number of modes; here, a fixed DCT resolution of 21 x 21 values is used. This generally results in some loss of image fidelity, but retains the main structure of the beam profile and some details. In principle, better reconstructions can be obtained by increasing the number of DCT modes. In this study, the limiting factors are expected to be the number of quadrupole scan steps, and the limited range of phase advances.

The 4D phase spaces obtained using this method are also stored in a compressed form, encoded with an equivalent DCT algorithm for higher-dimensional arrays. An image (or phase space distribution) can be retrieved from its respective DCT by applying the appropriate inverse DCT operation.

## MACHINE LEARNING TECHNIQUE

The ANN used in this study was implemented in Keras [12]. A resolution of 21 points per side was used for the DCTs of the 32 beam images (the input layer), and 19 points per side for the DCT of the 4D phase space (the output layer).

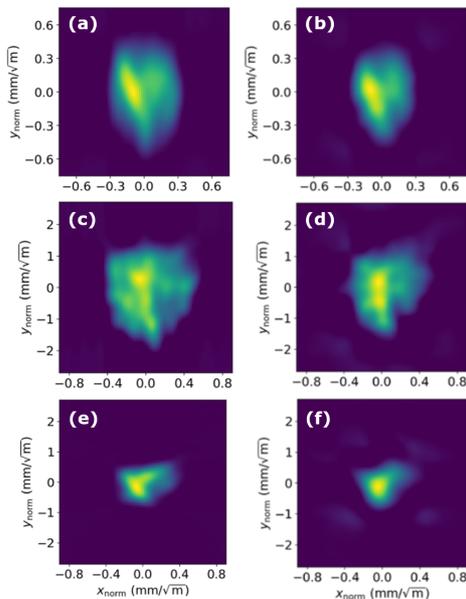


Figure 3: Validation images showing the observed (left) and reconstructed (right) beam profiles at the observation point, for different quadrupole scan steps. (a) and (b) show profiles for a 10 pC bunch charge, while (c) – (f) correspond to quadrupole scan steps with 100 pC bunch charge.

Between the input and output layers, there are two hidden layers, each followed by a dropout layer to limit overtraining.

Training data for the ANN was generated by superimposing 4D Gaussian distributions with random shapes, positions, and intensities to produce artificial normalised phase spaces. Corresponding beam images were obtained by transforming these distributions under phase space rotations (matching the steps of the quadrupole scan), and projecting them onto the  $(x_{\text{norm}}, y_{\text{norm}})$  plane. 3000 artificial phase spaces were generated as training data, of which 100 were reserved for model validation. Training the model takes several minutes on a standard laptop PC using the Adam optimiser [13].

After training, the ANN was able to reconstruct 4D phase space distributions when provided with the DCTs for a series of beam images. Validation of the model using simulated data, including non-Gaussian phase spaces, demonstrated its ability to reconstruct complex 4D distributions despite the limited range of training data (see [14] for further details).

## RESULTS AND DISCUSSION

The trained ANN was applied to quadrupole scan data taken on CLARA. Full details of the technique and results are published in [14]. Figure 2 shows projections of the 4D phase space, reconstructed at the exit of the linac using the ML tomography method. The technique produces high-fidelity images of the phase space projections, showing the rich beam substructure in the 100 pC case. For comparison, an equivalent reconstruction was obtained using Algebraic Reconstruction Tomography (ART). The two methods are broadly in good agreement, however, Fig. 2 contains fewer artefacts than the results from ART (see [14] for full details).

The tomography results can be validated by attempting to reconstruct the measured beam profiles at the observation point from the 4D phase space distribution. Figure 3 shows a comparison between several measured and reconstructed beam profiles, for bunch charges of 10 pC and 100 pC. Qualitatively, there is good agreement between pairs of images, with only some loss of fine detail in the reconstructed profiles. Similar results were obtained from the ART tomography.

For quantitative comparison, Fig. 4 shows the observed and reconstructed beam sizes across all 32 steps of the quadrupole scan. Two linear optics models, initialised with different covariance matrices, are also shown for comparison. The first (the “calibrated model”) is initialised using the measured Twiss functions at the reconstruction point. The second (“linear optics”) is initialised by fitting a covariance matrix to the 4D distributions obtained from tomography.

## CONCLUSIONS

A method for transverse phase space tomography using compressed beam images and machine learning has been demonstrated with experimental data. This method will be used to characterise the CLARA high repetition rate (100 Hz) gun [15], and for commissioning of the full accelerator at momentum up to 250 MeV/c.

While our tomography technique has certain advantages over conventional algorithms (such as ART), it is not necessarily the optimal implementation. In future, we plan to extend our method to more complex ANN architectures (or other sophisticated ML tools), which may yield better reconstruction of the 4D phase space. Using basis functions other than DCTs for image compression, or generating a greater variety of training data, may also be beneficial.

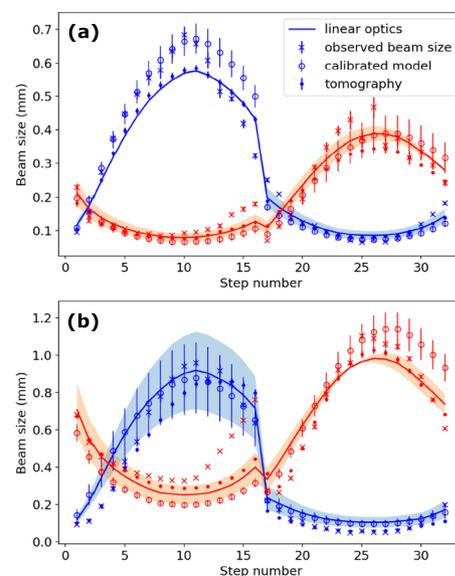


Figure 4: Horizontal (blue) and vertical (red) beam sizes at the observation point (a) 10 pC and (b) 100 pC bunch charges. The observed beam sizes are compared against the reconstructions from ML tomography, and linear optics models initialised with different beam covariance matrices.

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