# NUCLEON AND ANTINUCLEON PROCESSES

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The present report covers invited and contributed papers on several matters which are not obviously connected to each other, and have been given at two different but simultaneous sessions of the Conference. It is therefore relatively difficult to make an integrated presentation of the available material, although I here try to show the relation between the different approaches to the same problem. I am by no means certain to have been even partially successful in achieving this result.

Since it is always a good idea to begin with relatively simple considerations before entering into the more technical details, I shall start with the discussion of antinucleonic processes, for which there is at present relatively little to say from the theoretical point of view. I shall then present the more abundant contributions to the study of the nucleon-nucleon interaction.

## I. THE NUCLEON ANTINUCLEON INTERACTION

I shall limit my discussion to three points: The magnitude of total cross sections at high energy; the radius of the region of inelasticity; and the correlation between pions produced in the annihilation process.

### 1. Behavior of the total $p$-$\bar{p}$ cross section at high energy

You have seen the results on the behavior of ($p$-$p$ and ($p$-$\bar{p}$)) cross sections at high energies. These have been shown by Chamberlain. Beyond 2 BeV, the $p$-$p$ cross section remains practically constant, whereas the $p$-$\bar{p}$ cross section still seems to be decreasing rather rapidly. Its ratio to the $p$-$p$ cross section is approximately 2.0 at 1 BeV/c, 1.52 at 1.6 BeV/c and 1.34 at 10.7 BeV/c. The theoretical point which is relevant here is based on the theorem due to Pomeranchuk [1], which states that the cross sections for particle or anti-particle scattering on a given target should tend to each other asymptotically at high energy provided two conditions are satisfied: *(a)* each cross section should tend to a constant; *(b)* the target has a finite " effective radius " $R$ so that the scattering takes place only in states with orbital angular momentum $l<L\sim R/\lambda\!\!\!^{-}$. This theorem seems to be well satisfied by the $\pi^{\pm}$-$p$ cross sections beyond 2 BeV. It is easily satisfied already by the $\bar{K}$-$p$ and $K$-$p$ scattering cross sections at 8 BeV. It is therefore a little puzzling that there still should be such a difference at 10 BeV in the present case. The characteristic energy beyond which the Pomeranchuk theorem should be valid must be such that many channels have already been opened for inelastic processes so that it no longer matters very much that more of them come into play. In this respect it is strange that the limit for the $p$-$\bar{p}$ cross section is not already reached at 2 BeV (although we may have answers in the future on this point). One might argue that the energy available in the center of mass at 10 BeV is still not very large compared to the rest mass of the nucleons, but it seems more reasonable to compare the energy to the mass of the pion which determines the opening of new channels for the inelastic cross section. Probably the explanation lies in the fact that the pure annihilation process, which is a characteristic feature of the $p$-$\bar{p}$ interaction, is still quite important at high energy (this is connected with the problem of the radius of the region of inelasticity which we shall discuss next) and there the Pomeranchuk limit will be reached only when this process seems negligible compared to the multiple meson production. In the light of these ideas one might venture into making a breakdown of the 55 mb cross section at 10 BeV/c as follows: about 15 mb may come from annihilation (this is the dif-

ference between the $p$-$p$ and $p$-$\bar{p}$ cross section). Of the remaining 40 mb about 10 or 12 mb might come from the elastic cross section which at very high energy should be comparable to that of the $p$-$p$ process because it is mostly diffraction scattering; the rest would then be due to multiple meson production so that one would conclude that the ratio of the latter to straight annihilation might be roughly 2 to 1.

## 2. Radius of the region of inelasticity

Several theoretical investigations have been carried out on the $p$-$\bar{p}$ interaction, either phenomenologically or on the basis of meson theory. One of the most puzzling questions is the determination of the range of the annihilation region. Following the arguments of meson theory, the range is expected to be of the order of twice the nucleon Compton wave length (or about $0.4\times10^{-13}$ cm, which is also the radius of the repulsive core in the $N$-$N$ interaction) rather than of the pion Compton wave length. Since one cannot expect that the validity of a potential model or of static meson theory can be extended to the multi-BeV energy region, I would like to base my discussion of this problem on qualitative semi-classical concepts. In particular, a " maximum theorem " has been demonstrated years ago which relates the total and elastic cross section to the maximum number of partial waves $(L+1)$ which contribute to the scattering process. This theorem can be expressed as follows :

$$\frac{[\sigma_{\rm tot}]^2}{\sigma_{\rm el}}\leqslant 4\pi\lambda\!\!\!^{-2}(L+1)^2. \tag{1}$$

The limit is obtained by making all the phase shifts purely imaginary,

$$\delta_l = i\beta_l \qquad (l\leqslant L) \tag{2}$$

and varying the real parameters $\beta_l$ (scattering to be constant) so that the ratio of cross sections is maximized. In other words, the equality in the maximum theorem is obtained when the interaction takes place in a uniform purely absorbing region, the elastic scattering being solely shadow scattering. $(L+1)$ is therefore the number of partial waves which contribute to the inelastic process.

Now, if we consider all the experimental data between 200 MeV and 2 BeV, we find that within 20%, which is certainly of the order of the combined experimental error, the ratio $[\sigma_{\rm tot}]^2/\sigma_{\rm el}$ remains practically constant and equal to 300 mb. On the other hand, $(L+1)\lambda\!\!\!^{-}$ is roughly the radius $R$ of the interaction region. The inequality can then be understood as meaning that the size of the interaction region remains approximately constant in the above energy range, and that

$$300\,\text{mb}\leqslant 4\pi R^2$$

or

$$R > 1.5\times10^{-13}\,\text{cm}\,. \tag{3}$$

One can object to the above argument if there is an effective real potential outside the annihilation region so that the particles of high angular momenta which strike the target well outside this region will be curved in and thus annihilated, so that the number given for $R$ is really equal to the range of the real scattering potential. This argument, however, does not hold if $l$ is sufficiently large because the centrifugal barrier will prevent any such effect. The calculation of Ball and Chew, for example, shows that beyond $l=2$, the centrifugal barrier dominates the meson theoretical potential. Actually, if there is an effective potential $V(r)$, classical arguments show that the maximum radius of interaction will be given by the equation

$$R=(L+1)\lambda\!\!\!^{-}[1+\frac{|V(R)|}{E}]^{-1}. \tag{4}$$

However, the maxiumm theorem will be modified in this case because of the appearance of real scattering phases, but this can be shown to be merely a second order effect. If we assume that $V(r)$ is approximately constant for $r\geq R$, and if we replace the inequality of Eq. (1) by an approximate equality, we obtain by combining Eq. (1) and (4)

$$\frac{[\sigma_{\rm tot}]^2}{\sigma_{\rm el}}\simeq 4\pi R^2(1+\frac{|V|}{E}) \tag{5}$$

This dependence on $1/E$ agrees better with experiment than the previous result of a constant value for this ratio. One finds that the experimental data can be fitted quite well by choosing $R\simeq1.43\times10^{-13}$ cm and $|V|\simeq 38$ MeV, which are reasonable values. The interpretation of the above result can be entirely different if the inelastic cross section consists primarily of straight annihilation or if it is due more to multiple meson production. The rather poor information which is available at present can be interpreted as enforcing the first hypothesis. If our result remains valid up to 10 BeV, we can predict at that energy an elastic cross section of $12 \pm 4$ mb which is certainly compatible with the data obtained from $p$-$p$ scattering.

### 3. Correlation between pions in the annihilation process

Very little experimental work has been done so so far on the correlation between the pions produced in the $p$-$\bar{p}$ annihilation. This kind of information is, however, extremely important theoretically, because the final state of the process contains only particles of baryonic number zero. In particular, it is probably the best way to look for evidence of a pion-pion interaction. More specifically, if the annihilation takes place at rest in a hydrogen bubble chamber, we can be practically sure, using arguments similar to those of Day, Sucher and Snow [2] for $K^-$ absorption, that the annihilation takes place in an $S$ state. If we then select the events which correspond to two or three pions, the correlations between the pions predicted by the statistical theory are well known. If there is, in addition, a strong pion-pion interaction, it will almost certainly be seen in the angular distributions. Another means to detect an isotopic spin dependent pion-pion interaction such as the one which follows from the assumption of a resonance in the $J = 1$, $T = 1$ state is to compare the correlation between pions of like or unlike charges. A paper has been contributed to this conference by Pinski, Sudarshan and Mahantappa, who have investigated this last process. They make use of the Lorentz invariant form of the statistical theory and incorporate into it a pion-pion resonance by a covariant analogue of the Breit-Wigner formula, involving a constant $\lambda^2$ related to the strength of the resonance and assume it to take place in the $J = 1$, $T = 1$ state at a mass of $4\mu$ (where $\mu$ is the pion mass). In order to maintain the covariant characteristic of their calculation, they consider only the angular correlation due to the probability distribution of the effective mass square $M^2 = (p_1+p_2)^2$ of a pair of pions of known charges. They conclude that if there is no resonance ($\lambda^2 = 0$) the $M^2$ distribution is broad for both $(++)$ or $(+-)$ pion pairs; if $\lambda^2 \neq 0$ the $(++)$ mass distribution is still relatively broad but the $(+-)$ one exhibits a sharp peak at a value of $M$ slightly larger than 4. This effect remains when one computes the average $M^2$ distribution associated with the $\pi$ charges. It is the opinion of the rapporteur that this paper has the goal of stimulating a highly interesting type of experiment although some details will probably have to be changed when the data are available.

Before closing my discussion of the antinucleon processes, I would like to list a few of the experiments which, in my opinion are interesting from the theoretical point of view.

*(a)* Perform low energy $\bar{p}$-scattering

*(b)* At all energies obtain a breakdown of the inelastic cross section between annihilation and multiple meson production

*(c)* Extend the measurement of total $p$-$\bar{p}$ cross sections to the 20-30 BeV region.

*(d)* At very high energies measure the ratio $\sigma_{\text{tot}}/\sigma_{\text{el}}$ in addition to $\sigma_{\text{tot}}$

*(e)* Investigate the correlation between pions, especially in two and three pion events.

This completes the first part of my report concerning the $N$-$\bar{N}$ interaction.

## II. THE NUCLEON-NUCLEON INTERACTION

Up to the last one or two years, the analysis of the two nucleon problem was mostly confined to two different approaches. One could use a potential either purely phenomenologically or base it partly on meson theoretical arguments. One could also in an even less ambitious way limit oneself to a phase shift analysis of the data at a particular energy, if these were sufficiently accurate to make such an analysis at all possible. Let us first summarize rapidly the situation in these two ways dealing with the problem up to the last year.

### 1. The potential method

It is usually recognized that the analysis of the potential between two nucleons as a function of their separation can be divided into 3 regions, I, II, III, the division of course not being very sharply defined. In region I, which corresponds to $R > 1.5$ f the interaction is dominated by the one-pion exchange potential (OPEP), which is the only part of the interaction which is well known without any ambiguity. In region II, it is believed that a large contribution to the interaction comes from the exchange of two pions. However, this contribution cannot be calculated unambiguously in the whole of the range of the region II. Thus, the unknown effects due to the exchange of more pions, relativistic corrections, nuclear recoil, exchange of other particles, etc., undoubtedly come into play. It has not been possible to fit experimental data beyond 40 MeV by taking into account in region II only the potential coming from the exchange of two pions whatsoever method is used to define it. Finally, in region III, where practically nothing can be inferred from meson theory, the interaction can almost certainly be described phenomenologically by a fairly strong repulsion, a core remaining relatively hard up to 300 MeV at least for the lowest order partial waves. Its position can be found between 0.4 f and 0.6 f depending on the shape. Among the relatively successful attempts to fit the data in a wide energy range, the most extremely phenomenological is the 14 parameter potential of Gammel and Thaler [3] in which the interaction is completely determined empirically. The only allusion to meson theory is that sums of Yukawa potentials are used to represent the interaction. On the contrary, Signell and Marshak [4] and later Signell, Zinn and Marshak [5] took from field theory the maximum possible information (~Gartenhaus potential) adding only to region II a semi-phenomenological spin-orbit force, and of course, the repulsive core of region III. However, they do not find it possible to fit the experimental data beyond 150 MeV. More recently, the independent work of Bryan, Saylor and Marshak [6] has appeared. They keep only the OPEP from meson theory and calculate the rest of the interaction in such a way that the rather complete data available at 310 MeV are reasonably fitted. This potential seems also to fit well the recent data obtained at 210 MeV by the Rochester Group.

In summary, the history of the meson theoretical potential approach to nuclear forces has been one of alternative advances and retreats; the final settlement line being placed in such a way that nobody can decently be proud of it, namely at the border between regions I and II.

## 2. The phase shift analysis at discrete energies

As you know the rather complete set of data available on *p-p* scattering at 310 MeV (including differential cross sections, polarization and scattering parameters) made it possible for the first time in 1957 to make a full 14 parameter partial wave analysis which yielded eight different solutions (Stapp, Ypsilantis, and Metropolis) [7] (SYM). Later it was recognized by Moravcsik, MacGregor and Stapp [8] (MMS) that the earlier analysis could be greatly simplified if the partial waves higher than *F* were calculated from the Born approximated contribution to the *S*-matrix of one-pion exchange contribution (OPEC). Then it was possible at 300 MeV to reduce the number of free parameters to 9 and to show also that among the five " best " solutions of SYM one could be excluded on the grounds that the contribution of OPEP made things worse instead of improving them, and that the four others could fall eventually into two classes, probably corresponding to only two distinct solutions (they will be called in the following MMS1 and MMS2). This analysis was later extended to 210 MeV with very similar results, and also to 35 MeV with only some ambiguities concerning the $^1D_2$ phase shift. In a paper reported at this Conference by Tyapkin a group of Russian physicists have carried out a similar analysis at 95, 150 and 310 MeV, yielding very similar results. The only significant difference is that a new computing method due to Gelfand has been used which, it is claimed, reduces tremendously the labor and computing time and also eliminates the possibility of getting spurious fits corresponding to small secondary minima in the $\chi^2$ distribution surface. At a given energy, the computer is supposed to have found the eight SYM solutions in a single run of two hours. These eight solutions are divided into three separate regions at 310 MeV, and only two of them persist at low energies.

## 3. Energy dependent phase shift analysis

Although the phenomenological kind of investigation at a fixed energy has given some valuable information, it has been suggested that a simultaneous energy dependent analysis would be of great importance to choose between the various sets of phase shifts that have been proposed. One cannot, for example, decide that a solution at 95 MeV is " similar " to a solution at 310 MeV unless one has an idea of how each phase shift continuously varies with energy. Some of the sets found at a given energy might have such a strange energy variation that they could be excluded on various theoretical grounds, for instance, the causality requirements of Wigner, which give a lower limit on the energy variation of each phase shift. In the past few months, an energy dependent analysis from 0 to 345 MeV of *p-p* scattering has been carried out by two different groups using entirely different methods and has yeilded essentially one single energy dependent phase shift solution, which apart from minor differences, is the same for both groups. This is, in my opinion, a rather striking result and perhaps one of the most interesting to be reported at this Conference. I shall now proceed to give some details on this work. The most comprehensive analysis has been done at Yale by Breit and his collaborators. For phases higher than $F$ waves, the contribution of the relativistic form of OPEP was chosen up to values of $J$ of the order of 12 from 210 to 345 MeV, 10 from 118 to 172 MeV, etc., the criterion being that the last phase included should be less than 0.0005 radian. The remaining nine parameters (the ${}^1S_0$, ${}^1D_2$, ${}^3P_{0,1,2}$ and ${}^3F_{2,3,4}$ phases and the mixing parameter of ${}^3F_2$ with ${}^3P_2$) were determined at all energies by using 541 different pieces of data in six different searches for which a different testing curve of energy variation of the parameter was used in each case. In the searches called YRB 1, 2 and 3, the phases predicted by the Signell, Marshak potential (with a small phenomenological change introduced at Yale) were used up to 150 MeV. Beyond that, the SYL solutions 1, 2 and 3 were used, and a smooth approximate curve drawn through them. In the searches called YLA, YLAM 1 and 2, the phases of Gammel and Thaler were used as starting values. From then on, each parameter was continuously varied with the energy by a gradient method until the corresponding curve would reach a maximum. In those searches the $\pi$-$p$ coupling constant used for OPEP potential was taken as $g^2 = 14$. It was then concluded that the searches called YRB 1, YRB 3, and YLAM yielded a solution which should definitely be excluded.

A similar program has been carried out at Livermore-Berkeley by Moravcsik, Noyes and Stapp [9], using a somewhat smaller amount of data, namely 387 pieces, and is not so advanced as the Yale program. However, it is already clear that one energy dependent solution was found and that it agrees remarkably well with the Yale solution apart from small differences. The method employed is different, expecially with respect to the starting curve for which explicit forms of energy dependence were used that were mostly flexible generalizations of the low energy effective range formulae. These contain 6 parameters which are adjusted to give an approximate fit. The phase shifts were then varied by a step by step method until the $\chi^2$ values fall below an assigned maximum for the higher phases. They also used the OPEP with $g^2 = 14$.

I shall show now some of the phase shifts obtained by both investigations without going into details. The ${}^1S_0$ is particularly interesting in that the two curves are really quite close. For the first time also, the existence of a repulsive core is clearly and uniquely demonstrated. The ${}^3P_0$ phase shift shows a similar behavior. The only significant difference is in the ${}^3F_2$ phase shift for which one solution was found at Livermore-Berkeley which does not seem to vary with energy in a reasonable manner.

## 4. Charge independence

Let us now discuss the work on charge independence and on the shape of the one-pion exchange potential. An analysis of the *n-p* scattering data has also been carried out by the Yale group using the accepted value of the coupling constant for OPEP and the *p-p* phases in the appropriate states. The Yale group believes that they have obtained a vague knowledge of at least five phases, namely: ${}^3S_1$, ${}^3D_1$, ${}^1P_1$, ${}^3D_2$, and ${}^3G_4$. This result would then imply that charge independence at high energy has been varified for nucleon-nucleon scattering.

Another interesting result obtained at Yale concerns the mathematical form of the OPEP. This was obtained by modifying the form of the $(\sigma_1 \cdot \sigma_2)$ term in two different ways and by adding a purely central spin-independent term proportional to the singlet even OPEP. The phases corresponding to $J \leq 3$ were then subjected to a gradient search. The best fit for these modified forms did not differ significantly from the unmodified ones corresponding to $g^2 = 14$ except perhaps for the spin independent modification which can be interpreted as showing the influence of the two-pion exchange potential on these high $L$ phase values. Approximately the same value of the coupling constant was obtained from the high phases of *n-p* scattering with a possible difference of about 10%. This is another piece of evidence in favor of charge independence.

## 5. The meson theoretic potential

In spite of the well known difficulties in describing unambiguously the potential in region II, a group of Japanese physicists have started on an ambitious program to try to understand better the non-static effects which are coming into play in this region. Preliminary results have been reported to the conference by Machida. The one-pion and two-pion exchange potentials were calculated in a nonlocal form in momentum space without neglecting recoil or expanding in powers of the inverse nucleon mass. The Fourier transform of these nonlocal potentials were then approximated analytically by local functions of $r$ only. It is possible that the OPEP contains in its non-static form a quadratic spin orbit term which is supposed to have opposite sign for pseudoscalar (*ps*) or pseudovector (*pv*) coupling. More specifically, in the singlet even states of orbital angular momentum $l$, this term takes the form

$$V_{II}(x) = \mp 0.0018\mu l(l+1)\frac{e^{-x}}{x^3}\left(1+\frac{3}{x}+\frac{3}{x^2}\right) \qquad (6)$$

where the lower (upper) sign corresponds to *pv* (*ps*) coupling. This result is then compared to the phenomenological potential of Bryan who claims that a positive quadratic spin orbit term is necessary to fit the singlet even phases. One then concludes that the *pv* coupling is in better agreement with experiment than the *ps* coupling. This conclusion seems to the rapporteur to be rather strong compared to the evidence which is offered. Firstly, many experts on phenomenology will agree that a quadratic spin orbit term is not at all necessary in order to fit the data. Secondly, the theoretical result itself seems to be rather puzzling, because of the well known equivalence theorem. It seems to me that since a canonical transformation can change the *ps* second order term into a *pv* one, the difference in sign which has been obtained can probably be compensated by the inclusion of higher order terms. This kind of ambiguity is typical in the calculations of mesonic corrections to the OPEP, and thus serves to demonstrate once more their unreliability.

Another contribution to this conference on the form of the potential in the inner regions comes from Breit. He is concerned with the idea that the repulsive core and a singular short range spin orbit coupling can be obtained simultaneously by postulating the existence of a vector meson that can be related to a two-pion or three-pion unstable state which has been postulated in other connections. The difficulty there is that the mass of the postulated resonance is not comparable with the rather short range of the repulsive core and of the L-S coupling term. Breit points out that if this resonance has a short lifetime, as seems to be the case, it could not be reliably approximated by a particle except in very energetic collisions. The corresponding interaction would therefore be strongly energy dependent, and this might eliminate the contradiction. In this case, however, one does not know what to do. Besides, it is not absolutely clear to me that a spin-orbit potential can consistently be derived as a Thomas relativistic correction to a highly singular repulsive short-ranged potential.

For the sake of completeness, I shall mention the work that has been done on the boundary condition model for *p-p* scattering by Saylor, Bryan and Marshak on one hand, and by Lomon and co-workers on the other. The idea is to use in region I and II the relatively weak tails predicted by the one and two pion exchange potentials and to lump all the phenomenology (additional singular forces, spin orbit forces repulsive cores, etc.) into a set of boundary condition for the logarithmic derivative of the wave function at a given point. Although this approach has not yet progressed far enough with realistic potential tails, it is, in my opinion, an interesting and possibly more flexible method than the one based on the phenomenological potentials.

## 6. Dispersion theory

After the success of dispersion relations in the treatment of the meson-nucleon scattering, it was natural to try to apply it to the nucleon-nucleon problem, This program was started two or three years ago by Goldberger, Nambu, and Oehme [10]. Unfortunately, the two body problem does not possess the simplifying features which are brought into the pion-nucleon case by the existence of a strong resonance. Furthermore, the existence of two spins increases the mathematical complexity of the scattering matrix. The introduction of the Mandelstam representation with a certain amount of dynamical information has made it possible in the last year to make real progress in the calculation of the nucleon-nucleon scattering amplitude. Both MacDowell and Amati have reported to this conference the independent work of two groups located at Princeton and Berkeley on the one hand and CERN on the other. Although these contributions merely constitute a program which still has to be performed, there is some hope that one might obtain by this method a more satisfactory though still phenomenological understanding of the nucleon-nucleon problem. We have to be cautious; as some of you know very well, this problem has already defeated many great hopes that arose many times in the past.

The dispersion theoretical approach avoids entirely the concept of potential. Therefore, the ambiguities that I have mentioned earlier never arise. Furthermore, the strict enforcement of the unitarity condition has the advantage of limiting, at least in part, the contribution coming from the high energy regions where one cannot calculate. Whether the concept of potential, with all its limitations which we realize, is completely useless is a question on which people may differ. It seems to me that for the time being at least it is still a useful tool to correlate several problems of nuclear physics. Furthermore, dispersion theoreticians are also human beings; they make approximations. For example, the weight functions which enter the dispersion relations are calculated by an expansion in terms of the particle masses which contribute to the intermediate states. It is not obvious that in problems where a dominant state does not exist this expansion has any better chance to converge than the expansion of the potential in terms of the number of exchanged pions. At any rate, the potential remains as a good instrument to get some physical insight into the approximations made in the dispersion field. This will be made clear in a moment because I shall try to emphasize the relation between the present approach and the one I have discussed earlier.

Now the scattering matrix $T$ is a function of the four momenta $p_1$, $p_2$, $p'_1$, $p'_2$ of the initial and final particles. It can be seen that for each isotopic spin state, $T$ can be expressed as the sum of five invariants multiplied by the five corresponding scalar functions of the three variables $s$, $t$, and $u$ that were defined yesterday by Wick. The choice of these invariants constitutes a certain degree of arbitrariness, but it must satisfy several conditions:

*(a)* The corresponding scalar functions $F_i$ should not contain singularities other than the original scattering amplitude which satisfies the Mandelstam representation.

*(b)* These functions should have simple symmetry properties under the exchange of $t$ and $u$ so that the Pauli principle can be applied easily.

*(c)* They should also have simple crossing symmetries under the interchange of $s$ and $t$.

One can show that conditions $a$, $b$, and $c$ are incompatible. It is however, more important to satisfy conditions $a$ and $b$. If one also requires that the decomposition in the nucleon-nucleon channel be made in a simple fashion, so that the singlet, triplet separation can be effected easily, one is then led to a set of invariants which correspond exactly to those of a Fermi interaction, namely

$$\mathcal{O}_1 = 1^{(1)}1^{(2)},\ \mathcal{O}_2 = \gamma_\mu^{(1)}\gamma_\mu^{(2)},\ \text{etc.}$$

The scattering matrix is then written as follows:

$$\langle p'_1 p'_2 | T | p_1 p_2 \rangle = \sum_{\alpha=0,1} \sum_{i=0}^{5} F_i^{(\alpha)}(s,t,u)[U_i - \widetilde{U}_i]P_\alpha,$$

where

$$U_i = \bar{u}(p'_2)\bar{u}(p'_1)\mathcal{O}_i u(p_2)u(p_1),$$

and $P_\alpha$ is the projection operator for the isotopic spin state. The $F_i^{(\alpha)}$ are the ten scalar functions which describe the scattering and are supposed to obey a Mandelstam representation. The latter is then written as a one-dimensional representation by separating the terms in which the variable $t$ is dominant from those in which it is the variable $s$ that is the most important. This is the method outlined by Cini and reported by

Wick yesterday. We then have an equation of the form

$$F_i^{(\alpha)}(s,t,u)=C_i^{(\alpha)}\frac{g^2}{\mu^2-t}+\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\frac{\rho_i^{(\alpha)}(s,t')}{t'-t}+$$

$$+\frac{1}{\pi}\int_{4M^2}^{\infty}ds'\frac{\chi_i^{(\alpha)}(s',t)}{s'-s}+(-1)^i[s\to u], \qquad (10)$$

where $\rho_i^{(\alpha)}(s,t)$ has a cut in $s$ starting at $4M^2$ and $\chi_i^{(\alpha)}$ has a cut in $t$ starting at $9\mu^2$. This equation makes an explicit separation of the one-pion and two-pion contributions to the scalar functions $F_i$, all the remaining contributions being lumped into the last term, which in the present state of the theory will have to be treated phenomenologically. The $C_i$ are unimportant numerical coefficients. The contributions to $\rho_i^{(\alpha)}$ coming from the cut starting at $4M^2$ can be immediately reexpressed by unitarity in terms of the nucleon scattering amplitude, provided one neglects inelastic processes. This approximation already limits the validity of this approach to the energy region in which the concept of a potential is meaningful, i.e. below the threshold for real pion production. There is another contribution to $\rho_i^{(\alpha)}$ coming from the unphysical cut in $t$ which corresponds to the physical cut in the $N$-$\bar{N}$ scattering process. This contribution, which is the discontinuity of $F_i$ across the cut, is equal to the corresponding projection of the imaginary part of the $N$-$\bar{N}$ scattering amplitude. This can be explained as follows :

$$\text{Im}\langle N\bar{N}|T|N\bar{N}\rangle=\sum_n\langle N\bar{N}|T^\dagger|n\rangle\langle n|T|N\bar{N}\rangle\times$$

$$\times\,\delta(P-P_n), \qquad (11)$$

where the states $n$ consist of all the states of nucleon number zero. The first one is the one-pion state and is already included in the Born term. The second is the two-pion state to which we limit our attention. Now, the amplitude $\langle N\bar{N}|T|2\pi\rangle$, which corresponds to nucleon-antinucleon annihilation into two pions, can be obtained by analytical continuation of the pion-nucleon amplitude. A Mandelstam representation can be written for the latter, which is then transformed, by exactly the same method as above, into an integral equation which involves the $\pi$-$\pi$ scattering amplitude as a kernel. This equation has been solved approximately by Bowcock, Cottingham, and Lurié [11)], by replacing the pion-pion amplitude by a Breit-Wigner formula, corresponding to a resonance in the $T=1$, $J=1$ state. It involves three arbitrary constants which are completely determined by the $S$ wave pion nucleon scattering and the two nucleonic form factors. The substitution of the solution of BCL into the integral equation does not involve any new arbitrary constants. Such constants will appear when the necessary subtractions are made to insure convergence for the lowest order partial waves and also when the contributions coming from higher mass states are approximated phenomenologically.

It is now possible to compare what we have just done which the semi-phenomenological approach which we described earlier. The Born term is just the OPEP. The second term contains the complete contribution of two pions (TPEC). The natural way to improve the fit of the data which has been obtained so far, is to use this contribution to calculate more phases, namely the $^1D_2$ and $^3F_{2,3,4}$ phases, leaving probably only the $S$ and $P$ waves to be determined phenomenologically. This can be done by approximating the remaining part of the unphysical cut starting at $9\mu^2$ either by constants, or by a low order polynomial in $s$, or even by a suitably chosen number of poles. This is, of course, all the same thing if $s$ is small compared to $9\mu^2$. The introduction of a pole with the correct strength will correspond to introducing a set of repulsive Yukawa potentials which might be interpreted, according to the ideas of Breit, as combining the effect of a repulsive core and a singular spin-orbit coupling. The replacement of the unknown terms by a constant or equivalently by a pole at infinity is strictly equivalent to putting in a hard core. In order to treat the lowest waves a partial wave analysis of the Mandelstam representation has to be made. The partial wave amplitudes are found to have singularities as a function of the center of mass momentum $q^2$. These were thoroughly discussed yesterday. There is a physical cut for positive $q^2$ and a series of unphysical cuts corresponding to the one-pion, two-pion, etc. states on the negative real axis. A dispersion relation can therefore be written for each partial wave, which has to be transformed into a Fredholm integral equation by the method due to Chew and Mandelstam : each partial wave amplitude can be written as $h=N/D$, where $N$ has only a left hand cut and $D$ the right hand cut. Unitarity enables us to write $\operatorname{Im} D$ as a function of $N$, and $\operatorname{Im} N$ as an unknown function of $q^2$, which comes from the partial wave projections of the Mandelstam representation. The following set of dispersion relations can therefore be written;

$$N(q^2) = \int_{-\infty}^{-\mu^2/4} dp^2 \frac{R(p^2)D(p^2)}{p^2 - q^2}$$

$$D(q^2) = 1 - \frac{q^2}{\pi} \int_0^{\infty} dp^2 \frac{N(p^2)}{pE(p^2)(p^2 - q^2)}, \qquad (12)$$

where the arbitrary constant implied by the ratio has been determined by forcing $D(0) = 1$ by performing a subtraction. Substitution of $N$ into the $D$ equation yields the following nonsingular integral equation,

$$D(q^2) = 1 + q^2 \int_{-\infty}^{-\mu^2/4} dp^2 K(p^2, q^2) R(p^2) D(p^2), \qquad (13)$$

where $K$ is a simple function of the relative momenta. It is relatively easy to prepare a computing program to solve this integral equation for $N$ and $D$. The intention of the Livermore-Berkeley group is to introduce for the negative cut the calculated values coming from the OPEC and TPEC up to $q^2 = -9\mu^2/4$, and to replace the rest of the cut by an arbitrarily chosen set of poles which will represent perhaps four parameters to be fitted to the lowest wave data. Thus, at least for *p-p* scattering, we are now in position to achieve a unique and fairly satisfactory set of energy dependent phase shifts. This will constitute a rather interesting test of the calculation of the two-pion contribution. If this is successful, then one can probably be quite hopeful for the future of the two-body problem.

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## DISCUSSION

THORNDIKE: I have a question about the phase shifts in *p-p* analysis. Most of the analyses that you have described used as a starting point a solution of type 1. My question is what evidence is there for the uniqueness of, say, Breit's energy dependent phase-shift solution.

LÉVY: I would prefer that Breit answer that question.

BREIT: A solution that was referred to in the talk we just heard, YRB2, was based on the SYM solution 2 which was joined to the Signell-Marshak solution below 150 MeV. When this starting point was improved by searches, it settled down to the fit that was called YRB2. That fit showed a few features that appeared to exclude it as a reasonable fit to data. One of these features is a bump in the polarization versus angle curve at constant energy. This bump develops at about 150 MeV and becomes very strong at 276 MeV, decidedly outside experimental error, and persists up to 310 MeV. Another unreasonable feature is a wiggle that shows itself in the cross section curve at 45° as a function of the energy. The solution YRB2 has been investigated regarding its possible relationship to YRB1 and YLAM. This was done by employing a linear variation of the phase shifts. The curve that was obtained for the sum of the square of the variation plotted against the parameters in a linear variation formula showed a distinct decrease going down to a minimum. However, for a larger value of the parameter it increased to a maximum giving a plateau and then a rather rapid decrease from that plateau down to a minimum that was very close to the solution YLA, shown a year ago at the London Conference. YLA later changed to what is now called YLAM. On the other hand, a similar linear variation procedure showed that YLA and YLAM are connected to YRB1 without going over a plateau. This is the reason why one may say that the YRB1 and the YLAM are very close relatives while YRB2 is of a different species.

NOYES: I also wish to comment on the uniqueness of the phase shift fit. There is an independent piece of evidence that favors solution 1 over solution 2. This is our analysis of the 210 MeV data of Rochester, which has recently been extended to larger angles for the A parameter by Tinlot. We no longer find any really acceptable solutions, but the type 1 solution is very much favored over the others. Until the angular distribution upon which this analysis is based is redone at 210 MeV, this cannot be conclusive, but I feel that this is fairly strong evidence for solution 1 over solution 2.

BLOKHINTSEV: I would like to make some comments concerning the results of the quantitative investigations of $\pi$-$N$ and $N$-$N$ interactions in Dubna, reported here by Petrzilka and Veksler.

First, I would like to remind you of the picture of the distribution of the meson cloud charge in a nucleon which we have reported at the previous Conference held in Kiev [1]. We can distinguish three kinds of $\pi$-$N$ collisions according to the momentum transfer to the nucleons. *(a)* Collisions with small momentum transfer, corresponding to the remote region of a nucleon ($r > \hbar/\mu c$). This region is a region of a purely one-pion exchange and it is of great interest for the study of the $\pi$-$\pi$ interaction. But this region is almost empty—it is only a " stratosphere " of a nucleon (see Fig. 1). We attempted to determine the cross section for the $\pi$-$\pi$ interaction using the optical model and diagram techniques. We found by both methods for $\pi$-$N$ collisions, $\sigma_{\pi\pi}^{\text{tot}} \sim 80$ mb with $p_\pi \sim 7$ BeV/c (lab. system). However, the statistics are very poor for $\pi$-$N$ collisions with momentum transfer $q \ll \mu c$ and for an even number of generated pions. Therefore, $\sigma_{\pi\pi}$ is determined with a large statistical error (about 100%). *(b)* The second class of collisions are those with intermediate momentum transfer, taking place in the region $2\hbar/Mc < r < \hbar/\mu c$. It is very easy to estimate from the optical model that this region is the most important one. *(c)* Finally, the region $q > Mc$, $r \sim \hbar/Mc$ gives only a small contribution to the whole process on the basis of purely geometrical considerations ($\pi r^2$ is small), so I shall not discuss further this type of collision. I come back to the second type. My collaborator, Wang Yun [2], calculated the momentum transfer in high energy $\pi N$ collisions using one-pion exchange terms and obtained quite reasonable agreement with the experimental results for 7 BeV/c pions [3].

It is quite clear, then, that in this region the role of higher graphs could not be disregarded. Nevertheless, I think that the contribution of the higher graphs may be not large. This is just my point. To estimate the contribution from the higher graphs we can compare the graph of Fig. 2 *(a)*, representing an inelastic collision with the one-pion exchange graph. In this case the graph of Fig. 2 *(b)*, gives the diffraction scattering corresponding to the inelastic process (Fig. 2 *(a)*). But we know that in the high energy region this scattering is small compared with the inelastic one. Therefore I think that it is quite reasonable to use one-pion exchange terms for an estimation of inelastic events. Similar considerations are applicable to $N$-$N$ collisions [4]. It seems to me that this point of view is strongly supported by the experimental results obtained in Dubna as they were presented in Petrzilka's and Veksler's reports. The momentum transfer in high energy collisions in the 7-10 BeV region is indeed small.

SALZMAN: Consider a general binary collision where the two incident particles collide by exchange of a single virtual boson. The four momentum

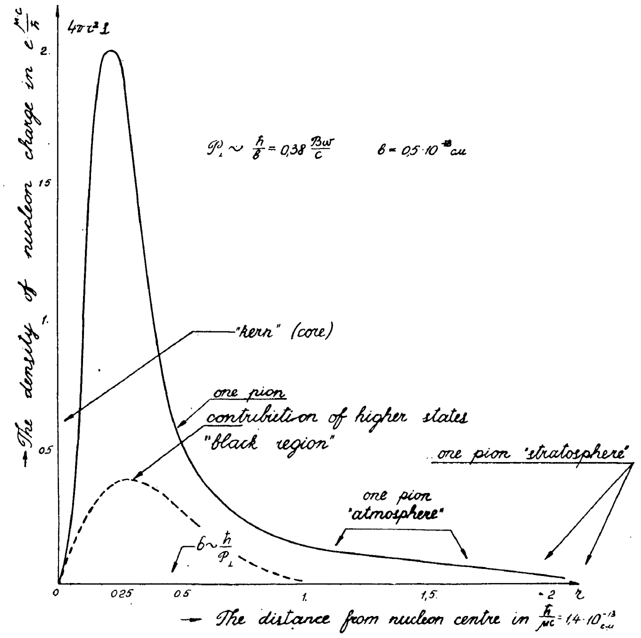


Fig. 1 The qualitative picture of a nucleon (proton).

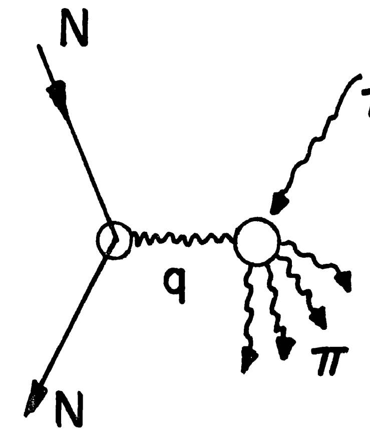


Fig. 2 *(a)* Pion pole graph.

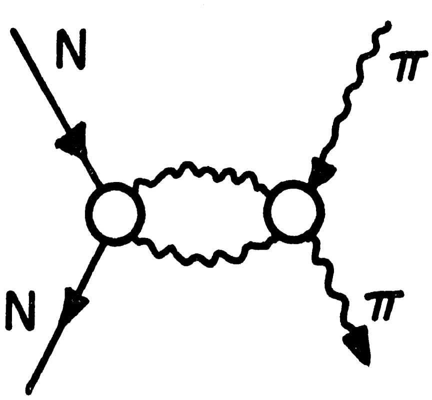


Fig. 2 *(b)* Multiple pion graph,

squared is $q^2$ and we are interested in the case that $q^2$ is small [$q^2$ for a real boson would be the boson mass squared]. For small $q^2$ the exchanged boson behaves as an incoming almost real boson in the rest system of the group of final state particles emerging from one vertex in the sense that in this system it carries energy *into* the vertex. At the same time it *also* behaves as an incoming almost real boson at the other vertex in the rest system of the group of particles emerging from it. This is a surprising fact which follows from just the kinematics, but it enables us to identify each vertex as the amplitude for the scattering of an almost real boson with one of the two incident particles. It is obvious that the formula resulting from this fact, given in detail in Session S2 should have wide applicability. I want to emphasize that for small $q^2$ one is close to a physical cross section at *each* vertex.

LINDENBAUM: I want to make a comment on the non-annihilation meson production in nucleon-antinucleon interactions since this is going to be a key factor in separating the value of the annihilation cross section from the total inelastic cross section. Assume that the difference in the numbers that Chamberlain reported, which, in the $\bar{n}p$ case was 20±9 and in the $\bar{p}p$ case was 5±1 mb, is a real difference. This is, of course, for the non-annihilation meson production. I think that there is a possible explanation for this difference, and the explanation indicates that these cross section separations have to be done very carefully. That is, the $\bar{n}p$ is a pure $T = 1$ state, and the $\bar{p}p$ is a mixture of $T = 0, 1$ in equal proportions. To the extent that these non-annihilation collisions might be thought of as occurring outside the annihilation region, the same characteristics of these two $T$ states would occur as in nucleon-nucleon collisions. These characteristics are quite different for the two different $T$ states; this is connected with the dominance of the $T = J = {}^3/_2$ state in the single pion production. It is estimated that there is little single pion production in the $T = 0$ state in the nucleon-nucleon case at these energies, but a very high cross section, about 30 mb, in $T = 1$. This would go in the right direction to explain Chamberlain's numbers and in fact the ratio would be understandable if one keeps in mind the experimental errors. The same line of reasoning would indicate that the way you detect these particular interactions has a bearing on the estimated total cross section of the process. Now, if you study these events in detail, there are unique predictions that can be carried over from the nucleon-nucleon case. For example at ~900 MeV the antiprotons should show a peak at ~90 MeV and the pions should show exactly the same energy spectrum as in nucleon-nucleon collisions at the same energy. Both the kinematics and the charge ratios of the various final states in $\bar{n}p$ can be quite different from those in $\bar{p}p$ and therefore estimates of total non-annihilation meson production cross sections obtained from measuring selected charge states will be influenced by these effects. Of course, when you get above several BeV you are in much better shape, because then the differences become smaller.

WEINBERG: I would like to make a comment on Pomeranchuk's theorem about the equality of the proton-proton and proton antiproton cross sections at infinite energies. As Lévy remarked, this theorem is based on the assumption that the cross sections become constant at high energy. This is a rather unnatural assumption to make and one that is difficult to justify, to say the least. Actually all that one really needs to assume is that the difference of the cross sections does not change sign an infinite number of times as the energy increases, and that the forces are of finite range. In fact, you can prove a slightly stronger theorem. The method of proof is similar to Pomeranchuk's, but it makes use of a mathematical trick from the analytic theory of continued fractions.

BREIT: On listening to the summary paper of Lévy, one might have obtained the impression that the next step in the understanding of the nucleon-nucleon problem most likely will have to be the connection between the pion-pion interaction and nucleon-nucleon scattering employing the dispersion relations. It is likely that Lévy did not really mean that. It is also highly probable that a satisfactory treatment along these lines would be most welcome. It does seem, however, that up to a point the rather elementary consideration of a process in space-time may be useful. The simplifying factor, that in these days of high powered methods people are ashamed to discuss, is the relatively strong centrifugal barrier that prevents the two particles from coming close together for large values of the orbital angular momentum. It is for that reason that OPEP works, and employing this view one can tell approximately with what distances one is dealing. This tells one that for distances greater than about 2.9 fermis, probably the one-pion exchange potential is the main thing. I should be quite cautious here, but it appears that the simple potential modifications to which Lévy referred are of about the right magnitude when compared to what appears to be a reliable meson-theoretic calculation of Gupta. Thus it seems to me that it might be possible to study the nucleon-nucleon system and to understand whether possibly the large distance behavior is due exclusively to pionic phenomena without being able to represent the whole of nucleon-nucleon scattering. It may not be necessary to develop dispersion theory up to a point where one can calculate the entire interaction in order to be sure of the underlying principles, just as we know the interaction of two lithium atoms is correctly described by the Coulomb potential even though we are not able to calculate the force.

MARSHAK: I just want to say that the one-pion exchange potential really does work well at large distances. It should also be pointed out that some recent work by Saylor, Bryan and myself at Rochester has shown that you can actually use the static one- and two-pion exchange potentials down to about half a pion Compton wave length. A velocity dependence is put in by assuming an energy independent boundary condition at this distance. This gives an excellent fit up to about 300 MeV. This work was developed as a result of the experience of Bryan that the spin-orbit potential is weak outside half a pion Compton wave length and becomes very strong inside. I mention this because in a sense it is evidence for the sort of argument that Breit is stressing, that some of the meson theoretic considerations could be pushed down to shorter distances. The success of this boundary condition model may have real significance for the more refined calculations.

LÉVY: I think that the boundary condition model is a very interesting method which lumps all the phenomenology quite simply into terms. It might be more flexible than the potential approach. Now, to Breit I would like to say that it is certainly clear that we are going to make more use of what happens in the inner regions. I think that no one can honestly say from where the agreement is going to come. For the time being we have to limit ourselves to OPEP type talk.

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# GENERAL ANALYTIC TECHNIQUES

**Rapporteur : K. Symanzik**

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My report is on general analytic techniques that have been reported or submitted in the Session S 2. The first five papers deal with perturbation theory, in the sense that they deal with the analytical properties of each term in the perturbation theoretical expansion of, say, a scattering amplitude, i.e. of each contributing Feynman graph. The first four papers are concerned with the region of analyticity in energy and/or momentum transfer of these functions or, what is the same, with the location of their singularities. The paper by Cutkosky studies these singularities in a more detailed fashion and in its spirit already leads beyond perturbation theory. The following papers discuss the consequences of unitarity for analytical continuations and special aspects thereof. The papers by Newton and by Fonda, Radicati, and Regge discuss analytical and other properties of a non-relativistic many-channel scattering matrix. The next two papers deal with more ethereal problems. Nishijima gives a new formulation of local field theory. Starting from an already well established formulation of such theories the other paper by Symanzik tries to work out some of the implications. A still more fundamental attitude is taken in the final two papers, since they deal with causality itself in the relativistic and nonrelativistic case, respectively.

Last year, at the Kiev Conference, Landau reported a method to locate the singularities of the functions represented by Feynman graphs. This method was the stimulus for a number of independent investigations, most of which, however, used an older and, as a matter of fact, more powerful technique developed by Eden as early as 1952. This method I shall now briefly sketch. I realize that this sketch can be appreciated only with some mathematical training. I shall, however, be back to more understandable topics in a few moments.

Consider a Feynman graph with altogether $n$ internal lines. Its contribution to the scattering matrix will be the integral (consider scalar particles only)

$$F(s,t) \sim \int dk_1 \ldots dk_L \prod_{i=1}^{n} (q_i^2 - m_i^2 + i\varepsilon)^{-1}, \qquad (1)$$

where $q_i$ are the momenta carried by the lines, which are linear combinations of the integration momenta and the external momenta, $p_1 \ldots p_4$, whose squares will be supposed to be always $m^2$ (Fig. 1). Feynman's parameterization method gives

$$F(s,t) \sim \int \ldots \int d\alpha_1 \ldots d\alpha_n \delta(1 - \Sigma\alpha_i) \frac{n(\alpha)}{[D_\varepsilon(\alpha', s, t)]^p}, \qquad (2)$$

where $p > 0$, $n(\alpha)$ is rational in $\alpha$, and

$$D_\varepsilon(s,t) = sf(\alpha) + tg(\alpha) - M^2 K(\alpha) + i\varepsilon, \qquad (3)$$