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Gravitational effects on quantum correlations in three-flavor neutrino oscillations

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Abstract In this paper, we explore the quantum properties of three-flavor neutrino propagating in a Schwarzschild metric. It is found that the different strength of gravitational effects are obtained by adjusting the magnitude of GM arising in the oscillation phase. Using the weak field approximations, we show that the gravitational effects can make the entanglement oscillates over a large rage when $\mathrm{GM} = 5.1 \times 10^8$ Km and $\mathrm{GM} = 7 \times 10^7$ Km, respectively, Moreover, for $\mathrm{GM} = 4.8\times10^8$ Km and $\mathrm{GM} = 9.3\times10^7$ Km, the suppression of the entanglement can be observed due to the gravitational effects. Meanwhile, in this case, the gravitational effects also make the distribution of entanglement tighter through investing the entanglement complete monogamy relation. Furthermore, we examine the gravitational effects on the violation of the Svetlichny inequality to study the nonlocality of the system. It is shown that when $\mathrm{GM} = 6\times10^8$ Km and $\mathrm{GM} = 7\times10^7$ Km, the gravitational effects make the Svetlichny parameters always greater than 4, implying that the genuine tripartite nonlocality of the system is always present. However, the gravitational effects also restrain the violation of the Svetlichny to make the regions of the absence of nonlocality increase. The gravitational effects on the monogamy property of nonlocality lies in the change of the effective bound of the maximum bipartite nonlocality of the neutrinos system. Therefore, our investigations may be helpful to understanding of quantumness of the neutrinos system in curved space-time.

1 Introduction

The unification of quantum mechanics with gravitation is one of the most challenging problems in theoretical physics. One of the main causes is the absence of experimental evidence of quantum features of gravitation. Only a few experiments have been used to detect quantum mechanics in a classical gravitational field. Collela et al. [1] first proposed an experiment in which gravity-induced quantum phase shift of neutrons is obtained to show the neutrons follow quantum mechanics in a gravitational field. Since then, Stodolsky [2] further studied the quantum mechanics in a gravitational, obtaining the quantum mechanics phase concerned with the propagating of a free particle in an external gravitational field using semiclassical approximation. Except for the interferometry of neutrons and photons, the other subject characterizing the coupling effect of gravitational effects and quantum mechanics is neutrino oscillations which is a well known phenomenon and establish the nonzero mass of neutrinos. Today it is generally accepted that neutrinos is weakly interacting particles with the ability to penetrate into ordinary matter with minimum interactions, and that they take the variation among three flavors v_e, v_μ, v_τ during propagation [3–7]. These properties render neutrinos most charming particles of the stand model and make them special probes for the applications of quantum scales.

In multipartite quantum system, one of the most important properties in studying multipartite entanglement is that entanglement is monogamous, which means that quantum entanglement in general cannot be shared by different parties. This limited shareability of entanglement was first quantified by Coffman et al. [8], using a three-qubit monogamy relation for the squared concurrence, $C^2_{A|BC} \geq C^2_{AB} + C^2_{AC}$, generally known as CKW inequality. Since then, several similar monogamy relations of other quantum entanglement mea-

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sures have been established [9–15]. Recently, Guo et al. [16] proposed a complete monogamy relation which adds the global entanglement in ABC and the bipartite entanglement in part BC that the original CKW inequality is missed. On the other hand, Bell nonlocality [17] as a property of quantum correlations have been studied by various avatars of Bell inequality in multipartite system, such as Mermin and Svetlichny inequalities. Such manifestation of Bell inequality plays an important role, for example, when deeling with the three-flavor neutrino oscillations. Similar to entanglement, the nonlocality cannot be arbitrarily shared among subsystems, which means that nonlocality is also monogamous. The notion of monogamy of nonlocality has motivated some theoretical research to explore how effectively bipartite nonlocality is shared in multipartite system [18–23].

One of the subjects in neutrino physics is investing quantum properties of mixed flavor state, which contribute to explore the possibility of utilizing neutrino as a resource in quantum information processing. In this line, harnessing the tools of quantum resource theory to explore the quantum nature of neutrino oscillations in terms of oscillation probabilities has been analyzed widely [24–41]. The effects of gravitational field on neutrino oscillation have been studied through reckoning the transformation of quantum-mechanical phase for neutrino propagation in curved space-time [42–44]. On the other hand, studying the effect of gravity on the phase shift of neutrino oscillation is also a fascinating topic [45–52]. Furthermore, one can investigate the gravitational effect on the quantumness of neutrino oscillation. In this context, Ettefaghi et al. [53] have studied the gravitational effects on quantum coherence in neutrino oscillation analyzing from the perspectives of qualitative (through investing the violation of Leggett–Garg inequality) and quantitative (by calculating l_1-norm coherence). In that work, they find that the gravitational effects change the ranges of time evolution of the violation of Leggett–Garg inequality but dose not alter the maximum amount of the quantum coherence, providing the signs of a nontrivial role of gravitational effects on quantum correlations in the context of neutrino oscillations. Besides, it is worth to explore gravitation effects on quantumness of neutrino oscillation from other types of quantum correlation measures.

Although the gravity induced neutrino oscillations has invited many investigations conducted from the different aspects of neutrino oscillations, there have been few researches concerning the gravity effect on the quantum nature of neutrino oscillation, which will be helpful to study the performances of the quantum resources in the curved space-time. This promoted us to analyze how various quantum correlations measures to be reassessed in the neutrino system via gravitational effects. In this paper, we focus on the gravitational effects on spatial correlations exhibited by the neutrino-system employing entanglement and nonlocality measures and their distributions in three-flavor neutrino oscillations in a Schwarzschild background via the plane wave approach. Here, by adjusting the gravity parameter arising in the oscillation phase to control different gravitational strength, we show that the value of entanglement and nonlocalty correlations shows different behaviors in the presence of gravitational effects. For example, in the case of the neutrinos propagating radially outwards the gravitational source, when GM $= 5.1 \times 10^8$ Km, the entanglement can oscillate over a wide range while there exist a decrease in its maximal value in comparison to that in the flat space-time when GM $= 4.8 \times 10^8$ Km. Moreover, when GM $= 6 \times 10^8$ Km, the Svetlichny parameters is always greater than 4 while the time regions of the Svetlichny parameters greater than 4 are decrease for GM $= 4.8 \times 10^8$ Km. The results show that the gravitational effects can facilitate and suppress the entanglement and nonlocality rather than just generate a phase shift in correlation measures. This work provides an insight into the better understanding of the role of gravitation effects in the study of quantumness of neutrino oscillations.

The plan of this paper is organized as follows: in Sect. 2 we give a brief description of the model of three-flavors neutrino oscillation in flat and curved space-time. In Sect. 3 we will study properties of tripartite quantum correlations in radial propagation of neutrinos in a Schwarzschild metric, where we use the tripartite entanglement of formation (EOF) and the complete monogamy relation to investigate the entanglement property of the system, and exploit the Svetlichny inequality and nonlocality monogamy to study the nonlocality property of the system. Finally in Sect. 4, we will discuss our result and make the conclusion.

2 Three-flavor neutrino oscillations in flat and curved space-time

In this section, we will briefly introduction the three-neutrino mixing framework considered in flat and curved space time. The flavor state $|\nu_\alpha\rangle$ ($\alpha = e, \mu, \tau$) of a neutrino emitted via weak interaction at a space-time point $A(t_A, \vec{x}_A)$ is obtained from a coherent superposition of mass eigenstates, $|\nu_k\rangle$ ($k = 1, 2, 3$),

$$|\upsilon_\alpha\rangle = \sum_k U^*_{\alpha k} |\upsilon_k\rangle , \quad (1)$$

where U is the unitary PMNS (Pontecorvo-Maki-Nakagawa-Sakata) mixing matrix characterized by three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and a charge conjugation and parity (CP) violating phrase δ_{cp} and in case of the three flavors it can be shown as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{cp}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta_{cp}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{cp}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{cp}} & -c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{cp}} & c_{13}c_{23} \end{pmatrix}, \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ $(i, j = 1, 2, 3)$. The CP violating can be ignored due to the failure of observing it experimentally. There exist a relative shift of mass eigenstate phases when they reach the detector located in space-time point $B\left(t_B, \vec{x}_B\right)$, which can be described within the plane wave approach

$$\left|\nu_k(t, \vec{x}_B)\right\rangle = e^{-i\Phi_k}\left|\nu_k\right\rangle, \tag{3}$$

where the relative phase shift is given by

$$\Phi_k = E_k\left(t_B - t_A\right) - \vec{p}_k \cdot (\vec{x}_B - \vec{x}_A). \tag{4}$$

Therefore, if a neutrino is produced with a given flavor α at the space-time point $A(t_A, \vec{x}_A)$, using the Eqs. (1) and (3), the probability detected with flavor β at the detection point $B\left(t_B, \vec{x}_B\right)$ is given by

$$\begin{aligned} P_{\alpha\to\beta} &= \left|\left\langle \nu_\alpha \mid \nu_\beta\left(t_B, \vec{x}_B\right)\right\rangle\right|^2 \\ &= \delta_{\alpha\beta} - 4\sum_{k>j}\mathrm{Re}\left(\hat{U}^*_{\alpha k}\hat{U}_{\beta k}\hat{U}_{\alpha j}\hat{U}^*_{\beta j}\right)\sin^2\left(\frac{\Phi_{kj}}{2}\right) \\ &\quad + 2\sum_{k>j}\mathrm{Im}\left(\hat{U}^*_{\alpha k}\hat{U}_{\beta k}\hat{U}_{\alpha j}\hat{U}^*_{\beta j}\right)\sin\left(\frac{\Phi_{kj}}{2}\right), \end{aligned} \tag{5}$$

where $\Phi_{kj} = \Phi_k - \Phi_j$. Note that the difference of phase shift incapably cause the decoherence of the flavor eigenstates due to the propagation which means that the coherence length is larger than the corresponding oscillation length.

For relativistic neutrinos in flat space-time described by the Minkowski metric, one can obtain that

$$\Phi_{kj} \simeq \frac{\Delta m^2_{kj}}{2E_0}\left|x_B - x_A\right|, \tag{6}$$

where $\Delta m^2_{kj} \equiv \left|m^2_k - m^2_j\right|$ is the mass-squared difference and E_0 is the energy of the massless neutrino measured by the observer at rest at infinity.

In the scenario where the above formalism generalize to a curved spacetime, the quantum phase given by Eq. (4) can rewriting as its covariant form

$$\Phi_k = \int_A^B p^{(k)}_\mu dx^\mu, \tag{7}$$

where $p^{(k)}_\mu = m_k g_{\mu\nu}\frac{dx^\nu}{ds}$ is the canonical conjugate momentum corresponding to the coordinate x^μ, with $g_{\mu\nu}$ the metric tensor and ds the line element. Now, we focus on the case of neutrino propagating in a gravitational field within a Schwarzschild spacetime metric

$$ds^2 = B(r)dt^2 - B(r)^{-1}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2, \tag{8}$$

where

$$B(r) = \left(1 - \frac{2GM}{r}\right) \tag{9}$$

Here G is the Newtonian constant and M represents the mass of the source of the gravitational field. Further, due to the gravitational field is isotropic, the motion of neutrinos may be selected on the equatorial plane ($\theta = \pi/2$ and $d\theta = 0$). Therefore, the accumulated phase of the each mass eigenstate that propagate from the production point $A\left(t_A, r_A, \phi_A\right)$ to the detection point $B\left(t_B, r_B, \phi_B\right)$ may be represented as

$$\Phi_k = \int_A^B\left[E_k dt - p_k(r)dr - J_k d\phi\right], \tag{10}$$

where $E_k \equiv p^{(k)}_t$, $p_r \equiv -p^{(k)}_r$, and $J_k(r) \equiv -p^{(k)}_\phi$ are the components of the canonical momentum $p^{(k)}_\mu$. In the presence of gravity, it is applicable to take the neutrino propagation over its proper distance L_p, which is in general taken the form

$$\begin{aligned} L_p &\equiv \int_{r_A}^{r_B}\sqrt{g_{rr}}dr \\ &= r_B\sqrt{1 - \frac{2GM}{r_B}} - r_A\sqrt{1 - \frac{2GM}{r_A}} \\ &\quad + 2GM\left[\ln\left(\sqrt{r_B - 2GM} + \sqrt{r_B}\right)\right. \\ &\quad \left. - \ln\left(\sqrt{r_A - 2GM} + \sqrt{r_A}\right)\right]. \end{aligned} \tag{11}$$

For the simplicity of following discussion, considering the weak field approximation where L_p is directly approximated to

$$L_p \simeq r_B - r_A + GM\ln\frac{r_B}{r_A}, \tag{12}$$

where r_A and r_B are the position of the source and detector relative to the reference frame of the gravitational source. In order to use this result, the propagation of neutrinos can then be studied in a radial direction in a Schwarzschild gravitational field, which corresponds to $d\phi = 0$ and the vanish of angular momentum. As for the flat-space time, applying the relativistic expansion $m_k \ll E_k$ and assuming $0 < B(r) \leqslant 1$, we can calculate the phase of the kth mass eigenstate defined in Eq. (10) as follows [42]

$$\Phi_k \simeq \frac{m_k^2}{2E_0} |r_B - r_A| . \tag{13}$$

The phase shift that are responsible for the oscillation is, therefore, obtained by subtraction from Eq. (13) the corresponding expression for the jth mass eigenstate as

$$\Phi_{kj} \simeq \frac{\Delta m_{kj}^2}{2E_0} |r_B - r_A| . \tag{14}$$

Note that the energy measured by the observer at the detector at r_B is not E_0, but rather the local energy $E_0^{\mathrm{loc}}(r_B)$, which is defined by the relation

$$E_0^{\mathrm{loc}}(r_B) = \left(1 + \frac{GM}{r_B}\right) E_0. \tag{15}$$

Inserting the Eqs. (12) and (15) into the Eq. (14), the resulting phase shift is given by

$$\Phi_{kj} \simeq \left(\frac{\Delta m_{kj}^2 L_p}{2E_0^{\mathrm{loc}}(r_B)}\right)\left[1 - GM\left(\frac{1}{L_p}\ln\frac{r_B}{r_A} - \frac{1}{r_B}\right)\right] \tag{16}$$

Note that the second square parenthesis stands for the correction due to the gravitational effects where gravity effect term GM = 0 reduce to the case of the flat space time. In the following, we wish to study the effects of gravitational on quantum entanglement and nonlocality and their distribution in three flavor neutrino oscillation. For an appropriate evaluation, the best values of three flavor oscillation parameters are given by

$$\begin{aligned} \Delta m_{21}^2 &= 7.50 \times 10^{-5} eV^2, \\ \Delta m_{31}^2 &= 2.457 \times 10^{-3} eV^2, \\ 20\Delta m_{32}^2 &= 2.382 \times 10^{-3} eV^2, \\ \theta_{12} = 33.48^\circ, &\quad \theta_{23} = 42.3^\circ, \quad \theta_{13} = 8.50^\circ. \end{aligned} \tag{17}$$

Consequently, when neutrino radially propagate in the gravitational field, using the Eqs. (1)–(3) and (16), the neutrino flavor states evolution can be described as

$$|\psi(L_p)\rangle_\alpha = a_{\alpha e}(L_p)|\nu_e\rangle + a_{\alpha\mu}(L_p)|\nu_\mu\rangle + a_{\alpha\tau}(L_p)|\nu_\tau\rangle , \tag{18}$$

where $a_{\alpha\beta}(L_p) = \sum_k U_{\alpha k}^* e^{-i\phi_k(L_p)} U_{\beta k}$ is the transition amplitude. Here we used the flavor-qubit correspondence $|\nu_e\rangle \equiv |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau$, $|\nu_\mu\rangle \equiv |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau$ and $|\nu_\tau\rangle \equiv |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau$.

3 Quantum correlations in radial propagation of neutrino in a schwarzschild metric

In this section we are going to study the gravitational effect on the quantum entanglement and nonlocality in NO, and analyse the distribution of them by using the corresponding monogamy relations. In order to better facilitate the gravitational effect on the entanglement and nonlocality, we will be considering the following two different situations that the neutrinos propagate radially outwards and towards the gravitational source:

(i) For the case of neutrino propagate radially outward the gravitational source, the distance r_B that detector with respect to the gravitational source, as defined in Eq. (12), can be expressed in the form

$$r_B = L_p + r_A - GM \ln\left(\frac{L_p}{r_A} + 1\right). \tag{19}$$

Therefore the oscillation phases may be expressed in the weak field approximation as

$$\Phi_{kj}(L_p) \simeq \frac{\Delta m_{kj}^2 L_p}{2E_0^{\mathrm{loc}}} \left[1 - GM\left(\frac{1}{L_p}\ln\left(\frac{L_p}{r_A} + 1\right) - \frac{1}{L_p + r_A}\right)\right]. \tag{20}$$

(ii) For the case of neutrino propagation radially towards the gravitational source, the radial distance of detector takes the form

$$r'_B = r'_A - L_p - GM \ln\left(1 - \frac{L_p}{r'_A}\right). \tag{21}$$

Here we use the notation r'_A and r'_B to represent the radial distance of the neutrino source and detector in this case, respectively. Therefore, we can again obtain the oscillation phase as

$$\Phi_{kj}(L_p) \simeq \frac{\Delta m_{kj}^2 L_p}{2E_0^{\mathrm{loc}}} \left[1 - GM\left(\frac{1}{L_p}\ln\left(\frac{L_p}{r'_A} + 1\right) - \frac{1}{L_p + r'_A}\right)\right]. \tag{22}$$

In order to obtain the entanglement and nonlocality in terms of the propel distance L_p, we take $E_0^{\mathrm{loc}}(r_B) = 3 \times 10^2$TeV, and assume that $r_A = 10^8$ Km and 2×10^8 Km $\leq L_p \leq 4 \times 10^8$ Km for neutrino propagating outwards and

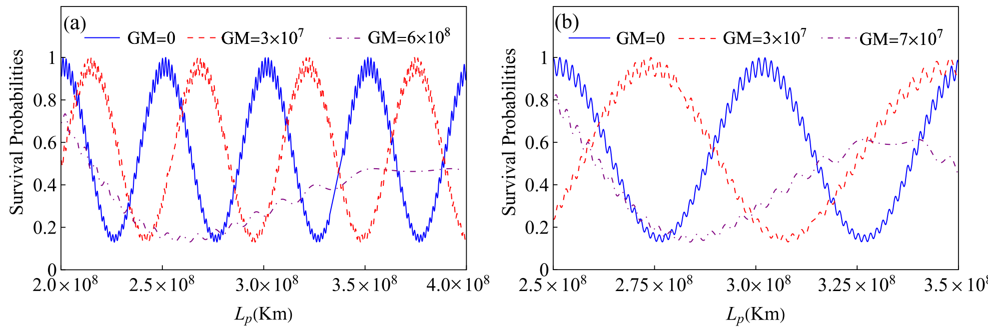

Fig. 1 The survival probabilities $P_{\nu_e\to\nu_e}$ as a function of L_p, for neutrinos radially propagating **a** outwards and **b** inwards to the gravitational source for different GM, where GM $= 0$ corresponds to the flat space time and nonzero GM corresponds to curved space time

$r'_A = 4\times 10^8$ Km and 2.5×10^8 Km $\leq L'_p \leq 3.5\times 10^8$ Km for neutrino propagating inwards.

3.1 Entanglement for neutrino oscillation in schwarzschild metric

Assuming a neutrino emission source and hypothetical detector respectively situated in the radial distance r_A and r_B from the center of the gravitational field source which is described by spherically symmetric Schwarzschild metric. Considering the case that the initial flavor state is $|\nu_e\rangle$ in which the evolution of the state can be obtained from Eq. (18) as

$$\left|\psi_e(L_p)\right\rangle = a_{ee}(L_p)|100\rangle + a_{e\mu}(L_p)|010\rangle + a_{e\tau}(L_p)|001\rangle, \tag{23}$$

Here the square of transition amplitude terms $a_{e\beta}(L_p)$ ($\beta = e, \mu, \tau$) represent the oscillation probabilities which are constrained by the normalization condition that $\sum_\beta P_{e\beta} = 1$, where the survival probabilities $P_{\nu_e\to\nu_e}$ versus the proper distance L_p with different values of GM are plotted in Fig. 1. It can be seen that for the case of the flat space-time, i.e. GM $= 0$, the survival probability can arrive at maximal value 1 as well as the case for the gravity effect term GM $= 3\times 10^7$ Km in both outwards and inwards propagation. However, for the case of GM $= 6\times 10^8$ Km and GM $= 7\times 10^7$ Km correspond to outwards and inwards propagation, respectively, there exists a damping in the maximum value of the survival probabilities, which implies that the gravitational effects suppress the survival probability respect to certain values of GM.

Now, to measure the entanglement of three flavor neutrino state, of form Eq. (23), we use the tripartite entanglement of formation (EOF) defined as follows [16]

$$E(\rho) = \frac{1}{2}\left[S(\rho_A) + S(\rho_B) + S(\rho_C)\right], \tag{24}$$

where $S(\rho_i) = -\mathrm{Tr}(\rho_i \log \rho_i)$ is the von Neumann entropy of the reduced density matrix $\rho_i = \mathrm{Tr}_{kj}(\rho)$ $(i, j, k \in \{A, B, C\})$ of the pure state ρ. Accordingly, we can calculate the systemic entanglement measured by tripartite EOF in terms of the transition probabilities, i.e.,

$$\begin{aligned} E(\rho^e_{e\mu\tau}) = -\frac{1}{2}\big[&P_{ee}(L_p)\log_2 P_{ee}(L_p) \\ &+ P_{e\mu}(L_p)\log_2 P_{e\mu}(L_p) + P_{e\tau}(L_p)\log_2 P_{e\tau}(L_p) \\ &+ \left(P_{e\mu}(L_p) + P_{e\tau}(L_p)\right)\log_2\left(P_{e\mu}(L_p) + P_{e\tau}(L_p)\right) \\ &+ \left(P_{ee}(L_p) + P_{e\tau}(L_p)\right)\log_2\left(P_{ee}(L_p) + P_{e\tau}(L_p)\right) \\ &+ \left(P_{e\mu}(L_p) + P_{ee}(L_p)\right)\log_2\left(P_{e\mu}(L_p) + P_{ee}(L_p)\right)\big]. \end{aligned} \tag{25}$$

Figure 2 shows the tripartite EOF versus the proper distance L_p for different gravity effect terms GM in both the case of neutrinos propagating radially outward and inward. We can observe that there exists a phase shift in the evolution of EOF in the curved space-time compared to the corresponding one in the flat space-time (GM $= 0$) due to the gravitational effects. This lead to some range of the parameter in which the value of entanglement in the curved space-time is larger than the one in the flat space-time. Nevertheless, it is interesting that, for the case of some certain values of GM, such as GM $= 5.1\times 10^8$ Km and GM $= 7\times 10^8$ Km for neutrinos radially propagating outwards and inwards, respectively, this behavior disappears and replaced by oscillating at some large value intervals of EOF, as observed numerically that the EOF is larger than 1.1 in the ranges $[2\times 10^8, 4\times 10^8]$ and $[2.6\times 10^8, 3.5\times 10^8]$ for neutrinos radially propagating outwards and inwards, respectively. A notable result is that the gravitational effects lead to a damping in the maximum values of the EOF in the curved space-time in comparison to the corresponding one in the flat space-time when GM $= 4.8\times 10^8$ Km and GM $= 9.3\times 10^7$ Km corresponding to outwards and inwards propagation respectively. The reason of causing above different cases is as follows: we treated neutrinos in the plane waves way in this study and the corresponding phases have been revised by the gravitational effects. Therefore, when we choose different values of GM, the phase of constructing entanglement presents two diverse cases that they are summed constructively or summed destructively, which causes different magnitudes of

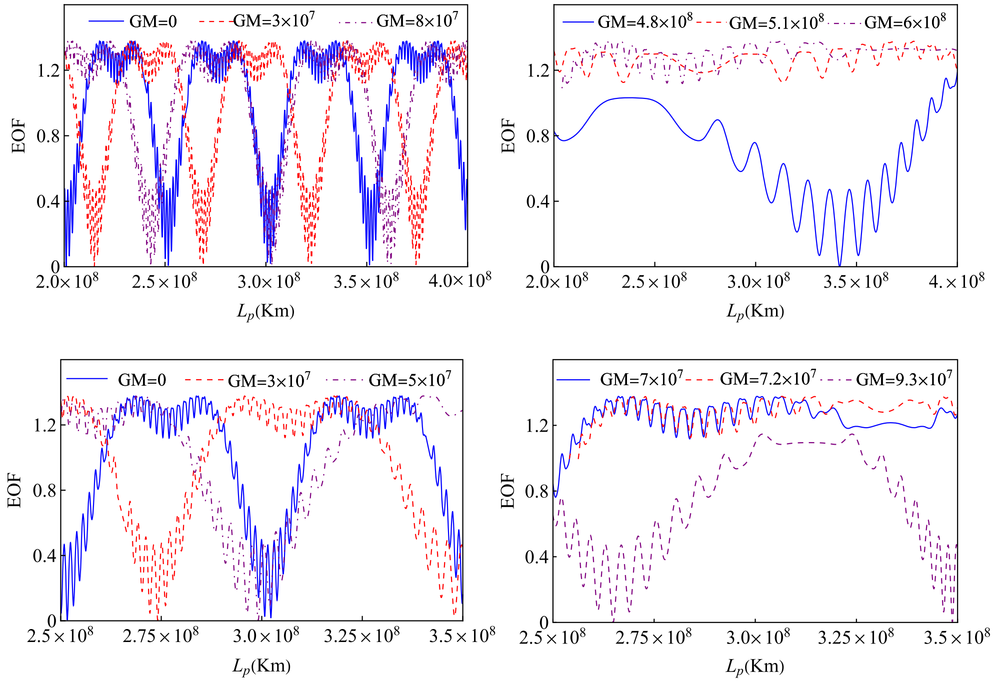

Fig. 2 Variations of tripartite EOF as a function of L_p for different GM, where GM $= 0$ corresponds to the flat space time and nonzero GM corresponds to curved space time. Top: corresponding to neutrinos radially propagating outwards to the gravitational source. Bottom: corresponding to neutrinos radially propagating inwards to the gravitational source

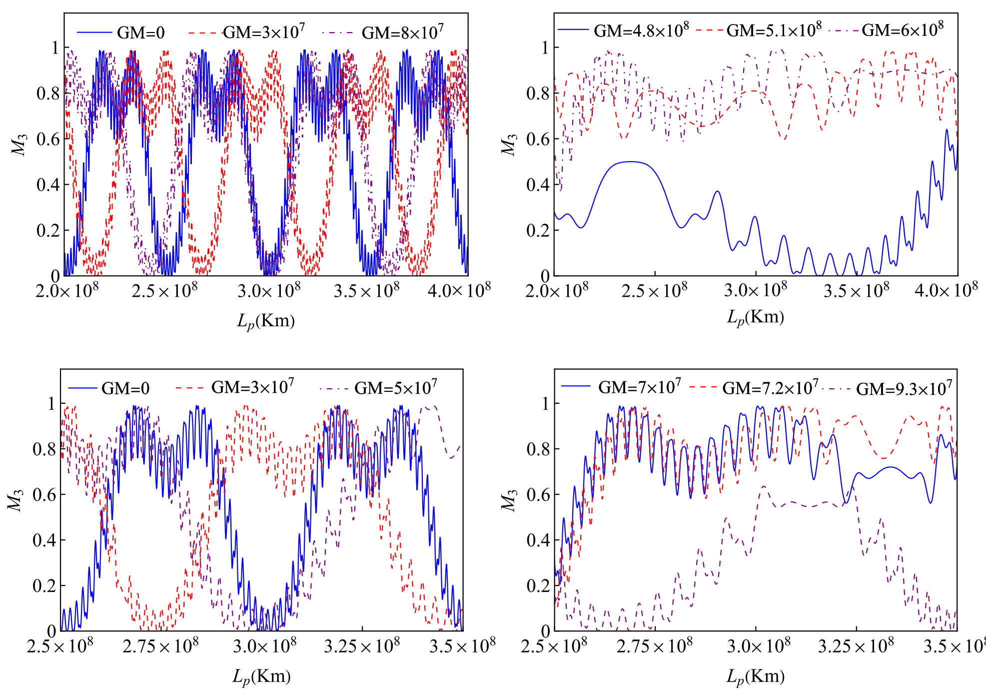

Fig. 3 Complete monogamy of entanglement with respect to the proper distance (L_p) traveled by neutrinos with different GM, where GM $= 0$ corresponds to the flat space time and nonzero GM corresponds to curved space time. Top: corresponding to neutrinos radially propagating outwards to the gravitational source. Bottom: corresponding to neutrinos radially propagating inwards to the gravitational source

the entanglement during propagation. The multipartite entanglement can also be seen from the complete monogamy relation, which reads [16]

$$E^2(\rho_{ABC}) \geqslant E^2(\rho_{AB}) + E^2(\rho_{AC}) + E^2(\rho_{BC}), \tag{26}$$

Here $E_{\rho_{AB}}$ is the bipartite EOF as well as $E_{\rho_{AC}}$ and $E_{\rho_{BC}}$, which is described as $E_f(\rho_{AB}) = \min_{\{p_i, |\phi_i\rangle_{AB}\}} \sum_i p_i E_f(|\phi_i\rangle_{AB})$. The minimum is taken over all decompositions of $\rho_{AB} = \sum_i p_i |\phi_i\rangle_{AB} \langle\phi_i|$ with $p_i \geq 0$, $\sum_i p_i = 1$. For two-qubit mixed state, there has an analytical formula of calculation of bipartite EOF that $E(\rho_{AB}) = H\left(\frac{1+\sqrt{1-|C(\rho_{AB})|^2}}{2}\right)$ with $H(x) = -x\log_2 x - (1-x)\log_2(1-x)$ the binary entropy and $C(\rho_{AB}) = \max\left\{\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0\right\}$ the quantity called concurrence of ρ_{AB}, and λ_is represent the eigenvalues of the matrix $(\sigma_y \otimes \sigma_y)\rho^*_{AB}(\sigma_y \otimes \sigma_y)$ with decreasing order. Note that the complete monogamy relation is saturated when tripartite EOF corresponds only to bipartite entanglement.

Therefore, the next multipartite entanglement for three-flavor neutrino system can be established based on the complete monogamy relation as

$$M_3 = E^2(\rho^e_{ABC}) - E^2(\rho^e_{AB}) - E^2(\rho^e_{AC}) - E^2(\rho^e_{BC}), \tag{27}$$

which is different from zero if we have entanglement beyond the entanglement of its pairs. Figure 3 shows the value of M_3 for the cases under study with different GM. We can observe that M_3 is always greater than or equal to zero, which further confirms the existence of genuine tripartite entanglement of the three-flavor neutrino system and suggests that the entanglement distribution follows in a complete monogamy way even in the presence of gravity effects. For neutrinos radially propagating outwards with GM $= 4.8 \times 10^8$ Km, M_3 oscillates between 0 and 0.646, and for the case of neutrinos radially propagating inwards with GM $= 9.3 \times 10^7$ Km, M_3 oscillates between 0 and 0.632. This implies that the entanglement distribution depending on complement monogamy relation can be constrained tighter due to the presence of gravitational effects.

3.2 Nonlocality for neutrino oscillation in schwarzschild metric

To probe the gravitational effects to genuine nonlocal correlations of three-flavor neutrino system when neutrinos travel over proper distance L_p in the gravity field, we consider the Svetlichny inequality, a generalized form of Bell inequality, which is based on the hybrid local nonlocal form of probability correlations, as follows:

$$P_M(a_1a_2a_3) = \sum_{k=1}^{3} P_k \int d\lambda \rho_{ij}(\lambda) P_{ij}(a_i a_j \mid \lambda) P_k(a_k \mid \lambda), \tag{28}$$

where λ is the shared local variable, and a_1, a_2, a_3 are the outcomes of the measurements. Here the subscript M stands for bipartition sections. If correlations fair to write in this form, then such correlations are considered to exhibit genuine tripartite nonlocality, which is referred to as Svetlichny nonlocality. For a three qubit system, the Svetlichny [54] parameter is defined as

$$\begin{aligned} S_3 = &ABC + AB'C + ABC' - AB'C' \\ &+ A'BC - A'B'C - A'BC' - A'B'C'. \end{aligned} \tag{29}$$

Here X and X' $(X = A, B, C)$ are two different measurements performing to each qubit. The classical bound of Svetlichny parameter is $S_3 \leq 4$ and its violation implies that the all the parties are are nonlocally correlated, i.e. the genuine tripartite nonlocality is exhibited, suggesting in turn the existence of genuine tripartite entanglement. Figure 4 depicts the Svetlichny parameter S_3 for the time evolution of an initial electron neutrino with respect to L_p with different values of GM. In the case of the flat spce-time, we find regions in the time evolution with S_3 fairs to cross the classical bound in our system, which means the absence of the genuine nonlocality, whereas for the curved space-time the evolution behavior of S_3 depends on the magnitude of GM, such that for neutrinos propagation outwards with GM $= 6 \times 10^8$ Km and inwards with GM $= 7 \times 10^7$ Km, S_3 is always larger than 4 (expect for few separable state). These results show that the genuine nonlocality is largely present in the time evolution, while for the GM $= 4.8 \times 10^8$ Km and GM $= 9.3 \times 10^7$ Km pertaining to outwards and inwards propagations, respectively. The gravitational effect surpasses the violation of the Svetlichny inequality, which is implied by the augmenting of the parameter range that the absence of genuine nonlocality in comparison to the case of the flat space-time. It is notable that since the violation of Svetlichny inequality is only a sufficient witness of genuine nonlocality but not a necessary condition, there exists the regions with absence of genuine nonlocality where the genuine tripartite entanglement is exhibited, as shown in Fig. 2.

Further, the monogamy of nonlocality is based on the maximun violation of Bell inequality, and the well known Bell inequality is Clauser–Horne–Shimony–Holt (CHSH) inequality (say for two-qubit state) represented as

$$|\mathrm{Tr}(\rho B_{CHSH})| \leqslant 2, \tag{30}$$

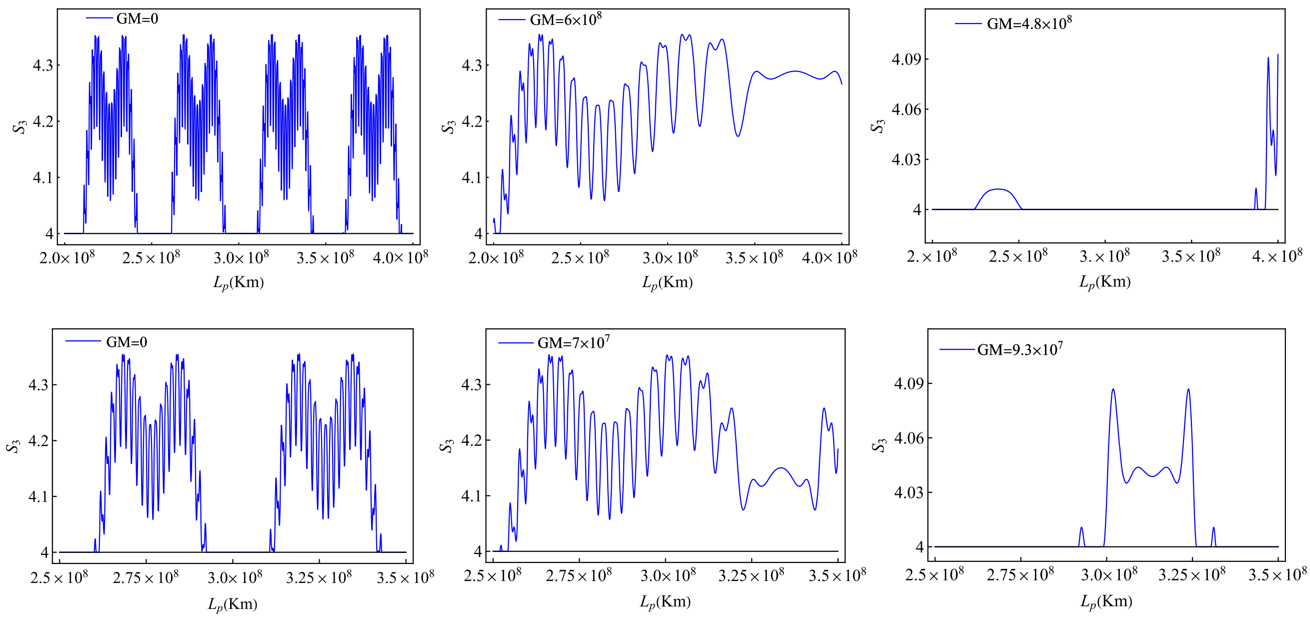

Fig. 4 Variations of the Svetlichny S_3 with respect to the proper distance (L_p) traveled by neutrinos with different GM, where GM $= 0$ corresponds to the flat space time and nonzero GM corresponds to curved space time. Top: corresponding to neutrinos radially propagating outwards to the gravitational source. Bottom: corresponding to neutrinos radially propagating inwards to the gravitational source. The system detects genuine multipartite nonlocality if the value of S_3 is above 3

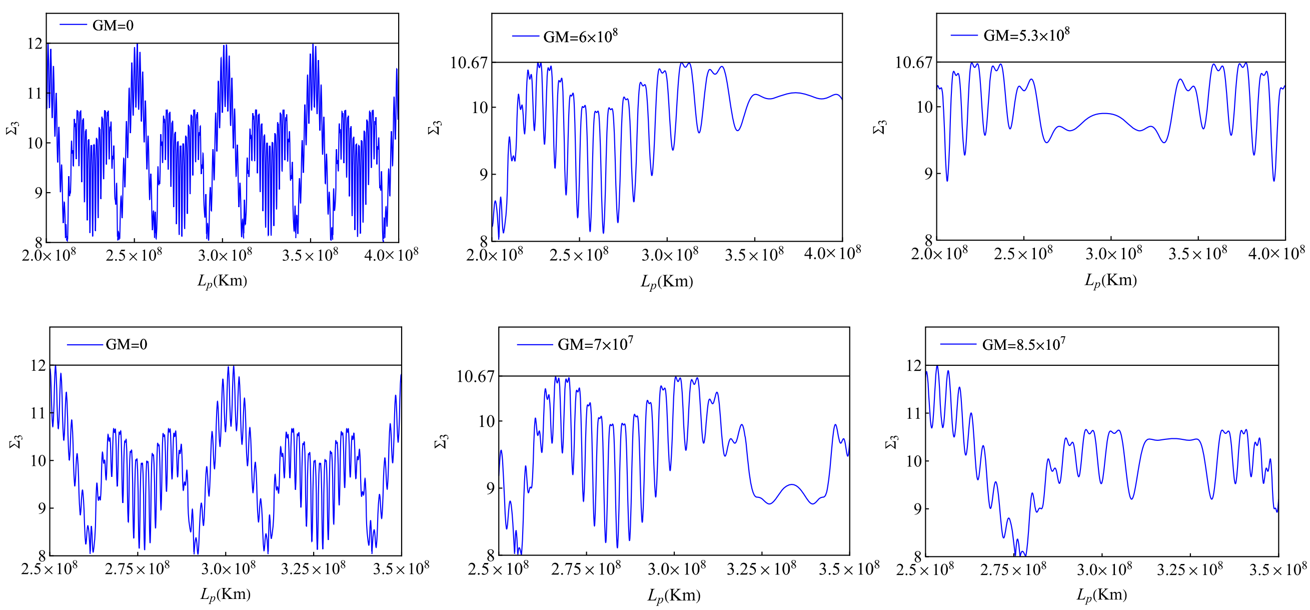

Fig. 5 Monogamy of nonlocality with respect to the proper distance (L_p) traveled by neutrinos with different GM, where GM $= 0$ corresponds to the flat space time and nonzero GM corresponds to curved space time. Top: corresponding to neutrinos radially propagating outwards to the gravitational source. Bottom: corresponding to neutrinos radially propagating inwards to the gravitational source

Here B_{CHSH} is the CHSH operator

$$B_{CHSH} = A_1 \otimes B_1 + A_1 \otimes B_2 + A_2 \otimes B_1 - A_2 \otimes B_2, \tag{31}$$

with $\mathbf{A_j}\left(\mathbf{B_j}\right) = \mathbf{a_j}\left(\mathbf{b_j}\right) \cdot \sigma$ are measurement settings pertaining to each qubit, where $a_j(b_j)$ are unit vectors in $\mathbb{R}^3$ and $\sigma = \left(\sigma_x, \sigma_y, \sigma_z\right)$ are Pauli matrices. The maximum violation of the CHSH inequality (30) is

$$\max\langle B_{CHSH}\rangle_\rho = \max|\mathrm{Tr}\,(\rho B_{\mathrm{CHSH}})| = 2\sqrt{M(\rho)}, \qquad (32)$$

where $M(\rho) = \max_{j<k}\left\{\mu_j + \mu_k\right\}$, $j, k \in \{1, 2, 3\}$, μ_j, μ_k are the two largest eigenvalues of the real symmetric matrix $T^T T$, Here the matric T with the entries $t_{ij} = \mathrm{Tr}\,\rho\left(\sigma_i \otimes \sigma_j\right)$. For a three qubit system the nonlocality monogamy measure [21] is defined as

$$\Sigma_3 = \max\langle B_{AB}\rangle^2 + \max\langle B_{AC}\rangle^2 + \max\langle B_{BC}\rangle^2, \qquad (33)$$

where B_{AB} is the CHSH operator for parties A and B, and some with B_{AC}, B_{BC}. Note that the quantity Σ_3 is always be bounded such as $0 \leq \Sigma_3 \leq 12$. The pairwise maximum violations of the CHSH Bell inequality of three-flavor neutrino system can be expressed in terms of the transition and survival probabilities among different flavor modes written in terms of propel distance L_p. Therefore, if the initial flavor state is $|\nu_e\rangle$, we have

$$\begin{aligned}
\max\left\langle B^e_{e\mu}\right\rangle &= 8P_{ee}(L_p)P_{e\mu}(L_p) \\
&\quad + 2max\left[4P_{ee}(L_p)P_{e\mu}(L_p), (2P_{e\tau}(L_p)-1)^2\right], \\
\max\left\langle B^e_{e\tau}\right\rangle &= 8P_{ee}(L_p)P_{e\tau}(L_p) \\
&\quad + 2max\left[4P_{ee}(L_p)P_{e\tau}(L_p), (2P_{e\mu}(L_p)-1)^2\right], \\
\max\left\langle B^e_{\mu\tau}\right\rangle &= 8P_{e\mu}(L_p)P_{e\tau}(L_p) \\
&\quad + 2max\left[4P_{e\mu}(L_p)P_{e\tau}(L_p), (2P_{ee}(L_p)-1)^2\right]. \qquad (34)
\end{aligned}$$

In Fig. 5, we depict the distribution of the monogamy measure Σ_3 for different values of GM. Although Σ_3 ranges from 0 to 12, the effective range is not start at 0. For the flat space-time, the effective range of Σ_3 is limited to [8, 12]. It is notable that the upper bound of Σ_3 decreases to around 10.67 in the presence of gravitational effects with GM $= 6 \times 10^8$ Km and GM $= 7 \times 10^7$ Km corresponding to outwards and inwards, respectively, while the genuine tripartite nonlocality is presented, as shown in Fig. 4. This shows the close relation between maximum bipartite nonlocality and global nonlocality. It is possible to find that when GM $= 5.3 \times 10^8$ Km for neutrinos inwards propagation, the effective range is limited to around [8.88, 10.67], where the effective lower bound has a decrease in comparison to other cases under study. Summing up, the gravitational effects can give rise to manifest change in the effective bounds of the nonlocal monogamy relation, where the low nonlocality monogamy measure $\max\left(\left\langle B_{e\mu}\right\rangle^2 + \langle B_{e\tau}\rangle^2 + \left\langle B_{\mu\tau}\right\rangle^2\right)$ correspondings to high bipartite correlations.

4 Conclusions

Gravity as one of the most fundamental physical effects has many potential applications in modern physics. Using quantum correlations can investigate the role of gravitational effect in quantum information. In this paper, we have studied the gravitational effect on entanglement, nonlocality, and their monogomy properties for three-flavor neutrino system. Here, we have considered the plane wave approach treated for neutrino oscillation and the wave packet decoherence effects have been ignored, which is applicable for our discussion because the modifications induced by neutrino localization (wave packet approach) only play a crucial role when the propagation length is of the order of the coherence one. We have shown that the gravitational effects cause the occurrence of phase shift of entanglement compared to one in flat space-time when GM $= 3 \times 10^7$ Km, leading to a local dependence in wave lengths of the oscillation for entanglement. Furthermore, the gravitational effects can not only make the entanglement oscillate around a large value but also suppresses the entanglement, which is witnessed in the form of the decrease in the maximum value of the entanglement in curved space-time when GM $= 4.8 \times 10^8$ Km and GM $= 9.3 \times 10^7$ Km, because of the constructive and destructive effects in some terms of entanglement. Also, we have analyzed the complete monogamy relation of entanglement to show the distribution of entanglement among neutrino system. It is found that the distributions of entanglement satisfy the complement monogamy relation for all cases and the limitation of the entanglement shareability will be tighter in the curved space-time for GM $= 4.8\times10^8$ Km and GM $= 9.3\times10^7$ Km corresponding to outwards and inwards propagation respectively. Furthermore, The gravitational effects facilitate the violation of Svetlichny inequality to make the genuine nonlocal correlations (nonlocality shared among all parties) always present for the full-time evolution when GM $= 6 \times 10^8$ Km and GM $= 7 \times 10^7$ Km. However, For GM $= 4.8 \times 10^8$ Km and GM $= 9.3 \times 10^7$ Km corresponding to outwards and inwards propagation, respectively, the gravitational effects suppress the violation of Svetlichny inequality, resulting that the regions of the absence of the genuine nonlocal correlations increase. On the other hand, the nonlocality monogamy relation shows that the effective bound of the maximum bipartite nonlocality of the neutrino system has been changed in curved space-time due to the gravitational effects. We hope that our investigations could provide a significant applications for using quantum resources to study the gravitational effects on the oscillations of neutainos in the future and facilitate the connection between the gravitational effects and quantum information.

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