Journal of Physics G: Nuclear and Particle Physics

PAPER • OPEN ACCESS

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To cite this article: A N Ivanov 2018 *J. Phys. G: Nucl. Part. Phys.* **45** 025004

OPEN ACCESS
IOP Publishing
Journal of Physics G: Nuclear and Particle Physics
J. Phys. G: Nucl. Part. Phys. **45** (2018) 025004 (19pp)
https://doi.org/10.1088/1361-6471/aa9107

Lorentz structure of the vector part of matrix elements of $n \longleftrightarrow p$ transitions, caused by strong low-energy interactions and hypothesis of conservation of charged vector current

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Received 2 June 2017, revised 20 September 2017
Accepted for publication 4 October 2017
Published 16 January 2018

Abstract

We analyse the Lorentz structure of the matrix elements of the 'neutron $\longleftrightarrow$ proton' transitions, induced by the charged hadronic vector current. We show that the term providing conservation of the charged hadronic vector current in the sense of the vanishing matrix element of the divergence of the charged hadronic vector current of the 'neutron $\longleftrightarrow$ proton' transitions even for different neutron and proton masses (see T Leitner *et al* 2006 *Phys. Rev.* C **73** 065502 and A M Ankowski 2016 arXiv:1601.06169 [hep-ph]) has a dynamical origin, related to the *G*-even first class current contribution. We show that because of the invariance of strong low-energy interactions under *G*-parity transformations, the *G*-odd contribution with Lorentz structure q_μ, where q_μ is a transferred momentum, does not appear in the matrix elements of the 'neutron $\longleftrightarrow$ proton' transitions.

Keywords: Lorentz structure, pion–nucleon interaction, G-parity

(Some figures may appear in colour only in the online journal)

1. Introduction

In the paper by Leitner *et al* [1] (see a.lso [2, 3]) the matrix element of the 'neutron $\longrightarrow$ proton' or $n \to p$ transition, induced by the charged hadronic vector current $V^{(+)}_\mu(0)$, has been written in the form

$$\langle p(k_p,\sigma_p)|V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle = \bar u_p(k_p,\sigma_p)\left(\left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right)F_1(q^2) + \frac{\mathrm{i}\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right)u_n(k_n,\sigma_n)$$
$$= \bar u_p(k_p,\sigma_p)\left(\gamma^\nu\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1(q^2) + \frac{\mathrm{i}\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right)u_n(k_n,\sigma_n), \quad (1)$$

where $\bar u_p(k_p,\sigma_p)$ and $u_n(k_n,\sigma_n)$ are the Dirac bispinor wave functions of the free proton and neutron in the final and initial states of the $n\to p$ transition, $m_N=(m_p+m_n)/2$ is a nucleon mass or an averaged nucleon mass, expressed in terms of the proton m_p and neutron m_n masses, $\eta_{\mu\nu}$ is the metric tensor of the Minkowski spacetime and γ_μ and $\sigma_{\mu\nu}=\frac{i}{2}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$ are the Dirac matrices [4]. Then, $q=k_p-k_n$ is the transferred momentum and $F_1(q^2)$ and $F_2(q^2)$ are the form factors. The second term in equation (1) describes the contribution of weak magnetism. The right-hand-side (r.h.s.) of equation (1) vanishes after multiplication by a transferred momentum q^μ, i.e.

$$q^\mu\langle p(k_p,\sigma_p)|V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle = 0, \quad (2)$$

even for $m_p\neq m_n$. Such a property of the matrix element of the $n\to p$ transition indicates conservation of the charged hadronic vector current $V^{(+)}_\mu(x)$, but only in the sense of the vanishing matrix element $\langle p(k_p,\sigma_p)|\partial^\mu V^{(+)}_\mu(x)|n(k_n,\sigma_n)\rangle=0$. This, of course, should not contradict the conservation of vector current or CVC hypothesis of Feynman and Gell-Mann [5]. Recently [6] we have shown that the term $(-q_\mu\not{q}/q^2)F_1(q^2)$ is the contribution of the first class current [7, 8].

This paper addresses the analysis of the dynamical nature of the term with Lorentz structure $q_\mu\not{q}$. As has been proposed in [6], the vector part of the matrix element of the $n\to p$ transition, caused by the contributions of the first class current only, should take the following general form:

$$\langle p(k_p,\sigma_p)|V^{(+)}_\mu(0)|n(k_n,\sigma_n)\rangle$$
$$= \bar u_p(k_p,\sigma_p)\left(\gamma_\mu F_1(q^2) + \frac{\mathrm{i}\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2) + \frac{q_\mu\not{q}}{m_N^2}F_4(q^2)\right)u_n(k_n,\sigma_n). \quad (3)$$

We show below that the appearance of the term with Lorentz structure $q_\mu\not{q}$ is fully caused by strong low-energy interactions.

The paper is organized as follows. In section 2 we propose to analyse the dynamical nature of the term with Lorentz structure $q_\mu\not{q}$ using a strongly coupled πN-system with linear pion–nucleon pseudoscalar interaction. We show that only the total hadronic isovector vector current, being the sum of the nucleon and mesonic currents, can be locally conserved. In section 3 we derive the Lorentz structure of the matrix element of the $n\to p$ transition using the path-integral technique. In section 4 we discuss the results obtained. In appendix A we calculate the cross sections for quasi-elastic electron neutrino-neutron scattering and for inverse β-decay. In order to illustrate the influence of the contributions of the term with Lorentz structure $(-q_\mu\not{q}/q^2)F_1(q^2)$ we neglect the contributions of weak magnetism and outgoing nucleon recoil, and the radiative corrections. In figure 1 we plot the relative contributions of the term $(-q_\mu\not{q}/q^2)F_1(q^2)$. We show that the processes of quasi-elastic electron neutrino-neutron scattering and of inverse β-decay are insensitive to the contributions of the term, responsible for the vanishing of the matrix elements $\langle p|\partial^\mu V^{(+)}_\mu(0)|n\rangle = \langle n|\partial^\mu V^{(-)}_\mu(0)|p\rangle = 0$ for different

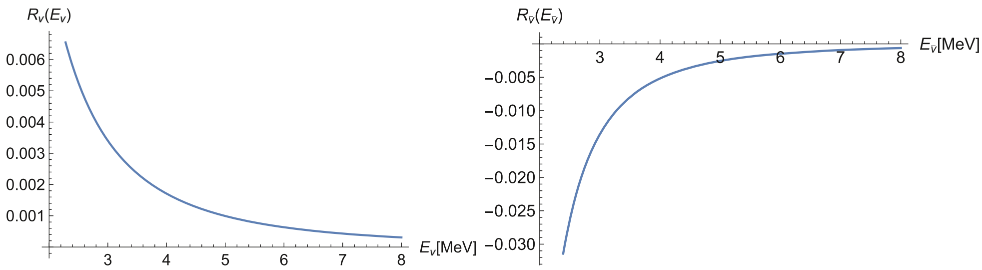

Figure 1. The relative contributions $R_\nu(E_\nu)$ (left) and $R_{\bar\nu}(E_{\bar\nu})$ (right) of the ECVC effect to the cross sections for the quasi-elastic electron neutrino-neutron and inverse β-decay in the neutrino and antineutrino energy regions $2\,\mathrm{MeV} \leqslant E_\nu \leqslant 8\,\mathrm{MeV}$ and $2\,\mathrm{MeV} \leqslant E_{\bar\nu} \leqslant 8\,\mathrm{MeV}$, calculated for $\lambda = -1.2750$ [24].

neutron and proton masses. In appendix B we analyse the dynamical nature of the Lorentz structure of the matrix element $\langle p|A_\mu^{(+)}(0)|n\rangle$ of the $n \to p$ transition, caused by the charged hadronic axial-vector current. We show that the linear pion–nucleon pseudoscalar interaction, used to analyse the dynamical nature of the Lorentz structure of the charged hadronic vector part of the $n \to p$ transition, allows the standard Lorentz structure of the axial-vector part of the hadronic $n \to p$ transition to be fully reproduced [1].

2. Hadronic vector current of a strongly coupled pion–nucleon system

As an example of a strongly coupled system we consider the πN-system with the simplest linear pseudoscalar interaction [9]. The Lagrangian of such a system is given by [9]

$$\mathcal{L}_{\pi N}(x) = \bar N(x)(\mathrm{i}\gamma^\mu\partial_\mu - m_N)N(x) + \frac{1}{2}\,\partial_\mu\vec\pi(x)\cdot\partial_\mu\vec\pi(x) - \frac{1}{2}\,m_\pi^2\,\vec\pi^{\,2}(x) + g_\pi\bar N(x)\mathrm{i}\gamma^5\vec\tau\cdot\vec\pi(x)N(x). \tag{4}$$

Here $N(x)$ is the nucleon isospin doublet with components $(p(x), n(x))$, where $p(x)$ and $n(x)$ are the proton and neutron field operators, $\vec\pi(x) = (\pi^+(x), \pi^0(x), \pi^-(x))$ is the pion field operator, m_N and m_π are the nucleon and pion masses, g_π is the pion–nucleon coupling constant, γ^5 is the Dirac matrix [4] and $\vec\tau = (\tau_1, \tau_2, \tau_3)$ is the Pauli isospin matrix [9].

The Lagrangian equation (4) is invariant under global isospin transformations [9]. This, according to Feynman and Gell-Mann [5], leads to the isovector hadronic vector current of the πN system given by

$$\vec V_\mu(x) = \frac{1}{2}\,\bar N(x)\vec\tau\gamma_\mu N(x) + \vec\pi(x)\times\partial_\mu\vec\pi(x), \tag{5}$$

local conservation of which one may check using the equations of motion. The Dirac equation for the nucleon and the Klein–Gordon equation for the pions are given by

$$(\mathrm{i}\gamma^\mu\partial_\mu - m_N + g_\pi\mathrm{i}\gamma^5\,\vec\tau\cdot\vec\pi(x))N(x) = 0,$$
$$(\Box + m_\pi^2)\vec\pi(x) - g_\pi\bar N(x)\mathrm{i}\gamma^5\vec\tau\,N(x) = 0. \tag{6}$$

Using the Dirac equation for the nucleon equation (6) one may show that the nucleon part of the isovector hadronic vector current equation (5) is not conserved

$$\partial^{\mu}\Big(\frac{1}{2}\,\bar{N}(x)\vec{\tau}\gamma_{\mu}N(x)\Big)=\frac{1}{2}\,\partial^{\mu}\bar{N}(x)\gamma_{\mu}\vec{\tau}\,N(x)+\frac{1}{2}\,\bar{N}(x)\vec{\tau}\gamma_{\mu}\partial^{\mu}N(x)$$
$$=-\vec{\pi}(x)\times g_{\pi}\,\bar{N}(x)\mathrm{i}\gamma^{5}\vec{\tau}\,N(x). \tag{7}$$

Hence, in the strongly coupled πN-system a strong non-conservation of the nucleon part of the isovector hadronic vector current is caused by strong low-energy interactions but not by isospin violation. The divergence of the mesonic part of the isovector hadronic vector current is equal to

$$\partial^{\mu}(\vec{\pi}(x)\times\partial_{\mu}\vec{\pi}(x))=\vec{\pi}(x)\times g_{\pi}\,\bar{N}(x)\mathrm{i}\gamma^{5}\vec{\tau}\,N(x). \tag{8}$$

Summing the contribution equations (7) and (8) we get $\partial^{\mu}\vec{V}_{\mu}(x)=0$. This means that in the strongly coupled πN-system only the total hadronic isovector vector current, being the sum of the nucleon and mesonic currents, can be locally conserved.

3. Dynamical Lorentz structure of the matrix element of the *n*→*p* transition, caused by the hadronic vector current equation (5)

The charged hadronic vector current responsible for the hadronic $n\to p$ transition is equal to [9]

$$V^{(+)}_{\mu}(x)=\bar{N}(x)\tau^{(+)}\gamma_{\mu}N(x)+2\,\varepsilon^{+bc}\,\pi^{b}(x)\partial_{\mu}\pi^{c}(x)=\bar{p}(x)\gamma_{\mu}n(x)$$
$$+\sqrt{2}\,\mathrm{i}(\pi^{0}(x)\partial_{\mu}\pi^{-}(x)-\pi^{-}(x)\partial_{\mu}\pi^{0}(x)), \tag{9}$$

where $\tau^{(+)}=(\tau^{1}+\mathrm{i}\tau^{2})/2$, $\varepsilon^{+bc}=(\varepsilon^{1bc}+\mathrm{i}\,\varepsilon^{2bc})/2$ and ε^{abc} is the Levi-Civita isotensor [9]. Now we may calculate the matrix element

$$\langle\mathrm{out},\,p(\vec{k}_{p},\,\sigma_{p})|V^{(+)}_{\mu}(0)|\mathrm{in},\,n(\vec{k}_{n},\,\sigma_{n})\rangle, \tag{10}$$

where $\langle\mathrm{out},\,p(\vec{k}_{p},\,\sigma_{p})|$ and $|\mathrm{in},\,n(\vec{k}_{n},\,\sigma_{n})\rangle$ are the wave functions of the free proton and neutron in the final (i.e. out-state at $t\to+\infty$) and initial (i.e. in-state at $t\to-\infty$) states, respectively [4]. Using the relation $\langle\mathrm{out},\,p(\vec{k}_{p},\,\sigma_{p})|=\langle\mathrm{in},\,p(\vec{k}_{p},\,\sigma_{p})|\mathbb{S}$, where $\mathbb{S}$ is the S-matrix, we rewrite the matrix element equation (10) as follows:

$$\langle\mathrm{out},\,p(\vec{k}_{p},\,\sigma_{p})|V^{(+)}_{\mu}(0)|\mathrm{in},\,n(\vec{k}_{n},\,\sigma_{n})\rangle=\langle\mathrm{in},\,p(\vec{k}_{p},\,\sigma_{p})|\mathbb{S}V^{(+)}_{\mu}(0)|\mathrm{in},\,n(\vec{k}_{n},\,\sigma_{n})\rangle. \tag{11}$$

Since the $n\to p$ transition is fully induced by strong low-energy interactions, we define the S-matrix only in terms of strong low-energy interactions. For simplicity we propose to use only the πN-system, the dynamics of which are determined by the Lagrangian equation (4). The corresponding S-matrix is given by [4]

$$\mathbb{S}=\mathrm{T}e^{\mathrm{i}\int d^{4}x\,\mathcal{L}_{\pi NN}(x)}, \tag{12}$$

where T is a time-ordering operator and $\mathcal{L}_{\pi NN}(x)$ is equal to

$$\mathcal{L}_{\pi NN}(x)=g_{\pi}\bar{N}(x)\mathrm{i}\gamma^{5}\vec{\tau}\cdot\vec{\pi}(x)N(x). \tag{13}$$

Plugging equation (12) into equation (11) we get

$$\langle\mathrm{in},\,p(\vec{k}_{p},\,\sigma_{p})|\mathbb{S}V^{(+)}_{\mu}(0)|\mathrm{in},\,n(\vec{k}_{n},\,\sigma_{n})\rangle=\langle\mathrm{in},\,p(\vec{k}_{p},\,\sigma_{p})|\mathrm{T}$$
$$\times\,(e^{\mathrm{i}\int d^{4}x\,\mathcal{L}_{\pi NN}(x)}V^{(+)}_{\mu}(0))|\mathrm{in},\,n(\vec{k}_{n},\,\sigma_{n})\rangle. \tag{14}$$

We determine the wave functions of the neutron and proton in terms of the creation operators (annihilation)

$$|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle = a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n,\, \sigma_n)|0\rangle,$$
$$\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)| = \langle 0|a_{p,\mathrm{in}}(\vec{k}_p,\, \sigma_p). \tag{15}$$

The operators $(a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n,\, \sigma_n),\, a_{p,\mathrm{in}}(\vec{k}_p,\, \sigma_p)$ and $(a_{n,\mathrm{in}}(\vec{k}_n,\, \sigma_n),\, a^{\dagger}_{p,\mathrm{in}}(\vec{k}_p,\, \sigma_p)$ obey standard anticommutation relations [4]

$$[a_{n,\mathrm{in}}(\vec{k}_n^{\,\prime},\, \sigma'_n),\, a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n,\, \sigma_n)] = (2\pi)^3\, 2E_n\, \delta^{(3)}(\vec{k}_n^{\,\prime} - \vec{k}_n)\delta_{\sigma'_n\sigma_n},$$
$$[a_{n,\mathrm{in}}(\vec{k}_n^{\,\prime},\, \sigma'_n),\, a_{n,\mathrm{in}}(\vec{k}_n,\, \sigma_n)] = [a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n^{\,\prime},\, \sigma'_n),\, a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n,\, \sigma_n)] = 0,$$
$$[a_{p,\mathrm{in}}(\vec{k}_p^{\,\prime},\, \sigma'_p),\, a^{\dagger}_{p,\mathrm{in}}(\vec{k}_p,\, \sigma_p)] = (2\pi)^3\, 2E_p\, \delta^{(3)}(\vec{k}_p^{\,\prime} - \vec{k}_p)\delta_{\sigma'_p\sigma_p},$$
$$[a_{p,\mathrm{in}}(\vec{k}_p^{\,\prime},\, \sigma'_p),\, a_{p,\mathrm{in}}(\vec{k}_p,\, \sigma_p)] = [a^{\dagger}_{p,\mathrm{in}}(\vec{k}_p^{\,\prime},\, \sigma'_n),\, a^{\dagger}_{p,\mathrm{in}}(\vec{k}_p,\, \sigma_p)] = 0. \tag{16}$$

We define the vacuum wave function as follows: $|0\rangle = |0_N\rangle|0_\pi\rangle$, where $|0_N\rangle$ and $|0_\pi\rangle$ are the vacuum wave functions of a nucleon and mesons, respectively. Since there are no mesons in the initial and final states of the $n \to p$ transition, we may rewrite the matrix element equation (14) as follows:

$$\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|\mathbb{S}V^{(+)}_\mu(0)|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle = {}_N\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|\langle 0_\pi|$$
$$\times\, \mathrm{T}(\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \mathcal{L}_{\pi NN}(x)}V^{(+)}_\mu(0))|0_\pi\rangle|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle_N. \tag{17}$$

The wave functions ${}_N\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|$ and $|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle_N$ mean that the operators $(a^{\dagger}_{n,\mathrm{in}}(\vec{k}_n,\, \sigma_n),\, a_{p,\mathrm{in}}(\vec{k}_p,\, \sigma_p)$ act only on the nucleon vacuum wave function $|0_N\rangle$. We calculate the vacuum expectation value $\langle 0_\pi|\mathrm{T}\Big(\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \mathcal{L}_{\pi NN}(x)}V^{(+)}_\mu(0)\Big)|0_\pi\rangle$ using the path-integral technique [10], rewriting the vacuum expectation value as follows:

$$\langle 0_\pi|\mathrm{T}(\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \mathcal{L}_{\pi NN}(x)}V^{(+)}_\mu(0))|0_\pi\rangle$$
$$= \mathrm{T}\bigg(\frac{1}{2}\,\bar{N}(0)\tau^{(+)}\gamma_\mu N(0)\int D\vec{\pi}\;\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\,\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x) - \frac{1}{2}\,m_\pi^2\,\vec{\pi}^{\,2} + g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}$$
$$+\, 2\,\varepsilon^{+bc}\int D\vec{\pi}\;\pi^b(z)\partial_\mu\pi^c(z)\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\,\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x) - \frac{1}{2}\,m_\pi^2\,\vec{\pi}^{\,2} + g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}\Big|_{z=0}\bigg). \tag{18}$$

The integrals are Gaussian. The calculation of the first integral runs as follows. We transcribe it into the form

$$\int D\vec{\pi}\;\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\,\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x) - \frac{1}{2}\,m_\pi^2\,\vec{\pi}^{\,2} + g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}$$
$$= \int D\vec{\pi}\;\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(-\frac{1}{2}\,\vec{\pi}\cdot(\Box + m_\pi^2 - \mathrm{i}0)\vec{\pi} + g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}. \tag{19}$$

Then, we make a shift

$$\vec{\pi}(x) \to \vec{\pi}(x) + \frac{1}{\Box_x + m_\pi^2 - \mathrm{i}0}\,g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot N(x) = \vec{\pi}(x)$$
$$+ \int \mathrm{d}^4y\;\Delta(x - y)g_\pi\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y), \tag{20}$$

where $\Delta(x-y)$ is the π-meson propagator [9]. The result of the integration is

$$\int D\vec{\pi}\ \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x)-\frac{1}{2}m_\pi^2\vec{\pi}^{\,2}+g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}$$
$$= \mathrm{e}^{\,\mathrm{i}\,\frac{1}{2}g_\pi^2\int \mathrm{d}^4x\mathrm{d}^4y\,\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)}. \tag{21}$$

To integrate the pionic part of the charge hadronic vector current we use the following procedure. We rewrite the path-integral, given by the second term in the r.h.s. of equation (18), with an external source $\vec{J}(x)$ of the π-meson field:

$$\varepsilon^{abc}\int D\vec{\pi}\ \pi^b(z)\partial_\mu\pi^c(z)\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x)-\frac{1}{2}m_\pi^2\vec{\pi}^{\,2}+g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}\Big|_{z=0}\to$$
$$\varepsilon^{abc}\int D\vec{\pi}\ \pi^b(z)\partial_\mu\pi^c(z)\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x)-\frac{1}{2}m_\pi^2\vec{\pi}^{\,2}+g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x)+\vec{J}(x)\cdot\vec{\pi}(x))}\Big|_{z=0}\to$$
$$\varepsilon^{abc}\int D\vec{\pi}\ \pi^b(z)\partial_\mu\pi^c(z)\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(-\frac{1}{2}\vec{\pi}(x)\cdot(\Box+m_\pi^2-\mathrm{i}0)\vec{\pi}(x)+g_\pi\bar{N}(x)i\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x)+\vec{J}(x)\cdot\vec{\pi}(x))}\Big|_{z=0}. \tag{22}$$

Then, we replace the pionic fields in the integrand by functional derivatives with respect to the external source:

$$\varepsilon^{abc}\int D\vec{\pi}\ \pi^b(z)\partial_\mu\pi^c(z)\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x)-\frac{1}{2}m_\pi^2\vec{\pi}^{\,2}+g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}\Big|_{z=0}\to$$
$$\to\varepsilon^{abc}(-\mathrm{i})\frac{\delta}{\delta J^b(z)}\frac{\partial}{\partial z^\mu}(-\mathrm{i})\frac{\delta}{\delta J^c(z)}$$
$$\int D\vec{\pi}\ \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(-\frac{1}{2}\vec{\pi}(x)\cdot(\Box+m_\pi^2-\mathrm{i}0)\vec{\pi}(x)+g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x)+\vec{J}(x)\cdot\vec{\pi}(x))}|_{z=0,\vec{J}=0}. \tag{23}$$

To calculate the integral over $\vec{\pi}$ we make a change of variables

$$\vec{\pi}(x)\to\vec{\pi}(x)+\frac{1}{\Box_x+m_\pi^2-\mathrm{i}0}(g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot N(x)+\vec{J}(x))$$
$$=\vec{\pi}(x)+\int \mathrm{d}^4y\ \Delta(x-y)(g_\pi\bar{N}(y)i\gamma^5\vec{\tau}N(y)+\vec{J}(y)). \tag{24}$$

As a result, for the integral over $\vec{\pi}$ we obtain the following expression:

$$\int D\vec{\pi}\ \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(-\frac{1}{2}\vec{\pi}(x)\cdot(\Box+m_\pi^2-\mathrm{i}0)\vec{\pi}(x)+g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x)+\vec{J}(x)\cdot\vec{\pi}(x))}$$
$$=\mathrm{e}^{\,\mathrm{i}\,\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^4y(g_\pi\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)+\vec{J}(x))\cdot\Delta(x-y)(g_\pi\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)+\vec{J}(y))}. \tag{25}$$

Plugging equation (25) into equation (23) and calculating the functional derivatives with respect to external sources we arrive at the expression

$$\varepsilon^{abc}\int D\vec{\pi}\ \pi^b(z)\partial_\mu\pi^c(z)\mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(\frac{1}{2}\partial_\alpha\vec{\pi}(x)\cdot\partial^\alpha\vec{\pi}(x)-\frac{1}{2}m_\pi^2\vec{\pi}^{\,2}+g_{\pi NN}\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}\cdot\vec{\pi}(x)N(x))}|_{z=0}$$
$$=\varepsilon^{abc}\,g_\pi^2\int \mathrm{d}^4x\ \Delta(x)\bar{N}(x)\mathrm{i}\gamma^5\tau^b\,N(x)\int d^4y\ \bar{N}(y)\mathrm{i}\gamma^5\tau^c\,N(y)\partial_\mu\Delta(-y)$$
$$\times\ \mathrm{e}^{\,\mathrm{i}\,\frac{1}{2}g_\pi^2\int \mathrm{d}^4x'\mathrm{d}^4y'\,\bar{N}(x')\mathrm{i}\gamma^5\vec{\tau}N(x')\cdot\Delta(x'-y')\bar{N}(y')\mathrm{i}\gamma^5\vec{\tau}N(y')}. \tag{26}$$

Thus, after calculating the vacuum expectation value equation (18) the matrix element equation (11) of the $n \to p$ transition becomes equal to

$$\langle \text{out}, p(\vec{k}_p, \sigma_p)|V^{(+)}_\mu(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle = {}_N\Big\langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathrm{T}\Big\{ (\bar{N}(0)\tau^{(+)}\gamma_\mu N(0)$$
$$+ 2\,\varepsilon^{+bc} g^2_\pi \int \mathrm{d}^4x \Delta(x)\bar{N}(x)\mathrm{i}\gamma^5\tau^b N(x)$$
$$\times \int \mathrm{d}^4x\, \bar{N}(y)\mathrm{i}\gamma^5\tau^c\, N(y)\partial_\mu\Delta(-y)\Big)$$
$$\times\ \mathrm{e}^{\,\mathrm{i}\,\frac{1}{2}\,g^2_\pi \int \mathrm{d}^4x'\mathrm{d}^4y'\, \bar{N}(x')\mathrm{i}\gamma^5\vec{\tau}N(x')\cdot\Delta(x'-y')\bar{N}(y')\mathrm{i}\gamma^5\vec{\tau}N(y')}\Big\}|\text{in}, n(\vec{k}_n, \sigma_n)\Big\rangle_N. \tag{27}$$

As the first step towards analysing the Lorentz structure of the matrix element of the $n \to p$ transition, given by equation (27), we propose to consider contributions of order g^2_π. We understand that the value of the coupling constant g_π is sufficiently large. Nevertheless, the Lorentz structure of the matrix element equation (27) can be fully understood to order g^2_π [11].

3.1. Lorentz structure of the matrix element equation (27) to order g^2_π, determined by the mesonic part of the charged hadronic vector current equation (9)

To order g^2_π the contribution of the mesonic part of the charged hadronic vector current is given by the expression

$$\begin{aligned}
&{}_N\langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathrm{T}\Big(2\,\varepsilon^{+bc} g^2_\pi \int \mathrm{d}^4x\Delta(x)\bar{N}(x)\mathrm{i}\gamma^5\tau^b N(x)\\
&\quad\times\int \mathrm{d}^4x\, \bar{N}(y)\mathrm{i}\gamma^5\tau^c\, N(y)\partial_\mu\Delta(-y)\Big)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle_N\\
&= \bar{u}_p(\vec{k}_p, \sigma_p)4\mathrm{i}g^2_\pi \int \mathrm{d}^4x\, \Delta(x)\mathrm{e}^{\,\mathrm{i}k_p\cdot x}\mathrm{i}\gamma^5 \int \mathrm{d}^4y(-\mathrm{i})\\
&\quad\times S_F(x-y)\mathrm{i}\gamma^5\, \partial_\mu\Delta(-y)\mathrm{e}^{-\mathrm{i}k_n\cdot y}\, u_n(\vec{k}_n, \sigma_n)\\
&\quad- \bar{u}_p(\vec{k}_p, \sigma_p)4\mathrm{i}g^2_\pi \int \mathrm{d}^4x\, \partial_\mu\Delta(-x)\mathrm{e}^{\,\mathrm{i}k_p\cdot x}\mathrm{i}\gamma^5 \int \mathrm{d}^4y(-\mathrm{i})\\
&\quad\times S_F(x-y)\mathrm{i}\gamma^5\, \Delta(y)\mathrm{e}^{-\mathrm{i}k_n\cdot y}\, u_n(\vec{k}_n, \sigma_n),
\end{aligned} \tag{28}$$

where $S_F(x-y)$ is the nucleon propagator [9]. To derive equation (28) we have used the relation $\varepsilon^{+bc}\tau^b\tau^c = 2\mathrm{i}\tau^{(+)}$. In the momentum representation the r.h.s. of equation (28) reads

$$\begin{aligned}
&{}_N\langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathrm{T}\Big(2\,\varepsilon^{+bc} g^2_\pi \int \mathrm{d}^4x\Delta(x)\bar{N}(x)\mathrm{i}\gamma^5\tau^b N(x)\\
&\quad\times\int \mathrm{d}^4x\, \bar{N}(y)\mathrm{i}\gamma^5\tau^c\, N(y)\partial_\mu\Delta(-y)\Big)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle_N\\
&= \bar{u}_p(\vec{k}_p, \sigma_p)\frac{g^2_\pi}{4\pi^2}\int \frac{\mathrm{d}^4p}{\pi^2\mathrm{i}}\,\frac{1}{m^2_\pi - (p-k_p)^2 - \mathrm{i}0}\\
&\quad\times \gamma^5\frac{1}{m_N - \hat{p} - \mathrm{i}0}\,\gamma^5\frac{(k_p + k_n - 2p)_\mu}{m^2_\pi - (p-k_n)^2 - \mathrm{i}0}\, u_n(\vec{k}_n, \sigma_n).
\end{aligned} \tag{29}$$

The integral is symmetric with respect to $k_p \longleftrightarrow k_n$ transformation. This means that the momentum integral possesses the Lorentz structure

$$\int \frac{d^4p}{\pi^2 i} \frac{1}{m_\pi^2 - (p - k_p)^2 - i0} \gamma^5 \frac{1}{m_N - \hat{p} - i0} \gamma^5 \frac{(k_p + k_n - 2p)_\mu}{m_\pi^2 - (p - k_n)^2 - i0}$$
$$= a\,\gamma_\mu + b\,P_\mu \not{P} + c\,q_\mu \not{q}, \tag{30}$$

which can be confirmed by a direct calculation of the integral, where $P = (k_p + k_n)/2$ and a, b and c are coefficients, which can be determined by a direct calculation of the integral. The symmetry of the integral with respect to $k_p \longleftrightarrow k_n$ transformation indicates that the term with Lorentz structure $q_\mu = (k_p - k_n)_\mu$, which is antisymmetric with respect to $k_p \longleftrightarrow k_n$ transformation, does not appear in the matrix element of the $n \to p$ transition in agreement with suppression of the contributions of the second class currents [7, 8].

As a consequence of the relations $m_N^2 \gg m_\pi^2 \gg q^2$ one may perform the calculation of the momentum integral equation (30) in the heavy baryon approximation [12]. Since we are interested in the term with Lorentz structure $q_\mu \not{q}$ only, skipping standard intermediate steps of the calculations for the coefficient c we obtain the result

$$c = \frac{1}{6 m_N m_\pi} \arctan\left(\frac{m_N}{m_\pi}\right), \tag{31}$$

where we have neglected contributions of order $O(1/m_N^2)$. Then, using the Dirac equation $\bar{u}_p \not{P} u_n = (m_p + m_n)/2 = m_N$ that does not violate the property of the term with Lorentz structure $P_\mu \not{P}$ to be a contribution of the first class current [6] and the Gordon identity [4]

$$\bar{u}_p(\vec{k}_p, \sigma_p) \frac{(k_p + k_p)_\mu}{2m_N} u_n(\vec{k}_n, \sigma_n) = \bar{u}_p(\vec{k}_p, \sigma_p)\gamma_\mu u_n(\vec{k}_n, \sigma_n) - \bar{u}_p(\vec{k}_p, \sigma_p) \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} u_n(\vec{k}_n, \sigma_n) \tag{32}$$

we transcribe equation (29) into the form

$$ {}_N\langle \mathrm{in}, p(\vec{k}_p, \sigma_p)| \mathrm{T}\Big(2\,\varepsilon^{+bc} g_\pi^2 \int d^4x \Delta(x) \bar{N}(x) i\gamma^5 \tau^b N(x)$$
$$\times \int d^4x\, \bar{N}(y) i\gamma^5 \tau^c\, N(y) \partial_\mu \Delta(-y)\Big)|\mathrm{in}, n(\vec{k}_n, \sigma_n)\rangle_N$$
$$= \bar{u}_p(\vec{k}_p, \sigma_p) \frac{g_\pi^2}{4\pi^2}\left(A_\pi\,\gamma_\mu + B_\pi \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} + C_\pi \frac{q_\mu \not{q}}{m_N^2}\right) u_n(\vec{k}_n, \sigma_n), \tag{33}$$

where $A_\pi = a + m_N b$, $B_\pi = -m_N b$ and $C_\pi = m_N^2 c$.

3.2. Lorentz structure of the matrix element equation (27) to order g_π^2, determined by the nucleon part of the charged hadronic vector current equation (9)

To order g_π^2 the dynamical contribution of the nucleon part of the charged hadronic vector current to the matrix element of the $n \to p$ transition, given by equation (27), is determined by the matrix element

$$
\begin{aligned}
&{}_N\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|\mathrm{T}\bigg(\bar{N}(0)\tau^{(+)}\gamma_\mu N(0)\mathrm{i}\,\frac{1}{2}\,g_\pi^2\int \mathrm{d}^4x\mathrm{d}^4y\;\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\\
&\quad\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)\bigg)|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle_N\\
&= \bar{u}_p(\vec{k}_p,\, \sigma_p)3\mathrm{i}g_\pi^2\int \mathrm{d}^4x\mathrm{d}^4y\;\gamma_\mu(-\mathrm{i})S_F(-x)\mathrm{i}\gamma^5(-\mathrm{i})\\
&\quad\times S_F(x-y)\Delta(x-y)\mathrm{i}\gamma^5\mathrm{e}^{-\mathrm{i}k_n\cdot y}\,u_n(\vec{k}_n,\, \sigma_n)\\
&\quad+\bar{u}_p(\vec{k}_p,\, \sigma_p)3\mathrm{i}g_\pi^2\int \mathrm{d}^4x\mathrm{d}^4y\;\mathrm{e}^{\,\mathrm{i}k_p\cdot x}i\gamma^5(-\mathrm{i})S_F(x-y)\\
&\quad\times\Delta(x-y)\mathrm{i}\gamma^5(-\mathrm{i})S_F(y)\gamma_\mu\,u_n(\vec{k}_n,\, \sigma_n)\\
&\quad-\bar{u}_p(\vec{k}_p,\, \sigma_p)\mathrm{i}g_\pi^2\int \mathrm{d}^4x\mathrm{d}^4y\;\mathrm{e}^{\,\mathrm{i}k_p\cdot x}\mathrm{i}\gamma^5(-i)S_F(x)\\
&\quad\times\gamma_\mu\Delta(x-y)(-\mathrm{i})S_F(-y)\mathrm{i}\gamma^5\,\mathrm{e}^{-\mathrm{i}k_n\cdot y}\,u_n(\vec{k}_n,\, \sigma_n),
\end{aligned}
\tag{34}
$$

where we have used the relations $\vec{\tau}^{\,2}=3$ and $\vec{\tau}\cdot\tau^{(+)}\vec{\tau}=-\tau^{(+)}$. The contributions of the first two terms in equation (34) can be removed by renormalization of the masses and wave functions of the neutron and proton, respectively [13, 14]. Thus, a non-trivial contribution comes from the third term only. In the momentum representation it reads

$$
\begin{aligned}
&{}_N\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|\mathrm{T}\bigg(\bar{N}(0)\tau^{(+)}\gamma_\mu N(0)\mathrm{i}\,\frac{1}{2}\,g_\pi^2\int \mathrm{d}^4xd^4y\;\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\\
&\quad\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)\bigg)|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle_N\\
&= \bar{u}_p(\vec{k}_p,\, \sigma_p)\frac{g_\pi^2}{16\pi^2}\int\frac{\mathrm{d}^4p}{\pi^2\mathrm{i}}\,\gamma^5\,\frac{1}{m_N-\hat{k}_p+\hat{p}-\mathrm{i}0}\,\gamma_\mu\,\frac{1}{m_N-\hat{k}_n+\hat{p}-\mathrm{i}0}\\
&\quad\times\gamma^5\,\frac{1}{m_\pi^2-p^2-\mathrm{i}0}u_n(\vec{k}_n,\, \sigma_n).
\end{aligned}
\tag{35}
$$

This integral is also symmetric with respect to $k_p\longleftrightarrow k_n$ transformation, so it should also have a structure

$$
\begin{aligned}
&\int\frac{\mathrm{d}^4p}{\pi^2\mathrm{i}}\,\gamma^5\,\frac{1}{m_N-\hat{k}_p+\hat{p}-\mathrm{i}0}\,\gamma_\mu\,\frac{1}{m_N-\hat{k}_n+\hat{p}-\mathrm{i}0}\gamma^5\,\frac{1}{m_\pi^2-p^2-i0}\\
&= a'\,\gamma_\mu+b'\,\not{P}P_\mu+c'\,q_\mu\not{q},
\end{aligned}
\tag{36}
$$

where the coefficients a', b' and c' are determined by direct calculation of the integral. Thus, the contribution of the term with Lorentz structure $q_\mu=(k_p-k_n)_\mu$, which is antisymmetric with respect to $k_p\longleftrightarrow k_n$ transformation, does not appear in the nucleon part of the charged hadronic vector current. Direct calculation of the integral in equation (36) gives the following value of the coefficient c': $c'=-1/24m_N^2$. Using the Dirac equation $\bar{u}_p\not{P}u_n=m_N$ that does not violate the property of the term with Lorentz structure $P_\mu\not{P}$ to be a contribution of the first class current [6] and the Gordon identity equation (32) we transcribe the r.h.s. of equation (35) into the form

$$
\begin{aligned}
&{}_N\langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathrm{T}\Big(\bar{N}(0)\tau^{(+)}\gamma_\mu N(0)\,\mathrm{i}\,\frac{1}{2}\,g_\pi^2\int \mathrm{d}^4x\mathrm{d}^4y\,\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\\
&\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)\Big)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle_N\\
&= \bar{u}_p(\vec{k}_p, \sigma_p)\frac{g_\pi^2}{4\pi^2}\Big(A_N\,\gamma_\mu + B_N\,\frac{\mathrm{i}\sigma_{\mu\nu}q^\nu}{2m_N} + C_N\,\frac{q_\mu \not{q}}{m_N^2}\Big)u_n(\vec{k}_n, \sigma_n),
\end{aligned}
\tag{37}
$$

where $A_N = (a' + m_N b')/4$, $B_N = -m_N b'/4$ and $C_N = m_N^2 c'$.

Summing the contributions of the nucleon and mesonic parts of the charged hadronic vector current for the vector part of the matrix element of the $n \to p$ transition equation (27), calculated to order g_π^2, we obtain the expression

$$
\begin{aligned}
&\langle \text{out}, p(\vec{k}_p, \sigma_p)|V_\mu^{(+)}(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle =_N \langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathrm{T}\\
&\times\Big\{\Big(\bar{N}(0)\tau^{(+)}\gamma_\mu N(0) + 2\,\varepsilon^{+bc}g_\pi^2\int \mathrm{d}^4x\Delta(x)\bar{N}(x)\mathrm{i}\gamma^5\tau^b N(x)\\
&\times\int \mathrm{d}^4x\,\bar{N}(y)\mathrm{i}\gamma^5\tau^c\,N(y)\partial_\mu\Delta(-y)\Big)\\
&\times \mathrm{e}^{\,\mathrm{i}\,\frac{1}{2}g_\pi^2\int \mathrm{d}^4x'\mathrm{d}^4y'\,\bar{N}(x')\mathrm{i}\gamma^5\vec{\tau}N(x')\cdot\Delta(x'-y')\bar{N}(y')\mathrm{i}\gamma^5\vec{\tau}N(y')}\Big\}|\text{in}, n(\vec{k}_n, \sigma_n)\rangle_N\\
&= \bar{u}_p(\vec{k}_p, \sigma_p)\Big(\Big(1 + \frac{g_\pi^2}{4\pi^2}(A_N + A_\pi)\Big)\gamma_\mu + \frac{g_\pi^2}{4\pi^2}(B_N + B_\pi)\frac{\mathrm{i}\sigma_{\mu\nu}q^\nu}{2m_N}\\
&+ \frac{g_\pi^2}{4\pi^2}(C_N + C_\pi)\frac{q_\mu \not{q}}{M_\pi^2}\Big)u_n(\vec{k}_n, \sigma_n).
\end{aligned}
\tag{38}
$$

Thus, we have shown that the matrix element of the $n \to p$ transition, calculated to order g_π^2, can be expressed in terms of three Lorentz structures, γ_μ, $\mathrm{i}\sigma_{\mu\nu}q^\nu$ and $q_\mu \not{q}$, which are induced by the first class current [6]. Indeed, the isovector hadronic vector current equation (5) has a positive G-parity and belongs to the first class current [6]

$$
\begin{aligned}
&\vec{V}_\mu(x) \xrightarrow{G} G\vec{V}_\mu(x)G^{-1} = G\Big(\bar{N}(x)\frac{1}{2}\,\vec{\tau}\,\gamma_\mu N(x)\Big)G^{-1} + G(\vec{\pi}(x)\times\partial_\mu\vec{\pi}(x))G^{-1}\\
&= N^T(x)C(-\mathrm{i})\tau_2\vec{\tau}\,\gamma_\mu \mathrm{i}\tau_2 C\bar{N}^T(x) + G\vec{\pi}(x)G^{-1}\times G\partial_\mu\vec{\pi}(x)G^{-1} = \vec{V}_\mu(x),
\end{aligned}
\tag{39}
$$

where T is a transposition and C is the charge conjugation matrix [4]. To derive the relation equation (39) we have used the relations $C\gamma_\mu C = \gamma_\mu^T$, $\mathrm{i}\tau_2\vec{\tau}\mathrm{i}\tau_2 = \vec{\tau}^T$, $N^T(x)\bar{N}^T(x) = -\bar{N}(x)N(x)$ [4] and $G\vec{\pi}(x)G^{-1} = -\vec{\pi}(x)$ [7, 8].

Since the coefficient C_N is much smaller than the coefficient C_π, the contribution of the Lorentz structure $q_\mu \not{q}$ to the matrix element of the $n \to p$ transition is practically determined by the mesonic part of the charged hadronic vector current equation (9). Of course, the coefficients A_π and A_N can depend on the ultra-violet cut-off Λ. However, such dependence can be removed by renormalization of the coupling constant g_π^2 [13, 14].

As strong low-energy interactions are invariant under G-parity transformation [8] (see also [6])

$$\mathcal{L}_{\pi NN}(x) = g_\pi \bar{N}(x) \mathrm{i}\gamma^5 \vec{\tau} N(x) \cdot \vec{\pi}(x) \xrightarrow{G} G(g_\pi \bar{N}(x) \mathrm{i}\gamma^5 \vec{\tau} N(x) \cdot \vec{\pi}(x)) G^{-1}$$
$$= \overline{N^G}(x) \mathrm{i}\gamma^5 \vec{\tau} N^G(x) \cdot \vec{\pi}^{\,G}(x)$$
$$= N^T(x) C(-\mathrm{i})\tau_2 \mathrm{i}\gamma^5 \vec{\tau} \mathrm{i}\tau_2 C \bar{N}^T(x) \cdot (-\vec{\pi}(x)) = \mathcal{L}_{\pi NN}(x), \quad (40)$$

where we have used the relation $C\gamma^5 C = -\gamma^{5T}$ [4], and the terms with Lorentz structures γ_μ, $\mathrm{i}\sigma_{\mu\nu}q^\nu$ and $q_\mu \not{q}$ possess positive G-parity, i.e. they are the contributions of the first class current [6], the term with Lorentz structure q_μ, having a negative G-parity [6], should not appear in the matrix element of the $n \to p$ transition equation (27) to any order of g_π^2-expansion. This allows us to write [6]

$$\langle \mathrm{out}, p(\vec{k}_p, \sigma_p) | V_\mu^{(+)}(0) | \mathrm{in}, n(\vec{k}_n, \sigma_n) \rangle = \bar{u}_p(\vec{k}_p, \sigma_p)$$
$$\times \left(F_1(q^2)\gamma_\mu + F_2(q^2) \frac{\mathrm{i}\sigma_{\mu\nu}q^\nu}{2m_N} + F_4(q^2) \frac{q_\mu \not{q}}{m_N^2} \right) u_n(\vec{k}_n, \sigma_n), \quad (41)$$

where $F_1(q^2)$, $F_2(q^2)$ and $F_4(q^2)$ are form factors, calculated to all orders of g_π^2-expansion.

4. Concluding discussion

We have analysed the Lorentz structure of the matrix element of the $n \to p$ transition, caused by the charged hadronic vector current. We have shown that in addition to the standard terms with Lorentz structure $F_1(q^2)\gamma_\mu$ and $F_2(q^2)\mathrm{i}\sigma_{\mu\nu}q^\nu/2m_N$, caused by the contributions of electric charge distribution and weak magnetism inside the hadron, one obtains the term with the structure $F_4(q^2)q_\mu \not{q}/m_N^2$. Using the simplest model of a strongly coupled πN-system with linear pion–nucleon pseudoscalar interaction we have shown that the contribution of the term with the Lorentz structure $F_4(q^2)q_\mu \not{q}/M^2$ is practically induced by the mesonic part of the hadronic isovector vector current. We have also shown that the term with Lorentz structure $F_3(q^2)q_\mu/m_N$, caused by the second class current [7, 8], cannot be, in principle, induced by strong low-energy interactions invariant under G-parity transformations.

A requirement of conservation of the charged hadronic vector current even for different masses of the hadrons in the initial and final states (see [1–3, 15, 16] and so on), in the sense of the vanishing of the matrix element $\langle p | \partial^\mu V_\mu^{(+)}(0) | n \rangle = 0$ of the hadronic $n \to p$ transition, leads to the relation $F_4(q^2) = -(m_N^2/q^2)F_1(q^2)$. Such a relation leads to the appearance of the term $(-q_\mu \not{q}/q^2)F_1(q^2)$ in the matrix element of the hadronic $n \to p$ transition.

For simplicity we have restricted our analysis to the simplest theory of πN strong interactions described by the Lagrangian equation (4) with linear pseudoscalar πNN-interaction [9, 13, 14]. However, one may assert that the result obtained, i.e. the existence of the term with Lorentz structure $q_\mu \not{q}$ and the suppression of the term with Lorentz structure q_μ, which are the contributions of the first and second class currents, respectively, should be valid in any theory of strong low-energy interactions [18–20], which are invariant under G-transformations [8] (see also [7]). Our assertion is based only on the G-invariance of such theories. Indeed, it is hardly possible to perform analytical calculations, which are similar to those we have carried out in this paper, within such complicated non-linear theories of meson–nucleon low-energy interactions as [18, 19] and Chiral perturbation theory [20].

In appendix A we have shown that the cross sections for electron neutrino-neutron scattering and for inverse β-decay, calculated in the non-relativistic approximation with respect to the outgoing hadron, are insensitive to the contributions of the term $(-q_\mu \not{q}/q^2)F_1(q^2)$. That is why one may assert that it is important to search for processes which are sensitive to the contributions of the term $(-q_\mu \not{q}/q^2)F_1(q^2)$.

In appendix B we have analysed the dynamical nature of the Lorentz structure of the matrix element $\langle p|A^{(+)}_\mu(0)|n\rangle$ of the $n \to p$ transition, caused by the charged hadronic axial-vector current $A^{(+)}_\mu(0)$. We have shown that the low-energy pion–nucleon interaction equation (13) allows us to reproduce fully the standard Lorentz structure of the axial part of the hadronic $n \to p$ transition [1].

The results obtained for the hadronic $n \to p$ transition [1] are of course fully valid for the hadronic $p \to n$ transition [17].

Acknowledgments

This work was supported by the Austrian 'Fonds zur Förderung der Wissenschaftlichen Forschung' (FWF) under Contracts I689-N16, P26781-N20 and P26636-N20 and 'Deutsche Förderungsgemeinschaft' (DFG) AB 128/5-2.

Appendix A. Cross sections for quasi-elastic electron neutrino-neutron scattering and for inverse β-decay

In this appendix we calculate the cross sections for quasi-elastic scattering $\nu_e + n \to p + e^-$ and inverse β-decay $\bar{\nu}_e + p \to n + e^+$ by taking into account the contributions of the term $-q_\mu \not{q}/q^2$ responsible for the constraint $\langle h'|\partial^\mu V^{(\pm)}_\mu(0)|h\rangle = 0$ even for different masses of incoming h and outgoing h' hadrons. Below the contributions of such a term we call the contributions of exact conservation of the charged weak hadronic vector current or the ECVC effect.

We define the amplitudes of quasi-elastic scattering $\nu_e + n \to p + e^-$ and inverse β-decay $\bar{\nu}_e + p \to n + e^+$ in the non-relativistic approximation for the outgoing nucleon. They are equal to

$$M(\nu_e n \to p\, e^-) = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, \langle p(\vec{k}_p, \sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle \times \left[\bar{u}_-(\vec{k}_-, \sigma_-)\gamma^\mu(1-\gamma^5)u_\nu\left(\vec{k}, -\frac{1}{2}\right)\right] \tag{A-1}$$

and

$$M(\bar{\nu}_e p \to n\, e^+) = +\frac{G_F}{\sqrt{2}}\, V_{ud}\, \langle n(\vec{k}_n, \sigma_n)|J^{(-)}_\mu(0)|p(\vec{k}_p, \sigma_p)\rangle \times \left[\bar{v}_{\bar{\nu}}(\vec{k}_{\bar{\nu}}, +\frac{1}{2})\gamma^\mu(1-\gamma^5)v_+(\vec{k}_+, \sigma_+)\right], \tag{A-2}$$

where G_F and V_{ud} are the Fermi weak coupling constant and the Cabibbo–Kobayashi–Maskawa matrix element [21], $\bar{u}_-(\vec{k}_-, \sigma_-)$ and $u_\nu(\vec{k}_\nu, -\frac{1}{2})$ are the Dirac wave functions of the free electron and electron neutrino with 3-momenta $\vec{k}_-$ and $\vec{k}_\nu$ and polarizations $\sigma_e = \pm 1$ and $-\frac{1}{2}$ [22–24], respectively, and (γ_μ, γ^5) are the Dirac matrices. Then, $\bar{v}_{\bar{\nu}}(\vec{k}_{\bar{\nu}}, +\frac{1}{2})$ and

$v_+(\vec{k}_+, \sigma_+)$ are the Dirac wave functions of the electron antineutrino and positron with 3-momenta $\vec{k}_{\bar{\nu}}$ and $\vec{k}_+$ and polarizations $\sigma_+ = \pm 1$ and $+\frac{1}{2}$ [22–24], respectively. We define the matrix elements $\langle p(\vec{k}_p, \sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle$ and $\langle n(\vec{k}_n, \sigma_n)|J^{(-)}_\mu(0)|p(\vec{k}_p, \sigma_p)\rangle$ of the hadronic $n \to p$ and $p \to n$ transitions as [1]

$$\langle p(\vec{k}_p, \sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle = \bar{u}_p(\vec{k}_p, \sigma_p)\left(F_1(q^2)\left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right) + F_A(q^2)\gamma_\mu\gamma^5\right)u_n(\vec{k}_n, \sigma_n) \tag{A-3}$$

and

$$\langle n(\vec{k}_n, \sigma_n)|J^{(-)}_\mu(0)|p(\vec{k}_p, \sigma_p)\rangle = \bar{u}_n(\vec{k}_n, \sigma_n)\left(F_1(q^2)\left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right) + F_A(q^2)\gamma_\mu\gamma^5\right)u_p(\vec{k}_p, \sigma_p), \tag{A-4}$$

where $J^{(\pm)}_\mu(0) = V^{(\pm)}_\mu(0) - A^{(\pm)}_\mu(0)$, $u_n(\vec{k}_n, \sigma_n)$ and $u_p(\vec{k}_p, \sigma_p)$ are the Dirac wave functions of the free neutron and proton with 3-momenta and polarizations $(\vec{k}_n, \sigma_n = \pm 1)$ and $(\vec{k}_p, \sigma_p = \pm 1)$. Then, $F_1(q^2)$ and $F_A(q^2)$ are the vector and axial-vector form factors [1]. The vector parts of the matrix element equations (A-3) and (A-4) obey the constraints

$$\langle p(\vec{k}_p, \sigma_p)|\partial^\mu V^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle = \langle n(\vec{k}_n, \sigma_n)|\partial^\mu V^{(-)}_\mu(0)|p(\vec{k}_p, \sigma_p)\rangle = 0 \tag{A-5}$$

even for different neutron and proton masses. In the matrix element equations (A-3) and (A-4) we have neglected the contributions of weak magnetism and one-pion exchange [1]. In the approximation, when the squared transferred momentum $q^2 = (\pm k_p \mp k_n)^2$ is much smaller than the M_V^2 and M_A^2 scales defining the effective radii of the vector and axial-vector form factors, the matrix element equations (A-3) and (A-4) can be reduced to the form

$$\langle p(\vec{k}_p, \sigma_p)|J^{(+)}_\mu(0)|n(\vec{k}_n, \sigma_n)\rangle = \bar{u}_p(\vec{k}_p, \sigma_p)\left(\gamma_\mu(1 + \lambda\gamma^5) - \frac{q_\mu \not{q}}{q^2}\right)u_n(\vec{k}_n, \sigma_n) \tag{A-6}$$

and

$$\langle n(\vec{k}_n, \sigma_n)|J^{(-)}_\mu(0)|p(\vec{k}_p, \sigma_p)\rangle = \bar{u}_n(\vec{k}_n, \sigma_n)\left(\gamma_\mu(1 + \lambda\gamma^5) - \frac{q_\mu \not{q}}{q^2}\right)u_p(\vec{k}_p, \sigma_p), \tag{A-7}$$

where $\lambda = -1.2750(9)$ is the axial coupling constant [26] (see also [22–25]).

In order to illustrate the contribution of the term $-q_\mu \not{q}/q^2$ responsible for the fulfilment of the constraints equation (A-5), we neglect the contributions of the weak magnetism, recoil and radiative corrections [24]. Skipping intermediate standard calculations [24] we obtain cross sections for quasi-elastic electron neutrino-neutron scattering $\sigma(E_\nu)$ and inverse β-decay $\sigma(E_{\bar{\nu}})$

$$\sigma(E_\nu) = \sigma_0(E_\nu) + \frac{G_F^2|V_{ud}|^2}{2\pi}\,k_-E_-\Bigg\{-\frac{m_e^2\Delta}{k_-E_-E_\nu}\Bigg[\ell n\Bigg(1 + \frac{2k_-E_\nu\beta_-}{m_e^2 - 2E_-E_\nu}\Bigg) - \ell n\Bigg(1 - \frac{2k_-E_\nu}{m_e^2 - 2E_-E_\nu}\Bigg)\Bigg] + \frac{2m_e^2\Delta^2}{(m_e^2 - 2E_-E_\nu)^2 - 4k_-^2E_\nu^2} - \frac{m_e^2\Delta^2}{4k_-E_-E_\nu^2}\Bigg[\ell n\Bigg(1 + \frac{2k_-E_\nu\beta_-}{m_e^2 - 2E_-E_\nu}\Bigg) - \ell n\Bigg(1 - \frac{2k_-E_\nu}{m_e^2 - 2E_-E_\nu}\Bigg) - \frac{4k_-E_\nu(m_e^2 - 2E_-E_\nu)}{(m_e^2 - 2E_-E_\nu)^2 - 4k_-^2E_\nu^2}\Bigg]\Bigg\}, \quad \text{(A-8)}$$

where $E_- = E_\nu + \Delta$, $k_- = \sqrt{E_-^2 - m_e^2}$ and $\beta_- = k_-/E_-$ are the energy, momentum and velocity of the electron, and

$$\sigma(E_{\bar\nu}) = \sigma_0(E_{\bar\nu}) + \frac{G_F^2|V_{ud}|^2}{2\pi}\,k_+E_+\Bigg\{\frac{m_e^2\Delta}{k_+E_+E_{\bar\nu}}\Bigg[\ell n\Bigg(1 + \frac{2k_+E_{\bar\nu}}{m_e^2 - 2E_+E_{\bar\nu}}\Bigg) - \ell n\Bigg(1 - \frac{2k_+E_{\bar\nu}}{m_e^2 - 2E_+E_{\bar\nu}}\Bigg)\Bigg] + \frac{2m_e^2\Delta^2}{(m_e^2 - 2E_+E_{\bar\nu})^2 - 4k_+^2E_{\bar\nu}^2} - \frac{m_e^2\Delta^2}{4k_+E_+E_{\bar\nu}^2}\Bigg[\ell n\Bigg(1 + \frac{2k_+E_{\bar\nu}}{m_e^2 - 2E_+E_{\bar\nu}}\Bigg) - \ell n\Bigg(1 - \frac{2k_+E_{\bar\nu}}{m_e^2 - 2E_+E_{\bar\nu}}\Bigg) - \frac{4k_+E_{\bar\nu}(m_e^2 - 2E_+E_{\bar\nu})}{(m_e^2 - 2E_+E_{\bar\nu})^2 - 4k_+^2E_{\bar\nu}^2}\Bigg]\Bigg\}, \quad \text{(A-9)}$$

where $E_+ = E_{\bar\nu} - \Delta$ and $k_+ = \sqrt{E_+^2 - m_e^2}$ are the energy and momentum of the positron. The cross sections $\sigma_0(E_\nu)$ and $\sigma_0(E_{\bar\nu})$ are given by [24]

$$\sigma_0(E_\nu) = (1 + 3\lambda^2)\frac{G_F^2|V_{ud}|^2}{\pi}\,k_-E_-, \qquad \sigma_0(E_{\bar\nu}) = (1 + 3\lambda^2)\frac{G_F^2|V_{ud}|^2}{\pi}\,k_+E_+. \quad \text{(A-10)}$$

In quasi-elastic electron neutrino-neutron scattering and inverse β-decay the neutrino and antineutrino energies vary in the regions $E_\nu \geqslant 0$ and $E_{\bar\nu} \geqslant (E_{\bar\nu})_{\rm thr} = ((m_n + m_e)^2 - m_p^2)/2m_p = 1.8061$ MeV [24]. The terms dependent on Δ are caused by the ECVC effect. The relative contributions of the ECVC effect to the cross sections under consideration we define as

$$R_\nu(E_\nu) = \frac{1}{2}\,\frac{1}{1+3\lambda^2}\Bigg\{-\frac{m_e^2\Delta}{k_-E_-E_\nu}\Bigg[\ell n\Bigg(1+\frac{2k_-E_\nu\beta_-}{m_e^2-2E_-E_\nu}\Bigg)-\ell n\Bigg(1-\frac{2k_-E_\nu}{m_e^2-2E_-E_\nu}\Bigg)\Bigg]$$
$$+\frac{2m_e^2\Delta^2}{(m_e^2-2E_-E_\nu)^2-4k_-^2E_\nu^2}$$
$$-\frac{m_e^2\Delta^2}{4k_-E_-E_\nu^2}\Bigg[\ell n\Bigg(1+\frac{2k_-E_\nu\beta_-}{m_e^2-2E_-E_\nu}\Bigg)-\ell n\Bigg(1-\frac{2k_-E_\nu}{m_e^2-2E_-E_\nu}\Bigg)$$
$$-\frac{4k_-E_\nu(m_e^2-2E_-E_\nu)}{(m_e^2-2E_-E_\nu)^2-4k_-^2E_\nu^2}\Bigg]\Bigg\}, \tag{A-11}$$

and

$$R_{\bar\nu}(E_{\bar\nu}) = \frac{1}{2}\,\frac{1}{1+3\lambda^2}\Bigg\{\frac{m_e^2\Delta}{k_+E_+E_{\bar\nu}}\Bigg[\ell n\Bigg(1+\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Bigg)-\ell n\Bigg(1-\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Bigg)\Bigg]$$
$$+\frac{2m_e^2\Delta^2}{(m_e^2-2E_+E_{\bar\nu})^2-4k_+^2E_{\bar\nu}^2}$$
$$-\frac{m_e^2\Delta^2}{4k_+E_+E_{\bar\nu}^2}\Bigg[\ell n\Bigg(1+\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Bigg)-\ell n\Bigg(1-\frac{2k_+E_{\bar\nu}}{m_e^2-2E_+E_{\bar\nu}}\Bigg)$$
$$-\frac{4k_+E_{\bar\nu}(m_e^2-2E_+E_{\bar\nu})}{(m_e^2-2E_+E_{\bar\nu})^2-4k_+^2E_{\bar\nu}^2}\Bigg]\Bigg\}, \tag{A-12}$$

where $R_\nu(E_\nu) = \Delta\sigma(E_\nu)/\sigma_0(E_\nu)$, $R_{\bar\nu}(E_{\bar\nu}) = \Delta\sigma(E_{\bar\nu})/\sigma_0(E_{\bar\nu})$ with $\Delta\sigma(E_\nu) = \sigma(E_\nu) - \sigma_0(E_\nu)$ and $\Delta\sigma(E_{\bar\nu}) = \sigma(E_{\bar\nu}) - \sigma_0(E_{\bar\nu})$, respectively. The cross section equations (A-8) and (A-9) are calculated in the laboratory frame in the non-relativistic approximation for outgoing hadrons. Since the most important region of the antineutrino energies for inverse β-decay is $2\ \mathrm{MeV} \leqslant E_{\bar\nu} \leqslant 8\ \mathrm{MeV}$ [24], in figure 1 we plot $R_\nu(E_\nu)$ and $R_{\bar\nu}(E_{\bar\nu})$ for E_ν and $E_{\bar\nu}$ varying over the regions $2\mathrm{MeV} \leqslant E_\nu \leqslant 8\ \mathrm{MeV}$ and $2\ \mathrm{MeV} \leqslant E_{\bar\nu} \leqslant 8\ \mathrm{MeV}$, respectively.

Our numerical analysis of the relative contributions of the ECVC effect to the cross sections for quasi-elastic electron neutrino-neutron scattering and for inverse β-decay shows that these processes are not sensitive to the ECVC effect. Indeed, the contribution of the ECVC effect to the cross section for quasi-elastic electron neutrino-neutron scattering is smaller than 0.7% at $E_\nu \simeq 2$ MeV and decreases by about two orders of magnitude at $E_\nu \simeq 8$ MeV. The cross section for inverse β-decay, applied to the analysis of the deficit of positrons induced by reactor electron antineutrinos [24, 27], should be averaged over the reactor electron antineutrino energy spectrum, which has a maximum at $E_{\bar\nu} \simeq 4\mathrm{MeV}$. According to figure 1, the contribution of the ECVC effect should decrease the yield of positrons Y_{e^+} by about 0.5%. Since such a contribution is smaller than the experimental error bars $Y_{e^+} = 0.943(23)$ [27], one may argue that inverse β-decay is insensitive to the contribution of the ECVC effect.

Appendix B. The Lorentz structure of the matrix element of the hadronic $n \to p$ transition, caused by the charged hadronic axial-vector current

In this appendix we analyse the Lorentz structure of the axial-vector part of the hadronic $n \to p$ transition, induced by the charged hadronic axial-vector current

$$A^{(+)}_{\mu}(x) = \bar{N}(x)\tau^{(+)}\gamma_{\mu}\gamma^5 N(x). \quad \text{(B-1)}$$

The matrix element of interest is

$$\langle \text{out}, p(\vec{k}_p, \sigma_p)|A^{(+)}_{\mu}(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle, \quad \text{(B-2)}$$

where $\langle \text{out}, p(\vec{k}_p, \sigma_p)|$ and $|\text{in}, n(\vec{k}_n, \sigma_n)\rangle$ are the wave functions of the free proton and neutron in the final (i.e. out-state at $t \to +\infty$) and initial (i.e. in-state at $t \to -\infty$) states, respectively [4]. Using the relation $\langle \text{out}, p(\vec{k}_p, \sigma_p)| = \langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathbb{S}$, where $\mathbb{S}$ is the S-matrix, we rewrite the matrix element equation (B-2) as follows:

$$\langle \text{out}, p(\vec{k}_p, \sigma_p)|A^{(+)}_{\mu}(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle = \langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathbb{S}A^{(+)}_{\mu}(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle. \quad \text{(B-3)}$$

Since the $n \to p$transition is fully induced by strong low-energy interactions, we define the S-matrix only in terms of the strong low-energy interaction described by the Lagrangian equation (4). It is given by equation (12). Plugging equation (12) into equation (B-3) we get

$$\langle \text{in}, p(\vec{k}_p, \sigma_p)|\mathbb{S}A^{(+)}_{\mu}(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle$$
$$= \langle \text{in}, p(\vec{k}_p, \sigma_p)|\text{T}(e^{\text{i}\int \text{d}^4x\, \mathcal{L}_{\pi NN}(x)} A^{(+)}_{\mu}(0))|\text{in}, n(\vec{k}_n, \sigma_n)\rangle. \quad \text{(B-4)}$$

After integrating over the pion-fields we arrive at the expression

$$\langle \text{out}, p(\vec{k}_p, \sigma_p)|A^{(+)}_{\mu}(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle =_N \Big\langle \text{in}, p(\vec{k}_p, \sigma_p)|\text{T}$$
$$\times\Big\{\bar{N}(0)\tau^{(+)}\gamma_{\mu}\gamma^5 N(0)\exp\Big(\text{i}\,\frac{1}{2}\,g^2_{\pi}\int \text{d}^4x'\text{d}^4y'\,\bar{N}(x')\text{i}\gamma^5\vec{\tau}N(x')\cdot$$
$$\Delta(x' - y')\bar{N}(y')\text{i}\gamma^5\vec{\tau}N(y')\Big\}|\text{in}, n(\vec{k}_n, \sigma_n)\Big)\Big\rangle_N. \quad \text{(B-5)}$$

To order g^2_{π} the dynamical contribution of the nucleon part of the charged hadronic axial-vector current to the matrix element of the $n \to p$ transition, given by equation (B-5), is determined by the matrix element

$$_N\langle \text{in}, p(\vec{k}_p, \sigma_p)|\text{T}\Big(\bar{N}(0)\tau^{(+)}\gamma_{\mu}\gamma^5 N(0)\text{i}\,\frac{1}{2}\,g^2_{\pi}\int \text{d}^4\text{x}\text{d}^4y\,\bar{N}(x)\text{i}\gamma^5\vec{\tau}N(x)$$
$$\cdot\Delta(x - y)\bar{N}(y)\text{i}\gamma^5\vec{\tau}N(y)\Big)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle_N$$
$$= [\bar{u}_p(\vec{k}_p, \sigma_p)\gamma_{\mu}\gamma^5 u_n(\vec{k}_n, \sigma_n)]\, 3(-\text{i})g^2_{\pi}\int \text{d}^4\text{x}\text{d}^4y$$
$$\times\, \text{tr}\{(-\text{i})S_F(y - x)\text{i}\gamma^5(-\text{i})S_F(x - y)\text{i}\gamma^5\}\,\Delta(x - y)$$
$$+ [\bar{u}_p(\vec{k}_p, \sigma_p)\text{i}\gamma^5 u_n(\vec{k}_n, \sigma_n)](-\text{i})g^2_{\pi}\int \text{d}^4\text{x}\text{d}^4y$$
$$\times\, \text{tr}\{\gamma_{\mu}\gamma^5(-\text{i})S_F(-x)\text{i}\gamma^5(-\text{i})S_F(x)\}\,\Delta(x - y)e^{\,\text{i}(k_p - k_n)\cdot y}$$
$$+ [\bar{u}_p(\vec{k}_p, \sigma_p)\text{i}\gamma^5 u_n(\vec{k}_n, \sigma_n)](-\text{i})g^2_{\pi}\int \text{d}^4\text{x}\text{d}^4y$$
$$\times\, \text{tr}\{\gamma_{\mu}\gamma^5(-\text{i})S_F(-y)\text{i}\gamma^5(-\text{i})S_F(y)\}\,\Delta(x - y)e^{\,\text{i}(k_p - k_n)\cdot x}. \quad \text{(B-6)}$$

In the momentum representation we get

$$
\begin{aligned}
&{}_N\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|\mathrm{T}\bigg(\bar{N}(0)\tau^{(+)}\gamma_\mu\gamma^5 N(0)\,\mathrm{i}\,\frac{1}{2}\,g_\pi^2\int \mathrm{d}^4x\mathrm{d}^4y\;\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\\
&\quad\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)\bigg)|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle_N\\
&= [\bar{u}_p(\vec{k}_p,\, \sigma_p)\gamma_\mu\gamma^5 u_n(\vec{k}_n,\, \sigma_n)]\;3\,g_\pi^2\,\delta^{(4)}(0)\int\frac{\mathrm{d}^4p d^4Q}{(2\pi)^4\mathrm{i}}\\
&\quad\times\mathrm{tr}\bigg\{\frac{1}{m_N-\not{p}-\mathrm{i}0}\,\gamma^5\frac{1}{m_N-\not{p}-\not{Q}-\mathrm{i}0}\,\gamma^5\bigg\}\,\frac{1}{m_\pi^2-Q^2-\mathrm{i}0}\\
&\quad+[\bar{u}_p(\vec{k}_p,\, \sigma_p)\gamma^5 u_n(\vec{k}_n,\, \sigma_n)]\,\frac{2g_\pi^2}{m_\pi^2-q^2-\mathrm{i}0}\int\frac{\mathrm{d}^4p}{(2\pi)^4\mathrm{i}}\\
&\quad\times\mathrm{tr}\bigg\{\gamma_\mu\gamma^5\frac{1}{m_N-\not{p}-\mathrm{i}0}\,\gamma^5\,\frac{1}{m_N-\not{p}-\not{q}-\mathrm{i}0}\bigg\},
\end{aligned}
\tag{B-7}
$$

where $q = k_p - k_n$. The contribution proportional to $\delta^{(4)}(0)$ can be in principle removed using the normal-ordered form of the four-nucleon operator of interaction [4]. Indeed, replacing $\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)$ by $:\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y):$ the vacuum expectation value of the operator $:\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y):$ is equal to zero. The contribution of the momentum integral of the second term is divergent and proportional to q_μ. As a result, we may define the matrix element equation (B-7) as follows:

$$
\begin{aligned}
&{}_N\langle \mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|\,\mathrm{T}\bigg(\bar{N}(0)\tau^{(+)}\gamma_\mu\gamma^5 N(0)\,\mathrm{i}\,\frac{1}{2}\,g_\pi^2\int \mathrm{d}^4\mathrm{x}\mathrm{d}^4y\;\bar{N}(x)\mathrm{i}\gamma^5\vec{\tau}N(x)\\
&\quad\cdot\Delta(x-y)\bar{N}(y)\mathrm{i}\gamma^5\vec{\tau}N(y)\bigg)|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle_N\\
&=\frac{g_\pi^2}{4\pi^2}\,D_N\,[\bar{u}_p(\vec{k}_p,\, \sigma_p)\gamma_\mu\gamma^5 u_n(\vec{k}_n,\, \sigma_n)]+\frac{g_\pi^2}{4\pi^2}\,\frac{2m_N q_\mu}{m_\pi^2-q^2-\mathrm{i}0}\\
&\quad\times E_N\,[\bar{u}_p(\vec{k}_p,\, \sigma_p)\gamma^5 u_n(\vec{k}_n,\, \sigma_n)].
\end{aligned}
\tag{B-8}
$$

Thus, for the matrix element equation (B-5), calculated to order g_π^2, we obtain the following expression:

$$
\begin{aligned}
&\langle \mathrm{out},\, p(\vec{k}_p,\, \sigma_p)|A_\mu^{(+)}(0)|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\rangle =_N \bigg\langle\,\mathrm{in},\, p(\vec{k}_p,\, \sigma_p)|\mathrm{T}\\
&\quad\times\bigg\{\bar{N}(0)\tau^{(+)}\gamma_\mu\gamma^5 N(0)\,\exp\bigg(\mathrm{i}\,\frac{1}{2}\,g_\pi^2\int \mathrm{d}^4x'\mathrm{d}^4y'\;\bar{N}(x')\mathrm{i}\gamma^5\vec{\tau}N(x')\cdot\\
&\Delta(x'-y')\bar{N}(y')\mathrm{i}\gamma^5\vec{\tau}N(y')\bigg\}\,|\mathrm{in},\, n(\vec{k}_n,\, \sigma_n)\bigg)\bigg\rangle_N\\
&=\bar{u}_p(\vec{k}_p,\, \sigma_p)\bigg(\bigg(1+\frac{g_\pi^2}{4\pi^2}\,D_N\bigg)\gamma_\mu\gamma^5+\frac{g_\pi^2}{4\pi^2}\,\frac{2m_N q_\mu}{m_\pi^2-q^2-i0}\,E_N\,\gamma^5\bigg)u_n(\vec{k}_n,\, \sigma_n).
\end{aligned}
\tag{B-9}
$$

Following [11] we may argue that the Lorentz structure of the matrix element equation (B-5), calculated to order g_π^2, should be valid to all orders of the g_π^2-expansion. The latter is, of course, because of the invariance of strong low-energy interactions under G-transformations. Thus, the matrix element equation (B-5) should have the Lorentz structure, induced by the first class axial-vector current [7]

$$\langle \text{out}, p(\vec{k}_p, \sigma_p)|A^{(+)}_\mu(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle = \bar{u}_p(\vec{k}_p, \sigma_p) \times \left(-F_A(q^2)\gamma_\mu\gamma^5 + \frac{2m_N q_\mu}{m_\pi^2 - q^2 - i0} F_P(q^2)\gamma^5\right) u_n(\vec{k}_n, \sigma_n), \tag{B-10}$$

where $F_A(q^2)$ and $F_P(q^2)$ are the axial-vector and pseudoscalar form factors [1]. Taking into account the PCAC hypothesis (or the hypothesis of Partial Conservation of Axial-vector Current) [28, 29] we may rewrite the r.h.s. of equation (B-10) as

$$\langle \text{out}, p(\vec{k}_p, \sigma_p)|A^{(+)}_\mu(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle = -\bar{u}_p(\vec{k}_p, \sigma_p) \times \left(\gamma_\mu\gamma^5 + \frac{2m_N q_\mu}{m_\pi^2 - q^2 - i0} \gamma^5\right) F_A(q^2) u_n(\vec{k}_n, \sigma_n), \tag{B-11}$$

where we have set $F_A(q^2) = -F_P(q^2)$. At $q^2 = 0$ we set $F_A(q^2) = \lambda$, where $\lambda = -1.2750(9)$ is the axial coupling constant [26] (see also [22–25]). In the chiral limit $m_\pi \to 0$ the matrix element equation (B-11) multiplied by the 4-momentum transferred q^μ vanishes

$$\lim_{m_\pi \to 0} q^\mu \langle \text{out}, p(\vec{k}_p, \sigma_p)|A^{(+)}_\mu(0)|\text{in}, n(\vec{k}_n, \sigma_n)\rangle = 0 \tag{B-12}$$

even for different neutron and proton masses, according to the PCAC hypothesis [28, 29] pointing out an operator relation $\partial^\mu \vec{A}_\mu(x) = m_\mu^2 F_\pi \vec{\pi}(x)$, where F_π is the pion-decay constant [21]. In the chiral limit $m_\pi \to$ we get $\partial^\mu \vec{A}_\mu(x) = 0$ which agrees well with equation (B-12).

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