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Probing light new physics via precision observables at low energies

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Given the lack of any positive detection of heavy new physics at high-energy experiments at colliders, a growing interest has encouraged the physics community to explore the possibility for physics beyond the standard model to be light and weakly coupled. The most suitable experiments where to look for this kind of new physics candidates are typically those -performed at low energy- where a large precision is achieved. In this talk, I will discuss the opportunity to probe light new physics candidates -such as axion-like particles (ALPs) - via different low-energy processes. A particular focus will be put on flavour-diagonal CP-violating observables, such as the electric dipole moments of either composite or elementary particles.

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1. Introduction

The standard model (SM) of particle physics is one of the most successful scientific theories ever constructed: not only it describes the interactions among all of the particles species so far discovered, but almost all of its predictions have been experimentally verified at extremely high accuracy levels. Nevertheless, it is a known fact that the SM itself cannot be the ultimate theory of nature, as it leaves several fundamental questions unanswered.

These can be of either observational nature, or of theoretical nature. For instance, regarding the first class of problems, the SM is unable to account for neutrino masses or for the observed dark matter abundance in the universe, nor it is able to explain the observed overabundance of matter over anti-matter in the universe. Within the second class of open questions one can include the flavour puzzle, regarding the origin of the observed structure of the SM Yukawa matrices, as well as some fine-tuning problems related to the unsettling smallness of some SM input parameters with respect to their expected size. These regard for instance the value of the Higgs mass parameter, associated with the hierarchy problem, or the value of the QCD theta angle, related to the so-called strong CP problem. Finally, the necessity to provide a coherent description of gravity at the smallest length scales necessarily requires extending the SM. All of these issues point at the existence of new physics (NP) beyond the SM (BSM) which can provide them with satisfactory answers. Despite the urgent necessity for NP, however, no experimental evidence of new states has so far come to light.

From a theoretical viewpoint, there exist both models putting forward heavy new states emerging at large energy scales, yet out of the reach of current collider experiments, and models featuring light and feebly interacting particles which have so far escaped detection. On the experimental side, a fervid research programme has been undertaken, aiming to search for NP signals both at the energy frontier and at the precision frontier. Whereas the first kind of experiments aim at probing new states by progressively increasing the center-of-mass energy available at colliders, possibly uncovering new states via direct detection, the latter have an altogether different approach. Indeed, they are rather rooted in the idea that an understanding on BSM physics can be gathered by considering the possible mismatch existing between precisely measured quantities and the corresponding accurate predictions from the SM. The resulting discrepancies can then be ascribed to the effect of virtual NP states.

Given the lack of any positive detection of any NP state, all of these possibilities, and the combinations thereof, should be taken into account on the same footing when trying to chart the landscape of BSM physics with an agnostic attitude. In this talk I will focus on a specific possibility, that is to probe light NP in precision observables at low energies. This possibility is particularly intriguing, both from the theoretical and the experimental point of view.

From the theoretical perspective, light NP candidates, if ever discovered, would allow us to probe new symmetries beyond the SM ones. Indeed, the lightness of these candidates has to be ensured by the presence of either global or local symmetries. A paradigmatic example of the first case is given by axion-like particles (ALPs), which are nothing but the pseudo Nambu-Goldstone bosons (pNGBs) associated with the spontaneous breaking of some symmetry beyond the SM. Local symmetries are instead naturally related to the existence of gauge vector bosons; a new exact gauge symmetry would predict a perfectly massless gauge boson, but these particles can acquire small mass terms if the aforementioned symmetry is spontaneously broken by, e.g., some Higgs-like mechanism beyond the SM. Experimentally, an appealing possibility to probe this kind of scenarios is provided by precision observables at low energies, such as leptonic electric and magnetic dipole moments, lepton flavour violating processes ($\mu \to eee$, $\mu \to e\gamma$ ...), or rare meson decays ($K \to \pi + \text{inv.}$, $B \to K + \text{inv.}$).

In this contribution, I will discuss the opportunity to probe light NP candidates via different low-energy probes. In order to illustrate the general approach to this kind of searches, I will focus on a paradigmatic example, that is the search for CP-violating ALPs by studying their impact on the electric dipole moments (EDMs) of either composite or fundamental particles.

2. CP-violating axion-like particles

Throughout this contribution, I will refer as "axion-like particles" to the pseudo Nambu-Goldstone bosons that emerge from the spontaneous symmetry breaking of some unspecified global symmetry above the electroweak scale. As such, they can be regarded as generalizations of the standard QCD axion, with two noteworthy differences. First of all, unlike the QCD axion, ALPs need not necessarily solve the strong CP problem. Moreover, their masses and coupling constants are arbitrary parameters to be either measured or bounded experimentally.

ALPs have been receiving considerable attention in the last decade, as they represent economical extensions of the SM which can be invoked to solve some open questions in particle physics. For instance, besides the QCD axion solution to the strong CP problem[1–4] –see [5] for a modern review on the topic–, ALP-based solutions have been proposed to address the hierarchy problem[6] or the flavour puzzle[7, 8]. In addition to this, in some specific regions of their parameter space, ALPs can be regarded as appealing dark matter candidates.

In parallel to theory developments, an intense and broad experimental program has been undertaken in the attempt to probe NP scenarios involving axions and ALPs [9, 10]. This program encompasses experiments that range from low-energy experiments, such as haloscopes, helioscopes and optical setups, which aim at probing light, "wave-like" ALPs with masses as low as few eVs, to beam dump and collider experiments, covering up mass regions as large as few GeVs. ALPs have been searched for also in rare flavour-violating decays, both in the hadronic [11] and in the leptonic sector[12]. Indeed, as no fundamental reason exists for ALPs to respect the accidental SM flavour group, there is also no reason why they should not induce flavour-changing neutral currents already at the tree level.

An intriguing possibility to probe ALPs is by taking into account their impact on CP-violating (CPV) observables, such as the EDMs of particles, nucleons, nuclei, atoms and molecules. Indeed, EDMs are flavour-diagonal, CPV observables which feature a basically negligible SM background, so that any positive detection of a non-null EDM in a wide variety of either fundamental or composite particles would necessarily imply the existence of BSM physics. In addition to this, an impressive research program is being carried out at several low-energy experiments, with the aim of improving the current EDM bounds by orders of magnitude.

In order to probe ALPs via EDMs, clearly, they must feature CPV interactions with SM fields. The latter can originate quite naturally in a wealth of different scenarios. For instance, they can be generated in strongly-coupled theories based on a confining gauge group beyond the SM one, with a dynamics that is analogous to the one providing the SM neutral pion with small CPV couplings.

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Alternatively, the mixing between an ALP state and the Higgs field in relaxion models can quite naturally supply the ALP with CPV interactions with SM fields. Finally, some well-motivated UV complete models exist predicting CPV light scalars. Regardless of the specific origin of their couplings, the interactions of a CPV ALP ϕ with SM fields are conveniently described by the following effective Lagrangian [13]:

$$\begin{aligned}\mathcal{L}_\phi = & e^2\frac{\tilde{C}_\gamma}{\Lambda}\,\phi\, F_{\mu\nu}\tilde{F}^{\mu\nu} + g_s^2\frac{\tilde{C}_g}{\Lambda}\,\phi\, G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a + i\, y_P^{ij}\,\frac{v}{\Lambda}\,\phi\,\bar{f}_i\gamma_5 f_j + \\ & e^2\frac{C_\gamma}{\Lambda}\,\phi\, F_{\mu\nu}F^{\mu\nu} + g_s^2\frac{C_g}{\Lambda}\,\phi\, G^a_{\mu\nu}G^{\mu\nu}_a + y_S^{ij}\,\frac{v}{\Lambda}\,\phi\,\bar{f}_i f_j\,,\end{aligned} \tag{1}$$

where Λ defines the cutoff scale of the effective field theory (EFT) description. Even if $\Lambda \gtrsim 1$ TeV is assumed, I will focus only on electromagnetic and strong interactions, since the role of weak interactions is subleading and can be neglected in a first approximation.$F_{\mu\nu}$ and $G^a_{\mu\nu}$ denote the SM field-strength tensors for the photon and the gluon field, respectively, and $\tilde{F}_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$ are their duals. f denotes a SM fermion in the mass basis, i and j are its flavour indices and the matrices y_P and y_S are hermitian by construction.

The operators in the first line possess opposite CP-transformation properties with respect to those in the second line, so that CP is violated regardless of the CP-nature of the ALP field ϕ. In addition to having different CP transformation properties, it is also interesting to notice that the two sets of operators display a different behaviour with respect to the (approximate) shift symmetry enjoyed by the ALP field. Indeed, whereas the operators in the first line respect the shift symmetry at the classical level [1], the ones in the second line break it explicitly. An interesting point to be made is then related to the expected hierarchy between operators breaking the shift symmetry and those preserving it, and consequently about the size of CP-violating interactions in comparison to those preserving CP. Indeed, whereas the first will have to feature a dependence on the product of Wilson coefficients related to both the set of operators ($\sim C_{\rm CPE}\, C_{\rm CPO}$), the second are naively going to be sensitive to the square of the largest Wilson coefficient in the two sectors ($\sim C^2_{\rm CPE} + C^2_{\rm CPO}$).

The distinction between operators possessing opposite CP transformation properties is thus closely related to that of operators behaving differently under the action of the ALP shift symmetry. This turns out being particularly useful when it comes to classifying the Jarlskog invariants of the theory, which represent a basis-independent measure of the amount of CP violation that is present in the theory under consideration. In our current setup, the Jarlskog invariants related to the Lagrangian in Eq. (1) are found to be:

$$C_a\tilde{C}_b\,,\quad y_S^{ii}\tilde{C}_a\,,\quad y_P^{ii}C_b\,,\quad y_S^{ii}y_P^{jj}\,,\quad y_S^{ik}y_{SM}^{kk}y_P^{ki}\,, \tag{2}$$

where $a = \{\gamma, g\}$ and y_{SM} is SM Yukawa matrix.

The phenomenology related to CPV ALPs crucially depends on their mass. With this respect, it is useful to distinguish three main regimes:

1) $m_\phi \gtrsim$ **few GeVs**. In this regime QCD is perturbative, and the original Lagrangian has to be evolved down to the ALP mass scale, where the ALP field is integrated out of the theory and a matching onto an appropriate low-energy description is performed. For details, see [13].

[1] The pseudoscalar Yukawa operator is indeed generated from the interaction term $\partial_\mu\phi\bar{f}\gamma^\mu\gamma_5 f$ upon integrating by parts and applying the equations of motion.

2) $m_\phi \ll$ **GeV**. In this case QCD confines before the ALP field is integrated out. It then remains a propagating degree of freedom, whose interactions with quarks and gluons have to be re-expressed in terms of low-energy interactions with hadrons. This can be done, as I will discuss in detail in the following, by resorting to non-perturbative techniques such as chiral perturbation theory (χpt)[14].

3) 1 **GeV** $\lesssim m_\phi \lesssim$ **few GeVs**. In this intermediate case the ALP mass lies in the same mass range where QCD resonances exist. QCD cannot be treated perturbatively, and χpt is out of its regime of validity. One has then to resort to other non-perturbative techniques.

In the following I will mainly focus on the first two scenarios; before delving into the details of the phenomenological analyses of the first two frameworks, an observation has to be made: regardless of the precise ALP mass value, the three possible scenarios require, up to a certain point, a similar EFT treatment. First of all, the renormalization of the Lagrangian in Eq. (1) has to be performed, together with the running of its Wilson coefficients from the EW scale, where $\mathcal{L}_\phi$ is defined, down to either the ALP mass scale or to the QCD confinement scale. At such scales indeed, $\mathcal{L}_\phi$ no longer represents a suitable effective descriptions of the CPV effects one would like to investigate. Different effective low-energy descriptions are then needed, upon which the evolved Lagrangian has to be properly matched. As the matching procedure is performed, one has then to construct the Jarlskog invariants appearing in the chosen low-energy description. Once a matching dictionary is established, these can be re-expressed in terms of the "high-energy" Jarlskog invariants in Eq. (2). This allows then to connect experimental constraints, which are going to be expressed in terms of the low-energy Jarlskog invariants of the theory, with the sources of CP violation in Eq. (1). Once a UV completion is specified, this would allow to directly constrain the fundamental sources of CP violation above the electroweak scale in terms of experimental bounds.

In the rest of this paper I will discuss the details of this procedure in the case of both "heavy" ALPs, with masses larger than a few GeVs, and "light" ALPs, characterized by a mass considerably lighter than the QCD confinement scale.

2.1 CP-violating ALP Lagrangian for "heavy" ALPs: $m_\phi \gtrsim$ few GeV

The first case I will consider corresponds to $m_\phi \gtrsim$ few GeV, a regime in which QCD can be treated perturbatively. After having properly evolved the Wilson coefficients of the Lagrangian in Eq. (1) down to the ALP mass scale, one has to integrate out the corresponding degree of freedom. The relevant low-energy Lagrangian describing the impact of new CPV sources on the EDMs of molecules, atoms, nuclei and nucleons featuring only SM fields reads [15]

$$\begin{aligned}\mathcal{L}_{\rm CPV} = &\sum_{i,j=u,d,e} C_{ij}(\bar{f}_i f_i)(\bar{f}_j i\gamma_5 f_j) + \alpha_s C_{Ge}\, GG\, \bar{e} i\gamma_5 e + \alpha_s C_{\tilde{G}e}\, G\tilde{G}\, \bar{e}e \\ &- \frac{i}{2}\sum_{i=u,d,e} d_i \bar{f}_i (F\cdot\sigma)\gamma_5 f_i - \frac{i}{2}\sum_{i=u,d} g_s d_i^C \bar{f}_i (G\cdot\sigma)\gamma_5 f_i + \frac{d_G}{3} f^{abc} G^a \tilde{G}^b G^c\,,\end{aligned} \tag{3}$$

where color-octet 4-quark operators were omitted, since they emerge only at one-loop level in the ALP framework under consideration. The dim-4 $G\tilde{G}$ operator was instead neglected under the

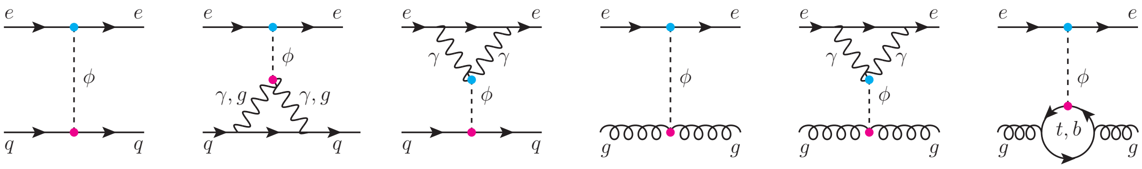

Figure 1: Leading contributions to the Weinberg operator. Figure from Ref. [13]

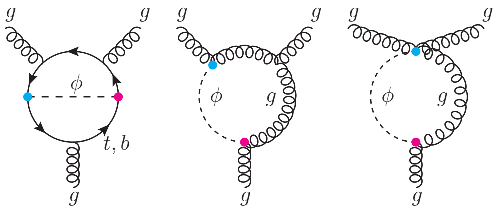

Figure 2: Leading contributions to the Weinberg operator. Figure from Ref.[13]

assumption of the existence of some UV mechanism solving the strong CP problem. Within the EFT approach I have been discussing, the low-energy Wilson coefficients C_{ij}, C_{Ge} and $C_{\tilde{G}e}$ are generated via a t-channel ALP exchange, see Fig. 1 and read

$$C_{ij} \simeq \frac{v^2}{\Lambda^2}\frac{y_S^{ii}y_P^{jj}}{m_\phi^2}\,, \qquad C_{Ge} = \frac{4\pi}{m_\phi^2}\frac{v}{\Lambda^2}C_g y_P^{ee}\,, \qquad C_{\tilde{G}e} = \frac{4\pi}{m_\phi^2}\frac{v}{\Lambda^2}\tilde{C}_g y_S^{ee}\,. \tag{4}$$

The last term in Eq. (3) describes the three-gluon CP-odd Weinberg operator which. Within the CPV ALP EFT, it is generated by the diagrams in Fig. 2. The related Wilson coefficient d_G is found to be:

$$d_G \simeq \frac{g_s\alpha_s}{(4\pi)^3}\sum_{i=t,b}\frac{v^2}{\Lambda^2}\frac{y_S^{ii}y_P^{ii}}{4m_i^2} + \frac{3g_s}{\pi^2}\frac{g_s^4 C_g\tilde{C}_g}{\Lambda^2}\log\frac{\Lambda}{m_\phi}\,, \tag{5}$$

where the first term stems from the two-loop diagram featuring a heavy top or botton quark loop. The second term of Eq. (5) is instead induced by the remaining one-loop diagrams in Fig. 2.

Finally, the virtual exchange of a CPV ALP can generate fermionic (C)EDMs. The leading-order contributions are due to Feynman diagrams in Fig. 3 and read

$$\begin{aligned}\frac{d_i}{e} \simeq &-\sum_k \frac{Q_k}{16\pi^2}\frac{m_k}{m_\phi^2}\frac{v^2}{\Lambda^2}\Re(y_S^{ik}y_P^{ki})\,\ell(x_k) - \sum_k \frac{N_c\,\alpha\,Q_i\,Q_k^2}{8\pi^3\,m_k}\frac{v^2}{\Lambda^2}\left(y_P^{ii}y_S^{kk}f(x_k) + y_S^{ii}y_P^{kk}g(x_k)\right)\\ &-\frac{Q_i}{2\pi^2}\frac{v}{\Lambda^2}e^2(y_S^{ii}\tilde{C}_\gamma - C_\gamma y_P^{ii})\log\frac{\Lambda}{m_\phi} - \frac{3\,\alpha\,Q_i^3}{\pi^3}\frac{m_i}{\Lambda^2}e^4 C_\gamma\tilde{C}_\gamma\log^2\frac{\Lambda}{m_\phi}\\ &-\delta_{qi}\frac{2\,\alpha_s\,Q_i}{\pi^3}\frac{m_i}{\Lambda^2}e^2 g_s^2(C_\gamma\tilde{C}_g + C_g\tilde{C}_\gamma)\log^2\frac{\Lambda}{m_\phi}\,,\end{aligned} \tag{6}$$

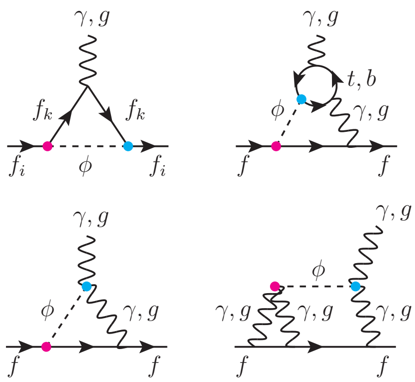

Figure 3: Leading contributions to the fermionic (C)EDMs. Figure from Ref.[13]

in the EDM case (with $i = e, u, d$, $q = u, d$, and $N_c = 3$) and

$$\begin{aligned} d_i^C \simeq &- \sum_k \frac{1}{16\pi^2} \frac{m_k}{m_\phi^2} \frac{v^2}{\Lambda^2} \Re(y_S^{ik} y_P^{ki})\, \ell(x_k) - \sum_k \frac{\alpha_s}{16\pi^3 m_k} \frac{v^2}{\Lambda^2} \left(y_P^{ii} y_S^{kk} f(x_k) + y_S^{ii} y_P^{kk} g(x_k) \right) \\ &- \frac{1}{2\pi^2} \frac{v}{\Lambda^2} g_s^2 (y_S^{ii} \tilde{C}_g - C_g y_P^{ii}) \log \frac{\Lambda}{m_\phi} - \frac{4\,\alpha_s}{\pi^3} \frac{m_i}{\Lambda^2} g_s^4 C_g \tilde{C}_g \log^2 \frac{\Lambda}{m_\phi} \\ &- \frac{3\,\alpha\, Q_i^2}{2\pi^3} \frac{m_i}{\Lambda^2} e^2 g_s^2 (C_\gamma \tilde{C}_g + C_g \tilde{C}_\gamma) \log^2 \frac{\Lambda}{m_\phi}\,, \end{aligned} \tag{7}$$

for CEDMs ($i = u, d$). The loop functions featured in these expressions read $\ell(x) = (3-4x+x^2+2\log x)/(1-x)^3$, where $x_k = m_k^2/m_\phi^2$ and, in the limit $x \gg 1$, $f(x) \approx (6\log x + 13)/18$ and $g(x) \approx (\log x + 2)/2$. All of the above results were first obtained in [13], by regularizing UV divergences with a hard cutoff and under the assumption of no significant cancellations with finite terms in the EFT description. With this information, it is then possible to constrain the various Jarlskog invariants directly in terms of experimental bounds.

Constraints on d_e, C_{ij} and C_{Ge} in Eq. (3), for instance, are better obtained by using the polar molecule ThO. It is a known fact that the electron spin-precession frequency $\omega_{\rm ThO}$ is affected by both d_e and CP-odd electron-nucleon (N) interactions [16]

$$\omega_{\rm ThO} = 1.2\,{\rm mrad/s} \left(\frac{d_e}{10^{-29}\, e\,{\rm cm}} \right) + 1.8\,{\rm mrad/s} \left(\frac{C_S}{10^{-9}} \right) .$$

The coefficient C_S related to the effective interaction $\mathcal{L} \supset -\frac{G_F}{\sqrt{2}} C_S \bar{N} N \bar{e} i\gamma_5 e$ [16] can be re-expressed in terms of the coefficients C_{ij} and C_{Ge} as $C_S/v^2 \simeq -17(C_{ue} + C_{de}) + 4.7\,{\rm GeV}\, C_{Ge}$. Another extremely constraining CPV observable which can be employed in order to put bounds on this scenario is the neutron EDM. It receives contributions from both quark (C)EDMs, the Weinberg operator and 4-quark operators [17–19]:

$$\begin{aligned} d_n = &\, 0.784(28)\, d_u - 0.204(11)\, d_d - 0.55(28)\, e\, d_u^C - 1.10(55)\, e\, d_d^C \\ &+ 50(40)\,{\rm MeV}\, e\, d_G + 30(20)\,{\rm MeV}\, e\,(C_{ud} - C_{du})\,. \end{aligned} \tag{8}$$

CP-violating invariant	Bound	Observable
$\lvert C_\gamma \tilde{C}_\gamma \rvert$	6.2×10^{-3}	$\omega_{\rm ThO}(d_e)$
$\lvert C_g \tilde{C}_g \rvert$	1.4×10^{-6}	$d_n(d_G)$
$\lvert C_\gamma \tilde{C}_g \rvert$	0.40	$d_{\rm Hg}(C_P, C_S)$
$\lvert C_g \tilde{C}_\gamma \rvert$	2.3×10^{-3}	$\omega_{\rm ThO}(C_S)$
$\lvert y_S^{ee} \tilde{C}_\gamma - y_P^{ee} C_\gamma \rvert$	6.9×10^{-11}	$\omega_{\rm ThO}(d_e)$
$\lvert y_S^{uu} \tilde{C}_g - y_P^{uu} C_g \rvert$	8.1×10^{-9}	$d_n(d_u^C)$
$\lvert y_S^{dd} \tilde{C}_g - y_P^{dd} C_g \rvert$	6.5×10^{-9}	$d_n(d_d^C)$
$\lvert y_S^{uu} y_P^{dd} - y_S^{dd} y_P^{uu} \rvert$	5.6×10^{-9}	$d_{\rm Hg}(C_{ud} - C_{du})$
$\lvert y_S^{ee}\, y_P^{ee} \rvert$	1.0×10^{-10}	$\omega_{\rm ThO}(d_e)$

Table 1: Bounds on CP-violating invariants for $\Lambda = 1$ TeV and $m_\phi = 5$ GeV. In the 3rd column we specify the observable and the leading operator setting the bound (in brackets).

The experimental bound $d_n < 1.8 \cdot 10^{-26}\, e$ cm (90% C.L.) [20, 21] can then be directly translated in constraints on the EW-scale Wilson coefficients in the Lagrangian in Eq. (1). Finally, the diamagnetic atom ^{199}Hg can be used to put bounds on both nuclear and leptonic CP-odd interactions [16, 18]

$$d_{\rm Hg} \simeq 4.0 \cdot 10^{-4} d_n - [2.8\, C_S - 2.1\, C_P]\, 10^{-22}\, e\,\text{cm}\,, \tag{9}$$

where $C_P \simeq C_P^{(0)} - C_P^{(1)}$ is defined via the Lagrangian $\mathcal{L} \supset -\frac{G_F}{\sqrt{2}} \bar{N}(C_P^{(0)} + \tau_3 C_P^{(1)}) i\gamma_5 N \bar{e}e$. Constraints are then placed by making utilizing $C_P/v^2 = 350(C_{eu} + C_{ed}) + 1.1\,\text{GeV}\, C_{\tilde{G}e}$ together with the experimental upper bound $d_{\rm Hg} < 6 \cdot 10^{-30} e$ cm (90% C.L.) [22]. In Table 1 some results for $m_\phi = 5$ GeV are reported, showing the sensitivity of EDM probes to the CPV Jarlskog invariants of the theory.

2.2 CP-violating ALP Lagrangian: $m_\phi \lesssim 1$ GeV

If CPV ALPs have sub-GeV masses, perturbative QCD techniques can no longer be employed and one has to resort to non-perturbative methods. In this section I will discuss the inclusion of a light CPV ALP in a chiral perturbation theory framework, see [14, 23] for more details. In order to understand how this can be done, it is useful to first consider the massless QCD Lagrangian:

$$\mathcal{L}^0_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R\,, \tag{10}$$

where $q = (u, d, s)^T$. This Lagrangian is invariant under the action of elements of the chiral symmetry group $G = SU(3)_L \times SU(3)_R$; chiral symmetry-breaking terms, such as masses or possibly interactions with non-QCD fields can be included in $\mathcal{L}^0_{\rm QCD}$ by introducing appropriate spurions (a_μ, v_μ, s, p) as external source fields:

$$\begin{aligned} \mathcal{L}^{\rm ext}_{\rm QCD} &= \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu(v_\mu + a_\mu\gamma_5)q + \bar{q}(s - ip\gamma_5)q \\ &= \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu(2r_\mu P_R + 2\ell_\mu P_L)q + \bar{q}(s - ip\gamma_5)q\,. \end{aligned} \tag{11}$$

The chiral counterpart of the QCD Lagrangian equipped with external sources then simply reads

$$\mathcal{L}^{\rm ext}_{\chi{\rm pt}} = \frac{f_\pi^2}{4} {\rm Tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma + \Sigma^\dagger \chi + \chi^\dagger \Sigma\right] + O(p^4)\,, \tag{12}$$

where $\Sigma(x) = \exp\left[i\lambda_a \pi_a(x)/f_\pi\right]$ is the matrix field collecting all the mesonic fields $\pi_a(x)$, which are to be identified as the Goldstone boson of the $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ spontaneous symmetry breaking pattern. $f_\pi = 92.4 \pm 0.3$ MeV is the pion decay constant. Under $SU(3)_L \times SU(3)_R$ the mesonic matrix field transforms as $\Sigma(x) \rightarrow R\Sigma(x)L^\dagger$. Moreover, in the previous expression we made use of the following definitions:

$$D_\mu \Sigma = \partial_\mu \Sigma - i r_\mu \Sigma + i \Sigma \ell_\mu \qquad \text{and} \qquad \chi = 2B_0\left(s + ip\right). \tag{13}$$

At low energies, the path-integral description of the generating functional $Z\left[v_\mu, a_\mu, s, p\right]$ allows one to establish a duality between the Lagrangians (11) and (12):

$$\exp(iZ) = \int \mathcal{D}q\, \mathcal{D}\bar{q}\, \mathcal{D}G_\mu\, \exp\left(i \int d^4x\, \mathcal{L}^{\text{ext}}_{\text{QCD}}\right) = \int \mathcal{D}\Sigma \exp\left(i \int d^4x\, \mathcal{L}^{\text{ext}}_{\chi\text{pt}}\right). \tag{14}$$

Such a duality can be then employed to construct the chiral counterparts to fermionic bilinears by taking functional derivatives of the QCD and the χpt actions with respect to the appropriate external sources. For example, let us consider the scalar Dirac bilinear $\bar{q}_i \mathcal{Z}_{ij} q_j$. It can be obtained from $\mathcal{L}^{\text{ext}}_{\text{QCD}}$ by taking a functional derivative with respect to the external source χ, conveniently specialized to $s = \mathcal{Z}$. To extract its chiral counterpart, all we have to do is then to take the same functional derivative on $\mathcal{L}^{\text{ext}}_{\chi\text{pt}}$:

$$\bar{q}_i \mathcal{Z}_{ij} q_j = -\mathcal{Z}_{ij} \frac{\partial \mathcal{L}^{\text{ext}}_{\text{QCD}}}{\partial \mathcal{Z}_{ij}} \equiv -\mathcal{Z}_{ij} \frac{\partial \mathcal{L}^{\text{ext}}_{\chi\text{pt}}}{\partial \mathcal{Z}_{ij}} = -\frac{f_\pi^2}{2} B_0 \text{Tr}\left[\mathcal{Z}(\Sigma + \Sigma^\dagger)\right]. \tag{15}$$

If instead one is interested in a quantity that can be obtained from the QCD Lagrangian by applying Noether's procedure, the quark-hadron duality of Eq. (14) simply instructs us that its chiral counterpart can be obtained by applying the same procedure to the χpt Lagrangian instead.

As far as the chiral counterpart to the CPV ALP Lagrangian is concerned, it is useful to first rewrite it as

$$\mathcal{L}^{\text{QCD}}_\phi = e^2 \frac{\tilde{C}_\gamma}{\Lambda} \phi F \tilde{F} + g_s^2 \frac{\tilde{C}_g}{\Lambda} \phi G \tilde{G} + \frac{\partial_\mu \phi}{\Lambda} \bar{q}\, \gamma^\mu Y_P \gamma_5\, q + e^2 \frac{C_\gamma}{\Lambda} \phi F F + g_s^2 \frac{C_g}{\Lambda} \phi G G + \frac{v}{\Lambda}\, \phi\, \bar{q}\, y_S\, q\,, \tag{16}$$

by integrating by parts and applying the equations of motion on the ALP interaction with the pseudoscalar Dirac bilinear. This will clearly induce shifts and redefinitions of the Wilson coefficients in the theory, which I leave here as implicit. Moreover, in the Lagrangian above, $q^T = (u, d)$, while Y_P and y_S are hermitian matrices.

The procedure I have outlined before can then be straightforwardly applied to find the chiral Lagrangian corresponding to Eq. (16). The resulting low-energy ALP effective Lagrangian describes the interaction of the CPV ALP with light SM fields, such as photons, leptons, pions and nucleons. It consists in two pieces possessing opposite CP-transformation properties, which, in a two-flavour setting, read

	c_γ	$y_{\ell,S}$	κ	$\mathcal{Z}$	$C_{\phi\mathrm{NN}}$
$\tilde{c}_\gamma$	$\tilde{c}_\gamma\, c_\gamma$	$\tilde{c}_\gamma\, y_{\ell,S}$	$\tilde{c}_\gamma\, \kappa$	$\tilde{c}_\gamma\, \mathcal{Z}$	$\tilde{c}_\gamma\, C_{\phi\mathrm{NN}}$
$y_{\ell,P}$	$y_{\ell,P}\, c_\gamma$	$y_{\ell,P}\, y_{\ell,S}$	$y_{\ell,P}\, \kappa$	$y_{\ell,P}\, \mathcal{Z}$	$y_{\ell,P}\, C_{\phi\mathrm{NN}}$
Δ^A_{ud}	$\Delta^A_{ud}\, c_\gamma$	$\Delta^A_{ud}\, y_{\ell,S}$	$\Delta^A_{ud}\, \kappa$	$\Delta^A_{ud}\, \mathcal{Z}$	$\Delta^A_{ud}\, C_{\phi\mathrm{NN}}$
$\tilde{C}_{\phi\mathrm{N}}$	$\tilde{C}_{\phi\mathrm{N}}\, c_\gamma$	$\tilde{C}_{\phi\mathrm{N}}\, y_{\ell,S}$	$\tilde{C}_{\phi\mathrm{N}}\, \kappa$	$\tilde{C}_{\phi\mathrm{N}}\, \mathcal{Z}$	$\tilde{C}_{\phi\mathrm{N}}\, C_{\phi\mathrm{NN}}$

Table 2: Jarlskog invariants emerging from the interactions in Eq. (17) and (18).

$$\mathcal{L}^{\chi\mathrm{pt}}_{\phi,\,\text{CP-even}} = e^2\frac{c_\gamma}{\Lambda}\,\phi\, F F + \frac{v}{\Lambda} y^{ij}_{\ell,S}\,\phi\,\bar{\ell}_i \ell_j + \kappa\,\frac{\phi}{\Lambda}\,\left[\partial^\mu\pi_0\partial_\mu\pi_0 + 2\partial^\mu\pi^+\partial_\mu\pi^-\right]$$
$$+\frac{\phi}{\Lambda}\,(\pi_0^2 + 2\pi^+\pi^-) m_\pi^2 \left[\frac{v}{2}\frac{\mathcal{Z}_u + \mathcal{Z}_d}{m_u + m_d} - 2\,\kappa\right] + \frac{\phi}{\Lambda} C_{\phi\mathrm{NN}} \bar{N} N \tag{17}$$

and

$$\mathcal{L}^{\chi\mathrm{pt}}_{\phi,\,\text{CP-odd}} = e^2\frac{\tilde{c}_\gamma}{\Lambda}\,\phi\, F\tilde{F} + i\frac{v}{\Lambda} y^{ij}_{\ell,P}\,\phi\,\bar{\ell}_i\gamma_5 \ell_j + \frac{\phi}{\Lambda}\,\pi_0(\pi_0^2 + 2\pi^+\pi^-)\left[\frac{m_\pi^2 m_\phi^2}{m_\pi^2 - m_\phi^2}\frac{\Delta^A_{ud}}{6 f_\pi \Lambda}\right] \tag{18}$$
$$+\frac{2}{3}\frac{\Delta^A_{ud}}{f_\pi\Lambda}\frac{m_\pi^2}{m_\pi^2 - m_\phi^2}\,\partial^\mu\phi\,(\pi_0\pi^-\partial_\mu\pi^+ + \pi_0\pi^+\partial_\mu\pi^- - 2\pi^+\pi^-\partial_\mu\pi^0) + \frac{\partial_\mu\phi}{\Lambda}\bar{N}\tilde{C}_{\phi\mathrm{N}}\gamma^\mu\gamma_5 N\,.$$

Details on the construction of the effective ALP couplings to nucleons and baryons can be found in [14]. The matching dictionary between such an effective low-energy Lagrangian and the one in Eq. (16) is provided in (20). In particular, the physical ALP mass after diagonalization of the ALP-pion mixing terms is found to be [14]

$$m_\phi^2 = M_\phi^2 + m_\pi^2(\lambda_g)^2\frac{m_u m_d}{(m_u+m_d)^2}\frac{f_\pi^2}{\Lambda^2} + m_\pi^2\frac{M_\phi^4}{(m_\pi^2 - M_\phi^2)^2}(\Delta^A_{ud})^2\frac{f_\pi^2}{\Lambda^2}\,, \tag{19}$$

where M_ϕ denotes the bare ALP mass and $\lambda_g = 32\pi^2\tilde{C}_g$.

The subdivision of the complete low-energy Lagrangian in two blocks possessing opposite CP transformation properties proves to be particularly useful when it comes to constructing the low-energy Jarlskog invariants. They are reported in Table 2. It is important to stress once more that the matching dictionary between the two descriptions in Eq. (16) and in the sum o Eq. (17) and (18) in coefficients in Table 2 can be directly related to those in Eq. (16) via the following relations [14]

$$c_\gamma = C_\gamma - \frac{\beta^0_{\mathrm{QED}}}{\beta^0_{\mathrm{QCD}}}C_g\,,\qquad \tilde{c}_\gamma = \tilde{C}_\gamma - 4N_c\,\mathrm{tr}\,(Q_A Q_q^2)\tilde{C}_g\,,\qquad \kappa = \frac{8\pi}{\alpha_s}\frac{g_s^2 C_g}{\beta^0_{\mathrm{QCD}}}\,,$$
$$\mathcal{Z} = y_S + \frac{g_s^2 C_g}{\beta^0_{\mathrm{QCD}}}\frac{8\pi}{\alpha_s}\frac{M_q}{v}\,,\qquad \Delta^A_{ud} = Y^u_P - Y^d_P - 16\pi^2\tilde{C}_g\frac{m_d - m_u}{m_d + m_u}\,,$$
$$\tilde{C}_{\phi\mathrm{p}} = Y^u_P\Delta_u + Y^d_P\Delta_d - 16\pi^2\tilde{C}_g\left[\frac{m_u\Delta_d}{m_u+m_d} + \frac{m_d\Delta_u}{m_u+m_d} - \frac{m_\phi^2\Delta^A_{ud}}{m_\pi^2 - m_\phi^2}\frac{\Delta_u - \Delta_d}{32\pi^2\tilde{C}_g}\right],$$

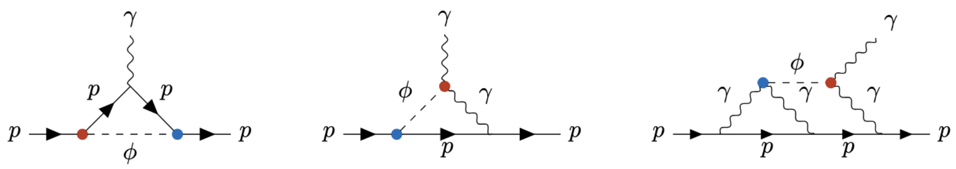

Figure 4: ALP-mediated Feynman diagrams contributing to the proton EDM, d_p. Figure from Ref. [14].

$$
\begin{aligned}
\tilde{C}_{\phi\mathrm{p}} &= Y_P^u \Delta_u + Y_P^d \Delta_d - 16\pi^2 \tilde{C}_g \left[\frac{m_u \Delta_d}{m_u + m_d} + \frac{m_d \Delta_u}{m_u + m_d} - \frac{m_\phi^2 \Delta_{ud}^A}{m_\pi^2 - m_\phi^2} \frac{\Delta_u - \Delta_d}{32\pi^2 \tilde{C}_g} \right] , \\
\tilde{C}_{\phi\mathrm{n}} &= Y_P^u \Delta_d - Y_P^d \Delta_u - 16\pi^2 \tilde{C}_g \left[\frac{m_u \Delta_u}{m_u + m_d} + \frac{m_d \Delta_d}{m_u + m_d} + \frac{m_\phi^2 \Delta_{ud}^A}{m_\pi^2 - m_\phi^2} \frac{\Delta_u - \Delta_d}{32\pi^2 \tilde{C}_g} \right] , \\
C_{\phi\mathrm{pp}} &= \frac{y_{q,S}^u \, v}{m_u} \sigma_u + \frac{y_{q,S}^d \, v}{m_d} \sigma_d + \frac{y_{q,S}^s \, v}{m_s} \sigma_s - \frac{32\pi^2 C_g}{9} \left(m_p - \sigma_u - \sigma_d - \sigma_s \right) , \\
C_{\phi\mathrm{nn}} &= \frac{y_{q,S}^u \, v}{m_d} \sigma_d + \frac{y_{q,S}^d \, v}{m_u} \sigma_u + \frac{y_{q,S}^s \, v}{m_s} \sigma_s - \frac{32\pi^2 C_g}{9} \left(m_p - \sigma_u - \sigma_d - \sigma_s \right) ,
\end{aligned}
\tag{20}
$$

where $\Delta_u = 0.858(22)$ and $\Delta_d = -0.418(22)$ [24, 25], and the values of σ_u, σ_d and σ_s can be found in Ref. [26].

2.2.1 ALP contribution to EDMs

The low-energy Lagrangian in Eq. (17) and (18) can be directly employed to calculate the contributions of a light ALP to the proton EDM, see Fig. 4 for the relevant topologies. According to the standard definition of fermionic EDMs in Eq. (3), one finds [14]

$$
\begin{aligned}
d_p \simeq -\frac{e\,Q_p}{4\pi^2 \Lambda^2} \Bigg[& C_{\phi\mathrm{pp}} \tilde{C}_{\phi\mathrm{p}} + e^2 m_p c_\gamma \tilde{C}_{\phi\mathrm{p}} \left(6 + 2 \ln \frac{\Lambda_{\mathrm{ren}}^2}{m_p^2} \right) + e^2 \tilde{c}_\gamma C_{\phi\mathrm{pp}} \left(2 + \ln \frac{\Lambda_{\mathrm{ren}}^2}{m_p^2} \right) \\
& + 3 \frac{Q_p^2}{\pi^2} m_p e^6 c_\gamma \tilde{c}_\gamma \ln^2 \frac{\Lambda_{\mathrm{ren}}}{m_\phi} \Bigg] ,
\end{aligned}
\tag{21}
$$

where the renormalization scale $\Lambda_{\mathrm{ren}} \simeq m_p$ and the result is here reported in the limit $m_\phi \ll m_p$.

Obtaining an estimate for the neutron EDM requires instead a different approach. Indeed, as they do not possess a minimal coupling to the electromagnetic field, neutrons are prevented from developing an EDM at order $O(p^2)$ in χpt. They do nonetheless possess non-minimal couplings to $F_{\mu\nu}$ at $O(p^4)$ in the χpt expansion [27, 28], which generate a non-null contribution to d_n. The relevant χpt Lagrangian at next-to-leading order reads

$$
\mathcal{L}_{\gamma\mathrm{N}}^{\mathrm{NLO}} = \frac{1}{4 m_N} \left[C_p \, \bar{p} \sigma^{\mu\nu} p + C_n \, \bar{n} \sigma^{\mu\nu} n \right] F_{\mu\nu} ,
\tag{22}
$$

where $C_p \simeq 1.79$ [25] and $C_n \simeq 1.91$ [25] are measured low-energy constants.

From these interaction terms, it is then possible to compute the corresponding contribution to d_n. The Feynman diagrams are the same as for the proton EDM, and an order-of-magnitude estimate for d_n is readily found to be [14]

$$d_n \sim -\frac{e\,C_n}{4\pi^2\Lambda^2}\left[-\frac{3}{8}C_{\phi\mathrm{nn}}\tilde{C}_{\phi\mathrm{n}}\frac{\Lambda^2_{\mathrm{ren}}}{m_n^2}+e^2(2m_n c_\gamma\tilde{C}_{\phi\mathrm{n}}+3\tilde{c}_\gamma C_{\phi\mathrm{nn}})\ln\frac{\Lambda^2_{\mathrm{ren}}}{m_n^2}\right]. \tag{23}$$

With these expressions at hand, one can then constrain the low-energy Jarlskog invariants of the theory by exploiting the current experimental bounds on $d_p < 2.1\times10^{-25}\,e$ cm [29] and $d_n < 1.8\times10^{-26}\,e$ cm (90% C.L.) [20, 21].

Considering just two couplings at a time, one can then easily find the constraints in Table 3.

	y_S^u	y_S^d	C_γ	C_g
y_P^u	2.8×10^{-12}	1.8×10^{-12}	8.0×10^{-8}	1.1×10^{-10}
y_P^d	9.9×10^{-12}	6.4×10^{-12}	1.1×10^{-7}	3.8×10^{-10}
$\tilde{C}_\gamma$	1.2×10^{-6}	1.8×10^{-6}	4.8×10^{-3}	7.1×10^{-5}
$\tilde{C}_g$	1.1×10^{-9}	7.4×10^{-10}	2.4×10^{-5}	4.4×10^{-8}

Table 3: Upper limits on Jarlskog invariants obtained from the bounds on the neutron and proton EDMs assuming $\Lambda = 1$ TeV [14]. To make contact with the notation in (16), here I have defined $y_P^q = -2\frac{m_q}{v}Y_P^q$.

2.3 Interplays with other constraints

In the previous sections I have discussed the possibility to probe CPV ALPs by studying their impact on the EDMs of particles, nucleons, nuclei and molecules. These observables are clearly the most interesting ones to be considered in order to constrain such a kind of hypothetical particle. However, when it comes to performing thorough phenomenological analyses regarding some NP candidate, it is important to consider all of the possible experimental probes and theoretical hints that can be disposed of to further constrain the available parameter space.

In particular, it is important to consider:

1) The interplay with other precision observables, as they can provide handles on the magnitude of individual Wilson coefficients, or on orthogonal combinations of Wilson coefficients with respect to those that are probed by the primary precision probe one has in mind. For instance, in the case of a CPV ALP the primary observable is given by EDMs. The same couplings that generate this kind of effects are expected to induce a non-null contribution to the magnetic dipole moment of the same particle species. Limits on these quantities can then be used to constrain combination of Wilson coefficients that are orthogonal to those appearing in the expressions for EDMs.

2) The interplay with direct searches. These are necessary to unambiguously establish the existence of any NP candidate and are seminal in assessing some of their fundamental properties.

3) The interplay with theoretical principles. Theoretical guidance from symmetry principles can be often exploited to significantly reduce the viable parameter space for NP candidates, or at least to identify its most promising regions to be experimentally explored.

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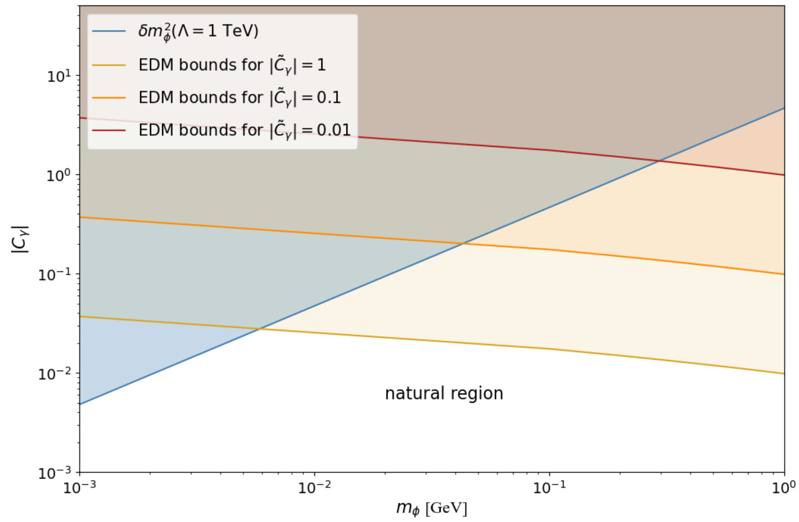

Figure 5: The interplay between EDM bounds and symmetry-based theoretical arguments can be used to narrow down the naturally available parameter space for NP candidates.

To be more concrete, I will discuss in some details the first and the last point wihin the scope of CPV ALPs. For instance, regarding the interplay between different low-energy probes, in the case of a light CPV ALP, an interesting complementary probes to EDMs is given by rare kaon decays to invisible states. Assuming that the ALP is light and long-lived enough to escape colliders, indeed, one can directly translate the experimental bounds on the decays $K^+ \to \pi^+ + \mathrm{inv.}$, $K^+ \to \pi^+\pi_0 + \mathrm{inv.}$, $K_L \to \pi_0 + \mathrm{inv.}$ and $K_L \to \pi_0\pi_0 + \mathrm{inv.}$ to theoretical constraints on some of the Wilson coefficients appearing in the theory.

To be more explicit, from the BNL result, $\mathrm{BR}(K^+ \to \pi^+ + \mathrm{inv}) < 7.3 \times 10^{-11}$ [30] and the KOTO one, $\mathrm{BR}(K_L \to \pi_0 + \mathrm{inv}) < 2 \times 10^{-9}$ [31], one can probe Y_V^{ds}:

$$|Y_V^{ds}| \lesssim 1.4 \times 10^{-9} \frac{\Lambda}{\mathrm{TeV}} \qquad |\mathrm{Im}\, Y_V^{ds}| \lesssim 3.6 \times 10^{-9} \frac{\Lambda}{\mathrm{TeV}} \tag{24}$$

Similarly, the E787 result $\mathrm{BR}(K^+ \to \pi_0\pi^+ + \mathrm{inv}) < 3.8 \times 10^{-5}$ [32] and the E391 one, $\mathrm{BR}(K_L \to \pi_0\pi_0 + \mathrm{inv}) < 0.7 \times 10^{-6}$ [33] can be used to probe the parameter Y_A^{ds}:

$$|Y_A^{ds}| \lesssim 1.1 \times 10^{-5} \frac{\Lambda}{\mathrm{TeV}} \qquad |\mathrm{Re}\, Y_A^{ds}| \lesssim 1.7 \times 10^{-6} \frac{\Lambda}{\mathrm{TeV}}\,. \tag{25}$$

Combining the information obtained from flavour probes thus offers a handle on some of the individual coefficients appearing in the Lagrangian in Eq. (16). These can be then combined with the bounds from EDMs to investigate the properties of the NP interactions with SM fields.

Another interesting point to be made regards the possibility to make use of theoretical guidance in further reducing the available parameter space, and hence signal favourable regions in the parameter space to be explored experimentally. More into detail, in the case of a CPV ALP, one can make use of a simple yet effective symmetry argument. As it was discussed before, in the Lagrangian in Eq. (1) there are two classes of operators: those that respect the ALP shift symmetry and those that break it. Radiative corrections to the ALP mass (a symmetry breaking term in the Lagrangian), can only be induced by shift-symmetry-breaking interactions. By requiring the radiative mass corrections to be at least as large as those induced by a single operator at a time, one can then further reduce the available parameter space to be probed by making use of EDMs. A typical situation is illustrated in Fig. 5, where the ALP mass corrections induced by the the shift-symmetry-breaking operator $\phi F F$ were considered. Based on naive dimensional anlysis, these amount to

$$\delta m_\phi^2 \sim 16\,\alpha_{\rm em}^4\,|C_\gamma|^2\Lambda^2\,. \tag{26}$$

A naturally favoured region for the CPV ALP to live in then corresponds to the one in which mass corrections at least as large as this one can be accommodated. The interplay with EDM bounds can then be used to further narrow down the available parameter space.

3. Conclusions

Several unanswered open questions in the standard model exist, pointing at the existence of new physics addressing them. Given the lack of any hint on its properties, it is important to approach searches for physics beyond the standard model with an agnostic attitude, systematically exploring all of the viable possibilities to probe it. In this talk, I have focused on the possibility to probe light new physics candidates by investigating their impact on low-energy precision observables. In order to illustrate how this kind of search can be performed, I have taken as an example the case of CP-violating axion-like particles. I have shown how precision observables can be employed to probe specific new physics scenarios: they are, for instance, the EDMs of particles, nucleons, nuclei and molecules in the case of CP-violating Axion-like particles. I have then stressed the importance of the interplay between different precision observables to constrain the available parameter space for light new physics candidates. I have also stressed the importance of symmetry-based theoretical arguments which can provide some guidance in identifying the most interesting regions in the available parameter space. That of CP-violating axion-like particles is however to be regarded as a paradigmatic example of a more general attitude towards the search of light new physics candidates. The same techniques I have been discussing can be easily applied to any low-energy search of light physics beyond the standard model.

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