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Search for the $a_0(980)$–$f_0(980)$ mixing in weak decays of D_s/B_s mesons

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ABSTRACT

Scalar mesons $a_0^0(980)$ and $f_0(980)$ can mix with each other through isospin violating effects, and the mixing intensity has been predicted at the percent level in various theoretical models. However the mixing has not been firmed established on the experimental side to date. In this work we explore the possibility to extract the a_0–f_0 mixing intensity using weak decays of heavy mesons: $D_s \to [\pi^0\eta, \pi^+\pi^-]e^+\nu$, $B_s \to [\pi^0\eta, \pi^+\pi^-]\ell^+\ell^-$ and the $B_s \to J/\psi[\pi^0\eta, \pi^+\pi^-]$ decays. Based on the large amount of data accumulated by various experimental facilities including BEPC-II, LHC, Super KEKB and the future colliders, we find that the a_0–f_0 mixing intensity might be determined to a high precision, which will lead to a better understanding of the nature of scalar mesons.

1. Introduction

Light scalar mesons below 1 GeV play an important role in understanding the QCD vacuum since they share the same quantum numbers J^{PC}. But due to the nonperturbative nature of QCD at low energy the internal structure of scalar mesons is extremely complicated and still under controversy. They have been interpreted as quark–antiquark, tetra-quarks, hadronic molecule, quark–antiquark–gluon hybrid, and etc. [1].

Among various phenomena, it is anticipated that the mixing between the $a_0^0(980)$ and $f_0(980)$ resonances may shed light on the nature of these two resonances, and therefore has been studied extensively on different aspects and in various processes. For an incomplete list of discussions in the literature, please see Refs. [2–25] and references therein. To date no firm experimental determination on this quantity is available yet. The possibility of extracting the $a_0^0(980)$–$f_0(980)$ mixing from the $J/\psi \to \phi a_0^0(980) \to \phi\eta\pi^0$ reaction has been explored in Refs. [17,18]. This reaction is an isospin breaking process with the initial state of isospin 0 and the final state of isospin 1. BES-III collaboration has used this process to determine the mixing [26]:

$$\xi_{fa}^{J/\psi} \equiv \frac{\mathcal{B}(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0)}{\mathcal{B}(J/\psi \to \phi f_0(980) \to \phi\pi^+\pi^-)} = (0.60 \pm 0.20 \pm 0.12 \pm 0.26)\%, \quad (1)$$

where the uncertainties are statistical, systematics due to the measurement and the parametrization, respectively. As one can see, the statistical significance is only about 3.4σ.

To more precisely determine the mixing intensity, two parallel researches can be conducted in the future. On the one hand, one may collect more data on the J/ψ (and ψ') and accordingly the errors in this quantity can be reduced significantly. On the other side, one may look for new channels that can be used to determine the mixing parameter. This will also provide a cross-check of the results derived from the J/ψ decays. In this work, we will focus on the latter category. Weak decays of heavy mesons are not only of great value to determine the standard model parameters (see Ref. [27] for a recent review), but can also provide an ideal platform to study hadron structures [28]. In the following, we will examine the possibility to extract the mixing intensity from the rare decays of D_s and B_s: $D_s \to [\pi^0\eta, \pi^+\pi^-]e^+\nu$, $B_s \to [\pi^0\eta, \pi^+\pi^-]\ell^+\ell^-$ and the $B_s \to J/\psi[\pi^0\eta, \pi^+\pi^-]$ decays. An advantage in these modes is that the lepton (or the J/ψ) is an iso-singlet system and thus there is a natural isospin filter. At the quark level, the intermediate state has $I = 0$. It should be noticed that the semileptonic D_s and B_s decays into the $\pi^+\pi^-$ via the $f_0(980)$ have already been observed by CLEO-c [29–31] and LHCb collaboration [32], respectively. The branching fraction

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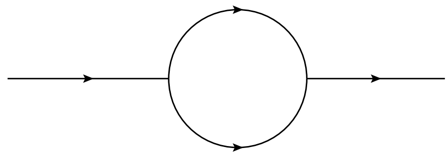

Fig. 1. One-loop corrections to two-point function.

of the $B_s \to J/\psi f_0(980) \to J/\psi\pi^+\pi^-$ is also measured in Refs. [33–41].

The rest of this paper is organized as follows. In Sec. 2, we will give a brief overview of the a_0–f_0 mixing mechanism. We will discuss the mixing effects in B_s and D_s decays in Sec. 3. A short summary is presented in the last section.

2. The $f_0(980)$–$a_0(980)$ mixing mechanism

For the nearly degenerate $a_0^0(980)$ with isospin 1 and $f_0(980)$ with isospin 0, both can couple to the $K\bar{K}$ state, but the charged and neutral kaon thresholds differ by about 8 MeV. This difference leads to the $a_0^0(980)$–$f_0(980)$ mixing. This mechanism was pioneered in Ref. [2] followed by many references. It has been recently confirmed by a recent analysis in the (unitarized) chiral perturbation theory [19], in which various mixing effects have been considered and the loop through $K\bar{K}$ is found dominance. In the following we will use the abbreviation a_0 and f_0 to denote the $a_0^0(980)$ and $f_0(980)$ for simplicity.

For illustration, we consider the propagation of the $f_0(980)$ and include the loop corrections through two pseudo-scalars M_1 and M_2. The one-loop corrections are shown in Fig. 1. If one sums these loop corrections in the chain approximation, the $f_0(980)$ propagator will become:

$$G(s) \equiv \frac{i}{D_f(s)} = \frac{i}{s-m_{f_0}^2} + \frac{i}{s-m_{f_0}^2}(-i\mathcal{M}^2)\frac{i}{s-m_{f_0}^2} + \ldots = \frac{i}{s-m_{f_0}^2-\mathcal{M}^2}, \quad (2)$$

with the loop corrections

$$-i\mathcal{M}^2 = ig_{f_0M_1M_2}ig^*_{f_0M_1M_2}\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_{M_1}^2}\frac{i}{(k-p)^2-m_{M_2}^2}. \quad (3)$$

Here the $g_{f_0M_1M_2}$ denotes the coupling of the f_0 with the M_1, M_2:

$$\mathcal{L} = ig_{f_0M_1M_2}f_0M_1M_2. \quad (4)$$

The real part of the $\mathcal{M}^2$ will renormalize the bare mass, leading to the pole in the propagator as the physical mass. The remanent multiplicative constant in the real part is absorbed by the field strength renormalization factor. On the other side, the effective Hamiltonian is guaranteed only when the exchanged intermediate states are not far from mass-shell. Thus it is more reliable to calculate the imaginary part of the $\mathcal{M}^2$ which will result in a nonzero mass-dependent decay width:

$$\Gamma_{12}^f(s) = -\frac{1}{\sqrt{s}}\mathrm{Im}[\mathcal{M}^2](s) = \frac{1}{16\pi\sqrt{s}}|g_{f_0M_1M_2}|^2\rho_{12}(s), \quad (5)$$

with $\rho_{bc}(s) = \sqrt{[1-(m_b-m_c)^2/s][1-(m_b+m_c)^2/s]}$.

With the incorporation of the mixing effects, we have the a_0/f_0 propagator:

$$G(s) = \frac{i}{D_f(s)D_a(s)}\begin{pmatrix} D_a(s) & D_{af}(s) \\ D_{af}(s) & D_f(s)\end{pmatrix}, \quad (6)$$

where D_a and D_f are the denominators of the resummed propagators for the $a_0^0(980)$ and $f_0(980)$, respectively:

$$D_a(s) = s-m_a^2+i\sqrt{s}[\Gamma^a_{\eta\pi}(s)+\Gamma^a_{K\bar{K}}(s)], \quad (7)$$

$$D_f(s) = s-m_f^2+i\sqrt{s}[\Gamma^f_{\pi\pi}(s)+\Gamma^f_{K\bar{K}}(s)]. \quad (8)$$

Since the mixing term is already small at leading order, it is not necessary to sum all order corrections. We have the expression for the D_{af}:

$$D_{af}(s) = i\frac{g_{a_0^0(980)K^+K^-}g_{f_0(980)K^+K^-}}{16\pi}\left\{\rho_{K^+K^-}(s)-\rho_{K^0\bar{K}^0}(s)\right\}, \quad (9)$$

where we have kept only the imaginary part in the loop correction.

One can investigate the mass-dependent $f_0 \to a_0$ mixing intensity $\xi(s)$:

$$\xi(s) = \left|\frac{D_{af}(s)}{D_a(s)}\right|^2 = \left|\frac{g_{a_0^0(980)K^+K^-}g_{f_0(980)K^+K^-}[\rho_{K^+K^-}(s)-\rho_{K^0\bar{K}^0}(s)]}{16\pi D_a(s)}\right|^2. \quad (10)$$

As one can see, the mixing parameter arises when the charged and neutral kaons have different masses. The results also rely on the couplings $g_{a_0^0(980)K^+K^-}$, $g_{f_0(980)K^+K^-}$ and the mass pole structures in the propagator. Various theoretical models predict different values for these quantities, and a thorough discussion has been presented in Refs. [17,18].

A few remarks are given in order.

- In the loop amplitude in Eq. (3) the imaginary part can be reliably calculated, while its real part suffers from large uncertainties. On the one hand, it may contain the ultraviolet divergence that needs the renormalization procedure. On the other hand, the effective Hamiltonian is valid only when the involved particles are not far from the mass-shell.
- There are ambiguities in the couplings of a scalar meson with two pseudo-scalar mesons. Instead of using the effective Hamiltonian in Eq. (4), one may adopt the following form:

$$\mathcal{L} = ig'_{f_0M_1M_2}f_0\partial^\mu M_1\partial_\mu M_2, \quad (11)$$

which is inspired by the chiral symmetry. The above interaction leads to a different result for the loop amplitude:

$$-i\mathcal{M}^2 = ig'_{f_0M_1M_2}ig'^*_{f_0M_1M_2}\times\int\frac{d^4k}{(2\pi)^4}[k\cdot(k-p)]^2\frac{i}{k^2-m_{M_1}^2}\frac{i}{(k-p)^2-m_{M_2}^2}. \quad (12)$$

Apparently the real part of the loop amplitude is dramatically different, even the ultraviolet structure. Fortunately, the imaginary part can still be reliably calculated base on the correspondence:

$$g_{f_0M_1M_2} = g'_{f_0M_1M_2}\frac{s-m_1^2-m_2^2}{2}. \quad (13)$$

We have the results for the width and the off-diagonal term in the mixed propagator as follows:

$$\Gamma_{12}^f(s) = \frac{1}{16\pi\sqrt{s}}|g'_{f_0M_1M_2}|^2\rho_{12}(s)\frac{(s-m_1^2-m_2^2)^2}{4}, \quad (14)$$

$$D_{af}(s) = i\frac{g'_{a_0^0K^+K^-}g'_{f_0K^+K^-}}{16\pi}\left\{\frac{[s-2m_{K^+}^2]^2}{4}\rho_{K^+K^-}(s) - \frac{[s-2m_{K^0}^2]^2}{4}\rho_{K^0\bar{K}^0}(s)\right\}. \quad (15)$$

Table 1
Meson masses (in units of MeV) and couplings (in units of GeV) predicted by various models or determined by experimental measurements. The mixing intensity $\xi_{fa}(s)$ (in unit of %) is evaluated at $\sqrt{s} = 991.3$ MeV, which is at the center the K^+K^- and $K^0\bar{K}^0$ threshold. The integrated mixing intensity $\bar{\xi}_{fa}$ (in unit of %) is evaluated by Eq. (25) with the kinematics in Eqs. (26) and (27).

model/experiment	m_{a_0}	$g_{a_0\pi\eta}$	$g_{a_0K^+K^-}$	m_{f_0}	$g_{f_0\pi^0\pi^0}$	$g_{f_0K^+K^-}$	$\xi_{fa}(\%)$	$\bar{\xi}_{fa}(\%)$
$q\bar{q}$ model [43]	983	2.03	1.27	975	0.64	1.80	2.2	0.9
$q^2\bar{q}^2$ model [43]	983	4.57	5.37	975	1.90	5.37	6.5	1.8
$K\bar{K}$ model [44–46]	980	1.74	2.74	980	0.65	2.74	20.1	11.1
SND [47,48]	995	3.11	4.20	969.8	1.84	5.57	8.5	2.6
KLOE [49,50]	984.8	3.02	2.24	973	2.09	5.92	3.2	0.8
BNL [51,52]	1001	2.47	1.67	953.5	1.36	3.26	1.8	0.5
CB [42,53]	999	3.33	2.54	965	1.66	4.18	2.6	0.7

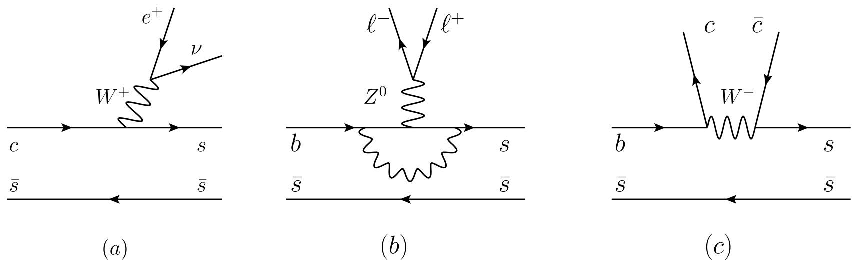

Fig. 2. Feynman diagrams for the D_s and B_s decays into the $f_0(980)$ with the $\bar{s}s$ component at the quark level. The panel (a) denotes the semileptonic D_s decay, in which the lepton pair $e^+\nu$ is emitted. One and typical Feynman diagram for the semileptonic $B_s \to f_0\ell^+\ell^-$ $(\ell = e, \mu, \tau)$ decay are given in panel (b). The last panel (c) corresponds to the nonleptonic B_s decay into the J/ψ.

- It is necessary to stress that the mass-dependent mixing intensity $\xi(s)$ in Eq. (10) can not be directly measured in experiment. On experimental side, the measured observable $\bar{\xi}$ is a ratio of branching fractions as in Eq. (1). But as we will show later, one may derive the partial widths of several heavy B_s/D_s meson decays into the $\pi^0\eta$, $\pi^+\pi^-$ in terms of the propagators as in Eqs. (20) and (21) based on the factorization approach.
- As illustrated in Refs. [17,18], results for $|\xi(s)|^2$ have a peak in the region between the two thresholds for the charged and neutral kaon systems. The peak value is in the range of 0.01 to 0.2. Adopting the effective Hamiltonian in Eq. (4) and with the meson masses (in units of MeV) taken from Particle Data Group [1]

$$m_{K^+} = 493.677, \quad m_{K^0} = 497.614,$$
$$m_{\pi^0} = 134.9766, \quad m_\eta = 547.862, \tag{16}$$

  we update the predictions for the mixing intensity $\xi_{fa}(s)$ at $\sqrt{s} = 991.3$ MeV in Table 1. In the calculation, the isospin symmetry has been used for the $\pi^+\pi^-$ system.
- Different predictions by various models indicate that the f_0–a_0 mixing depends on the nature of the scalars. As one can see from Table 1, the $K\bar{K}$ molecule gives the largest mixing followed by the four quark picture. However, one should keep in mind that the absolute value for the mixing from each model is quite model-dependent and suffers sizable uncertainty, which may make it difficult to discriminate between various pictures.

Despite the above ambiguities and uncertainties, a reliable measurement of the mixing will be very useful to constrain model parameters and ultimately understand the nature of these scalars. Present available experimental measurements on the coupling constants of $g_{a_0^0K^+K^-}$, $g_{f_0K^+K^-}$ and $g_{a_0^0\pi^0\eta}$ cannot give reliable determination of the a_0–f_0 and hence cannot give much constraint on theoretical models. Direct precise measurements of the mixing parameter are needed to provide a useful test of these model predictions and also cross-checks of previous measurements.

3. Mixing effects in the B_s and D_s decays

In this section, we will analyze the mixing intensity in the semileptonic decays of B_s and D_s mesons. More explicitly, the considered decay processes include

$$D_s \to \pi^0\eta e^+\nu, \quad D_s \to \pi^+\pi^- e^+\nu,$$
$$B_s \to \pi^0\eta\ell^+\ell^-, \quad B_s \to \pi^+\pi^-\ell^+\ell^-,$$
$$B_s \to \pi^0\eta J/\psi, \quad B_s \to \pi^+\pi^- J/\psi. \tag{17}$$

We will take the D_s decay as the example, whose Feynman diagram is shown in the panel (a) of Fig. 2. After emitting the off-shell W-boson, the hadronic sector is the $\bar{s}s$ which will couple to the iso-singlet component $f_0(980)$. Then the decay amplitudes for the $D_s \to \pi^+\pi^- e^+\nu \equiv D_s \to f_0 e^+\nu \to \pi^+\pi^- e^+\nu$ and $D_s \to \pi^0\eta e^+\nu \equiv D_s \to f_0 e^+\nu \to a_0^0 e^+\nu \to \pi^0\eta e^+\nu$ are given as

$$\mathcal{A}(D_s \to \pi^+\pi^- e^+\nu) = \hat{A}\left\{\frac{i}{D_{f_0}} \times i g_{f_0\pi^+\pi^-}\right\},$$
$$\mathcal{A}(D_s \to \pi^0\eta e^+\nu) = \hat{A}\left\{\frac{i}{D_{f_0}D_a} D_{fa} \times i g_{a_0\pi\eta}\right\}, \tag{18}$$

where the amplitude $\hat{A}$ can be expressed in terms of the transition form factors:

$$\langle f_0(p_{f_0})|\bar{s}\gamma_\mu\gamma_5 c|D_s(p_{D_s})\rangle = -i\Big\{F_1(q^2)\Big[P_\mu - \frac{m_{D_s}^2 - m_{f_0}^2}{q^2}q_\mu\Big] + F_0(q^2)\frac{m_{D_s}^2 - m_{f_0}^2}{q^2}q_\mu\Big\}. \tag{19}$$

The double differential decay width is then derived as

$$\frac{d\Gamma(D_s \to \pi^+\pi^- e^+\nu)}{ds dq^2} = \frac{\lambda^{3/2}G_F^2|V_{cs}|^2}{192 m_{D_s}^3\pi^3}F_1^2(q^2).\frac{\sqrt{s}}{\pi|D_f(s)|^2}\Gamma^f_{\pi^+\pi^-}(s), \tag{20}$$

$$\frac{d\Gamma(D_s\to\pi^0\eta e^+\nu)}{dsdq^2} = \frac{\lambda^{3/2}G_F^2|V_{cs}|^2}{192m_{D_s}^3\pi^3}F_1^2(q^2).\frac{\sqrt{s}|D_{af}(s)|^2}{\pi|D_f(s)D_a(s)|^2}\Gamma^a_{\pi\eta}(s), \tag{21}$$

where q^2 is the invariant mass of the lepton pair, and the s is the invariant mass square of the two pseudo-scalars. Here G_F is the Fermi constant, V_{cs} is the CKM matrix element, and the Källen function λ is: $\lambda = m_{D_s}^4 + s^2 + (q^2)^2 - 2(m_{D_s}^2 q^2 + m_{D_s}^2 s + sq^2)$.

Since in this work we are interested in the mixing intensity in the $a_0(980)$–$f_0(980)$ resonance region, one may integrate out the q^2 first, leading to

$$\frac{d\Gamma(D_s\to\pi^+\pi^-e^+\nu)}{ds} = C\frac{\sqrt{s}}{\pi|D_f(s)|^2}\Gamma^f_{\pi^+\pi^-}(s), \tag{22}$$

$$\frac{d\Gamma(D_s\to\pi^0\eta e^+\nu)}{ds} = C\frac{\sqrt{s}|D_{fa}(s)|^2}{\pi|D_f(s)D_a(s)|^2}\Gamma^a_{\pi\eta}(s), \tag{23}$$

where the coefficient C is obtained via the integration over q^2. The mass-dependent mixing intensity can be defined as

$$\xi^{D_s}_{fa}(s) \equiv \frac{d\Gamma(D_s\to\pi^0\eta e^+\nu)/ds}{d\Gamma(D_s\to\pi^+\pi^-e^+\nu)/ds} = \frac{|D_{af}(s)|^2\Gamma^a_{\pi\eta}(s)}{|D_a(s)|^2\Gamma^f_{\pi^+\pi^-}(s)}, \tag{24}$$

while in experiments one can directly measure the integrated mixing intensity:

$$\overline{\xi}^{D_s}_{fa} \equiv \frac{\Gamma(D_s\to\pi^0\eta e^+\nu)}{\Gamma(D_s\to\pi^+\pi^-e^+\nu)} \equiv \frac{\int_{s'_{min}}^{s'_{max}} ds d\Gamma(D_s\to\pi^0\eta e^+\nu)/ds}{\int_{s_{min}}^{s_{max}} ds d\Gamma(D_s\to\pi^+\pi^-e^+\nu)/ds} = \int_{s'_{min}}^{s'_{max}} ds\frac{\sqrt{s}|D_{fa}(s)|^2}{|D_f(s)D_a(s)|^2}\Gamma^a_{\pi\eta}(s)\Big/\int_{s_{min}}^{s_{max}} ds\frac{\sqrt{s}}{|D_f(s)|^2}\Gamma^f_{\pi^+\pi^-}(s). \tag{25}$$

Here the $s^{(\prime)}_{min}$ and $s^{(\prime)}_{max}$ denote the lower and upper invariant mass cuts. In the previous BES-III analysis of the mixing intensity using the J/ψ decays [26], the mass of the mixing signal is set to 991.3 MeV at the center of charged and neutral kaon thresholds, and the width of the mixing signal is set to 8 MeV. It corresponds to

$$s'_{min} = [(991.3-4)\ \text{MeV}]^2, \quad s'_{max} = [(991.3+4)\ \text{MeV}]^2. \tag{26}$$

For the $f_0(980)$, one may follow the BES-III analysis of the $J/\psi\to\phi\pi^+\pi^-$ [42]:

$$s_{min} = [900\ \text{MeV}]^2, \quad s_{max} = [1000\ \text{MeV}]^2. \tag{27}$$

Based on various models and measurements, we show the results for the integrated quantity $\overline{\xi}_{fa}$ with the kinematics in Eqs. (26) and (27) in Table 1. From this table, one can observe the result is at the order $\mathcal{O}(1\%)$.

On experimental side, the CLEO collaboration has firstly measured the branching fraction [30]:

$$\mathcal{B}(D_s\to f_0(980)(\to\pi^+\pi^-)e^+\nu_e) = (2.0\pm0.3\pm0.1)\times10^{-3}, \tag{28}$$

but a recent analysis based on the CLEO-c data gives a similar result with a smaller central value [31]:

$$\mathcal{B}(D_s\to f_0(980)(\to\pi^+\pi^-)e^+\nu_e) = (1.3\pm0.2\pm0.1)\times10^{-3}. \tag{29}$$

In near future the BES-III collaboration will collect about 3 fb^{-1} data in e^+e^- collision at the energy around 4.18 GeV [54]. This corresponds to a few times 10^6 events of the D_s mesons and accordingly a few thousand events for the $D_s\to\pi^+\pi^-e^+\nu$ decay before any kinematics cut. As we can see if the mixing intensity is at the percent level, there is a promising prospect to measure/constrain the mixing by BES-III collaboration using the $D_s\to[\pi^0\eta,\pi^+\pi^-]e^+\nu$.

The analysis of the $B_s\to[\pi^0\eta,\pi^+\pi^-]\ell^+\ell^-$ and $B_s\to[\pi^0\eta,\pi^+\pi^-]J/\psi$ (with $\ell=e,\mu,\tau$) is also similar. For instance in the semileptonic decay, one can study the mass-dependent and integrated mixing intensity which is defined as

$$\xi^{B_s}_{fa}(s) \equiv \frac{d\Gamma(B_s\to\pi^0\eta\ell^+\ell^-)/ds}{d\Gamma(B_s\to\pi^+\pi^-\ell^+\ell^-)/ds} = \frac{|D_{af}(s)|^2\Gamma^a_{\pi\eta}(s)}{|D_a(s)|^2\Gamma^f_{\pi^+\pi^-}(s)}, \tag{30}$$

$$\overline{\xi}^{B_s}_{fa} \equiv \frac{\Gamma(B_s\to\pi^0\eta\ell^+\ell^-)}{\Gamma(B_s\to\pi^+\pi^-\ell^+\ell^-)} = \int_{s'_{min}}^{s'_{max}} ds\frac{\sqrt{s}|D_{fa}(s)|^2}{|D_f(s)D_a(s)|^2}\Gamma^a_{\pi\eta}(s)\Big/\int_{s_{min}}^{s_{max}} ds\frac{\sqrt{s}}{|D_f(s)|^2}\Gamma^f_{\pi^+\pi^-}(s). \tag{31}$$

For the rare decay $B_s\to f_0(\to\pi^+\pi^-)\mu^+\mu^-$, the LHCb collaboration has performed a detailed analysis with the result [32]:

$$\mathcal{B}(B_s\to f_0(980)(\to\pi^+\pi^-)\mu^+\mu^-) = (8.3\pm1.7)\times10^{-8}. \tag{32}$$

This has already triggered some theoretical interpretations using two-meson light-cone distribution amplitudes (LCDAs) [55,56]. The LHCb collaboration has also systematically studied the $B_s\to J/\psi\pi^+\pi^-$ decays [33–38], and some implications on the structure of scalar mesons have been explored in Refs. [57–59]. The averaged branching fraction is given as [1]

$$\mathcal{B}(B_s\to J/\psi f_0(980)(\to\pi^+\pi^-)) = (1.35\pm0.16)\times10^{-4}. \tag{33}$$

Since much more data will be collected by experimental facilities including the LHCb detector [60] the Super-B factory at the KEK [61], it is likely to precisely derive the $a_0(980)$ and $f_0(980)$ mixing from these weak decays of heavy mesons.

Apart from the a_0–f_0 mixing, it is necessary to discuss other mechanisms for the B_s/D_s decays into the $\pi^0\eta$ in (17), and a few of them are given as follows.

- A potential source may arise form the isospin violating effects from the K^0. Taking the D_s decay as the example, one has the branching fraction [1]:

$$\mathcal{B}(D_s\to K^0e^+\nu_e) = (3.7\pm1.0)\times10^{-3}, \tag{34}$$

which is comparable to the branching ratio of the $D_s\to f_0e^+\nu$. However since the K^0 has a very tiny width and the $a_0(980)$ lies very far from the K^0, its contribution to $\pi^+\pi^-$ or $\pi^0\eta$ at the $a_0(980)$ mass is negligibly small.

- Rescattering effects from intermediate states like $K\bar{K}^*$ have been explored in the J/ψ decays into the $\phi\pi^0\eta$ in Ref. [17], whose contributions are found comparable with the a_0–f_0 mixing. As an estimate, we may also expect that rescattering effects from channels like $D_s \to K\bar{K}^*/(K\bar{K})e^+\nu \to \pi^0\eta e^+\nu$ are at the same level with the $D_s \to f_0 e^+\nu \to a_0^0 e^+\nu \to \pi^0\eta e^+\nu$. One notable feature for the a_0–f_0 mixing contribution is that it has a narrow peak at the a_0 mass region. In the previous BES-III analysis [26], the mass of the mixing signal is set to 991.3 MeV, while the width of the mixing signal is set to 8 MeV. However the rescattering contributions from $D_s \to K\bar{K}^*/(K\bar{K})e^+\nu \to \pi^0\eta e^+\nu$ spread in a relatively large mass region, and accordingly its contribution in this kinematics window should be very small. Therefore by separating the narrow peak from the rescattering contributions, one may obtain a very precise measurement of the a_0–f_0 mixing.
- In the D_s/B_s decays, the $f_0(980)$ can decay into the $\pi^0\pi^0$ and subsequently one neutral π^0 converts into η. Such contribution $D_s \to f_0 e^+\nu \to \pi^0\pi^0 e^+\nu \to \pi^0\eta e^+\nu$ depends on the π^0–η mixing. Since the $\pi^0\eta$ is from the $f_0(980)$ resonance, it is difficult to distinguish from the a_0–f_0 mixing contribution. If the π^0–η mixing intensity were comparable with the a_0–f_0 mixing, one should simultaneously consider both contributions in reactions including the J/ψ and the B_s/D_s decays. This will be left for a future publication. Nevertheless, even in this case, the heavy B_s/D_s decays are certainly of great value towards a precise determination of the mixing.

4. Summary

To understand the internal structure of light scalar mesons is a long-standing problem in hadron physics. It is expected that some aspects can be unraveled by the study of $a_0^0(980)$–$f_0(980)$ mixing. The two scalar mesons can couple to the K–$\bar{K}$ and will mix with each other due to the different masses for the charged and neutral kaons. The mixing intensity has been predicted at the percent level in various theoretical models. A number of processes have been proposed to study the mixing, but to date there is no firm evidence on the experimental side.

In this work we have proposed to use the weak decays of the B_s and D_s mesons to study the a_0–f_0 mixing. We have studied the semileptonic decays of heavy mesons, $D_s \to [\pi^0\eta, \pi^+\pi^-]e^+\nu$, $B_s \to [\pi^0\eta, \pi^+\pi^-]\ell^+\ell^-$ and the $B_s \to J/\psi[\pi^0\eta, \pi^+\pi^-]$ decays. Based on the large amount of data accumulated by various experimental facilities including BEPC-II, Super KEKB, LHC and the future colliders like the High Intensity Electron Positron Accelerator (HIEPA) expected running at 2–7 GeV with the designed luminosity of 10^{35} cm^{-2} s^{-1}, the Z-factory running at Z-pole and the circular electron–positron collider (CEPC), it is very likely that the a_0–f_0 mixing intensity can be determined to a high precision, which will lead to a better understanding of the nature of scalar mesons.

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References

[1] K.A. Olive, et al., Particle Data Group Collaboration, Chin. Phys. C 38 (2014) 090001, http://dx.doi.org/10.1088/1674-1137/38/9/090001.

[2] N.N. Achasov, S.A. Devyanin, G.N. Shestakov, Phys. Lett. B 88 (1979) 367, http://dx.doi.org/10.1016/0370-2693(79)90488-X.

[3] N.N. Achasov, S.A. Devyanin, G.N. Shestakov, Yad. Fiz. 33 (1981) 1337, Sov. J. Nucl. Phys. 33 (1981) 715.

[4] N.N. Achasov, G.N. Shestakov, Phys. Rev. D 56 (1997) 212, http://dx.doi.org/10.1103/PhysRevD.56.212, arXiv:hep-ph/9610409.

[5] O. Krehl, R. Rapp, J. Speth, Phys. Lett. B 390 (1997) 23, http://dx.doi.org/10.1016/S0370-2693(96)01425-6, arXiv:nucl-th/9609013.

[6] B. Kerbikov, F. Tabakin, Phys. Rev. C 62 (2000) 064601, http://dx.doi.org/10.1103/PhysRevC.62.064601, arXiv:nucl-th/0006017.

[7] F.E. Close, A. Kirk, Phys. Lett. B 489 (2000) 24, http://dx.doi.org/10.1016/S0370-2693(00)00951-5, arXiv:hep-ph/0008066.

[8] A.E. Kudryavtsev, V.E. Tarasov, JETP Lett. 72 (2000) 410, Pisma Zh. Eksp. Teor. Fiz. 72 (2000) 589, http://dx.doi.org/10.1134/1.1335118, arXiv:nucl-th/0102053.

[9] V.Y. Grishina, L.A. Kondratyuk, M. Buescher, W. Cassing, H. Stroher, Phys. Lett. B 521 (2001) 217, http://dx.doi.org/10.1016/S0370-2693(01)01210-2, arXiv:nucl-th/0103081.

[10] F.E. Close, A. Kirk, Phys. Lett. B 515 (2001) 13, http://dx.doi.org/10.1016/S0370-2693(01)00799-7, arXiv:hep-ph/0106108.

[11] A.E. Kudryavtsev, V.E. Tarasov, J. Haidenbauer, C. Hanhart, J. Speth, Phys. Rev. C 66 (2002) 015207, http://dx.doi.org/10.1103/PhysRevC.66.015207, arXiv:nucl-th/0203034.

[12] L.A. Kondratyuk, E.L. Bratkovskaya, V.Y. Grishina, M. Buescher, W. Cassing, H. Stroher, Phys. At. Nucl. 66 (2003) 152, Yad. Fiz. 66 (2003) 155, http://dx.doi.org/10.1134/1.1540670, arXiv:nucl-th/0207033.

[13] N.N. Achasov, A.V. Kiselev, Phys. Lett. B 534 (2002) 83, http://dx.doi.org/10.1016/S0370-2693(02)01696-9, arXiv:hep-ph/0203042.

[14] N.N. Achasov, G.N. Shestakov, Phys. Rev. Lett. 92 (2004) 182001, http://dx.doi.org/10.1103/PhysRevLett.92.182001, arXiv:hep-ph/0312214.

[15] V.Y. Grishina, L.A. Kondratyuk, M. Buescher, W. Cassing, Eur. Phys. J. A 21 (2004) 507, http://dx.doi.org/10.1140/epja/i2004-10004-2, arXiv:nucl-th/0402093.

[16] N.N. Achasov, G.N. Shestakov, Phys. Rev. D 70 (2004) 074015, http://dx.doi.org/10.1103/PhysRevD.70.074015, arXiv:hep-ph/0405129.

[17] J.J. Wu, Q. Zhao, B.S. Zou, Phys. Rev. D 75 (2007) 114012, http://dx.doi.org/10.1103/PhysRevD.75.114012, arXiv:0704.3652 [hep-ph].

[18] J.J. Wu, B.S. Zou, Phys. Rev. D 78 (2008) 074017, http://dx.doi.org/10.1103/PhysRevD.78.074017, arXiv:0808.2683 [hep-ph].

[19] C. Hanhart, B. Kubis, J.R. Pelaez, Phys. Rev. D 76 (2007) 074028, http://dx.doi.org/10.1103/PhysRevD.76.074028, arXiv:0707.0262 [hep-ph].

[20] J.J. Wu, X.H. Liu, Q. Zhao, B.S. Zou, Phys. Rev. Lett. 108 (2012) 081803, http://dx.doi.org/10.1103/PhysRevLett.108.081803, arXiv:1108.3772 [hep-ph].

[21] F. Aceti, W.H. Liang, E. Oset, J.J. Wu, B.S. Zou, Phys. Rev. D 86 (2012) 114007, http://dx.doi.org/10.1103/PhysRevD.86.114007, arXiv:1209.6507 [hep-ph].

[22] L. Roca, Phys. Rev. D 88 (2013) 014045, http://dx.doi.org/10.1103/PhysRevD.88.014045, arXiv:1210.4742 [hep-ph].

[23] V.E. Tarasov, W.J. Briscoe, W. Gradl, A.E. Kudryavtsev, I.I. Strakovsky, Phys. Rev. C 88 (2013) 035207, http://dx.doi.org/10.1103/PhysRevC.88.035207, arXiv:1306.6618 [hep-ph].

[24] T. Sekihara, S. Kumano, Phys. Rev. D 92 (3) (2015) 034010, http://dx.doi.org/10.1103/PhysRevD.92.034010, arXiv:1409.2213 [hep-ph].

[25] F. Aceti, J.M. Dias, E. Oset, Eur. Phys. J. A 51 (4) (2015) 48, http://dx.doi.org/10.1140/epja/i2015-15048-5, arXiv:1501.06505 [hep-ph].

[26] M. Ablikim, et al., BESIII Collaboration, Phys. Rev. D 83 (2011) 032003, http://dx.doi.org/10.1103/PhysRevD.83.032003, arXiv:1012.5131 [hep-ex].

[27] W. Wang, Int. J. Mod. Phys. A 29 (2014) 1430040, http://dx.doi.org/10.1142/S0217751X14300403, arXiv:1407.6868 [hep-ph].

[28] E. Oset, et al., arXiv:1601.03972 [hep-ph].

[29] J. Yelton, et al., CLEO Collaboration, Phys. Rev. D 80 (2009) 052007, http://dx.doi.org/10.1103/PhysRevD.80.052007, arXiv:0903.0601 [hep-ex].

[30] K.M. Ecklund, et al., CLEO Collaboration, Phys. Rev. D 80 (2009) 052009, http://dx.doi.org/10.1103/PhysRevD.80.052009, arXiv:0907.3201 [hep-ex].

[31] J. Hietala, D. Cronin-Hennessy, T. Pedlar, I. Shipsey, Phys. Rev. D 92 (1) (2015) 012009, http://dx.doi.org/10.1103/PhysRevD.92.012009, arXiv:1505.04205 [hep-ex].

[32] R. Aaij, et al., LHCb Collaboration, Phys. Lett. B 743 (2015) 46, http://dx.doi.org/10.1016/j.physletb.2015.02.010, arXiv:1412.6433 [hep-ex].

[33] R. Aaij, et al., LHCb Collaboration, Phys. Lett. B 698 (2011) 115, http://dx.doi.org/10.1016/j.physletb.2011.03.006, arXiv:1102.0206 [hep-ex].

[34] R. Aaij, et al., LHCb Collaboration, Phys. Lett. B 707 (2012) 497, http://dx.doi.org/10.1016/j.physletb.2012.01.017, arXiv:1112.3056 [hep-ex].

[35] R. Aaij, et al., LHCb Collaboration, Phys. Rev. D 86 (2012) 052006, http://dx.doi.org/10.1103/PhysRevD.86.052006, arXiv:1204.5643 [hep-ex].

[36] R. Aaij, et al., LHCb Collaboration, Phys. Rev. D 87 (5) (2013) 052001, http://dx.doi.org/10.1103/PhysRevD.87.052001, arXiv:1301.5347 [hep-ex].

[37] R. Aaij, et al., LHCb Collaboration, Phys. Rev. D 89 (9) (2014) 092006, http://dx.doi.org/10.1103/PhysRevD.89.092006, arXiv:1402.6248 [hep-ex].
[38] R. Aaij, et al., LHCb Collaboration, Phys. Rev. D 90 (1) (2014) 012003, http://dx.doi.org/10.1103/PhysRevD.90.012003, arXiv:1404.5673 [hep-ex].
[39] J. Li, et al., Belle Collaboration, Phys. Rev. Lett. 106 (2011) 121802, http://dx.doi.org/10.1103/PhysRevLett.106.121802, arXiv:1102.2759 [hep-ex].
[40] T. Aaltonen, et al., CDF Collaboration, Phys. Rev. D 84 (2011) 052012, http://dx.doi.org/10.1103/PhysRevD.84.052012, arXiv:1106.3682 [hep-ex].
[41] V.M. Abazov, et al., D0 Collaboration, Phys. Rev. D 85 (2012) 011103, http://dx.doi.org/10.1103/PhysRevD.85.011103, arXiv:1110.4272 [hep-ex].
[42] M. Ablikim, et al., BES Collaboration, Phys. Lett. B 607 (2005) 243, http://dx.doi.org/10.1016/j.physletb.2004.12.041, arXiv:hep-ex/0411001.
[43] N.N. Achasov, V.N. Ivanchenko, Nucl. Phys. B 315 (1989) 465, http://dx.doi.org/10.1016/0550-3213(89)90364-7.
[44] N.N. Achasov, V.V. Gubin, Phys. Rev. D 56 (1997) 4084, http://dx.doi.org/10.1103/PhysRevD.56.4084, arXiv:hep-ph/9703367.
[45] J.D. Weinstein, N. Isgur, Phys. Rev. D 27 (1983) 588, http://dx.doi.org/10.1103/PhysRevD.27.588.
[46] J.D. Weinstein, N. Isgur, Phys. Rev. D 41 (1990) 2236, http://dx.doi.org/10.1103/PhysRevD.41.2236.
[47] M.N. Achasov, et al., Phys. Lett. B 485 (2000) 349, http://dx.doi.org/10.1016/S0370-2693(00)00705-X, arXiv:hep-ex/0005017.
[48] M.N. Achasov, et al., Phys. Lett. B 479 (2000) 53, http://dx.doi.org/10.1016/S0370-2693(00)00334-8, arXiv:hep-ex/0003031.
[49] A. Aloisio, et al., KLOE Collaboration, Phys. Lett. B 536 (2002) 209, http://dx.doi.org/10.1016/S0370-2693(02)01821-X, arXiv:hep-ex/0204012.
[50] A. Aloisio, et al., KLOE Collaboration, Phys. Lett. B 537 (2002) 21, http://dx.doi.org/10.1016/S0370-2693(02)01838-5, arXiv:hep-ex/0204013.
[51] S. Teige, et al., E852 Collaboration, Phys. Rev. D 59 (1999) 012001, http://dx.doi.org/10.1103/PhysRevD.59.012001, arXiv:hep-ex/9608017.
[52] B.S. Zou, D.V. Bugg, Phys. Rev. D 48 (1993) 3948, http://dx.doi.org/10.1103/PhysRevD.48.R3948.
[53] D.V. Bugg, V.V. Anisovich, A. Sarantsev, B.S. Zou, Phys. Rev. D 50 (1994) 4412, http://dx.doi.org/10.1103/PhysRevD.50.4412.
[54] D.M. Asner, et al., Int. J. Mod. Phys. A 24 (2009) S1, arXiv:0809.1869 [hep-ex].
[55] W.F. Wang, H.n. Li, W. Wang, C.D. Lü, Phys. Rev. D 91 (9) (2015) 094024, http://dx.doi.org/10.1103/PhysRevD.91.094024, arXiv:1502.05483 [hep-ph].
[56] W. Wang, R.L. Zhu, Phys. Lett. B 743 (2015) 467, http://dx.doi.org/10.1016/j.physletb.2015.03.011, arXiv:1502.05104 [hep-ph].
[57] M. Bayar, W.H. Liang, E. Oset, Phys. Rev. D 90 (11) (2014) 114004, http://dx.doi.org/10.1103/PhysRevD.90.114004, arXiv:1408.6920 [hep-ph].
[58] F.E. Close, A. Kirk, Phys. Rev. D 91 (11) (2015) 114015, http://dx.doi.org/10.1103/PhysRevD.91.114015, arXiv:1503.06942 [hep-ex].
[59] T. Sekihara, E. Oset, Phys. Rev. D 92 (5) (2015) 054038, http://dx.doi.org/10.1103/PhysRevD.92.054038, arXiv:1507.02026 [hep-ph].
[60] R. Aaij, et al., LHCb Collaboration, Eur. Phys. J. C 73 (4) (2013) 2373, http://dx.doi.org/10.1140/epjc/s10052-013-2373-2, arXiv:1208.3355 [hep-ex].
[61] T. Aushev, et al., arXiv:1002.5012 [hep-ex].