QUARK MODELS OF HADRONIC INTERACTIONS

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1.0 MODELLING QUANTUM CHROMODYNAMICS

1.1 Models Of QCD

Although quantum chromodynamics is generally accepted as the fundamental theory of strongly interacting elementary particles, there exist no exact solutions to the theory. Important general properties of the theory have been derived, such as asymptotic freedom, the running of the coupling constant, and (plausibly) color confinement. A small (but very impressive) amount of information has been extracted from Monte Carlo lattice gauge theory (LGT) calculations. Such results are limited (in their accuracy and the number of properties calculated) by computer size and time considerations. To date, such calculations include only gluonic interactions, and do not allow for quark-antiquark virtual excitations.

At another level of sophistication is modelling. The exact theory is replaced by a mathematically simpler model (or effective theory) which incorporates as many features of the exact theory as possible. A small set of parameters may be fixed by (for example) LGT calculations and experiment. The utility of any model is then determined by its accuracy in describing other experimental data and predicting new phenomena. Thus modelling provides a bridge between fundamental theory and experiment. Comparison with experiment not only tests the model but, more importantly, provides a method to test the fundamental theory.

The MIT bag[1] was one of the earliest and most successful models of QCD, imposing confinement _a priori_ and including perturbatively interactions between quarks and gluons. An evolution of the MIT bag came with the introduction of the chiral[2] and cloudy[3] bags, which treat pions as elementary particles. The pion is an "anomalously" light hadron, and in a chirally invariant QCD should emerge as the massless Goldstone boson. Non-relativistic potential models have also had remarkable success, and have the advantage of being amenable to dynamic calculations.

The soliton model proposed by Friedberg and Lee[4,5] is particularly attractive. It is a covariant field theory and sufficiently general so that, for certain limiting cases of the adjustable parameters, it can describe either the MIT or SLAC (shell) bags. The confinement mechanism appears as a dynamic field. This allows non-static processes, such as bag oscillations and collisions (including bag creation and fission) to be calculated utilizing the well-developed techniques of nuclear many-body theory.

1.2 The Soliton Bag Model Of Friedberg And Lee

In the soliton model, the (effective) Lagrangian density is

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_\sigma + \mathcal{L}_{q\sigma} + \mathcal{L}_G + \text{(counter terms, Higgs fields, etc)}, \quad (1.1)$$

where the individual terms have the following interpretation:

$\mathcal{L}_q = \Sigma_f \, \bar{\psi}_f(\gamma \cdot p - m_f)\psi_f$ describes the quarks as Dirac particles of mass m_f, where f is the flavor. We take $m_u = m_d = 0$.

$\mathcal{L}_\sigma = \frac{1}{2}(\partial\sigma)^2 - U(\sigma)$ describes the scalar soliton field σ, which represents the complex structure of the vacuum, arising from virtual gluons and quark-pairs interacting among themselves. The momentum operator conjugate to σ is $\pi = \dot{\sigma}$, and the two satisfy the canonical equal-time commutation relations

$$[\sigma(\vec{r},t), \pi(\vec{r}\,',t)] = i\delta^3(\vec{r} - \vec{r}\,') \quad (1.2)$$

The non-linearity of the soliton field enters through the self-interaction function (see Fig. 1)

$$U(\sigma) = \frac{1}{2} a \sigma^2 + \frac{1}{3!} b \sigma^3 + \frac{1}{4!} c \sigma^4 + B. \quad (1.3)$$

The polynomial terminates in fourth order to ensure renormalizability. $U(0) = B$ is to be identified with the "bag constant" or volume energy density of a cavity. With a suitable adjustment of the constants, the function has two minima, one at $\sigma = 0$, and another, lower minimum, at $\sigma = \sigma_v$. The physical vacuum corresponds to the second minimum, and the constant B is chosen so that $U(\sigma_v) = 0$.

The quarks interact with the soliton field through the term $\mathcal{L}_{q\sigma} = g\bar{\psi}\sigma\psi$. In the presence of (real) quarks, the sum $U(\sigma) + g\bar{\psi}\sigma\psi$ may have a minimum (depending on the parameters) near $\sigma = 0$ (the perturbative vacuum). This leads to a cavity in the σ-field, which is called the "bag."

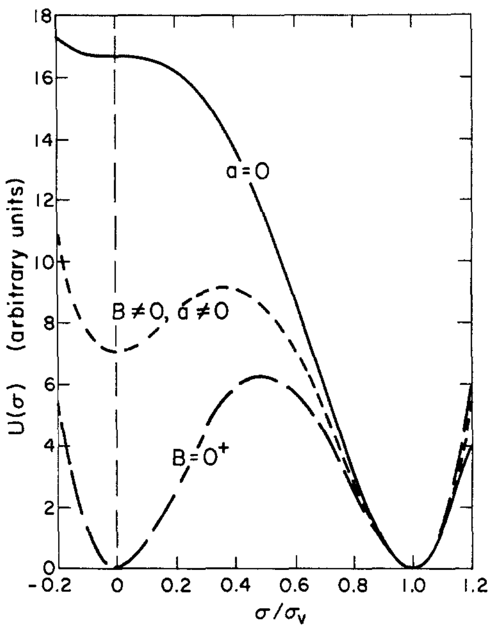

Fig. 1. Three forms for $U(\sigma)$.

Color gluon fields are introduced as in QCD, except that they interact with the soliton field through a dielectric function $\kappa(\sigma)$, chosen such that $\kappa(0) = 1$ and $\kappa(\sigma_v) = 0$. A convenient (but not unique) form is

$$\kappa(\sigma) = (\sigma/\sigma_v - 1)^2 \tag{1.4}$$

The magnetic susceptibility is $\mu = \kappa^{-1}$. The gluon part of the Lagrangian is written

$$\mathcal{L}_G = \frac{-\kappa}{4} F^c_{\mu\nu} F^{c\mu\nu} - g_s \bar{\psi} \gamma_\mu \frac{\lambda^c}{2} A^{c\mu} \psi \tag{1.5}$$

where g_s is the magnitude of the color charge. The strong coupling constant is $\alpha_s = g_s^2/4\pi$. The λ^c are the SU(3) color matrices; $c = 1....8$. The ψ functions have 4 (Dirac) times 3 (color) times n (flavor) components.

The requirements on κ yield color confinement. This can be seen easily if one keeps only terms linear in the gluon field. Then Gauss's law gives

$$\vec{\nabla}\cdot\vec{D}^c = \rho^c , \tag{1.6}$$

where c is the color index. If the total color charge does not vanish within some finite cavity, the D-field will fall off as r^{-2} as $r \rightarrow \infty$, and the color electric energy in medium

$$\frac{1}{2}\int d^3r\ D^2/\kappa(r) \tag{1.7}$$

will be infinite because $\kappa \rightarrow 0$ (exponentially in the model) as $r \rightarrow \infty$.

As long as one calculates diagrams only through order of one gluon exchange, there is no problem of double counting: the soliton field represents at least two-gluon structures. If higher order diagrams are calculated, the coefficients in the effective Lagrangian must be readjusted at each stage to compensate.

There are five parameters in the model: a, b, c, g and α_s. The first four involve only the soliton field and quarks; α_s is the quark-gluon coupling constant. The following key data may be used to help fix these parameters:

1.3 Soliton-quark Parameters

From LGT calculations we have available the following pieces of data.

{1} The bag constant[6] B = (220+20) MeV/fm^3. This is considerably larger than the MIT value[8] of 57 MeV/fm^3.

In the MIT model (if we neglect for the moment gluon effects), the bag energy is given by

$$E = \frac{2.04\ N_q\ \hbar c}{R} + \frac{4}{3}\pi\ R^3 B\ , \tag{1.8}$$

where N_q is the number of quarks (in the lowest $s_{\frac{1}{2}}$-state). Minimization with respect to R yields

$$R = \left(\frac{2.04\ N_q\ \hbar c}{4\pi B}\right)^{\frac{1}{4}} \tag{1.9}$$

Because of the appearance of $B^{\frac{1}{4}}$ in the denominator, this is the quantity often quoted. The discrepancy between the LGT and MIT values, in terms of $B^{\frac{1}{4}}$, is then only 40% rather than a factor of 4.

{2} m_{GB} = (720+40) MeV, the mass of the glueball state (0^{++}) [7]. We identify this state with an excitation of the pure soliton field, such that $U''(\sigma_V) = m_{GB}^2$.

The errors quoted in {1} and {2} above are presumably statistical only, and do not account for systematic effects of the lattice size or the omission of dynamical interactions with quarks.

From experiment we select the subset

{3} The mean of the nucleon and delta masses, $\bar{m} = (m_N + m_\Delta)/2$ = 1.087 GeV.

{4} The proton size $\langle r_p^2 \rangle$ = 0.83 fm.

{5} The proton magnetic moment, μ = 2.7925 nuclear magnetons.

{6} The ratio of the axial to vector coupling constants, g_A/g_V = 1.25.

1.4 The Strong Coupling Constant

α_s is a "running" constant, and is evaluated for the regime of hadronic sizes. To fix it, we may utilize

{7} The delta-nucleon mass difference, $m_\Delta - m_N$ = 297 MeV.

{8} The "string constant, which is the coefficient of the linear term in the potential between massive quarks, such as charmed quarks. This has been fit phenomenologically to the charmonium spectrum using a non-relativistic potential model[9] and has also been obtained in LGT calculations[7]; the value is $t \cong 1$ GeV/fm. The string constant can be calculated in the MIT model as follows. The energy per unit length in a uniform tube is

$$E/L = (B + \frac{1}{2}\vec{E}^c \cdot \vec{E}^c)\, A \tag{1.10}$$

where A is the cross sectional area. The flux of color electric field emanating from a source and passing through the cross section of the tube is

$$A\, E^c = \frac{1}{2} Q\, \lambda^c , \tag{1.11}$$

hence

$$t \equiv E/L = B\, A + \frac{1}{8} Q^2\, \lambda^c \cdot \lambda^c /A \,. \tag{1.12}$$

But $<\lambda^c\cdot\lambda^c> = 16/3$ and $\alpha_s = Q^2/4\pi$. This gives

$$t = B\,A + \frac{8}{3}\,\pi\,\alpha_s\,/A. \tag{1.13}$$

Minimization with respect to A yields the result

$$t = (32\,\pi\,\alpha_s\,B/3)^{\frac{1}{2}} \tag{1.14}$$

There is much more data to fit once the parameters are determined. These include hadronic spectra and resonances, decay widths, various reactions (especially nucleon-nucleon scattering), etc. There are many more experimental data of high and low quality, but the data from LGT calculations are still few and of uncertain accuracy.

In adjusting the model parameters, the fitting is dependent on the level of sophistication of the model calculations. Whenever more diagrams are included, one must readjust the model parameters. For example, if we were able to calculate all gluon interactions exactly, then presumably the effects of the soliton field would disappear completely.

2.0 THE MEAN FIELD APPROXIMATION

If we neglect gluon interactions, the Hamiltonian can be written

$$H = \int d^3r\ \{\tfrac{1}{2}[\pi^2 + |\vec{\nabla}\sigma|^2] + U(\sigma) + \psi^\dagger(\vec{\alpha}\cdot\vec{p}+\beta g\sigma)\ \psi\}\ . \tag{2.1}$$

As a preliminary step to dynamical calculations, we consider first static solutions to the field equations. The simplest of these is the mean field approximation (MFA). We set

$$\sigma = \sigma_0(\vec{r}) + \sigma_1;\quad \pi = \pi_0 + \pi_1\ , \tag{2.2}$$

where $\sigma_0(\vec{r})$ is a time-independent c-number and σ_1 is the quantum fluctuation operator. Because σ_0 is static, $\pi_0 = 0$. σ_1 and π_1 satisfy

$$[\sigma_1(\vec{r},t),\ \pi_1(\vec{r}',t)] = [\sigma(\vec{r},t),\ \pi(\vec{r}',t)] = i\ \delta^3(\vec{r}-\vec{r}')\ . \tag{2.3}$$

Similarly, we represent the quark field operators by

$$\psi = \Sigma_k\ C_k\ \psi_k\ \ , \tag{2.4}$$

where the ψ_k are a complete set of spinor-color-flavor vectors and

$$\{C_k^\dagger\ ,C_{k'}\} = \delta_{\vec{k},\vec{k}'}\ \ . \tag{2.5}$$

In the MFA, we consider a fixed occupation of quarks (3 quarks for nucleons; a quark-antiquark pair for mesons) and neglect the σ_1 field. The energy, relative to the vacuum, is then

$$E = \int d^3r\{\tfrac{1}{2}|\vec{\nabla}\sigma_0|^2 + U(\sigma_0) + \sum_{k\text{-occ}} \psi_k^\dagger(\vec{\alpha}\cdot\vec{p}+\beta g\sigma_0)\ \psi_k\}\ . \tag{2.6}$$

Extremization of E with respect to ψ_k (subject to the normalization constraint) and with respect to σ_0 leads to the coupled set of equations

$$(\vec{\alpha}\cdot\vec{p} + \beta g\sigma_0)\ \psi_k = \varepsilon_k\ \psi_k\ , \tag{2.7a}$$

$$-\ \nabla^2\sigma_0 + U'(\sigma_0) + g \sum_{k\text{-occ}} \bar{\psi}_k\ \psi_k = 0\ \ , \tag{2.7b}$$

where $U' = dU/d\sigma_0$. The first is a linear eigenvalue problem (if σ_0 is given); the second is a non-linear inhomogeneous differential equation (if ψ_k is given). The equations are solved alternately until self-consistency obtains.

The cycling proceeds as follows. Let m be the cycle number. We solve (2.7a) for $\psi_k^{(m)}\ \{\sigma_0^{(m-1)}\}$. Then we solve (2.7b) for $\sigma_0^{(m)}\{\psi_k^{(m)}\}$. Simple iteration was found not to converge, so a cycle convergence factor f was introduced, so that the replacement $f\sigma_0^{(m)} + (1-f)\sigma_0^{(m-1)} \to \sigma_0^{(m)}$ was made before advancing m. Rapid convergence was achieved for f = 0.5.

There are many techniques for solving (2.7a). I will outline here the method described in ref. [5]. Let $\sigma_0(r)$ be spherically symmetric. Suppressing the cycle index m, the Dirac function is written

$$\psi_k = \begin{pmatrix} u_k(r) \\ i\vec{\sigma}\cdot\hat{r}\ v_k(r) \end{pmatrix} \mathcal{Y}_{\kappa m} \quad , \tag{2.8}$$

where $\mathcal{Y}_{\kappa m} = \mathcal{Y}^{\ell}_{jm}$ is a Pauli spinor. The Dirac quantum number $\kappa = (j+\frac{1}{2})(-1)^{j-\ell+\frac{1}{2}}$. Here k stands for the set of all quantum numbers. The u and v satisfy

$$du_k/dr = -(g\sigma_0 + \varepsilon_k)\ v_k \ , \tag{2.9a}$$

$$dv_k/dr + 2v_k/r = (-g\sigma_0 + \varepsilon_k)\ u_k \quad . \tag{2.9b}$$

We begin with a previous guess of the eigenvalue, $\varepsilon_k^{(n)}$ and integrate the radial equations outward (satisfying the boundary condition at r = 0) to some match point R. Similarly the equations are integrated inward from some large starting radius to the same point R. Unless $\varepsilon_k^{(n)}$ is a true eigenvalue, it is not possible to match both u and v at R. The functions are scaled so that $u_k(R^-) = u_k(R^+)$. Then an improved estimate for the eigenvalue is

$$\varepsilon_k^{(n+1)} = \frac{\int \psi_k^{(n)\dagger}(\vec{\alpha}\cdot\vec{p} + \beta g\sigma_0)\psi_k^{(n)} d^3r}{\int \psi_k^{(n)\dagger}\psi_k^{(n)}\ d^3r}$$

$$= \varepsilon_k^{(n)} + \frac{u_k^{(n)}(R)\,(v_k^{(n)}(R^+) - v_k^{(n)}(R^-))\ R^2}{\int r^2 dr\,(u_k^{(n)\,2} + v_k^{(n)\,2})} \tag{2.10}$$

The process is iterated until the desired accuracy is achieved before going on to the solution of the σ_0-equation.

The non-linear σ_0 differential equation is solved by the very effective iterative method of Henyey and Wilets[10]. The radial differential equation (2.7b) can be written ($f = r\sigma_0$)

$$d^2f/dr^2 + G(f) = 0 \ . \tag{2.11}$$

Let $f^{(n)}$ be the approximation resulting from the n-th iteration. Then set

$$f^{(n+1)}(r) = f^{(n)}(r) + y^{(n)}(r) . \tag{2.12}$$

Then to first order in $y^{(n)}$, (2.11) becomes

$$d^2y^{(n)}/dr^2 + G'(f^{(n)})y^{(n)} = - [d^2f^{(n)}/dr^2 + G(f^{(n)})], \tag{2.13}$$

which is a linear, inhomogeneous differential equation. $f^{(n)}$ is required to satisfy the proper boundary conditions for each n. In the present case,

$$f^{(n)}(0) = 0, \quad f^{(n)}(r) \to r\sigma_v \text{ for } r \to \infty . \tag{2.14a}$$

Therefore

$$y^{(n)}(0) = 0, \quad y^{(n)}(r) \to 0 \text{ for } r \to \infty . \tag{2.14b}$$

Eq. (2.13) can be solved for $y^{(n)}$ by various, standard techniques. The accuracy of the result, however, is determined by the accuracy of the approximation used to represent the inhomogeneous term $d^2f^{(n)}/dr^2$. What we have done was to approximate the non-linear differential equation (2.11) by the Noumerov method. With $f_i = f(i\Delta r)$, we set

$$(f_{i+1} - 2 f_i + f_{i-1})/(\Delta r)^2 + \frac{1}{12}(G(f_{i+1}) + 10G(f_i) + G(f_{i-1})) = 0 , \tag{2.15}$$

then linearize the equations to obtain a band matrix operating on $y_i^{(n)}$. The matrix equation can be solved as follows (drop superscript "n"): Let

$$\alpha_i y_{i+1} + \beta_i y_i + \gamma_i y_{i-1} + H_i = 0 . \tag{2.16a}$$

Ansatz:

$$y_i = A_i y_{i+1} + B_i . \tag{2.16b}$$

Then

$$A_i = -\frac{\alpha_i}{\beta_i+\gamma_i A_{i-1}}, \quad B_i = -\frac{\gamma_i B_{i-1}+H_i}{\beta_i+\gamma_i A_{i-1}}. \tag{2.16c}$$

(2.16c) gives upward recursion relations for the A_i and B_i. Check boundary conditions: For $y_0 = 0$, we may begin with $A_0 = B_0 = 0$. At some maximum r (I_{max}), there is another boundary condition, for example $y_{Imax} = 0$. Then one can recur downward to obtain all of the y_i.

The numerical procedures work extremely well, and usually converge when a solution exists.

A non-exhaustive search of parameters to fit data {1} through {8} above has been made. It should be stressed again that the parameter set obtained depends on the order of the calculation. The result quoted here was obtained in the mean field approximation, including recoil corrections (which will be discussed below). Up to this point, gluon exchange has not been included. We will return later to interaction of the gluon field.

We find a "reasonable" parameter set which gives a moderately good fit to experimental data:

a = 236.13, b = -11 614, c = 180 000, g = 25.

The structure of the soliton field and the quark wave function for the nucleon are shown in Fig. 2. Other hadronic properties are summarized in the following table, which includes recoil effects discussed below[11,12]:

Table 1.

Bag characteristics for the parameters listed in the text.

	Model	Experiment	LGT
$\langle r_p^2\rangle^{1/2}$	0.83 fm	0.83	
$\bar{m}$	1,011 MeV	1,087	
μ_p	2.53 nm	2.7925	
g_A/g_V	1.04	1.25	
B	23.6 MeV/fm^3		220
m_{GB}	3,500 MeV		720
$\varepsilon(1s)$	287 MeV		

Our value of B is an order of magnitude smaller than the LGT calculations and somewhat smaller than the MIT value, indicating that surface tension plays an important role in confinement. We could "tolerate" an even smaller value. However, as mentioned above, the number usually quoted is B , which makes discrepancies among the various values appear to be less pronounced. We also find a larger glueball mass than that given by LGT calculations. It must be emphasized that the LGT calculations are incomplete in that $q\bar{q}$ excitations are not included.

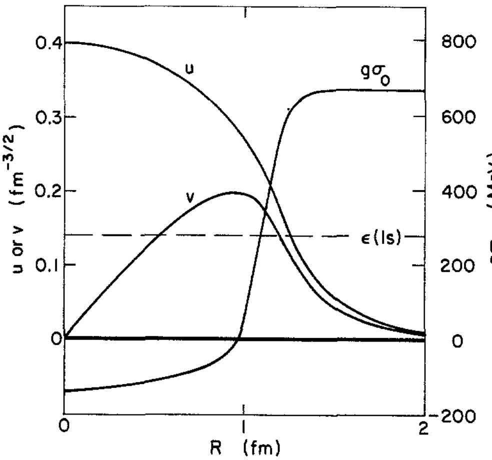

Fig. 2. Numerical results for the parameter set given in text above.

3.0 QUANTUM ALTERNATIVES TO THE MEAN FIELD

For handling the quantum fluctuations σ_1 and also for treating σ_0 as a quantum field, we now turn to the expansion of the field operators σ and π in terms of modes. We consider two approaches: (1) the coherent state and (2) Q-space.

3.1 The Coherent State

Instead of the division $\sigma = \sigma_0 + \sigma_1$, where $\sigma_0(r)$ is a c-number, we rather expand σ and π in a full set of modes,

$$\sigma = \sigma_v + \sum_n \left(\frac{1}{2\omega_n}\right)^{\frac{1}{2}} (a_n^\dagger s_n^* + a_n s_n) ,$$

$$\pi = i \sum_n \left(\frac{\omega_n}{2}\right)^{\frac{1}{2}} (a_n^\dagger s_n^* - a_n s_n) , \qquad (3.1)$$

where the ω_n are as yet undetermined and the $\{s_n\}$ are a complete orthonormal set. $\sigma(\vec{r},t)$ and $\pi(\vec{r},t)$ satisfy the usual equal time commutation relations if

$$[a_n(t), a^\dagger_{n'}(t)] = \delta_{nn'} ,$$
$$[a_n(t), a_{n'}(t)] = [a^\dagger_n(t), a^+_{n'}(t)] = 0 . \quad (3.2)$$

A "coherent" state in one mode, say $n = 0$, is obtained by the construction

$$|\lambda> = e^{\lambda a_0^\dagger} |0> , \quad (3.3)$$

where $a_n|0> = 0$ for all n. The coherent state satisfies

$$a_0|\lambda> = \lambda|\lambda> \quad (3.4)$$

and

$$<\lambda|\lambda> = e^{|\lambda|^2} \quad (3.5)$$

There is no loss of generality in choosing λ real. We consider now the case where s_0 is real.

To study the energy of the system relative to the "vacuum" <u>for these modes</u>, $|0>$, we normal order the Hamiltonian, :H:. In calculating the expectation value of :H:, we will need the following

$$<\lambda|: (a_0^\dagger + a_0)^n :|\lambda> = (2\lambda)^n<\lambda|\lambda> ,$$

$$<\lambda|: (a_0^\dagger - a_0)^n :|\lambda> = 0 \text{ for } n > 0 . \quad (3.6)$$

Expectation values involving a_i or $a_i^\dagger$ $(i > 0)$ vanish. We write the state vector in product form

$$|\Psi> = |\lambda>|q>/<\lambda|\lambda>^{\frac{1}{2}}$$

where $|\lambda>$ is the soliton coherent state and $|q>$ is the normalized quark state vector; let $\lambda = \xi \sqrt{\omega_0/2}$.

$$<\Psi|:H:|\Psi> =$$

$$\int d^3r\{\sum_k \psi_k^\dagger (\vec{\alpha}\cdot\vec{p} + \beta g\sigma_0)\psi_k + \frac{1}{2}|\nabla\sigma_0|^2 +$$

$$\frac{a'}{2}\sigma_0{}^2 + \frac{b}{3!}\sigma_0{}^3 + \frac{c}{4!}\sigma_0{}^4 + B'\} . \quad (3.7)$$

The coefficients a' and B' differ from a and B by (infinite) constants arising from normal ordering. Except for the renormalization of the coefficients a and B, (3.7) is identical with the mean field approximation expression for the energy (2.6) if we identify

$$\sigma_0(\vec{r}) \equiv \xi s_0(\vec{r}) + \sigma_v = \langle\Psi|\sigma|\Psi\rangle \tag{3.8}$$

with the mean field quantity. Note that the quantities ω_k do not enter explicitly in the final result.

Multiple-mode coherent state wave functions can be constructed by taking a product of the exponential operators:

$$|\lambda_1\lambda_2\ldots\rangle = \prod_n e^{\lambda_n a_n^\dagger}|0\rangle \tag{3.9}$$

but this may also be regarded as a single mode function!

3.2 Q-space

We proceed again with an orthonormal mode analysis, but rather than utilizing annihilation and creation operators, we introduce the operators P and Q :

$$\begin{aligned} \sigma &= \sigma_v + \Sigma\, s_n Q_n \,, \\ \pi &= \quad\;\; \Sigma\, s_n P_n \,, \end{aligned} \tag{3.10}$$

where

$$[Q_n, P_{n'}] = i\,\delta_{nn'} \,. \tag{3.11}$$

Once again we restrict consideration to one mode, and ignore cross coupling to other modes. We drop the subscript "0". The state vector is now written

$$|\Psi\rangle = \Phi(Q)|q\rangle \tag{3.12}$$

and we use the representation

$$P = -i\,\partial/\partial Q \,. \tag{3.13}$$

Before deriving the equation for (Q), we note that in the one mode approximation this approach differs from the coherent state in that the latter has an explicit form for occupation numbers. A more general form would be some arbitrary function, $F(a^{\dagger})|0\rangle$. The Q representation allows for the most general form of F; it gives the exact one mode (Tomanaga) formulation of the problem. In principle, we could normal order the $a \equiv (\omega^{-\frac{1}{2}}Q + i\omega^{-\frac{1}{2}}P)2^{-\frac{1}{2}}$ and $a^{\dagger}$ operators, but that would lead to a fourth order differential equation for $\Phi(Q)$. Rather, we keep the original ordering, and later subtract the vacuum energy associated with the mode(s) considered.

With $|\Psi\rangle$ given by (3.12), we find

$$\langle\Psi|H|\Psi\rangle = \int d^3r\{\sum_k \psi_k^{\dagger}(\vec{\alpha}\cdot\vec{p} + g\beta(\sigma_v + s\langle Q\rangle))\psi_k + s^2\langle P^2\rangle + |\vec{\nabla}s|^2\langle Q^2\rangle + \langle U(\sigma_v + sQ)\rangle\} , \tag{3.14}$$

where

$$\langle P^2\rangle \equiv - \int \Phi^* \frac{d^2}{dQ^2} \Phi \, dQ ,$$

$$\langle Q^n\rangle \equiv \int \Phi^* Q^n \Phi \, dQ . \tag{3.15}$$

We extremize $\langle H\rangle$ with respect to $\psi_k(\vec{r})$, $s(\vec{r})$ and $\Phi(Q)$, subject to normalization constraints on all three functions to obtain the three coupled equations

$$[\vec{\alpha}\cdot\vec{p} + \beta g(\sigma_v + s\langle Q\rangle - \varepsilon_k]\psi_k(\vec{r}) = 0 , \tag{3.16a}$$

$$[- \langle Q^2\rangle\nabla^2 + \langle P^2\rangle + \langle QU'(\sigma_v + sQ)\rangle + \langle Q\rangle g\Sigma \bar{\psi}_k\psi_k - \lambda]s(\vec{r}) = 0 , \tag{3.16b}$$

$$[\int d^3r(- s^2 \frac{d^2}{dQ^2} + |\vec{\nabla}s|^2Q^2 + U(\sigma_v + sQ) + g \sum_k \bar{\psi}_k s\psi_k Q) -E_Q]\Phi(Q) = 0. \tag{3.16c}$$

These equations have been solved self-consistently. Eqs. (a) and (c) are linear eigenvalue equations. Eq. (c) is a second order differential equation with a "potential" which is a fourth order polynomial in Q. Eq. (b) is a non-linear, inhomogeneous differential equation of the same form as the σ_0 equation (2.7b). In solving (3.16b), however, the Lagrange multiplier λ must be varied until a normalized function s is obtained. The three equations are cycled through (with a convergence factor on s) until self-consistency obtains.

Physical quantities are obtained by subtracting out the no-quark, same one-mode state quantities. Thus the s obtained by solving Eqs. (3.16a-b) with N quarks is now used to solve only Eq. (3.16c) with the term containing $\bar{\psi}_k s \psi_k$. absent. Comparisons have been made with the mean field approximation for the energy and $\langle\sigma\rangle$. The results agree (for the standard parameter set) to about 20%. Another measure of the validity of the mean field approximation is the quantum fluctuation in Q. We find

$$\frac{\langle Q\rangle}{\langle Q^2\rangle^{\frac{1}{2}}} \simeq 0.78 \tag{3.17}$$

The results of this section were obtained by Dethier.

4.0 SMALL AMPLITUDE OSCILLATIONS

We return to the mean field representation $\sigma = \sigma_0 + \sigma_1$, $\pi = \pi_1$. The Hamiltonian is given by

$$\begin{aligned} H = {} & \mathcal{E}_{\sigma_0} + \sum_k{}' \varepsilon_k c_k^\dagger c_k \\ & + \int d^3r \{ \tfrac{1}{2}(\pi_1^2 + |\vec{\nabla}\sigma_1|^2 + U''(\sigma_0)\sigma_1^2) + \tfrac{1}{6} U'''(\sigma_0)\sigma_1^3 + \tfrac{1}{24} c\sigma_1^4 \\ & \qquad + g \sum_{k\ell}{}' \bar{\psi}_k \sigma_1 \psi_\ell c_k^\dagger c_\ell \} . \end{aligned} \tag{4.1}$$

where $\mathcal{E}_{\sigma_0} = \int d^3r \; \mathcal{H}_\sigma (\sigma_0(r))$.

The primes on the sums denote subtraction of the same terms for k (and ℓ) occupied. Note the vanishing of terms linear in σ_1, if all quarks are in occupied states. The quantum part of the soliton field can be expanded in an orthonormal set:

$$\sigma_1 = \sum_n \left(\frac{1}{2\omega_n}\right)^{\frac{1}{2}} (a_n^\dagger s_n^* + a_n s_n) , \tag{4.2a}$$

$$\pi_1 = i \sum_n \left(\frac{\omega_n}{2}\right)^{\frac{1}{2}} (a_n^\dagger s_n^* - a_n s_n) . \tag{4.2b}$$

The Hamiltonian (4.1) simplifies if we <u>choose</u> $s_n(r)$ and ω_n to satisfy

$$(-\nabla^2 + U''(\sigma_0) - \omega_n^2)\, s_n(\vec{r}) = 0 \; ; \tag{4.3}$$

then

$$H = \mathcal{E}_{\sigma_0} + \Sigma_k' \; \varepsilon_k \, c_k^+ c_k + \Sigma_n \, \omega_n (a_n^\dagger a_n + \tfrac{1}{2}) + H_1 + H_3 + H_4 , \tag{4.4}$$

with

$$H_1 = g \sum_{k\ell n}' \int d^3r \; \bar{\psi}_k \, s_n \, \psi_\ell \, \left(\frac{1}{2\omega_n}\right)^{\frac{1}{2}} a_n + \text{h.c.} , \tag{4.5a}$$

$$H_3 = \frac{1}{6} \int d^3r \, (b + c\sigma_0)\, \sigma_1^3 \; , \tag{4.5b}$$

$$H_4 = \frac{c}{24} \int d^3r \; \sigma_1^4 \; , \tag{4.5c}$$

where σ_1 in H_3 and H_4 is represented by (4.2a). The diagrammatic meaning of these terms is shown in Fig. 3.

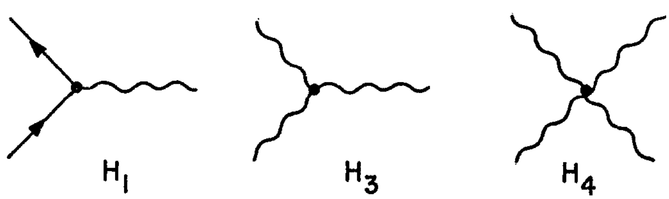

Fig. 3

Eq. (4.3) defines normal modes for oscillations about the mean field solution. Since we have $\sigma_0(r)$ to be spherically symmetric, we can set

$$s_n(\vec{r}) \equiv s_{\ell mn}(\vec{r}) = r^{-1} \, u_{\ell n}(r) \; Y_{\ell m}(\theta, \phi) \; , \tag{4.6}$$

whence

$$\left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + U'' - \omega_{\ell n}^2\right) u_{\ell n} = 0 \quad . \tag{4.7}$$

$U''(\sigma_0(r))$ has a sharp dip in the vicinity of the bag surface, as can be seen in Fig. 4. A rough estimate of the eigenfrequencies can be obtained by setting

$$U'' = U''(\sigma_0(r_0)) + (r-r_0)^2\,\Omega^2 , \tag{4.8}$$

where r_0 is the location of the minimum and

$$\Omega^2 = \frac{1}{2}\,\frac{d^2}{dr^2}\,U''(\sigma_0(r))\Big|_{r=0} \tag{4.9}$$

Then

$$\omega_{\ell n}^{\;2} \simeq U''(\sigma_0(r_0)) + \frac{\ell(\ell+1)}{r_0^2} + \Omega(2n+1) . \tag{4.10}$$

For the lowest state, using our standard parameters, we find $\omega_{00} \simeq$ 1400 MeV. The quark and soliton (σ_1) spectra are displayed schematically in Fig. 5.

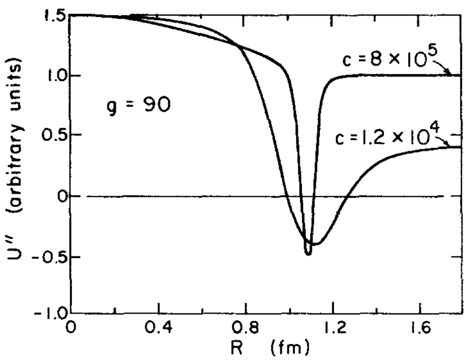

Fig. 4. $U''(\sigma(r))$ showing surface dip (B=0).

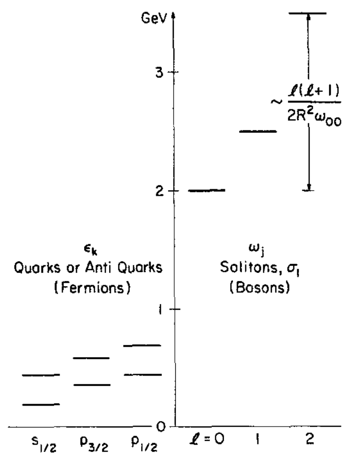

Fig. 5. Schematic representation of quark and soliton spectra.

Because of the sharp dip in U'' near the surface, the lowest normal modes are surface modes.

The soliton-quark interaction terms, H_1, couples the σ_1 excitations to quark particle-hole pairs. $q\bar{q}$ virtual excitations are interpreted as giving rise to a "meson" cloud surrounding the bag or, more specifically, the nucleon. I will return to the meson cloud later.

Although we began with a covariant Lagrangian, the MFA destroyed covariance by the selection of a preferred frame. Inclusion of the σ_1 part of the soliton field can restore covariance. For example, the localized MFA bag has $\langle\vec{P}^2\rangle \neq 0$. A state with P=0 is spread out over all space. Thus $\langle r^2\rangle$ will increase (ultimately to ∞) as the approximations are improved, and we are alerted to the fact that $\langle r^2\rangle$ is not a measure of nucleon size. We return to this in Section 6.

The terms H_3 and H_4 involve only σ_1 operators and lead to a restructuring of the soliton (σ_1) spectra. They can be handled in the one mode or uncoupled mode approximations straightforwardly in the Q-space formalism. For present purposes, we will neglect these terms and assume that their effects can be absorbed into the effective parameters. There may, however, be important physical effects in these terms.

5.0 LARGE AMPLITUDE MOTION AND COLLISIONS

5.1 The Generator Coordinate Method.

The method of generator coordinates (GCM) is being applied to large amplitude bag with particular application to N-N scattering. Consider a parameter or set of parameters which describe the static configuration of a system of quarks and the soliton field and let $|\alpha,n$ denote a set of basis states which is complete for any α. A method of obtaining these basis states is described below. The GCM state vector is written

$$|\Psi\rangle = \sum_n \int \phi_n(\alpha)\,|\alpha,n\rangle d\alpha \;. \tag{5.1}$$

Since the set is complete for each α, the expansion is overcomplete. In practice, this causes no problem since the sum is truncated to a small number of terms. In what follows, we consider only a single term and suppress n; actually, several configurations may be required. The generalization is straightforward.

The weight function $\phi(\alpha)$ is obtained by extremizing the expectation value of the Hamiltonian

$$\int d\alpha\phi^*(\alpha)\;\langle\alpha|H|\alpha'\rangle\;\phi(\alpha')\;d\alpha'$$

subject to the normalization constraint. Then

$$\int d\alpha'\;\langle\alpha|H-E|\alpha'\rangle\;\phi(\alpha') = 0 \tag{5.2}$$

This is the basic GCM integral equation for $\phi(\alpha)$. Depending upon whether the spectrum is discrete or continuous, it is either an eigenvalue or a scattering equation. Although we can work with the integral equation, it is instructive to consider the approximate differential equation which it satisfies. For a system which has well developed collective motion, we expect $\langle\alpha|H-E|\alpha'\rangle$ to fall off rapidly as a function of $\alpha - \alpha'$. To utilize that property, it is convenient to introduce the mean and relative parameters

$$\bar{\alpha} = \frac{1}{2}\,(\alpha+\alpha')\ , \quad \delta = (\alpha-\alpha') \tag{5.3}$$

Then

$$\begin{aligned}\langle\Psi|H-E|\Psi\rangle &= \int d\bar{\alpha}\int d\delta\phi^*(\bar{\alpha}+\tfrac{1}{2}\delta)\langle\bar{\alpha} + \tfrac{1}{2}\delta|H-E|\bar{\alpha} - \tfrac{1}{2}\delta\rangle\phi(\bar{\alpha} - \tfrac{1}{2}\delta)\\ &= \int d\bar{\alpha}\int d\delta[\phi^*(\bar{\alpha}) + \tfrac{1}{2}\delta\phi'^*(\bar{\alpha}) + \tfrac{1}{8}\,\delta^2\phi''^*(\alpha) + \ldots.]\times\\ &\quad\langle\bar{\alpha} + \tfrac{1}{2}\delta|H-E|\bar{\alpha} - \tfrac{1}{2}\delta\rangle\times\\ &\quad[\phi(\bar{\alpha}) - \tfrac{1}{2}\delta\ \phi'(\alpha) + \tfrac{1}{8}\delta^2\phi''(\alpha) + \ldots..]\ .\end{aligned} \tag{5.4}$$

Now $\langle\bar{\alpha} + \frac{1}{2}\sigma|H - E|\bar{\alpha} - \frac{1}{2}\delta\rangle$ is even in δ. The only integrals which survive are of the form

$$\mathcal{O}_n(\bar{\alpha}) = \int\langle\bar{\alpha} + \tfrac{1}{2}\delta|\mathcal{O}|\bar{\alpha} - \tfrac{1}{2}\delta\rangle\delta^n d\delta \tag{5.5}$$

for n even. Here $\mathcal{O}$ is either H or N = 1. Through order ϕ'' we have

$$\begin{aligned}&\langle\psi|H-E|\psi\rangle =\\ &\int d\bar{\alpha}\{\phi^*(H_0-E\ N_0)\phi + \tfrac{1}{4}(H_2-E\ N_2)(-\phi'^*\phi' + \tfrac{1}{2}\phi^{*''}\phi + \tfrac{1}{2}\phi^*\phi'')\}\ .\end{aligned} \tag{5.6}$$

This may be cast into the more familiar form

$$\int\tilde{\phi}^*(\,-\,\frac{d}{d\bar{\alpha}}\ \frac{1}{2B(\bar{\alpha})}\ \frac{d}{d\bar{\alpha}} + V(\bar{\alpha}) - E)\tilde{\phi}\ d\bar{\alpha} \tag{5.7}$$

with

$$V(\bar{\alpha}) = \frac{H_0}{N_0} + \frac{1}{2N_0^{\frac{1}{2}}} \frac{d}{d\bar{\alpha}} (H_2 - EN_2) \frac{d}{d\bar{\alpha}} \frac{1}{N_0^{\frac{1}{2}}} , \tag{5.8a}$$

$$B(\bar{\alpha}) = - \frac{N_0}{H_2-EN_2} , \tag{5.8b}$$

and

$$\tilde{\phi} = N_0^{\frac{1}{2}} \phi . \tag{5.8c}$$

Below we define α such that $\alpha \longrightarrow r$ as $r \longrightarrow \infty$, where r is (say) the separation of two bags. However, $V(\bar{\alpha})$ has no simple interpretation as a potential until $B(\bar{\alpha})$ is determined!

Following Mosel[13], we now introduce a change of variable from $\bar{\alpha}$ to x such that

$$x = X(\bar{\alpha}) = \int^{\bar{\alpha}} [B(\alpha')/\mu]^{\frac{1}{2}} d\alpha' , \tag{5.9a}$$

$$\tilde{\phi}(\bar{\alpha}) = f(\bar{\alpha})\psi(x) , \tag{5.9b}$$

$$f(x) = \text{const } [\mu B(\bar{\alpha})]^{\frac{1}{4}} \tag{5.9c}$$

where $\mu = M/2$ is the reduced nucleon mass. Then variation of Eq.(5.7) yields

$$\left(- \frac{1}{2\mu} \frac{d^2}{dx^2} + V + V_1 - E\right)\psi(x) = 0 \tag{5.10}$$

with

$$V_1 = \frac{1}{8} \frac{B''}{B^2} - \frac{1}{32} \frac{(B')^2}{B^3} \tag{5.11}$$

Since we choose α (or $\bar{\alpha}$) so that for separated bags $\alpha \rightarrow r$ (the bag separation) this is consistent with $B \rightarrow \mu$, $\alpha \rightarrow x$.

5.2 Generating The State Vector

We determine $|\alpha\rangle$ by minimizing the expectation value of the total Hamiltonian, $\langle\alpha|H|\alpha\rangle$ with respect to a variational mean field wave function for the quarks and a coherent state wave function for the soliton field subject to a constraint

$$\langle\alpha|Q|\alpha\rangle = Q_0(\alpha) \tag{5.12}$$

where Q is some moment of the quark distribution

$$Q = \int\bar{\psi}\, q(\vec{r})\psi\, d^3r \ . \tag{5.13}$$

The constrained mean field equations now assume the form

$$[\vec{\alpha}\cdot\vec{p} + \beta[g\sigma_0(\vec{r}) - \lambda q(\vec{r})] - \varepsilon_k]\psi_k = 0 \ , \tag{5.14a}$$

$$- \nabla^2\sigma_0 + U'(\sigma_0) + g \sum_{k\text{-occ}} \bar{\psi}_k\psi_k = 0 \ , \tag{5.14b}$$

where λ is a Lagrange multiplier. Instead of specifying the constraint function $q(\vec{r})$ explicitly and solving the pair of equations (5.14 a&b), it is more physical to specify the function in square brackets

$$[g\sigma_0(\vec{r}) - \lambda q(\vec{r})] \equiv \mathcal{V}(\vec{r}) \ , \tag{5.15}$$

then solve for the $\psi_k(\vec{r})$, and then for $\sigma_0(\vec{r})$; there is <u>no</u> iteration involved. The self consistency is implicit. The constraint can now be "discovered" by solving

$$\lambda q = q\sigma_0 - \mathcal{V}. \tag{5.16}$$

We parameterize $\mathcal{V}(\vec{r}) = \mathcal{V}(\alpha,\vec{r})$ by a folding procedure. We consider two spheres of radius R with centers separated by a distance α , as shown in Fig. 5. For $\alpha > 0$, we define a function $\Theta(\vec{r})$ equal to unity if $\vec{r}$ lies in the interior of either sphere and zero otherwise. If $\alpha < 0$, we choose $\Theta(r)$ to equal unity only in the intersecting, lens-shaped volume; this gives a natural continuation of α from positive values (prolate shapes) to negative values (oblate shapes). The radii of the spheres are chosen so that the enclosed volume is independent of α, and equals $2x(4/3)\ \pi R^3$, where R is the radius of a three quark bag. (The constant volume approximation, which is valid for a relativistic Fermi gas, can be relaxed to, say, minimize the energy as a function of R for each α, or to let R be another shape parameter.) α can assume all values, $-\infty < \alpha < +\infty$, and is equal to the separation of isolated bags for $\alpha > 2R$.

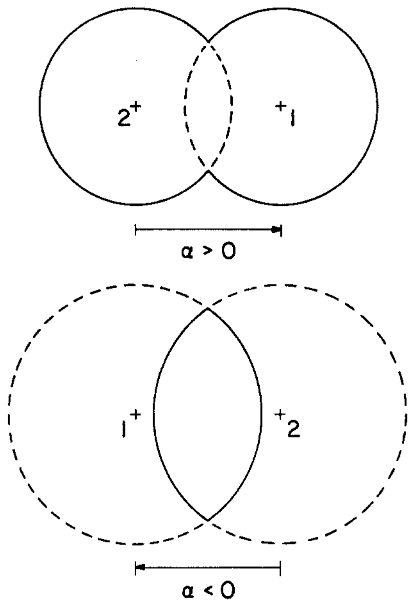

Fig. 5. Geometric shapes used to define $\Theta(\vec{r})$ and α.

Into this geometric shape is folded a Yukawa smoothing function, yielding

$$\mathcal{V}(\vec{r}) = g\sigma_V[1 - \frac{\gamma^2}{4\pi}\int \frac{e^{-\gamma|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\ \Theta(\vec{r}\,')\ d^3\vec{r}\,'] \tag{5.17}$$

Note that $\mathcal{V} \rightarrow g\sigma_V$ as $r \rightarrow \infty$ and $\mathcal{V} \cong 0$ for $\vec{r}$ well inside the geometric volume. It is adjusted to approximate the self consistent, unconstrained spherical solution for isolated bags.

The method of solution of equations (18 a&b) is a generalization of that described in ref. [5]. Here, however, both $\mathcal{V}(\vec{r})$ and $\sigma_0(\vec{r})$ are expanded in terms of even Legendre polynomials, and the quark functions are expanded in terms of Dirac spinors of good quantum number κ. An example of the shape of $\mathcal{V}(\vec{r})$ is shown in Fig. 6. Low lying eigenvalues of the Dirac equation as a function of α are shown in Fig. 7. Parity is a good quantum number for the quark functions, and we note that for well-separated bags the eigenvalues become doubly degenerate with respect to the two parities, corresponding to degenerate left and right states. All of the GCM calculations described here were performed by A. Schuh, and the work is continuing.

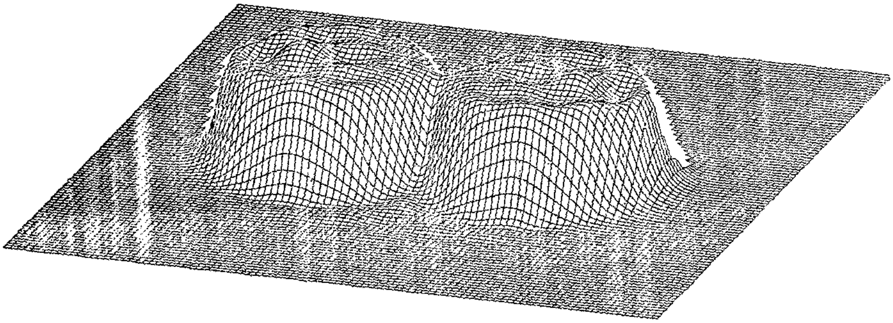

Fig. 6. The function $-\mathcal{V}(r)$ for $\alpha = 2$ fm and R = 1 fm.

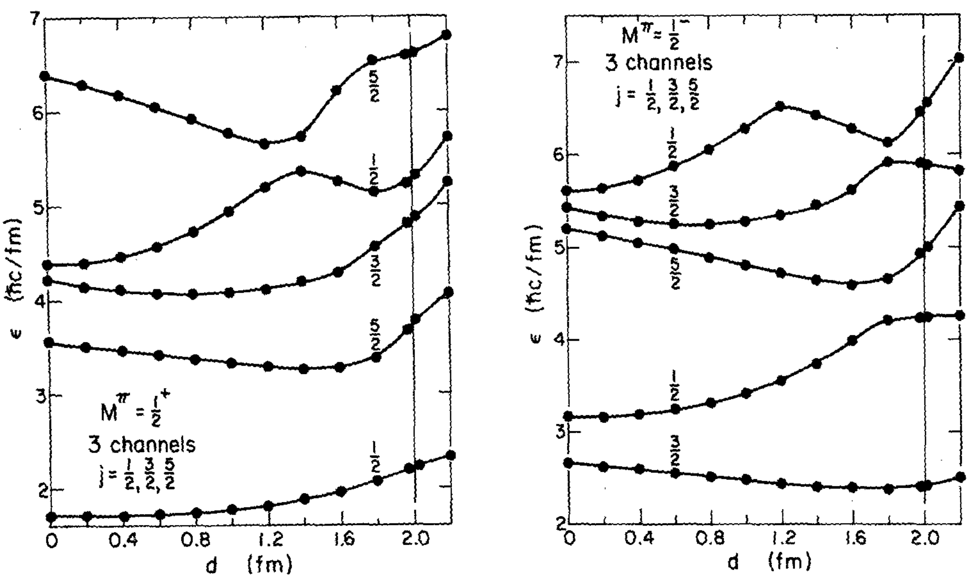

Fig. 7. Low lying quark eigenenergies calculated coupling three κ-states. Calculations coupling six states become flat, and exhibit degeneracy for + and - parities, for $\alpha > 2$ fm.

5.3 The State Vector in the Coherent State Representation

In order to implement the GCM, it is necessary to represent the state vector $|\alpha>$ as a fully quantal state. For this we use the coherent state representation of σ rather than mean field interpretation of the σ_0 obtained in solving (5.1). We need boson operators which are independent α. Corresponding in (3.1) we set

$$\sigma = \sigma_V + \sum \left(\frac{1}{2\omega_k V}\right)^{\frac{1}{2}} (a_k^\dagger e^{-i\vec{k}\cdot\vec{r}} + a_k e^{i\vec{k}\cdot\vec{r}})$$

$$\pi = i \sum \left(\frac{\omega_k}{2V}\right)^{\frac{1}{2}} (a_k^\dagger e^{-i\vec{k}\cdot\vec{r}} - a_k e^{i\vec{k}\cdot\vec{r}}) \tag{5.18}$$

where V is the box-normalization volume (which will go out). Then

$$|\alpha> = \prod_n C_n^\dagger(\alpha)\ e^{\sum\left(\frac{\omega_k}{2}\right)^{\frac{1}{2}} f_k(\alpha) a_k^\dagger} |0> N_\alpha^{-\frac{1}{2}} \tag{5.19}$$

where

$$N_\alpha = e^{\frac{1}{2}\sum\omega_k |f_k(\alpha)|^2} \tag{5.20}$$

and the f_k are the Fourier transforms of the field $\sigma_0(\vec{r},\alpha)$:

$$f_k(\alpha) = V^{-\frac{1}{2}} \int (\sigma_0(\vec{r},\alpha) - \sigma_v)\ e^{-i\vec{k}\cdot\vec{r}}\ d^3r \tag{5.21}$$

Although it is not essential to do so, a great <u>conceptual</u> simplification occurs if we <u>choose</u> $\omega_k = \Omega$, independent of k. In section 6.3 we face this question again and there consider the "natural" choice $\omega_k = (k^2 + U''(\sigma_v))^{\frac{1}{2}}$, and work with the f_k. However, with the choice $\omega_k = \Omega$, all required expressions can be evaluated in configuration space. Recall that because we normal order our Hamiltonian, a_k brings down an $f_k(\alpha)(\Omega/2)^{\frac{1}{2}}$ and $a_k^\dagger$ an $f_k^*(\alpha')(\Omega/2)^{\frac{1}{2}}$. We obtain

$$<\alpha'|\alpha> = \prod_n \left[\int \psi_n^\dagger(r,\alpha')\psi_n(\vec{r},\alpha)d^3r\right] e^{-\frac{\Omega}{4}\int[\sigma(\vec{r},\alpha') - \sigma(\vec{r},\alpha)]^2\ d^3r} \tag{5.22}$$

N.B. In the quark overlap functions we have assumed the very special case that $\int \psi_n^\dagger(\vec{r},\alpha)\psi_{n'}(\vec{r},\alpha)d^3r = 0$ for $n \neq n'$ because of spin, parity, j_z and color symmetries. In more complex configurations, the quark overlap integral will contain exchange terms. The normalization factors have been used explicitly so that $<\alpha|\alpha> = 1$.

Now one can calculate $<\alpha|:H:|\alpha'>$, which enters in (5.2) in the following conceptually simple manner. Let the Hamiltonian density [see (2.1)] be written

$$\mathcal{H} = \mathcal{H}(\psi^\dagger, \psi, \sigma, \pi). \tag{5.23}$$

Then

$$<\alpha'|:H:|\alpha> = <\alpha'|\alpha> \int d^3r\ \mathcal{H}(\psi_n^\dagger(\alpha),\ \ \psi_n(\alpha'), \sigma_{\alpha\alpha'}\ \pi_{\alpha\alpha'}) \tag{5.24}$$

with the arguments of $\mathcal{H}$ now functions rather than operators:

$$\sigma_{\alpha\alpha'}(\vec{r}) = \frac{1}{2}[\sigma_0(\vec{r},\alpha) + \sigma_0(\vec{r},\alpha']$$

$$\pi_{\alpha\alpha'}(\vec{r}) = \frac{i}{2}\ \Omega[\sigma_0(\vec{r},\alpha) - \sigma_0(\vec{r},\alpha')] \tag{5.25}$$

It is worth noting that $<\alpha|:\pi^2:|\alpha'>$ vanishes only for $\alpha' = \alpha$, which is also the case of the static bag.

The choice of Ω remains to be determined. Recall that it did not enter in the static, $\alpha' = \alpha$, calculations. There are at least two possibilities:

First, Ω is not a physical quantity, but the choice of value does affect our renormalization scheme. According to Stevenson's principle[14,15] of minimum sensitivity, such a parameter should be varied to find the value which extremizes or insensitizes the physical quantity of interest. In that case, we could even have $\Omega = \Omega(\bar{\alpha})$. One possible quantity of "interest" is $H_0(\bar{\alpha})$.

Second, one can argue that one should stick to $\omega_k = (k^2 + U''(\sigma_v))$ in order to nearly diagonalize the normal modes of the vacuum.

This matter is currently under study.

I conclude this section by emphasizing that in applying the GCM one need not go to the Schroedinger-like differential equation if the expansion in (5.4) is not valid. While the Schroedinger form has obvious conceptual advantages, the integral equation (5.2) is more generally valid.

6.0 RECOIL AND PROJECTION

A composite structure localized in a particular reference frame must be a wave packet containing a distribution of momentum components. This is the case for a bag described in the MFA. Denoting the total momentum operator by

$$\vec{P} = -\frac{i}{2}\int [\psi^{\dagger} \overleftrightarrow{\nabla} \psi + \pi \overleftrightarrow{\nabla} \phi]\, d^3r \tag{6.1}$$

and by $|B\rangle$ a localized bag state, then $\langle B|\vec{P}|B\rangle = 0$ but $\langle B|\vec{P}^2|B\rangle > 0$. Since the full theory is Lorentz invariant, corrections beyond the MFA should tend to produce a state of good $\vec{P}$ and the lowest would correspond to $\vec{P} = 0$. Such a state would be spread over all space. Clearly $\langle r^2\rangle$ is not an appropriate measure of hadronic size. We now consider two approaches to eliminate the effects of the spurious center-of-mass motion (momentum).

6.1 Center-of-energy Operator

A relativistic generalization of the center-of-mass operator is the center-of-energy operator

$$\vec{R} = \frac{\int d^3x\, \vec{x}\, \mathcal{H}(x)}{E} \quad , \quad E = \langle H\rangle \ , \tag{6.2}$$

first proposed by Fokker[16] and discussed extensively by Pryce[17]. The numerator in (6.2) is the exact boost operator, which we will return to in section 7. The operator $\vec{R}$ satisfies the commutation relations

$$[R_i, P_j] = i\delta_{ij}\, H/E \ , \tag{6.3}$$

$$\frac{d\vec{R}}{dt} = i[H,\vec{R}] = \vec{P}/E \ , \tag{6.4}$$

and

$$[R_i, R_j] = -\varepsilon_{ijk}\, M_k/E^2 \tag{6.5}$$

where $\vec{M}$ is the total angular momentum operator. (6.3) is the condition that $\vec{R}$ be canonically conjugate to P, which is satisfied in its expectation value or when operating on an eigenstate. (6.4) is a "very pleasant" and nontrivial result, which is valid for Bose fields, Dirac fields and for interacting systems. Recall that for the Dirac operator $\vec{r}_{op} = \int \psi^{\dagger}\vec{r}\psi d^3r$ we have

$$\frac{d\vec{r}_{op}}{dt} = \int \psi^{\dagger}\vec{\alpha}\psi d^3r \tag{6.6}$$

which has eigenvalues equal, in magnitude, only to the speed of light. Wave packets satisfy (6.4), and so does the position operator in the Foldy-Wouthuysen Representation[18]. The operator $\vec{R}$ is closely related to the F-W position operator. Actually, the $\vec{Q}$ operator proposed by Pryce [17] is essentially the F-W position operator, but it is more cumbersome than the $\vec{R}$-operator. The $\vec{R}$-operator is not part of a Lorentz four-vector, and the failure of the components of $\vec{R}$ to commute are shortcomings in $\vec{R}$ as a center-of-energy operator. Indeed, there is no completely satisfactory candidate for a center-of-energy.

We consider the mean field approximation, where quarks move independently in a scalar field, and assume further that there are A quarks in the same spacial state of eigenvalue ε. Then

$$<(\vec{r} - \vec{R})^2> = [1 - \frac{2\varepsilon}{E} + \frac{A\varepsilon^2}{E^2}] <\vec{x}^2> + \frac{3A}{4E^2} , \tag{6.7}$$

where E is the total energy (mass) of the system. Here $<x^2>$ is the quark mean-square size measured with respect to the bag center. The last term in (6.7) arises from the zitterbewegung of each quark; it is the effective mean-square size of point quark. It is recognizeable as the factor in the Darwin term. Even for a single point Dirac particle we would find the term $3/4E^2$ when measured by (6.7). The experimental analysis does not include this term and we therefore identify $<(\vec{r}-\vec{R})^2>$ with $<r^2> + 3/4E^2$, where $<r^2>$ is the intrinsic size. Then

$$\begin{aligned} <r^2> &\equiv <(\vec{r}-\vec{R})^2> - 3/4E^2 \\ &= [1 - \frac{2\varepsilon}{E} + \frac{A\varepsilon^2}{E^2}] <x^2> + \frac{3(A-1)}{4E^2} \end{aligned} \tag{6.8}$$

In the MIT model, $\varepsilon = 2.04/R$ and $<x^2> = 0.532\ R^2$. This leads to

$$<r^2> = [1 - \frac{15}{16A} + \frac{0.1907(A-1)}{A^2}] <x^2> , \text{ MIT} \tag{6.9a}$$

$$= 0.7007 \langle x^2 \rangle \, , \quad A = 2 \text{ (i.e. meson)} \tag{6.9b}$$

$$= 0.7298 \langle x^2 \rangle \, , \quad A = 3 \text{ (i.e. nucleon)} \; . \tag{6.9c}$$

The recoil corrections are similar in the soliton model. For the standard parameters, we calculate

$$\langle r^2 \rangle = 0.737 \langle x^2 \rangle \, , \quad A = 3 \; . \tag{6.10}$$

Since we fit the proton size, the effect of these corrections is to increase the required static bag radius by about 15%. This also increases the calculated magnetic moment (which scales as a length) by the same fraction.

6.2 Energy Corrections [10]

The Einstein relation is

$$m^2 = H^2 - \vec{P}^2 \tag{6.11}$$

where $\vec{P}$ is the total momentum. A bound on the lowest mass of the system is

$$m_0^2 \leqslant \langle H^2 \rangle - \langle \vec{P}^2 \rangle \; . \tag{6.12}$$

We approximate

$$m_0 \simeq \sqrt{\langle H \rangle^2 - \langle \vec{P}^2 \rangle} \; , \tag{6.13}$$

where the last term subtracts out the spurious square-momentum of the wave packet. In the mean field approximation

$$\langle \vec{P}^2 \rangle = \langle (\sum_i \vec{P}_i)^2 \rangle = \sum_i \langle \vec{P}_i^{\,2} \rangle \; . \tag{6.14}$$

In the MIT bag model, because of the discontinuity in the wave function at the boundary, $\langle \vec{P}_i^{\,2} \rangle$ is infinite for each quark; so also is $\langle H^2 \rangle$ infinite.

The corrections for the standard parameters of the soliton model is a decrease in the bag mass by a factor 0.84.

The results listed in Table 1 include the recoil corrections.

6.3 Projection

Given a state vector $|\vec{Z}>$ for a wave packet centered about the point $\vec{Z}$, one can construct an unnormalized vector of good momentum $|\vec{P}>$ by setting

$$|\vec{P}> = \int e^{i\vec{P}\cdot\vec{Z}}|\vec{Z}>d^3Z. \tag{6.15}$$

It is well known that there are difficulties with this procedure, because the set of states so produced are not boosted states of one another. The states, in general, differ not only in total momentum but also in intrinsic excitation. To calculate the mass of the system, we consider the state $|\vec{P}=0>$ and assume that of the set it has the lowest intrinsic excitation. Since the Hamiltonian is translationally invariant, we need only project on one side to evaluate

$$\frac{\int<\vec{Z}=0|H|\vec{Z}>d^3Z}{\int<\vec{Z}=0|\vec{Z}>d^3Z} = \frac{\int<-\frac{1}{2}\vec{Z}|H|\frac{1}{2}\vec{Z}>d^3Z}{\int<-\frac{1}{2}\vec{Z}|\vec{Z}>d^3Z} \tag{6.16}$$

The mean field approach cannot be used directly to evaluate (6.16), because the representation of the σ-field is not the same for different $\vec{Z}$. We use here the mode-expansion Sec. 3.1, and then use the one-mode coherent state approximation which is closely related to the mean field approximation. Thus we set

$$\begin{aligned}\sigma &= \sigma_V + \sum \frac{1}{\sqrt{2\omega_k V}}(a_k e^{i\vec{k}\cdot\vec{r}} + a_k^\dagger e^{-i\vec{k}\cdot\vec{r}}) , \\ \pi &= -i\sum\sqrt{\frac{\omega_k}{2V}}\,(a_k e^{i\vec{k}\cdot\vec{r}} - a_k^\dagger e^{-i\vec{k}\cdot\vec{r}}) .\end{aligned} \tag{6.17}$$

where V is normalizing box volume. The soliton part of the coherent state vector is

$$|f> = \exp\left(\sum_k \sqrt{\frac{\omega_k}{2}}\, f_k\, a_k^\dagger\right)|0> . \tag{6.18}$$

This is a one mode state of the form $\exp(\lambda A_0^\dagger)|0>$ with $\lambda A_0^\dagger \equiv \sum\sqrt{\frac{\omega_k}{2}}\, f_{\vec{k}} a_{\vec{k}}^\dagger$, where in general $A_i^\dagger = \sum_j U_{ij}^* a_J^\dagger$. We have

$$a_{\vec{k}}|f> = \sqrt{\frac{\omega_k}{2}}\, f_{\vec{k}}|f> \tag{6.19}$$

and

$$<f|\sigma|f> = \sigma_v + V^{-\frac{1}{2}} \Sigma\, f_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} , \tag{6.20}$$

where we have used $f_{\vec{k}} = f^*_{-\vec{k}}$. We normal order the Hamiltonian in the operator $a_k^\dagger$ and a_k, which also means normal ordering with respect to the A_i and $A_i^\dagger$. Everything goes through as in Sec. 3.1, including renormalized coefficients a, b and c. We have replaced a_0 of Sec. 3.1 with A_0). We can now identify

$$V^{-\frac{1}{2}} \Sigma f_{\vec{k}}\, e^{i\vec{k}\cdot\vec{r}} = \sigma_0(\vec{r}) - \sigma_v , \tag{6.21}$$

where $\sigma_0(\vec{r})$ solves the mean field equations.

A bag state displaced from the origin by $\vec{Z}$ (i.e., s_0 $(\vec{r}-\vec{Z})$) is described by

$$f_{\vec{k}}(\vec{Z}) = f_{\vec{k}}(0)\ e^{-i\vec{k}\cdot\vec{Z}} . \tag{6.22}$$

Our state vector

$$|\vec{Z}> = \prod_{n=1}^{3} \psi_n(\vec{r}_n-\vec{Z})\ \exp\ (\sum \sqrt{\frac{\omega_k}{2}}\, f_{\vec{k}}(\vec{Z})\, a_{\vec{k}}^{\dagger})\,|0> , \tag{6.23}$$

and we need to evaluate the following quantities:

$$N_\sigma(Z) = <f(-\tfrac{1}{2}\vec{Z})|f(\tfrac{1}{2}\vec{Z})> = \exp\ (\sum \frac{\omega_k}{2}\ f_k^*(-\tfrac{1}{2}\vec{Z}) f_k(\tfrac{1}{2}\vec{Z})$$

$$= \exp\left(\frac{\omega}{2}\int[\sigma_0(\vec{r}-\tfrac{1}{2}\vec{Z}) - \sigma_v][\sigma_0(\vec{r}+\tfrac{1}{2}\vec{Z}) - \sigma_v]d^3r\right); \tag{6.24}$$

$$<f(-\tfrac{1}{2}\vec{Z})|:\ \sigma^n\ :|f(+\tfrac{1}{2}\vec{Z})>$$

$$= [\sigma_Z(\vec{r})]^n\ N_\sigma(Z)\ N_q(Z) ; \tag{6.25}$$

$$\sigma_Z(\vec{r}) = \tfrac{1}{2}[\sigma_0(\vec{r}-\tfrac{1}{2}\vec{Z}) + \sigma_0(\vec{r}+\tfrac{1}{2}\vec{Z})] ; \tag{6.26}$$

$$N_q(Z) = \prod_n \int \psi_n^\dagger (\vec{r} - \tfrac{1}{2}\vec{Z})\psi_n(\vec{r} + \tfrac{1}{2}\vec{Z})d^3r \; . \tag{6.27}$$

The expectation value of $H_q + H_{q\sigma}$ is given simply by

$$\frac{\langle P=0|H_q+H_{q\sigma}|P=0\rangle}{\langle P=0|P=0\rangle} =$$

$$\frac{\sum\int \langle -\tfrac{1}{2}\vec{Z}|\psi_n^\dagger(\vec{r}-\tfrac{1}{2}\vec{Z})[\vec{\alpha}\cdot\vec{p}+\tfrac{1}{2}\beta g(\sigma_0(\vec{r}-\tfrac{1}{2}\vec{Z}) + \sigma_0(\vec{r}+\tfrac{1}{2}\vec{Z})]\psi(\vec{r}+\tfrac{1}{2}\vec{Z})|\tfrac{1}{2}\vec{Z}\rangle d^3Z}{\int \langle -\tfrac{1}{2}\vec{Z}|\tfrac{1}{2}\vec{Z}\rangle d^3Z} = \sum_n \varepsilon_n \; . \tag{6.28}$$

The final term is given by

$$\frac{\langle P=0|H_\sigma|P=0\rangle}{\langle P=0|P=0\rangle} = \frac{\int \mathcal{E}_\sigma(Z)N_\sigma(Z)N_q(Z)d^3Z}{\int N_\sigma(Z)N_q(Z)d^3Z} \tag{6.29}$$

where

$$\mathcal{E}_\sigma(Z) = \int [\tfrac{1}{2}|\vec{\nabla}\sigma_Z(\vec{r})|^2 + \tfrac{1}{2}\pi_Z^2 + U(\sigma_Z(\vec{r}))]d^3r \tag{6.30}$$

where

$$\pi_Z = i\,\frac{\omega}{2}\,[\sigma_0(\vec{r} - \tfrac{1}{2}\vec{Z}) - \sigma_0(\vec{r} + \tfrac{1}{2}\vec{Z})] \; . \qquad (6.31$$

The program to project the wave function and evaluate the expectation value of the Hamiltonian has been completed by G. Lübeck. We do not quote numerical results here because it appears that further variation of the wave functions <u>after</u> projection is desirable (perhaps necessary) to obtain reliable results. We propose to introduce scale parameters into the soliton Fourier components and quark wave functions (not full functional variation) in order to extremize (not minimize) the energy.

6.4 Boosting the Bag

The exact boost operator, acting on a zero momentum state, for an interacting relativistic field theory leads to the Lorentz translated state

$$|\vec{v}\rangle = e^{i\vec{v}\cdot\vec{K}}|\vec{P} = 0\rangle, \tag{6.32}$$

where the boost operator is

$$\vec{K} = \int d^3x\ \mathcal{H}(\vec{x})\ \vec{x}\ . \tag{6.33}$$

If

$$M_0 \simeq \langle\vec{P}=0|H|\vec{P}=0\rangle \tag{6.34}$$

is a good approximation, then we can construct the momentum state

$$|\vec{P}\rangle = e^{i\vec{P}\cdot\vec{K}/(M_0^2\ +\ \vec{P}^2)^{\frac{1}{2}}}|P=0\rangle \tag{6.35}$$

where $\vec{K}/(M_0^2+\vec{P}^2)^{\frac{1}{2}} \simeq \vec{R}$ [c.f. Eq. (6.2)], the center-of-energy operator. This boost operator was exploited by Betz and Goldflam [12] to study corrections to the electric and magnetic form factors. For the electric rms size, they obtained results in agreement with Eq. (6.7). They found a small reduction in the magnetic moment.

The present program differs from that of Betz and Goldflam [12] in that they boosted the MFA localized bag (which is not an eigenstate of $\vec{P}$), while we boost the projected, P=0 eigenstate.

7.0 GLUONS AND COLOR

Color is the keystone of QCD. Although the full nonlinearity of QCD could be incorporated in the effective Lagrangian, the objective in modelling QCD is to circumvent such extreme complexities by simulating the nonlinear effects through the scalar, color-singlet σ-field. At present, we restrict consideration to one-gluon exchange diagrams, which also implies linearization of the gluon field equations; the non-Abelian nature is preserved.

7.1 One Gluon Exchange in the MIT Bag

De Grand et al[8] in their classic 1975 paper fit a large body of hadronic spectra in the context of the MIT bag model, including a controversial gluon zero-point energy correction, and one-gluon exchange (OGE). The results involving only u and d (zero mass) quarks are presented in table 2. There are three adjustable parameters in their model. Four masses are well fitted; they had modest success with other properties. I count the degenerate ρ-ω pair a single success. The η-meson was not calculated.

Table 2.

One gluon exchange corrections in the MIT model, from De Grand, Jaffe, Johnson and Kiskis[8]. All energies are in GeV; the bag radius is in fm. There are no recoil corrections in these calculations.

	M_{exp}	M_{Bag}	R_0(fm)	E_0	E_V	E_Q	E_M
Δ	1.236	1.233	1.081	-.336	.308	1.119	.141
p	0.938	0.938	.987	-.367	.234	1.226	-.151
ω	0.783	0.783	.929	-.390	.196	.868	.110
ρ	0.77	0.783	.929	-.390	.196	.868	.110
π	0.139	0.280	.659	-.549	.070	1.222	-.462

$$E_0 = \frac{-Z_0}{R} \qquad Z_0 = 1.84\,\hbar c = 0.363 \text{ GeV-fm}$$

$$E_V = \frac{4}{3}\pi R^3 B \qquad B = 57.5 \text{ MeV/fm}^3$$

$$E_Q = \frac{A\ 2.04}{R}\hbar c$$

$$E_M \propto \frac{\alpha_s}{R} \qquad \alpha_s = 4\alpha_c = 2.2$$

The π splits off and is driven down by the OGE, but still appears at roughly twice the physical value. This we regard as a success, not a failure. We expect the OGE contributions to be qualitatively similar to the soliton model as in the MIT model.

De Grand et al[8] did not include recoil corrections in their original works, although there has been subsequent work on this problem in the context of the MIT model. Our own estimates, based on the considerations of Sec. 6.1 and ref. [11] lead to a significant reduction in the pion mass. Within a range of "acceptable" model parameters we can fit the pion mass, obtain zero, or even obtain a negative m^2. We await evaluation of the energy obtained with projected wave functions and the consistent inclusion of the OGE energies in the soliton model.

7.2 The Linear Gluon Propagator in Media

The linearized gluon field equations are essentially identical to Maxwell's equations. The chromo-electric and magnetic fields, in the Coulomb gauge, can be written

$$\vec{E}^c = -\vec{\nabla}A_0^c \ , \ \vec{B} = \vec{\nabla} \times \vec{A}^c \ ,$$
$$\vec{D}^c = \kappa E^c \ , \quad \vec{H} = \mu^{-1}\vec{B} = \kappa\vec{B} \ , \tag{7.1}$$

since $\kappa\mu = 1$. Then

$$-\vec{\nabla}\cdot\kappa\vec{\nabla}A_0^c = j_0^c$$
$$(\partial^2/\partial t^2 + \vec{\nabla}\times\kappa\vec{\nabla}\times)\vec{A}^c = \vec{j}_t^c \ . \tag{7.2}$$

where subscript "t" means the transverse component.

I will only outline the solution for the scalar part of the gluon propagator (Green function) here. The mathematical formulation and computer program for the general scalar and vector frequency-dependent propagators in an inhomogeneous frequency-dependent medium has been completed by M. Bickeboeller and is being written up as a Diplom thesis for the University of Bonn.

$\kappa(\sigma(\vec{r}))$ is an operator, but is treated in the MFA as a c-number. Present calculations are restricted to axially and reflectionally symmetric functions. This is not a fundamental restriction to the method. The scalar propagator is defined by

$$\vec{\nabla}\cdot\kappa\vec{\nabla}\, G_{00}(\vec{r},\vec{r}\,') = \delta^3(\vec{r}-\vec{r}\,') \ . \tag{7.3}$$

Note that in the Coulomb gauge, G_{00} is time (frequency) independent. Let

$$G_{00}(\vec{r},\vec{r}\,') = \kappa(r)^{-\frac{1}{2}}\mathcal{G}(\vec{r},\vec{r}\,')\kappa(r')^{-\frac{1}{2}} \tag{7.4}$$

Then

$$[-\nabla^2 + W(\vec{r})]\,\mathcal{G}(\vec{r},\vec{r}\,') = \delta^3(\vec{r},\vec{r}\,') \tag{7.5}$$

where

$$W(\vec{r}) = \frac{|\nabla\kappa|^2}{4\kappa^2} - \frac{1}{2}\,\frac{\nabla^2\kappa}{\kappa} \ . \tag{7.6}$$

We write

$$\mathcal{G}(\vec{r},\vec{r}\,') = \sum_{\alpha\alpha'} C_{\alpha\alpha'}\, J^{\alpha}(\vec{r}_<)\, N^{\alpha'}(\vec{r}_>) \ , \tag{7.7}$$

where $J^{\alpha}(\vec{r})$ and $N^{\alpha}(\vec{r})$ each satisfy the homogeneous differential equation (7.5) with regular boundary conditions at $r=0$ and $r=\infty$ respectively. For numerical reasons, we take $\kappa \to \varepsilon$ (a small constant) rather than zero as $r \to \infty$. Here $\vec{r}_< = \vec{r}$ and $\vec{r}_> = \vec{r}\,'$ if $|\vec{r}| < |\vec{r}\,'|$ and conversely. The set of constant coefficients $C^{\alpha\alpha'}$ are determined by the conditions on $\mathcal{G}(\vec{r},\vec{r}\,')$ near $|\vec{r}| = |\vec{r}\,'| =$ any arbitrary matching radius, r_m).

$$\mathcal{G}(r_m+\delta,\theta\phi;\, r_m,\theta',\phi') = \mathcal{G}(r_m-\delta,\theta,\phi;r_m,\theta',\phi') \tag{7.8}$$

and

$$-\int d\vec{s}\cdot\vec{\nabla}\mathcal{G} = 1 \tag{7.9}$$

for a small surface enclosing $\vec{r}\,'$.

$J^{\alpha}(\vec{r})$ and $N^{\alpha}(\vec{r})$ are expanded in terms of spherical harmonics, e.g.

$$J^{\alpha}(\vec{r}) = r^{-1}\sum_{\ell n} j^{\alpha}_{\ell m}(r)\; Y_{\ell m}(\theta,\phi) \ , \tag{7.10}$$

where the $j^{\alpha}(r)$ satisfy the coupled equations

$$\left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2}\right)j^{\alpha}_{\ell m} + \sum_{\ell'} \langle\ell m|W|\ell' m\rangle j^{\alpha}_{\ell' m} = 0 \ . \tag{7.11}$$

There is no coupling among functions of different m or parity. The index α labels the various linearly independent solutions. For example, let α stand for the set (L,m). Then independent solutions can be generated by starting at the origin with

$$j^{\alpha}_{\ell m} = Cr^{\ell}\ \delta_{\ell,L}\ . \tag{7.12}$$

Similar conditions can be applied for large r.

The vector propagator is complicated by requiring an expansion in terms of vector spherical harmonics. The requirement of transverse vector functions leads to a third order differential equation (for the electric modes) and the inhomogeneous term is a transverse vector delta function. As noted, Bickeboeller has solved these problems.

Thus a tensor linear gluon propagator $G_{\mu\nu}(\{\kappa\},\vec{r},\vec{r}\,',\omega)$ is constructed which is a functional of $\kappa(\sigma(\vec{r}))$ and the frequency $G_{\mu\nu}$ is diagonal in the color, and is in fact color-independent since κ is a color singlet.

The OGE interaction between quarks is then written

$$\frac{q_s^2}{4}\int d^3r_1 d^3r_2 (\bar{\psi}\lambda^c\gamma^{\mu}\psi)_1\ G_{\mu\nu}(\vec{r}_1,\vec{r}_2)\,(\bar{\psi}\lambda^c\gamma^{\nu}\psi)_2 \tag{7.13}$$

where q_s is the color charge. The λ-matrices act on the 3-color components of the quark spinors. For use in the GCM, ω is the energy difference in the quark states and κ may be evaluated at the mean deformation $\bar{\alpha}$.

8.0 TRANSITION FROM NUCLEAR MATTER TO THE QUARK PLASMA

The study of the N-N interaction gives insight into the change in the structure of the quark wave functions during the collision process. An alternate, albeit simplistic approach, is to consider nuclear matter as a collection of bags like the holes in Swiss cheese, and to study the structure of the system as it is compressed. We anticipate that the interstices between the bags--the physical vacuum--should disappear as the density is increased leading ultimately to quark plasma. Is the transition continuous or discontinuous? What is the order of the "phase" transition?

Our initial studies make rather drastic assumptions, although such approximations have proved useful in solid and fluid state physics to calculate equations of state. The approximations certainly need to be improved.

We replace the moving fluctuation bags by a regular, period fcc lattice of bags, characterized by the lattice displacement vectors $\vec{a}_n$.

We take $\sigma_0(\vec{r})$ to be periodic:

$$\sigma_0(r) = \sigma_0(\vec{r} + \vec{a}_n) \tag{8.1}$$

and to contain the reflectional and discrete rotational symmetries of a regular fcc lattice.

In the absence of OGE interactions, the quark functions satisfy the Bloch theorem

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}\ \psi_k(\vec{r}) \ , \tag{8.2}$$

where $\vec{k}$ is a continuous vector and $\psi_{\vec{k}}(\vec{r})$ is periodic,

$$\psi_{\vec{k}}(\vec{r}) = \psi_k(\vec{r} + \vec{a}_n) \ , \tag{8.3}$$

although it need not possess the other symmetries of σ_0. The ψ_k satisfy the Dirac equation

$$(\vec{\alpha}\cdot(\vec{p}+\vec{k}) + g\beta\sigma_0(\vec{r}))\psi_{\vec{k}} = \varepsilon_{\vec{k}}\ \psi_{\vec{k}} \ . \tag{8.4}$$

The ε_k have the characteristic band spectrum of a crystal.

At low density--well separated bags--the self-consistent solutions for σ and ψ are those of isolated bags; the energy spectrum is discrete. As the bags are moved closer together, the ε_k spread out into bands.

Although the lattice calculation is feasible (by choosing selected $\vec{k}$-vectors) we choose to use the Wigner-Seitz spherical cell approximation. A single bag is enclosed in a sphere of radius, r_0 such that its volume is the same as that ascribed to each bag in the crystal. Because of the assumed spherical symmetry, the lowest band assumes the form for s-states; s- can be represented by (2.8), so that

$$du_k/dr = (g\sigma_0 + \varepsilon_k)v_k \ , \tag{8.5a}$$

$$dv_k/dr + 2v_k/r = (-g\sigma_0 + \varepsilon_k)u_k \ , \tag{8.5b}$$

and

$$-\nabla^2\sigma_0 + U'(\sigma_0) + \frac{9g}{\bar{k}^3}\int_0^{\bar{k}} k^2 dk(u_k^2 - v_k^2). \tag{8.5c}$$

The quark functions are normalized to

$$\int_0^{r_0} r^2 dr(u_k^2 + v_k^2) = 1 \ . \tag{8.6}$$

The boundary conditions on σ are

$$\sigma'(0) = \sigma'(r_0) = 0 \ . \tag{8.7}$$

The lowest member of the quark band satisfies the boundary condition

$$v_{bot}(r_0) = 0 \Rightarrow u_{bot}(r_0) = 0 \ . \tag{8.8}$$

At the top of the band, we have

$$u_{top}(r_0) = 0 \ . \tag{8.9}$$

Using these boundary conditions and a given $\sigma_0(r)$, we can solve for the corresponding ε_{bot} and ε_{top}. The intermediate ε's lie in the continuum of the band and do not require the solution of an eigenvalue problem. Rather ε_k is to be specified and Eqs. (8.5a&b) integrated forthright. Let $t = k/k_{top}$ (k_{top} is inversely proportional to the lattice spacing); $t_{top} = 1$. We make the reasonable Ansatz that

$$\varepsilon(t) = \varepsilon_{bot} + (\varepsilon_{top} - \varepsilon_{bot})(1 - \cos\pi t)/2 \tag{8.10}$$

The inhomogeneous term in (5.6c) can be rewritten

$$\frac{9g}{\bar{t}^3} \int_0^{\bar{t}} t^2 dt(u_t^2 - v_t^2) \tag{8.11}$$

The factor $9/\bar{t}^3$ assures 3 quarks per bag irrespective of $\bar{t}$. The spin-flavor-color degeneracy of each state is 12-fold. Thus the band is only 1/4 filled.

We pack the band to a value $\bar{t} = (1/4)^{1/3}$. The self consistent solution of Eqs. (8.5) leads to a phase transition from bags to uniform quark (v=0) and σ fields at $r_0 \simeq 1.7$ fm, well above normal nuclear density. Gluon exchange is needed to promote clustering into three-quark structures.

9.0 SUMMARY AND PROSPECTS

The soliton model represents an extension of the MIT bag model to allow for the dynamical degrees of freedom associated with the confinement mechanism. The soliton model has 5 parameters, MIT has 3, but the soliton model has the flexibility, by choice of the parameters, to reproduce either the MIT or the SLAC bags. With appropriate choice of parameters and inclusion of one gluon exchange, the resulting hadronic spectra is similar to the MIT model.

Because the model can be cast in Hamiltonian form, dynamical processes can be calculated using techniques developed for nuclear collective motion. This permits calculation of N-N collisions, recoil corrections and the construction of bag states of good momentum. The last is essential for the proper calculation of electromagnetic form factors.

In this paper, the pion has been alluded to frequently. It is currently being studied actively in the context of the soliton model. The pion appears here as an anomalously light particle, split off and pushed down from the meson multiplet by OGE. The nucleon bag should be soft to $q\bar{q}$, virtual excitation with pion quantum numbers. In the soliton model, these virtual excitations are to be identified with the pion cloud. One can also calculate pi-nucleon coupling and the weak decay of the pion, $\pi \rightarrow \mu + \bar{\nu}_\mu$. Indeed, bags can be created and destroyed in the model.

This description of pion physics begins with a Lagrangian which does not respect chiral invariance and seeks to achieve PCAC from dynamics. The more fashionable approach is to begin with a chirally invariant Lagrangian from which the pion emerges as a massless Goldstone boson; somewhere, the pion must be given a mass and CAC broken. In all models, effective fields (σ or π or both [20] are introduced to describe degrees of freedom which are too difficult to handle explicitly. It seems prudent to pursue various models, since each has its advantage for particular physical phenomena.

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