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Antisymmetrization of the wave functions consisting of spin-isospin and hyperspherical parts

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Abstract. In order to investigate few-particle hypernuclei within the framework of the Hyperspherical Function Method, it is necessary to obtain full wave functions that are anti-symmetric under particle interchange. These wave functions must include not only hyperspherical, but also spin-isospin parts. According to the Parentage Scheme of Symmetrization, N-particle hyperspherical functions (N=3,4,5,6…) symmetrized with respect to (N-1)-particles, can be obtained from the N-particle hyperspherical functions with arbitrary quantum numbers by the use of the transformation coefficients related with the permutations of the last two particles. This article explains how to obtain fully antisymmetrized wave functions consisting of spin-isospin and hyperspherical parts. It is demonstrated that there are sixteen possible combinations in (3+1) configuration and 12 possible combinations in (2+2) configuration when spin and isospin functions are represented by [4], [31], and [22] representations of the four-particle permutation group S_4. A complete set of the fully antisymmetrized four-particle wave functions is obtained. It is demonstrated that proposed mathematical formalism can be easily generalized to obtain fully antisymmetrized wave functions for the systems of five and more particles.

1 Introduction

When investigating the dynamics of the few-body nuclear and hypernuclear systems within the framework of the Hyperspherical Function Method (HFM), the problem of the construction of the fully antisymmetrized wave functions under particle interchange arises. Traditional methods of obtaining antisymmetric Hyperspherical Functions (HF) involve either some approximations [1] or applications of antisymmetrization projectors to the HF [2-3]. These projectors involve sums over all particle permutations, and calculations of matrix elements of the interactions between such states is rather complex. Unlike existing methods, proposed method of antisymmetrization of HF is simple, does not involve complex calculations, does not use any approximations, and can be easily generalized for the systems with any number of particles.

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The microscopic approach to the study of the structure and decays of few-body hypernuclei by the use of the HFM in momentum space was developed and structure characteristics and decay rates were obtained in [4-10]. Four-body HF symmetrized with respect to three identical particles were introduced in [4] and Four-body symmetrized HF with given quantum numbers were constructed in [9]. Microscopic approach to the solution of five- body problem in hypernuclear physics within the framework of the HFM in momentum representation was developed and the dependence of the structure characteristics and wave functions on the types of nucleon-nucleon and hyperon-nucleon interaction potentials was studied in [10]. Parentage Scheme of Symmetrization (PSS) to the N-body symmetrized basis construction necessary for the description of the structural characteristics and decay reactions of the hypernuclear and nuclear systems with arbitrary number of particles was introduced in [9, 11]. The PSS allows to construct N-particle symmetrized hyperspherical functions on the bases of N-particle hyperspherical functions symmetrized with respect to (N–1)-particles by the use of the transformation coefficients related with the (N-1)-th and N-th particle permutations [11].

Next section explains how to construct fully antisymmetrized wave functions consisting of spin, isospin and hyperspherical parts for four-body systems in (3+1) and (2+2) configurations and derives complete sets of these functions.

2 Constructing fully antisymmetrized four-body wave functions

Fully antisymmetrized four-body wave functions can be constructed using different combinations of the spin, isospin, and hyperspherical parts. There are sixteen possible ways of construction of the fully antisymmetric four-body wave functions if spin and isospin functions are related with the following Young schemes: [4], [31], and [22].

If we consider (3+1) configuration, then spin and isospin functions for this configuration can be represented as:

$$\chi_s = [[\chi^1(12)\chi(3)]^{3/2}\chi(4)]^2; \qquad \chi_\alpha = [[\chi^0(12)\chi(3)]^{1/2}\chi(4)]^1$$
$$\chi_e = [[\chi^0(12)\chi(3)]^{1/2}\chi(4)]^0\,; \qquad -\chi_\beta = [[\chi^1(12)\chi(3)]^{1/2}\chi(4)]^1$$
$$\chi_f = [[\chi^1(12)\chi(3)]^{1/2}\chi(4)]^0\,; \qquad \chi_\gamma = [[\chi^1(12)\chi(3)]^{1/2}\chi(4)]^1$$

where $e=(2121)$, s=(1111), $\bar{s}$=(4321), f=(2211), α=(1121), β=(1211), γ=(2111), $\bar{\alpha}$=(3211), $\bar{\beta}$=(3121), $\bar{\gamma}$=(1321).

We can represent a product of the [31] and [31] configurations as the sum of the following representations of the four-particle permutation groups:

$$[31]\times[31] = [4]+[31]+[22]+[211] \qquad (1)$$

where $s=\frac{1}{\sqrt{3}}(\alpha_1\alpha_2+\beta_1\beta_2+\gamma_1\gamma_2)$ corresponds to the fully symmetrized four-particle permutation group [4]. The four-particle permutation group [31] includes α=(1121), β=(1211), γ=(2111), permutation group [211] includes $\bar{\alpha}$=(3211), $\bar{\beta}$=(3121), $\bar{\gamma}$=(1321), and permutation group [22] is represented by $e=(2121)$ and f=(2211):

$$\alpha = \frac{1}{\sqrt{3}}\alpha_1\gamma_2+\frac{1}{\sqrt{6}}\gamma_1\alpha_2-\frac{1}{\sqrt{3}}\alpha_1\beta_2-\frac{1}{\sqrt{3}}\beta_1\alpha_2$$
$$\beta = -\frac{1}{\sqrt{3}}\alpha_1\alpha_2+\frac{1}{\sqrt{3}}\beta_1\beta_2+\frac{1}{\sqrt{6}}\beta_1\gamma_2+\frac{1}{\sqrt{2}}\gamma_1\beta_2$$

$$\gamma=\frac{1}{\sqrt{6}}\alpha_1\alpha_2+\frac{1}{\sqrt{6}}\beta_1\beta_2-\frac{2}{\sqrt{6}}\gamma_1\gamma_2$$
$$\bar{\alpha}=\frac{1}{\sqrt{2}}\gamma_1\beta_2-\frac{1}{\sqrt{2}}\beta_1\gamma_2$$
$$\bar{\beta}=\frac{1}{\sqrt{2}}\alpha_1\gamma_2-\frac{1}{\sqrt{2}}\gamma_1\alpha_2$$
$$\bar{\gamma}=-\frac{1}{\sqrt{2}}\alpha_1\beta_2+\frac{1}{\sqrt{2}}\beta_1\alpha_2$$
$$e={}^{1}/_{\sqrt{6}}\{-\alpha_1\beta_2-\beta_1\alpha_2-\sqrt{2}\alpha_1\gamma_2-\sqrt{2}\gamma_1\alpha_2\}$$
$$f=1/\sqrt{6}\{\alpha_1\alpha_2-\beta_1\beta_2+\sqrt{2}\beta_1\gamma_2+\sqrt{2}\beta_2\gamma_1\} \quad (2)$$

The product of two [211] configurations can be represented as $[211]\times[211]=[211]+[22]+[31]+[4]$. The formulas for the [211], [31], and [22] configurations can be easily obtained from (2) by replacing α *=(1121),* β*=(1211),* γ*=(2111)* with $\bar{\alpha}$*=(3211),* $\bar{\beta}$*=(3121),* $\bar{\gamma}$=(1321) correspondingly. All other products of the four-body permutation groups can be reduced to the following sums:

$$[22]\times[22]=[4]+[22]+[1111]$$
$$[31]\times[22]=[31]+[211]$$
$$[211]\times[22]=[31]+[211]$$
$$[31]\times[1111]=[211]$$
$$[211]\times[1111]=[31]$$
$$[22]\times[1111]=[22] \quad (3)$$

A complete set of sixteen possible four-particle wave functions, anti-symmetric under particle permutations and consisting of spin, isospin and hyperspherical parts can be constructed in (3+1) configuration taking into consideration formulas (2) and (3). Below are presented the formulas expressing twelve possible combinations of the spin-isospin and hyperspherical parts in (3+1) configuration:

$$[4]\times[4]\times[1111]$$
$$\chi_s(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_s$$
$$[31]\times[31]\times[1111]$$
$$\frac{1}{\sqrt{3}}[\chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{s}}+\chi_\beta(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{s}}+\chi_\gamma(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{s}}]$$
$$[31]\times[31]\times[31]$$
$$\frac{1}{\sqrt{6}}[\chi_\gamma(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\alpha-\chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\alpha+\chi_\alpha(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\beta-\chi_\gamma(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\beta-\chi_\alpha(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\gamma+\chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\gamma]$$
$$[31]\times[31]\times[22]$$
$$\frac{1}{2\sqrt{3}}[-\chi_\alpha(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_f-\chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_f-\sqrt{2}\chi_\alpha(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_f-\sqrt{2}\chi_\gamma(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_f-\chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_e+\chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_e+\sqrt{2}\chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_e+\sqrt{2}\chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_e]$$
$$[31]\times[31]\times[211]$$

$$\frac{1}{3}[\frac{1}{\sqrt{2}}\chi_\alpha(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\alpha}}+\frac{1}{\sqrt{2}}\chi_\gamma(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\alpha}}-\chi_\alpha(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\alpha}}-\chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\alpha}}$$
$$-\chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\beta}}+\chi_\beta(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\beta}}+\frac{1}{\sqrt{2}}\chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\beta}}$$
$$+\frac{1}{\sqrt{2}}\chi_\gamma(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\beta}}+\frac{1}{\sqrt{2}}\chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\beta}}$$
$$+\frac{1}{\sqrt{2}}\chi_\beta(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\gamma}}-\sqrt{2}\chi_\gamma(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\gamma}}]$$

$$[22]\times[22]\times[1111]$$
$$\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{s}}+\chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_{\bar{s}}]$$

$$[22]\times[22]\times[4]$$
$$\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_{\bar{s}}-\chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{s}}]$$

$$[22]\times[22]\times[22]$$
$$\frac{1}{2}[\chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_f+\chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_f-\chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_l+\chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_l]$$

$$[22]\times[4]\times[22]$$
$$\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_f-\chi_f(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_e]$$

$$[22]\times[31]\times[31]$$
$$\frac{1}{\sqrt{3}}[\frac{1}{2}\chi_e(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\alpha+\frac{1}{\sqrt{2}}\chi_f(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\alpha+\frac{1}{2}\chi_f(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\alpha+\frac{1}{\sqrt{2}}\chi_e(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\beta$$
$$+\frac{1}{2}\chi_f(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\beta-\frac{1}{2}\chi_e(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\beta+\frac{1}{\sqrt{2}}\chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\gamma]$$

$$[22]\times[31]\times[211]$$
$$\frac{1}{\sqrt{3}}[-\frac{1}{2}\chi_f(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\alpha}}+\frac{1}{\sqrt{2}}\chi_e(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\alpha}}+\frac{1}{2}\chi_e(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\alpha}}-\frac{1}{\sqrt{2}}\chi_f(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\beta}}$$
$$+\frac{1}{2}\chi_e(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\beta}}+\frac{1}{2}\chi_f(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\beta}}+\frac{1}{\sqrt{2}}\chi_e(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\gamma}}$$
$$-\frac{1}{\sqrt{2}}\chi_e(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\gamma}}]$$

$$[31]\times[4]\times[211]$$
$$\frac{1}{\sqrt{3}}\left[\chi_\alpha(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_{\bar{\alpha}}+\chi_\beta(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_{\bar{\beta}}+\chi_\gamma(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\gamma}}\right] \quad (4)$$

where $\chi(\boldsymbol{\sigma}), \boldsymbol{\chi}(\boldsymbol{\tau})$ and $\boldsymbol{\phi}$ are spin, isospin and hyperspherical parts. The remaining four combinations can be easily constructed by switching spin and isospin functions $\boldsymbol{\sigma} \leftrightarrow \boldsymbol{\tau}$. Namely:

$$[31]\times[22]\times[31]=[22]\times[31]\times[31]\ \boldsymbol{\sigma}\leftrightarrow\boldsymbol{\tau}$$
$$[31]\times[22]\times[211]=[22]\times[31]\times[211]\ \boldsymbol{\sigma}\leftrightarrow\boldsymbol{\tau}$$
$$[4]\times[22]\times[22]=[22]\times[4]\times[22]\ \boldsymbol{\sigma}\leftrightarrow\boldsymbol{\tau}$$
$$[4]\times[31]\times[211]=[31]\times[4]\times[211]\ \boldsymbol{\sigma}\leftrightarrow\boldsymbol{\tau} \quad (5)$$

Formulas (4) and (5) represent complete set of fully antisymmetrized wave functions in (3+1) configuration consisting of spin, isospin, and hyperspecial parts.

While (3+1) configuration is widely used for the solution of the few-body problems where separation of the fourth particle is necessary, (2+2) configuration is preferable whenever we need to describe the movement of the center of mass of the first and last pairs of the particles. We have the following functions in (2+2) configuration:

$$a \equiv [31]as = (1121)$$
$$b \equiv [31]sa = \sqrt{\frac{2}{3}}(2111) + \sqrt{\frac{1}{3}}(1211)$$
$$c \equiv [31]ss = \frac{1}{\sqrt{3}}(2111) - \sqrt{\frac{2}{3}}\ (1211)$$
$$\bar{a} \equiv [211]sa = (3211)$$
$$\bar{b} \equiv [211]as = \sqrt{\frac{2}{3}}(1321) + \sqrt{\frac{1}{3}}(3121)$$
$$\bar{c} \equiv [211]aa$$
$$= \sqrt{\frac{1}{3}}(1321) - \sqrt{\frac{2}{3}}(3121) \qquad (6)$$

Spin and isospin functions in (2+2) configuration are expressed as follows:

$$\begin{array}{ll} \chi_e = \chi^0(12)\chi^0(34) & \chi_a = \chi^0(12)\chi^1(34) \\ \chi_f = [\chi^1(12)\chi^1(34)]^0 & \chi_b = \chi^1(12)\chi^0(34) \\ \chi_s = [\chi^1(12)\chi^1(34)]^2 & \chi_c = [\chi^1(12)\chi^1(34)]^1 \end{array} \qquad (7)$$

Taking into consideration (6) and (7), we can obtain the following twelve fully antisymmetrized four-body wave functions consisting of spin, isospin and hyperspherical parts in (2+2) configuration:

$$[4] \times [4] \times [1111]$$
$$\chi_s(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_{\bar{s}}$$
$$[31] \times\ [31] \times [1111]$$
$$\frac{1}{\sqrt{3}}[\chi_a(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_b(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_c(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_{\bar{s}}]$$
$$[31] \times\ [31] \times [31]$$
$$\frac{1}{\sqrt{6}}[\chi_c(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_a - \chi_b(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_a + \chi_a(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_b - \chi_c(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_b + \chi_b(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_c - \chi_a(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_c]$$
$$[31] \times\ [31] \times [22]$$
$$\frac{1}{2}[\frac{1}{\sqrt{3}}\chi_a(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_e + \frac{1}{\sqrt{3}}\chi_b(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_e - \frac{2}{\sqrt{3}}\chi_c(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_e + \chi_a(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_f + \chi_b(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_f]$$
$$[31] \times\ [31] \times [211]$$
$$1/\sqrt{6}[\chi_a(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{a}} + \chi_c(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{a}} - \chi_b(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_b - \chi_c(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_b + \chi_a(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{c}} + \chi_b(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{c}}]$$
$$[22] \times\ [22] \times [1111]$$
$$\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_{\bar{s}}]$$
$$[22] \times\ [22] \times [4]$$

$$\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_s - \chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_s]$$

$$[22]\times[22]\times[22]$$

$$\frac{1}{2}[\chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_e + \chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_e - \chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_f - \chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_f]$$

$$[22]\times[31]\times[31]$$

$$\frac{1}{2}[\frac{1}{\sqrt{3}}\chi_e(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_a + \chi_f(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_a + \chi_f(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_b + \frac{1}{\sqrt{3}}\chi_e(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_b - \frac{2}{\sqrt{3}}\chi_e(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_c]$$

$$[22]\times[31]\times[211]$$

$$\frac{1}{2}[-\frac{1}{\sqrt{3}}\chi_f(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{a}} + \chi_e(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{a}} + \chi_e(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{b}} - \frac{1}{\sqrt{3}}\chi_f(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{b}} + \frac{2}{\sqrt{3}}\chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{c}}]$$

$$[22]\times[4]\times[22]$$

$$\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_f - \chi_f(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_e]$$

$$[31]\times[4]\times[211]$$

$$\frac{1}{\sqrt{3}}[\chi_a(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_{\bar{a}} + \chi_b(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_{\bar{b}} + \chi_c(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_{\bar{c}}] \quad (8)$$

Formulas (4-5) and (8) represent complete sets of fully antisymmetrized four-body wave functions consisting of spin-isospin and hyperspherical parts.

3 Conclusion

The problem of the construction of the fully antisymmetrized four-body wave functions consisting of spin-isospin and hyperspherical parts has been solved first time in both (3+1) and (2+2) configurations. Complete sets of possible combinations of spin, isospin and hyperspherical parts have been obtained in (3+1) and (2+2) configurations. Proposed mathematical formalism is simple, does not involve complex calculations, and can easily be generalized for the systems with five and more particles.

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