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Dynamics of quantum entanglement in de Sitter spacetime and thermal Minkowski spacetime

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Abstract

We investigate the dynamics of entanglement between two atoms in de Sitter spacetime and in thermal Minkowski spacetime. We treat the two-atom system as an open quantum system which is coupled to a conformally coupled massless scalar field in the de Sitter invariant vacuum or to a thermal bath in the Minkowski spacetime, and derive the master equation that governs its evolution. We compare the phenomena of entanglement creation, degradation, revival and enhancement for the de Sitter spacetime case with that for the thermal Minkowski spacetime case. We find that the entanglement dynamics of two atoms for these two spacetime cases behave quite differently. In particular, the two atoms interacting with the field in the thermal Minkowski spacetime (with the field in the de Sitter-invariant vacuum), under certain conditions, could be entangled, while they would not become entangled in the corresponding de Sitter case (in the corresponding thermal Minkowski case). Thus, although a single static atom in the de Sitter-invariant vacuum responds as if it were bathed in thermal radiation in a Minkowski universe, with the help of the different dynamic evolution behaviors of entanglement for two atoms one can in principle distinguish these two universes.

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1. Introduction

Entanglement as a quantum property of compound systems means that the global states of a composite system cannot be written as a product of the states of its corresponding individual subsystems. It is the most fascinating nonclassical manifestation of quantum formalism and plays a significant role in quantum information and quantum computation [1,2]. Recently, most of the tasks such as quantum teleportation [3,4], quantum dense coding [5,6], quantum computational speedups [7], quantum error correction [8], cryptographic key distribution [9] have been implemented with entanglement. However, due to the unavoidable interaction with environment, quantum systems may be subjected to decoherence and dissipation, and correspondingly its entanglement may decrease and even experience sudden death [10–17]. Decoherence thus is one of the main obstacles to the realization of quantum information technologies. Besides, when two atoms are immersed in a common thermal bath, the indirect interactions between the otherwise independent atoms, as a result of the field correlation, may lead to some interesting phenomena, such as the revival of destroyed entanglement and the creation of entanglement in initially separable states [18–25]. The relevant investigations could guide us to effective quantum state preparation, storage and protection, effective implementation of quantum information tasks, and even the understanding of the property of external environments. Therefore, the entanglement dynamics of open quantum systems is an important issue in quantum information science and is worthy for us to study in different scenarios, e.g., relativistic framework [26–32].

On the other hand, de Sitter spacetime is a very simple curved background that has the same degree of symmetry as the Minkowski background, both having ten Killing vectors. It is also an important model of our universe in the far past and the far future, as verified by our current observations and the theory of inflation [33]. Besides, it is found that a single particle interacting with a conformally coupled massless scalar field in the de Sitter invariant vacuum state behaves exactly the same way as the one coupled to thermal bath in Minkowski spacetime [34–41]. Therefore, it is worthy to ask whether it is possible to distinguish de Sitter spacetime from the thermal Minkowski spacetime, i.e., which universe the inhabitants are exactly in. In this regard, let us note that Refs. [42–46] investigated this issue by considering different entangling power of these two universes. Moreover, it is shown that the Casimir–Polder interaction between atoms behaves quite differently in these two universes and thus one can in principle distinguish these two universes with the different behaviors of Casimir–Polder interaction [47,48].

In this paper, we study the dynamics of entanglement for two-atom system coupled with a massless scalar field in the de Sitter invariant vacuum and with a thermal bath in Minkowski universe. We first treat the two atoms as an open quantum system and obtain its master equation by tracing over the degree of freedom of quantum field. Then we discuss the evolution behaviors of entanglement between the two atoms in the de Sitter spacetime and Minkowski background. We find that for different initial states, the atomic entanglement evolves quite differently with respective to time or other parameters, such as the interatomic distance and temperature of thermal bath. Besides, the entanglement dynamics of atoms in de Sitter spacetime are also different from that of the corresponding thermal Minkowski spacetime case. We thus arrive at the conclusion that with the help of the different dynamic evolution behaviors of entanglement for two atoms one can in principle distinguish the de Sitter spacetime from the thermal Minkowski one. Note that the open quantum system approach applied in current paper is different from that in Refs. [42,45,46], and allows us to examine the entanglement dynamics, i.e., the entanglement evolution with respective to time. Besides, unlike the previous studies [42,45,43,46] where the authors only discussed the creation of entanglement for the atoms with special initial state, i.e.,

$|01\rangle$, and confined their analyzation to the entanglement remaining in the late asymptotic times, our work investigate the entanglement behavior of two atoms with different initial states, e.g., the X-type state, and at any time. As a consequence of that, a lot of new phenomena, such as sudden death of quantum entanglement, and the revival of destroyed entanglement, are presented in our paper, which provides more abundant criterion to tell the de Sitter spacetime from the thermal Minkowski spacetime, and more physics of entanglement for relativistic quantum information science [49].

The structure of this paper is as follows. Sec. 2 introduces briefly the physical model of two-atom system coupled with quantized massless scalar field. In Sec. 3, we explore dynamical behaviors of entanglement for two-atom system interacting with a massless scalar field in de Sitter and thermal Minkowski universe. A brief conclusion is given in the last section.

2. Physical model

We consider that two identical and mutually independent atoms weakly couples with quantized massless scalar field in its vacuum state. The total Hamiltonian of such a system is of the form

$$H = H_S + H_F + H_I. \tag{1}$$

H_S is the Hamiltonian of the two atoms,

$$H_S = \frac{\omega}{2}\sigma_3^{(1)} + \frac{\omega}{2}\sigma_3^{(2)}, \tag{2}$$

where $\sigma_i^{(1)} = \sigma_i \otimes \sigma_0$, $\sigma_i^{(2)} = \sigma_0 \otimes \sigma_i$, with σ_i $(i = 1, 2, 3)$ being the Pauli matrices and σ_0 being the 2×2 identity matrix. It is assumed that the two atoms have the same internal energy ω. H_F is the Hamiltonian of the scalar field. H_I is interaction Hamiltonian between atoms and scalar field, and can be expressed as

$$H_I(\tau) = \mu \left[\sigma_2^{(1)}\Phi(\mathbf{x}_1(\tau)) + \sigma_2^{(2)}\Phi(\mathbf{x}_2(\tau))\right], \tag{3}$$

where μ denotes the coupling constant and is assumed to be small, and τ is the proper time of the atoms.

We assume that the two atoms are uncorrelated with the external field at the beginning, and thus the initial state of the whole system is of the form: $\rho_{tot}(0) = \rho(0) \otimes |0\rangle\langle 0|$, with $\rho(0)$ and $|0\rangle$ being the initial state of the two-atom system and the initial state of the scalar field, respectively. The time evolution of the total system governed by the von Neumann equation $\frac{\partial \rho_{tot}(\tau)}{\partial \tau} = -i[H, \rho_{tot}(\tau)]$. In the weak-coupling limit (under Born–Markov approximation), by tracing over the field degrees of freedom, the evolution of the two-atom system can be written in the Kossakowski–Lindblad form [50–52]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -i[H_{\text{eff}}, \rho(\tau)] + \mathcal{L}[\rho(\tau)], \tag{4}$$

where the effective Hamiltonian is given by

$$H_{\text{eff}} = H_S - \frac{i}{2} \sum_{\alpha,\beta=1}^{2} \sum_{i,j=1}^{3} H_{ij}^{(\alpha\beta)} \sigma_i^{(\alpha)} \sigma_j^{(\beta)}, \tag{5}$$

and

$$\mathcal{L}[\rho] = \frac{1}{2}\sum_{\alpha,\beta=1}^{2}\sum_{i,j=1}^{3} C_{ij}^{(\alpha\beta)}[2\sigma_j^{(\beta)}\rho\sigma_i^{(\alpha)} - \sigma_i^{(\alpha)}\sigma_j^{(\beta)}\rho - \rho\sigma_i^{(\alpha)}\sigma_j^{(\beta)}]. \tag{6}$$

There are two terms in Eq. (4). The first term is dominated by the effective Hamiltonian, which describes the unitary evolution of the two-atoms system. The second term is dissipative term, which describes transition, dissipation and decoherence as a result of the interaction between the atom system and the quantum field.

$C_{ij}^{(\alpha\beta)}$ are the coefficients of the Kossakowski matrix which can be written as

$$C_{ij}^{(\alpha\beta)} = A^{(\alpha\beta)}\delta_{ij} - iB^{(\alpha\beta)}\epsilon_{ijk}\delta_{3k} - A^{(\alpha\beta)}\delta_{3i}\delta_{3j}, \tag{7}$$

with

$$A^{(\alpha\beta)} = \frac{\mu^2}{4}[\mathcal{G}^{(\alpha\beta)}(\omega) + \mathcal{G}^{(\alpha\beta)}(-\omega)],$$

$$B^{(\alpha\beta)} = \frac{\mu^2}{4}[\mathcal{G}^{(\alpha\beta)}(\omega) - \mathcal{G}^{(\alpha\beta)}(-\omega)]. \tag{8}$$

Replacing $\mathcal{G}^{(\alpha\beta)}(\omega)$ with $\mathcal{K}^{(\alpha\beta)}(\omega)$ in the above equations, $H_{ij}^{(\alpha\beta)}$ can be obtained in the same way as $C_{ij}^{(\alpha\beta)}$. $\mathcal{G}^{(\alpha\beta)}(\omega)$ and $\mathcal{K}^{(\alpha\beta)}(\omega)$ represent Fourier and Hilbert transforms respectively, defined as

$$\mathcal{G}^{(\alpha\beta)}(\lambda) = \int_{-\infty}^{\infty} d\Delta\tau e^{i\lambda\Delta\tau} G^{(\alpha\beta)}(\Delta\tau), \tag{9}$$

$$\mathcal{K}^{(\alpha\beta)}(\lambda) = \frac{P}{\pi i}\int_{-\infty}^{\infty} d\omega \frac{\mathcal{G}^{(\alpha\beta)}(\omega)}{\omega - \lambda}, \tag{10}$$

where P is the principal value, and

$$G^{(\alpha\beta)}(\tau - \tau') = \langle \Phi(\tau, \mathbf{x}_\alpha)\Phi(\tau', \mathbf{x}_\beta)\rangle \tag{11}$$

are the field correlation functions.

3. Evolution of entanglement in de Sitter and thermal Minkowski spacetime

In this section, we investigate the dynamics of entanglement for two atoms in two scenarios. One is that the two-atom system interacts with a massless scalar field in de Sitter invariant vacuum. The other is that the two-atom system is coupled to a thermal bath in Minkowski spacetime. We mainly focus on the difference of entanglement behavior for these two cases.

de Sitter background is a solution of the Einstein equations with the cosmological constant Λ, and it can be represented as the surface of the hyperboloid

$$z_0^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 = -\alpha^2, \tag{12}$$

embedded in the five dimensional Minkowski spacetime with the metric [35]

$$ds^2 = dz_0^2 - dz_1^2 - dz_2^2 - dz_3^2 - dz_4^2, \tag{13}$$

where $\alpha = \sqrt{3/\Lambda}$. By applying the following parametrization

$$
\begin{aligned}
z_0 &= \sqrt{\alpha^2 - r^2}\sinh t/\alpha, \\
z_1 &= \sqrt{\alpha^2 - r^2}\cosh t/\alpha, \\
z_2 &= r\cos\theta, \\
z_3 &= r\sin\theta\cos\phi, \\
z_4 &= r\sin\theta\sin\phi,
\end{aligned} \tag{14}
$$

we can obtain the static de Sitter metric

$$
\mathrm{d}s^2 = \left(1 - \frac{r^2}{\alpha^2}\right)\mathrm{d}t^2 - \left(1 - \frac{r^2}{\alpha^2}\right)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{15}
$$

It is obvious that a coordinate singularity exists at $r = \alpha$ where the so called cosmological horizon is. Note that in curved spacetime, how to determine the vacuum state of the quantum field is a delicate issue. Here we choose the de Sitter-invariant vacuum state as the state of the conformally coupled massless scalar field, since it is an analogous state to the Minkowski vacuum in flat spacetime, and it is considered to be a natural vacuum [53]. Correspondingly, the Wightman function for the massless scalar field is of the form [35,36]

$$
G^+(x, x') = -\frac{1}{4\pi^2}\frac{1}{(z_0 - z_0')^2 - \Delta z^2 - i\epsilon}, \tag{16}
$$

where $\Delta z^2 = (z_1 - z_1')^2 + (z_2 - z_2')^2 + (z_3 - z_3')^2 + (z_4 - z_4')^2$ and ϵ is an infinitesimal constant. We assume that the two atoms are located at the same distance with respect to the original point $r = 0$ but with different polar angles, i.e., the two atoms are static at (r, θ, ϕ) and (r, θ', ϕ), respectively. For this case, the corresponding Wightman function along the trajectories of the two atoms could be calculated by substituting the atomic trajectories into Eq. (16). It is given by [48]

$$
\begin{aligned}
G^{(11)}(x, x') &= G^{(22)}(x, x') = -\frac{1}{16\pi^2\kappa^2\sinh^2(\frac{\Delta\tau}{2\kappa} - i\epsilon)}, \\
G^{(12)}(x, x') &= G^{(21)}(x, x') = -\frac{1}{16\pi^2\kappa^2}\frac{1}{\sinh^2(\frac{\Delta\tau}{2\kappa} - i\epsilon) - \frac{r^2}{\kappa^2}\sin^2\frac{\Delta\theta}{2}},
\end{aligned} \tag{17}
$$

where we have used the definitions: $\kappa = \sqrt{g_{00}}\alpha = \sqrt{1 - r^2/\alpha^2}\alpha = \sqrt{\alpha^2 - r^2}$, and $\Delta\tau = \tau - \tau' = \sqrt{g_{00}}\Delta t = \sqrt{g_{00}}(t - t')$ with τ being the proper time of the atoms. According to Eq. (9), one can then calculate the Fourier transforms of the above field correlation function straightly:

$$
\begin{aligned}
\mathcal{G}^{(11)}(\lambda) &= \mathcal{G}^{(22)}(\lambda) = \frac{1}{2\pi}\frac{\lambda}{1 - e^{-2\pi\kappa\lambda}}, \\
\mathcal{G}^{(12)}(\lambda) &= \mathcal{G}^{(21)}(\lambda) = \frac{1}{2\pi}\frac{\lambda}{1 - e^{-2\pi\kappa\lambda}} f(\lambda, L/2),
\end{aligned} \tag{18}
$$

where

$$
f(\lambda, z) = \frac{\sin\left[2\kappa\lambda \mathrm{arcsinh}(z/\kappa)\right]}{2z\lambda\sqrt{1 + z^2/\kappa^2}}, \tag{19}
$$

and $L = 2r\sin(\Delta\theta/2)$ is the usual Euclidean distance between the two points $(r,\ \theta,\ \phi)$ and $(r,\ \theta',\ \phi)$, i.e., the distance between the two static atoms in de Sitter spacetime.

Substituting the above Fourier transforms into Eq. (8), we can obtain the coefficients $C_{ij}^{(\alpha\beta)}$ of Eq. (6) for the de Sitter spacetime case

$$A^{(11)} = A^{(22)} = A_1 = \frac{\Gamma}{4}\coth(\pi\kappa\omega), \quad A^{(12)} = A^{(21)} = A_2 = \frac{\Gamma}{4}\coth(\pi\kappa\omega) f(\omega, L/2),$$
$$B^{(11)} = B^{(22)} = B_1 = \frac{\Gamma}{4}, \quad B^{(12)} = B^{(21)} = B_2 = \frac{\Gamma}{4} f(\omega, L/2), \tag{20}$$

where $\Gamma = \mu^2\omega/2\pi$ is the spontaneous emission rate. Note that for convenience in the following, we will introduce a parameter $T = \frac{1}{2\pi\kappa}$ which is the temperature of thermal bath felt by the static observers in de Sitter spacetime [37–40].

For the thermal massless scalar field in the Minkowski spacetime with temperature $T = 1/2\pi\kappa$, the field correlation functions are given by

$$G^{(11)}(x, x') = G^{(22)}(x, x') = -\frac{1}{4\pi^2}\sum_{n=-\infty}^{+\infty}\frac{1}{(\Delta\tau - in/T - i\epsilon)^2},$$
$$G^{(12)}(x, x') = G^{(21)}(x, x') = -\frac{1}{4\pi^2}\sum_{n=-\infty}^{+\infty}\frac{1}{(\Delta\tau - in/T - i\epsilon)^2 - L^2}, \tag{21}$$

whose Fourier transforms can be gained with residue theorem

$$\mathcal{G}^{(11)}(\lambda) = \mathcal{G}^{(22)}(\lambda) = \frac{1}{2\pi}\frac{\lambda}{1 - e^{-\frac{\lambda}{T}}},$$
$$\mathcal{G}^{(12)}(\lambda) = \mathcal{G}^{(21)}(\lambda) = \frac{1}{2\pi}\frac{\lambda}{1 - e^{-\frac{\lambda}{T}}}\frac{\sin(L\lambda)}{L\lambda}. \tag{22}$$

Similarly, according to Eq. (8), we can obtain the coefficients $C_{ij}^{(\alpha\beta)}$ of Eq. (6)

$$A_1 = \frac{\Gamma}{4}\coth(\frac{\omega}{2T}), \quad A_2 = \frac{\Gamma}{4}\coth(\frac{\omega}{2T})\frac{\sin(L\lambda)}{L\lambda}, \quad B_1 = \frac{\Gamma}{4}, \quad B_2 = \frac{\Gamma}{4}\frac{\sin(L\lambda)}{L\lambda}. \tag{23}$$

Now, we consider how to solve the master equation (4). We assume that the initial state for the atoms is X-type states, which is an important class of quantum states and has been discussed extensively [54–62]. In terms of the Pauli matrices, arbitrary two-qubit states can be written as

$$\rho = \frac{1}{4}\sum_{i=0}^{3}\sum_{j=0}^{3} a_{i,j}(\tau)\sigma_i \otimes \sigma_j. \tag{24}$$

By substituting Eq. (24) into Eq. (4), we can obtain the non-trivial coupled differential equations

$$a'_{0,0}(\tau) = 0, \tag{25}$$
$$a'_{0,3}(\tau) = -4A_1 a_{0,3}(\tau) - 4B_1 a_{0,0}(\tau) - 2B_2 a_{1,1}(\tau) + 2B_2 a_{2,2}(\tau),$$
$$a'_{1,1}(\tau) = -4A_1 a_{1,1}\tau + 4A_2 a_{3,3}(\tau) + 2B_2\left(a_{0,3}(\tau) + a_{3,0}(\tau)\right),$$
$$a'_{2,2}(\tau) = -4A_1 a_{2,2}(\tau) - 4A_2 a_{3,3}(\tau) - 2B_2\left(a_{0,3}(t) + a_{3,0}(\tau)\right),$$
$$a'_{3,0}(\tau) = -4A_1 a_{3,0}(\tau) - 4B_1 a_{0,0}(\tau) - 2B_2 a_{1,1}(\tau) + 2B_2 a_{2,2}(\tau),$$
$$a'_{3,3}(\tau) = 4A_2 a_{1,1}(\tau) - 4A_2 a_{2,2}(\tau) - 8A_1 a_{3,3}(\tau) - 4B_1\left(a_{0,3}(\tau) + a_{3,0}(\tau)\right). \tag{26}$$

Note that in the above differential equations, we just list the evolution equations for the elements of the X-type states, while the evolution equations of the other elements are not shown because they are trivial due to the initial condition and symmetry of density operator.

To measure the entanglement between the two atoms, we take concurrence [63] as the measurement. For the X-type states, the concurrence is analytically given by

$$C(\rho) = \max[0, \frac{1}{2}|a_{1,1}(\tau) + a_{2,2}(\tau)| - M_1, \frac{1}{2}|a_{1,1}(\tau) - a_{2,2}(\tau)| - M_2], \tag{27}$$

where $M_1 = \frac{1}{2}\sqrt{(1 + a_{0,3}(\tau) - a_{3,0}(\tau) - a_{3,3}(\tau))(1 - a_{0,3}(\tau) + a_{3,0}(\tau) - a_{3,3}(\tau))}$ and $M_2 = \frac{1}{2}\sqrt{(1 - a_{0,3}(\tau) - a_{3,0}(\tau) + a_{3,3}(\tau))(1 + a_{0,3}(\tau) + a_{3,0}(\tau) + a_{3,3}(\tau))}$.

Before the detailed investigation of the time evolution of entanglement, let us first examine the behaviors of the asymptotic state that can be obtained by assuming the rates of change of the coefficients in Eq. (26) to be zero. If the two atoms are separated with a distance L, we can solve the relevant equations straightly and get

$$a_{1,1}(\infty) = a_{2,2}(\infty) = 0, a_{0,3}(\infty) = a_{3,0}(\infty) = \frac{1 - e^{\omega/T}}{e^{\omega/T} + 1}, a_{3,3}(\infty) = \frac{\left(e^{\omega/T} - 1\right)^2}{\left(e^{\omega/T} + 1\right)^2}. \tag{28}$$

In this case, $M_1(\infty) = M_2(\infty) = -\frac{1}{\cosh(\omega/T)+1}$, which implies that in de Sitter spacetime and thermal Minkowski spacetime, the spatially separated atoms can not extract the entanglement in the infinite time limit, besides, the atoms with entanglement will get disentangled within a finite time. However, let us note that when the interatomic separation is vanishing, the asymptotic state is the initial state dependent, and in this case entanglement can be generated between the two atoms [26].

In the following, we will study the dynamics of the entanglement of the two atoms with different initial states in de Sitter and thermal Minkowski spacetime. We first analyze the separable initial state to find out whether the two atoms can get entangled during their evolution. Then in the case of the entangled initial state, we want to investigate how the entanglement between the two atoms decays. Note that from Eqs. (20) and (23) we can see that the distance-dependent parameters for the de Sitter spacetime case and that for the thermal Minkowski spacetime case are different. It implies that the relevant dynamics of the two-atom system should behave differently for these two cases. Thus, although a single detector in de Sitter spacetime behaves the same as that in thermal Minkowski spacetime [38–40], i.e., by using a single detector one cannot tell the de Sitter spacetime from the thermal Minkowski spacetime, the two-atom system may enable us to do that.

3.1. Two atoms initially prepared in ground state $|00\rangle$

To examine the creation of entanglement between two atoms, we assume that the two atoms are initially prepared in the ground state, i.e., $|00\rangle$. Let us first consider the case that the distance between the two atoms is extremely small ($L \to 0$). We can see from Eqs. (20) and (23) that in this case, the relevant parameters involved to the atomic state evolution are the same due to $A_1 = A_2$ and $B_1 = B_2$. Thus, the dynamics of the two-atom system behave the same for the de Sitter spacetime and the thermal Minkowski spacetime cases. Under this condition, Eq. (26) can be solved analytically

$$a_{0,3}(\tau) = \frac{P_1\left(e^{\frac{\omega}{2T}} + e^{\frac{\omega}{T}} + 1\right)\left(e^{\frac{\omega}{2T}} + 1\right) - P_2\left(2e^{\frac{\omega}{2T}} - 2e^{\frac{\omega}{T}} + e^{\frac{3\omega}{2T}} - 1\right) - 2P_3\left(e^{\frac{2\omega}{T}} - 1\right)}{2P_3\left(e^{\frac{\omega}{T}} + e^{\frac{2\omega}{T}} + 1\right)},$$

$$a_{1,1}(\tau) = \frac{P_4[2P_5 - e^{2\Gamma\tau\operatorname{csch}\left(\frac{\omega}{2T}\right)} + e^{2\Gamma\tau\operatorname{csch}\left(\frac{\omega}{2T}\right)+\frac{3\omega}{2T}} - e^{\frac{3\omega}{2T}} - 1]}{2P_6^4[2\cosh\left(\frac{\omega}{T}\right) + 1]},$$

$$a_{2,2}(\tau) = \frac{P_4[-2P_5 + e^{2\Gamma\tau\operatorname{csch}\left(\frac{\omega}{2T}\right)} - e^{2\Gamma\tau\operatorname{csch}\left(\frac{\omega}{2T}\right)+\frac{3\omega}{2T}} + e^{\frac{3\omega}{2T}} + 1]}{2P_6^4[2\cosh\left(\frac{\omega}{T}\right) + 1]},$$

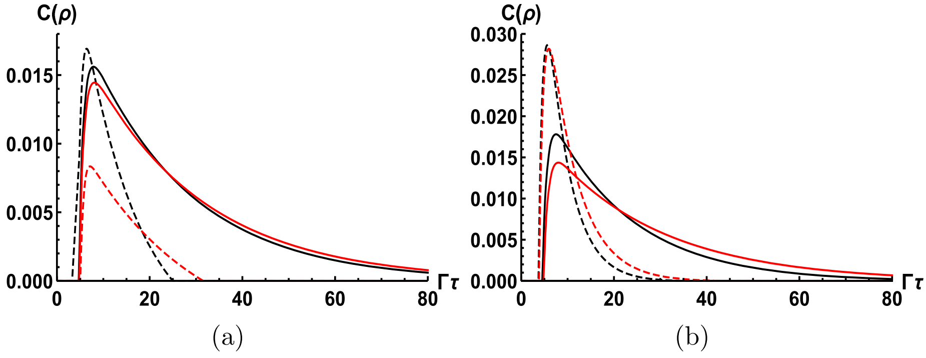

Fig. 1. Concurrence as a function of $\Gamma\tau$ for the two atoms initially prepared in ground state $|00\rangle$: (a) $T/\omega = \frac{1}{18}$ (solid lines) and $T/\omega = \frac{1}{6}$ (dashed lines) with $\omega L = \frac{1}{2}$, and (b) $\omega L = \frac{1}{2}$ (solid lines) and $\omega L = 1$ (dashed lines) with $T/\omega = \frac{1}{10}$. The black lines and red lines correspond to the de Sitter spacetime case and the thermal Minkowski spacetime case, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$a_{3,0}(\tau) = \frac{P_1\left(e^{\frac{\omega}{2T}} + e^{\frac{\omega}{T}} + 1\right)\left(e^{\frac{\omega}{2T}} + 1\right) - P_2\left(2e^{\frac{\omega}{2T}} - 2e^{\frac{2\omega}{T}} + e^{\frac{3\omega}{2T}} - 1\right) - 2P_3\left(e^{\frac{2\omega}{T}} - 1\right)}{2P_3\left(e^{\frac{\omega}{T}} + e^{\frac{2\omega}{T}} + 1\right)},$$

$$a_{3,3}(\tau) = \frac{P_7(-e^{\frac{3\omega}{2T}} + 1) + P_8\left(e^{\frac{3\omega}{2T}} + 1\right) - e^{\frac{\omega}{T}} + e^{\frac{2\omega}{T}} + 1}{e^{\frac{\omega}{T}} + e^{\frac{2\omega}{T}} + 1}, \tag{29}$$

where $P_1 = e^{\Gamma\tau[2\coth(\frac{\omega}{2T}) + \mathrm{csch}(\frac{\omega}{2T})] + \frac{\omega}{T}}$, $P_2 = e^{\Gamma\tau[2\cosh(\frac{\omega}{2T}) - 1]\mathrm{csch}(\frac{\omega}{2T}) + \frac{\omega}{T}}$, $P_3 = e^{4\Gamma\tau\coth(\frac{\omega}{2T})}$, $P_4 = e^{\Gamma\tau[2\cosh(\frac{\omega}{2T}) - 1]\mathrm{csch}(\frac{\omega}{2T})}$, $P_5 = e^{\Gamma\tau[2\coth(\frac{\omega}{2T}) + \mathrm{csch}(\frac{\omega}{2T})]}$, $P_6 = \cosh[\Gamma\tau\coth(\frac{\omega}{2T})] + \sinh[\Gamma\tau\coth(\frac{\omega}{2T})]$, $P_7 = e^{\Gamma\tau[\mathrm{csch}(\frac{\omega}{2T}) - 2\coth(\frac{\omega}{2T})] + \frac{\omega}{T}}$, $P_8 = e^{\frac{\omega}{T} - \Gamma\tau[2\coth(\frac{\omega}{2T}) + \mathrm{csch}(\frac{\omega}{2T})]}$. By substituting the above state parameters (29) into the measurement of entanglement (27), we can find that there is no generated entanglement for the two atoms when their distance vanishes. Let us note that the similar conclusion has been reported for the two accelerated atoms initially prepared at the ground state with vanishing distance [31]. Besides, when $L \to \infty$ ($L \gg \frac{1}{\omega}$), we find $A_2 = B_2 = 0$ for both the de Sitter spacetime case and the thermal Minkowski spacetime case. It implies that the two atoms respectively interact with two identical and independent thermal bath of quantum field with temperature $T = \frac{1}{2\pi\kappa}$. In this case, there is still no entanglement created between the atoms.

When the distance for the two atoms is not vanishing, we can solve Eq. (26) numerically. In Fig. 1, for the fixed temperature and distance we plot the atomic entanglement as a function of the evolution time. It's interesting that at some time the atoms could suddenly get entangled. The entanglement delay sudden birth derives from the interaction between atoms and massless scalar field, in the dark period when there is generated non-classic correlation but the states are still separable states. The entanglement sudden birth is associated with entanglement sudden death [64]. The lifetime of the existence of entanglement is related to the atomic distance and the temperature of thermal bath. Besides, we can see from the figure that the conditions for creation of entanglement, the maximum and the lifetime of created entanglement are different for the two spacetime cases. To detailedly address the issue of entanglement creation for both the de Sitter spacetime case and the thermal Minkowski spacetime case, in Fig. 2 we show the range of temperature and two-atom distance within which entanglement can be generated for two atoms in

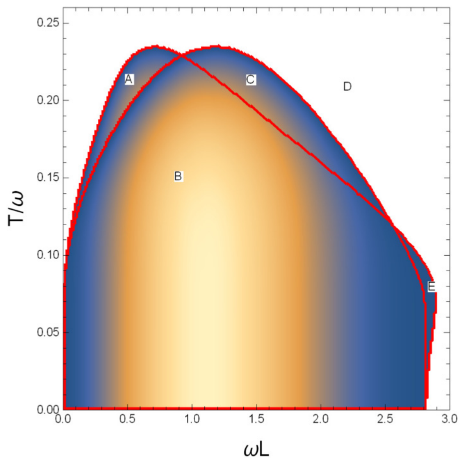

Fig. 2. Entanglement profile for two-atom systems initially prepared in $|00\rangle$. Region A: two atoms in de Sitter spacetime can get entangled while two atoms in thermal Minkowski spacetime can not. Region B: two atoms in both de Sitter spacetime and thermal Minkowski spacetime can get entangled. Region C: two atoms in thermal Minkowski spacetime can get entangled while two atoms in de Sitter spacetime can not. Region D: neither two atoms in de Sitter spacetime nor two atoms in thermal Minkowski spacetime can get entangled. Region E: two atoms in de Sitter spacetime can get entangled while two atoms in thermal Minkowski spacetime can not.

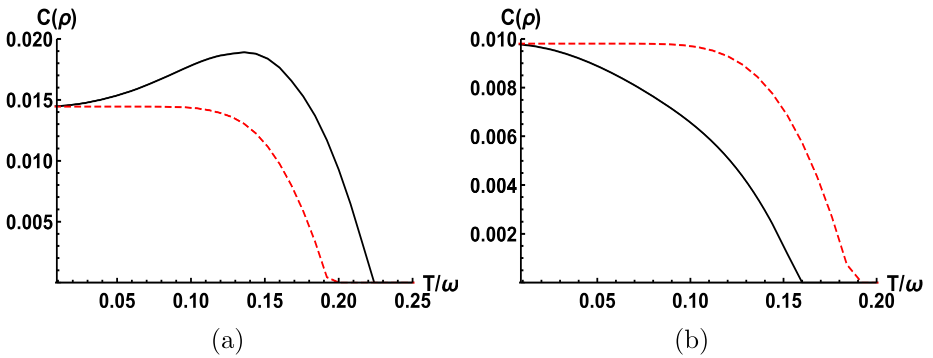

Fig. 3. The maximum of concurrence during evolution for two atoms in de Sitter spacetime (solid black lines) and thermal Minkowski spacetime (dashed red lines) initially prepared in $|00\rangle$ with $\omega L = \frac{1}{2}$ (a) and $\omega L = 2$ (b). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

de Sitter spacetime and thermal Minkowski spacetime initially prepared in state $|00\rangle$. We can see that the two atoms could get entangled only when the temperature and two-atom distance are relatively small, i.e., there exists upper bounds of temperature and interatomic separation larger than which entanglement cannot be generated. In addition, Fig. 2 shows another fact that the possible region of entanglement generation for two atoms in de Sitter spacetime does not completely overlap with that for two atoms in thermal Minkowski spacetime. That is, for certain conditions, two atoms in de Sitter spacetime can get entangled while two atoms in thermal Minkowski spacetime can not and vice versa.

In Fig. 3 we study the maximum of the created entanglement when the two atoms are initially prepare in the ground state. We find that for two atoms in the thermal Minkowski spacetime,

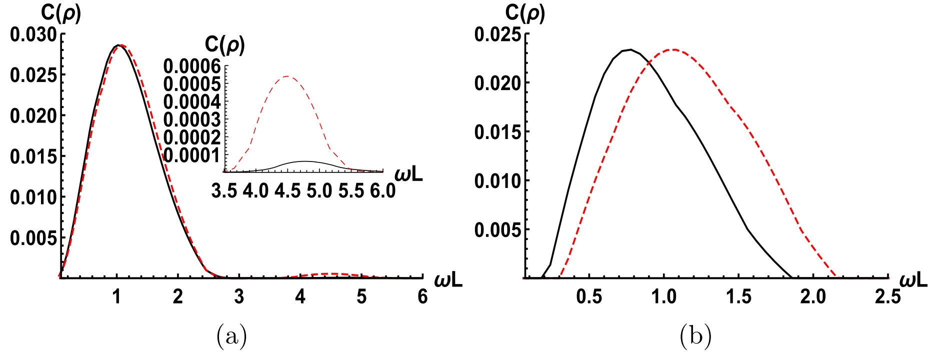

Fig. 4. The maximum of concurrence during evolution for two atoms in de Sitter spacetime (solid black lines) and thermal Minkowski spacetime (dashed red lines) initially prepared in $|00\rangle$ with $T/\omega = \frac{1}{18}$ (a) and $T/\omega = \frac{1}{6}$ (b). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the maximum of concurrence at fixed interatomic distance always decreases as the temperature grows. When the temperature is relatively small, the maximum of concurrence varies very slowly with temperature. However, for two atoms in de Sitter spacetime, the maximal concurrence does not always decrease with temperature, the increase or decrease of maximal concurrence with temperature depends on the specific value of the two-atom distance. Furthermore, the maximal concurrence of two atoms in thermal Minkowski spacetime may exceed that of two atoms in de Sitter spacetime. We give a brief approximate analysis of how this happens when the temperature is small. In the limit of small temperature, the spontaneous excitations can be neglected and the factor $\coth(\frac{\omega}{2T})$ approximates to 1, which leads to

$$A_1 = B_1 = \frac{\Gamma}{4}, \quad A_2 = B_2 = \frac{\Gamma}{4}m, \tag{30}$$

where m is the modulating function. Then we can solve the differential Eq. (26) straightly. For simplicity, we ignore the specific solution here. For the case of atoms in de Sitter spacetime, $m = f(\omega, L/2)$ is temperature dependent, thus the maximum of concurrence can either increase or decrease with temperature depending on the specific value of L. While for the case of atoms in thermal Minkowski spacetime, $m = \frac{\sin(L\lambda)}{L\lambda}$ is temperature independent, which accounts for why the maximum of concurrence is almost a constant for small temperature.

In Fig. 4, we plot the dependence of created entanglement on the interatomic distance. It is shown that there always exists a minimum and a maximum interatomic separation within which the atoms can be entangled for the atoms both in de Sitter spacetime and in thermal Minkowski spacetime. When $T/\omega = \frac{1}{18}$, there is a dark interval that entanglement cannot be created, as shown in Fig. 4(a). After the dark interval, there exists a distance where entanglement revives slightly, and revival amplitude of two atoms in de Sitter spacetime is smaller than that of two atoms in thermal Minkowski spacetime. Moreover, when $T/\omega = \frac{1}{6}$, it is found that the critical interatomic distances at which the sudden birth or the sudden death of entanglement occur are quite different for these two spacetime cases.

3.2. Two atoms initially prepared in entangled states

In order to study how the initial entanglement between two atom evolves in the de Sitter spacetime and thermal Minkowski spacetime, we assume that the two atoms are initially prepared in

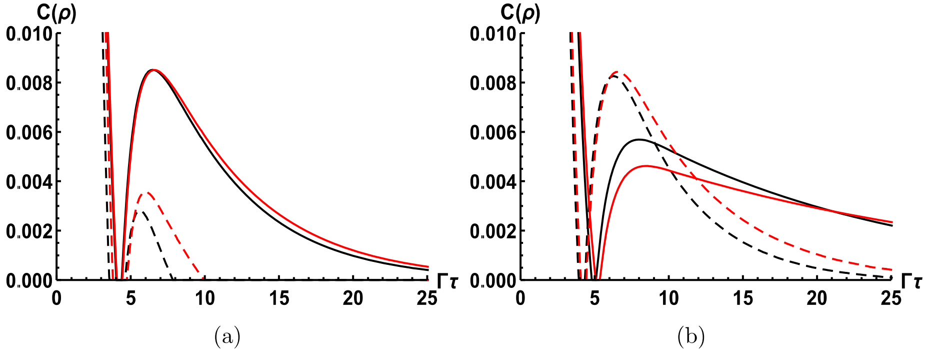

Fig. 5. Concurrence as a function of $\Gamma\tau$ when the two atoms are initially prepared in state $\frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$: (a) $T/\omega = \frac{1}{18}$ (solid lines) and $T/\omega = \frac{1}{6}$ (dashed lines) with $\omega L = 1$, and (b) $\omega L = \frac{1}{2}$ (solid lines), $\omega L = 1$ (dashed lines) with $T/\omega = \frac{1}{10}$. The black lines and red lines correspond to de Sitter spacetime and thermal Minkowski spacetime cases, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

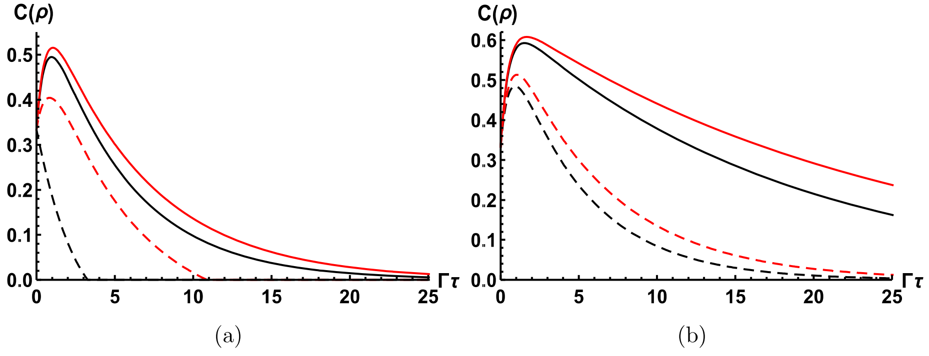

Fig. 6. Concurrence as a function of $\Gamma\tau$ when the two atoms are initially prepared in state $\frac{1}{\sqrt{3}}|\phi_3\rangle + \sqrt{\frac{2}{3}}|\phi_4\rangle$: (a) $T/\omega = \frac{1}{12}$ (solid lines) and $T/\omega = \frac{1}{3}$ (dashed lines) with $\omega L = 1$, and (b) $\omega L = \frac{1}{2}$ (solid lines), $\omega L = 1$ (dashed lines) with $T/\omega = \frac{1}{10}$. The black lines and red lines correspond to de Sitter spacetime and thermal Minkowski spacetime cases, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

an entangled state $\frac{1}{\sqrt{3}}|00\rangle + \sqrt{\frac{2}{3}}|11\rangle$. In Fig. 5 we plot the relevant dynamics of entanglement. It is found that the destroyed entanglement can be revived for the atoms both in de Sitter spacetime and in thermal Minkowski spacetime and entanglement experiences sudden birth and sudden death. More specifically, the entanglement between two atoms first decreases with evolution time to zero, and vanishes for a short time, then it suddenly appears and grows to a maximal value, finally it decays with evolution time again. Similarly, it can be observed that the lifetime of entanglement decreases as temperature and interatomic separation rise.

Instead of the basis, $|00\rangle$ and $|11\rangle$, here we use the Bell state, $|\phi_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ and $|\phi_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, as the basis to construct the entangled initial state for the two atoms, $\frac{1}{\sqrt{3}}|\phi_3\rangle + \sqrt{\frac{2}{3}}|\phi_4\rangle$. In Fig. 6, we plot the dynamics of entanglement for this initial state case.

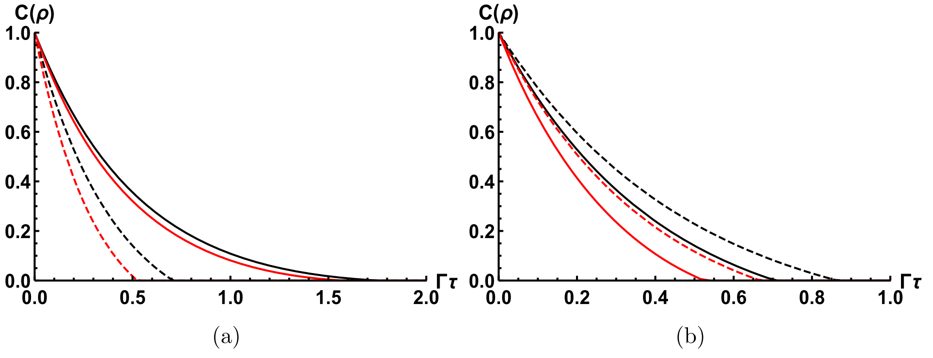

Fig. 7. Concurrence as function of $\Gamma\tau$ when the two atoms are initially prepared in state $|\phi_3\rangle$: (a) $T/\omega = \frac{1}{5}$ (solid lines) and $T/\omega = \frac{1}{2}$ (dashed lines) with $\omega L = 1$, and (b) $\omega L = 1$ (solid lines), $\omega L = 2$ (dashed lines) with $T/\omega = \frac{1}{2}$. The black lines and red lines correspond to de Sitter spacetime and thermal Minkowski spacetime cases, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

It can be seen that the entanglement firstly undergoes a enhancement of a short time, and then degrades with evolution time. However, when $T/\omega = \frac{1}{3}$, two atoms in de Sitter spacetime does not exist growth for $L = 1$ case (see Fig. 6(a)). In addition, the lifetime of entanglement decreases with increasing temperature and two-atom distance. The entanglement for two atoms in the de Sitter spacetime decays more faster than that for two atoms in the thermal Minkowski spacetime.

We assume the two atoms are at the maximally entangled state $|\phi_3\rangle$ and show the relevant dynamic evolution of entanglement in Fig. 7. We can see that entanglement decays rapidly with the evolution time for two atoms in both spacetimes. The larger the temperature is, the faster the entanglement decays, but the variation of entanglement with the interatomic distance is not the case. Besides, the entanglement for two atoms in the de Sitter spacetime decays more slowly than that for two atoms in the thermal Minkowski spacetime. For small temperature, the entanglement dynamics of the two atoms behaves quite similarly for the two spacetime cases (see Fig. 7(a)). We can arrive at this conclusion by expanding and analyzing the factor $f(\omega, L/2)$ (19) with respect to a quite small temperature T,

$$f(\omega, L/2) = \frac{\sin(L\omega)}{L\omega} - \left(\frac{1}{6}\pi^2 L^2 \cos(L\omega) + \frac{\pi^2 L \sin(L\omega)}{2\omega}\right) T^2 + O\left(T^4\right). \tag{31}$$

Obviously, the zeroth-order term is exactly the same form as the case of the thermal Minkowski spacetime. Thus, as the temperature increases, the entanglement behaviors of two atoms in the de Sitter spacetime becomes more distinguishable from that of two atoms in the thermal Minkowski spacetime. Note that the same situation happens for other initial states (see Fig. 4(a) and Fig. 5(a)).

In Fig. 8 we compare the entanglement dynamics for different initial entangled state cases when the interatomic distance $L \to 0$. It is found that: (1) when atoms is initially prepared in state $\frac{1}{\sqrt{3}}|\phi_3\rangle + \sqrt{\frac{2}{3}}|\phi_4\rangle$, entanglement is enhanced and develops to a stable value; (2) when two atoms initial state is $\frac{1}{2}|\phi_4\rangle + \frac{\sqrt{3}}{2}|\phi_3\rangle$, entanglement first degrades to zero and then is generated. Besides, higher temperature will induce the entanglement to disappear more earlier, and be cre-

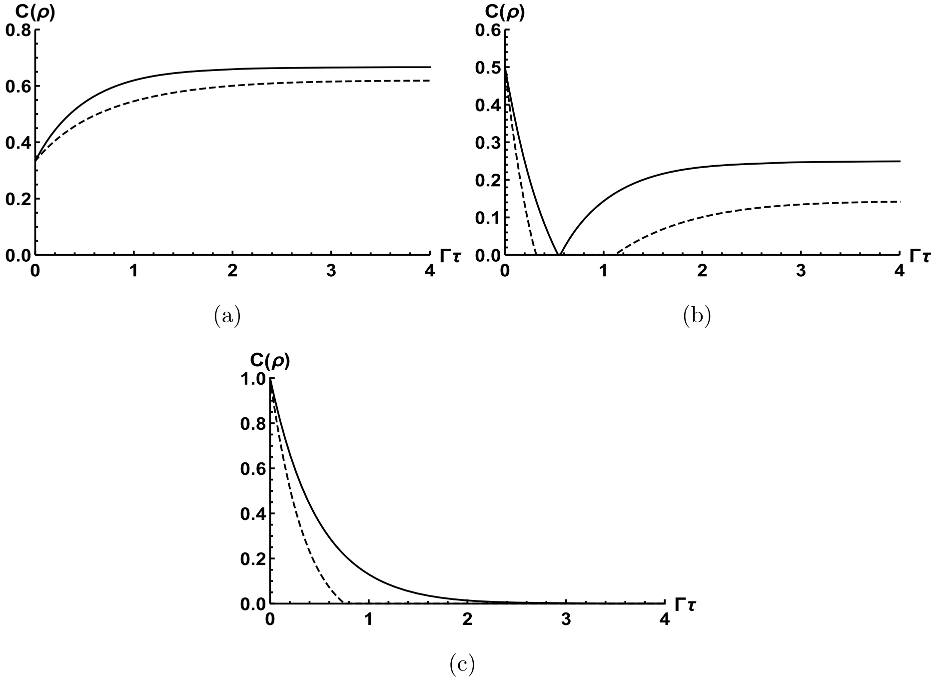

Fig. 8. Concurrence as function of $\Gamma\tau$ for $T/\omega = \frac{1}{10}$ (solid lines) and $T/\omega = \frac{1}{3}$ (dashed lines) with $L \to 0$ for different initial state cases: (a) $\frac{1}{\sqrt{3}}|\phi_3\rangle + \sqrt{\frac{2}{3}}|\phi_4\rangle$, (b) $\frac{1}{2}|\phi_4\rangle + \frac{\sqrt{3}}{2}|\phi_3\rangle$ and (c) $|\phi_3\rangle$. Note that in the limit $L \to 0$, the entanglement dynamics of two atoms in de Sitter spacetime and Minkowski spacetime behave the same.

ated later; (3) for the maximal initial entangled state $|\phi_3\rangle$, entanglement reduces monotonously with evolution time.

3.3. Two atoms initially prepared in the Werner state

Now, we consider that the initial state of two atoms is the Werner state $p|\phi_1\rangle\langle\phi_1| + (1-p)\frac{I}{4}$, $p \in [0, 1]$, where $|\phi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Werner state [65] is an important kind of X states and is discussed by many literatures [55,66–69]. Werner state is separable for $p \leq \frac{1}{3}$, but is quantum correlated except for $p = 0$ [55,67].

In Fig. 9, we plot the entanglement as a function of the evolution time by fixing the temperature and the interatomic distance. We can see that in this case the entanglement could be generated for initial separable state case and the lifetime of entanglement shortens as the temperature and the two-atom distance increase. Furthermore, with the increase of the evolution time the created entanglement for two atoms in the de Sitter spacetime decays more faster than that for two atoms in the thermal Minkowski spacetime. Similar to the analysis for the state $|00\rangle$ case in Fig. 2, we also show the range of temperature and interatomic distance within which entanglement can be generated for two atoms in de Sitter spacetime and thermal Minkowski spacetime initially prepared in the Werner state. It is found that the two atoms in de Sitter spacetime and thermal Minkowski spacetime have different entangled regions. When the temperature and in-

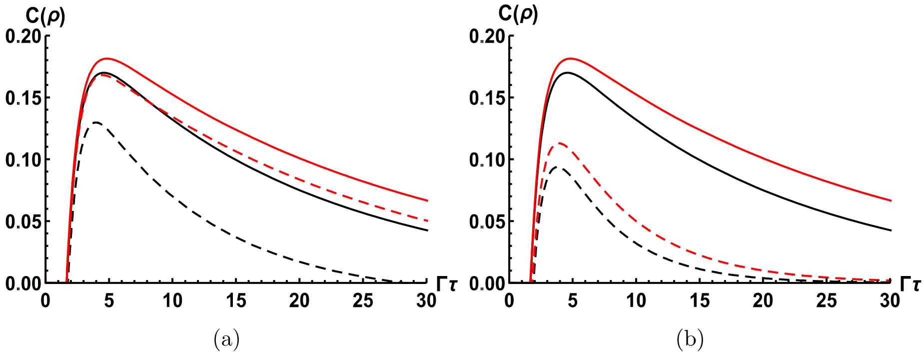

Fig. 9. Concurrence as function of $\Gamma\tau$ for the initial Werner state $p|\phi_1\rangle\langle\phi_1| + (1-p)\frac{I}{4}$ case when $p = \frac{1}{10}$: (a) $T/\omega = \frac{1}{10}$ (solid lines) and $T/\omega = \frac{1}{5}$ (dashed lines) with $\omega L = \frac{1}{2}$; and (b) $\omega L = \frac{1}{2}$ (solid lines) and $\omega L = 1$ (dashed lines) with $T/\omega = \frac{1}{10}$. The black lines and red lines correspond to de Sitter spacetime and thermal Minkowski spacetime cases, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

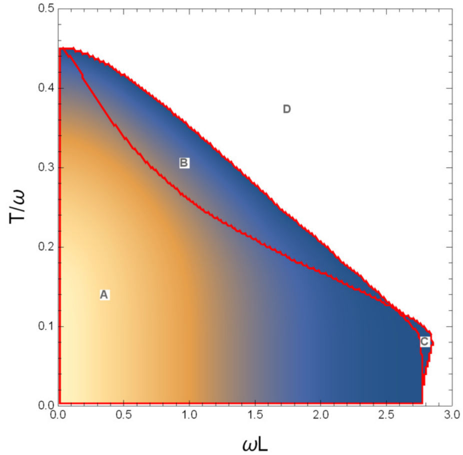

Fig. 10. Entanglement profile for two-atom systems initially prepared in the Werner state $p|\phi_1\rangle\langle\phi_1| + (1-p)\frac{I}{4}$ when $p = \frac{1}{10}$. Region A: two atoms in de Sitter spacetime and thermal Minkowski spacetime can get entangled. Region B: two atoms in thermal Minkowski spacetime can get entangled while two atoms in de Sitter spacetime can not. Region C: two atoms in de Sitter spacetime can get entangled while two atoms in thermal Minkowski spacetime can not. Region D: neither two atoms in de Sitter spacetime nor two atoms in thermal Minkowski spacetime can get entangled.

teratomic distance exceed the certain threshold value, entanglement can not be generated. (See Fig. 10.)

Unlike the separable state $|00\rangle$, Fig. 11 shows that for the separable Werner state $p|\phi_1\rangle\langle\phi_1| + (1-p)\frac{I}{4}$ with $p = \frac{1}{10}$, after one dark period, entanglement is generated suddenly and evolves to a stable value for the case of two atoms separation being very small ($L \to 0$).

4. Conclusion

We have studied the entanglement dynamics of two atoms in de Sitter spacetime and thermal Minkowski spacetime for various initial states. It is found that: (1) for the initial separable state,

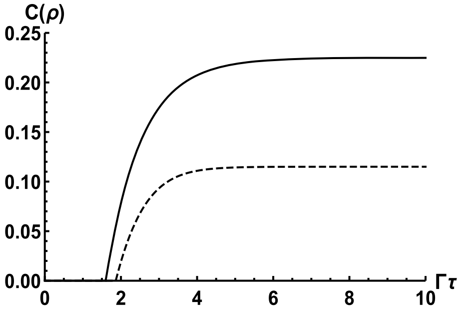

Fig. 11. Concurrence as function of $\Gamma\tau$ for $T/\omega = \frac{1}{10}$ (solid lines) and $T/\omega = \frac{1}{3}$ (dashed lines) with $L \to 0$ for initial Werner state $p|\phi_1\rangle\langle\phi_1| + (1-p)\frac{I}{4}$ when $p = \frac{1}{10}$ of two atoms in de Sitter spacetime and thermal Minkowski spacetime.

entanglement can be generated, and the lifetime of the created entanglement is temperature- and interatomic distance-dependent; (2) for some initial superposition entangled states, entanglement appears revival, delayed sudden birth and enhancement phenomena, and the lifetime of entanglement shortens with increasing temperature and interatomic separation; (3) for the initial maximal entangled state, entanglement decays quickly with the evolution time. Furthermore, the entanglement dynamics of two atoms for these two spacetime cases behave quite differently. In particular, the two atoms interacting with the field in the thermal Minkowski spacetime (with the field in the de Sitter-invariant vacuum), under certain conditions, could be entangled, while they would not become entangled in the corresponding de Sitter case (in the corresponding thermal Minkowski case). Thus, using the different dynamic evolution behaviors of entanglement for two atoms, one can in principle distinguish the de Sitter universe from the thermal Minkowski universe. If we obtain some concurrence data about the evolution of atoms, through these data, we can judge whether there exists entanglement under specified conditions to determine which spacetime the atoms are.

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