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Superconducting Cosmic Strings With Massive Fermions

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Abstract

Fermions which couple to the fields comprising a Nielsen–Olesen vortex (cosmic string) can generally exist as *massive* bound modes on the string. If these fermions carry electromagnetic charge then the string will act as a conducting wire with capacitance. The interaction of such cosmic strings with external electromagnetic fields is significantly altered from the superconducting string scenario of Witten. The current in the string oscillates at a characteristic resonant frequency, $\sim \mu$, and therefore produces a signature of narrow band radiation. The current is generally limited, $\sim e^2 E/\mu$, for strings of fixed length.

I. Introduction

Spontaneous symmetry breaking of gauge theories can give rise to objects such as solitons, vortices, and monopoles. These objects are typically non-trivial topological configurations of gauge and scalar fields. Cosmic strings are presumeably Nielsen–Olesen flux tubes which arise when the symmetry breaking has a nontrivial first homotopy group, $\Pi_1(G/H)$, [1]. Fermions can couple to these fields and, in particular, if fermions obtain a mass by coupling to the complex scalar field of an Abelian vortex, massless states localized on the vortex, known as Jackiw–Rossi zero-modes, can exist [2].

Cosmic strings have received much attention in the astrophysics community as it is believed that they may serve as the seeds for galaxy formation. Until recently, cosmic strings were thought to be important only through gravitational effects. However, Witten [3] has demonstrated that if there are fermionic zero-modes on the string which carry electric charge then the string can act as superconducting wire. The simplest model which exhibits this phenomenon is a $U(1) \times U(1)'$ gauge theory with $U(1)'$ broken to form the flux tube and the other $U(1)$ representing electromagnetism. Currents in these strings can be spectacular and offer a number of interesting possibilities for detection and for galaxy formation [4] (we shall not discuss the bosonic case in this paper). Presently, however, we will offer a slight variation on this simplest model, which we feel is a more general possibility, yet it will drastically alter the superconducting string scenarios and signatures.

The superconductivity of a flux tube is a consequence of the existence of the anomaly in an ungauged current (the gauged currents must be anomaly free, as usual [3]). In $1+3$ dimensions there occurs an anomaly involving a mixture of the field strength, $F_{\mu\nu}$, of the $U(1)$ group and the dual field strength, $\tilde{F}'_{\mu\nu}$, of the $U(1)'$ group of the form:

$$\partial^\mu j_\mu = \frac{e'e}{4\pi^2} F^{\mu\nu} \tilde{F}'_{\mu\nu} + \text{mass terms} = \frac{e'e}{\pi^2} \vec{E} \cdot \vec{B}' + \text{mass terms}. \tag{1.1}$$

In the simplest $U(1) \times U(1)'$ model j_μ is a vector current; the theory conserves parity (though in the vortex one can define a parity which is violated); each Weyl spinor contributes strength $ee'/32\pi^2$ to the usual anomaly; there are four Weyl spinors and we get an additional $2\times$ from the interchange of $F_{\mu\nu} \leftrightarrow F'_{\mu\nu}$.

In the vortex there is a B' field of the form $\sim \hat{z}2n/e'\pi r_c{}^2$; $r < r_c$ where n is the *vorticity* of the flux tube and r_c the core radius. The low energy fermionic zero modes in the vortex may be described by an effective $1+1$ dimensional chiral Lagrangian on the worldsheet of the flux tube, obtained essentially by integrating the $1+3$ Lagrangian in these modes over the cross-sectional area of the string. Defining an *axial* current in this effective theory as $\tilde{j}^a_\mu = \int dA\ j_\mu \approx \pi r_c{}^2 j_\mu$ we find from eq.(1.1) that in the flux tube $\tilde{j}^a_\mu$ has an anomaly of the form:

$$\partial^\mu \tilde{j}^a_\mu = \frac{ne}{\pi}\epsilon_{\mu\nu}F^{\mu\nu} + \text{mass terms} = \frac{2ne}{\pi}E + \text{mass terms}. \tag{1.2}$$

Here the contribution of a pure left–mover to the anomaly would be $e\tilde{F}/4\pi = eE/2\pi$. The axial current in $1+1$ arises from a vector current in $1+3$ because the coupling of the Higgs in the vortex to the fermions is chiral, the vortex spontaneously breaks parity and produces from a Dirac spinor in $1+3$ chiral zero modes in $1+1$. Since the effective low–energy theory must produce this identical result by direct evaluation of a $1+1$ Feynman diagram, we see that there must exist $N = 2n$ low energy modes in the effective theory (corresponding in the present case to n right–movers and n left–movers; see the explicit construction below). This may be viewed as a simple demonstration of the Atiyah–Singer index theorem for the flux tube (a more constructive approach is developed by E. Weinberg [5]). Note that this discussion is somewhat orthogonal to that of Callan and Harvey [6] in that here the anomaly in $1+1$ matches an anomaly in $1+3$, whereas in ref.[6] there is no corresponding anomaly in $1+3$ to the occurence of one in $1+1$. Thus the true electromagnetic current in ref.[6] must be anomaly free and we discover the occurence of additional terms involving the axion field which conduct the current in from the surrounding vacuum and cancel the anomaly.

Of course, the consistency of the anomalies between eq.(1.1) and eq.(1.2) is essentially a UV statement in both the effective $1+1$ theory and the $3+1$ theory. We can add various mass terms to the rhs of eq.(1.1) and eq.(1.2) and the index theorem counts the light modes at any given energy scale. Indeed, the "threshold modes" of Nohl and DeVega [7] have arbitrary masses but still obey the counting demonstrated here. In the present paper we will be concerned with the physical effects of the mass term corrections to eq.(1.2).

The actual physics of the superconductivity follows from eq.(1.2). The axial charge density, j_0^a is dual to the electromagnetic vector current, j_1^v, i.e. $j_\mu^a = \epsilon_{\mu\nu} j_\nu^v$. For a string of length L the axial charge, $Q^a = \int dl j_0^a = L j_1^v$ has eigenvalue N which is the number of right moving zero–modes minus the number of left moving zero–modes on the string (or plus the number of left moving anti–zero–modes on the string). States consisting of $N/2$ right movers and $N/2$ anti–left–movers are electric charge neutral but they correspond to a non–zero electric current flowing in the loop. Also, it is important to realise that only the axial anomaly leads to such states, since the electric current operator makes only pairs of right (left) – moving particles and antiparticles (e.g., the anti–particle of a right–mover is also a right–mover). Since $\epsilon_{\mu\nu} F^{\mu\nu}$ is just an applied electric field tangentially to the string, eq.(1.1) states that after some time, t, an applied electric field E produces in a string of length L an axial charge, $Q^3 = L j_1^v$ given by $Q^3 = (NLe/\pi)Et$ or:

$$j_1^v = \frac{Ne}{\pi} Et. \tag{1.3}$$

Once the electric field is removed the current remains fixed; it cannot relax by electromagnetic annihilation of right and left movers since these are independent (non-conjugate) particles (of course the current can relax through the anomaly of eq.(1.3)). This same argument is inherent in the more intuitive discussion given by Witten in terms of the rearrangement of the Dirac sea of left and right movers in the presence of the applied electric field [3].

In this letter, we consider the possibility that the fermionic modes localized at the string are not exactly zero modes but have a residual non-zero mass. This is fully compatible with the symmetries of the theory with a slight generalization of the original model of Witten, and may easily occur in a wide class of unified models. Indeed, even the minimal model is a special case without these effects which can readily occur there. In the absence of this residual mass there is an unbroken chiral symmetry in the effective $1+1$ theory, but this does not correspond to an unbroken chiral symmetry in the $1+3$ parent theory. Indeed, the breaking of the chiral symmetry in the $1+1$ theory can arise from ordinary KM flavor mixing–like effects in the parent theory.

The residual mass term acts like a *capacitance* in the equation for the electric current along the string and the resulting physics is dramatically different than that arising in the pure superconductivity case, even for extremely tiny residual masses (e.g for typical current epoch astrophysical parameters we find that no more than one Ampere of current occurs for $\mu > 10^{-16}$ ev). (This is actually an analogue to an electrical capacitance and is an "axial capacitance" in the $1+1$ model; due to the violation of parity the capacitance is a tensor).

We first present the simplest model Lagrangian which gives rise to massive charged modes trapped on a string. The effective theory of 1+1 dimensional massive fermions coupled to 3+1 dimensional electromagnetic fields is studied using the bosonized equations of motion. We then discuss how currents are generated in a constant electric field and how existing currents decay in the absence of external fields. Finally, we sketch the relevance of this model for cosmology.

II. The Model

The cosmic string is a non-trivial axially symmetric configuration of a complex scalar field ϕ and a gauge field A. Throughout this work, these fields will be taken as fixed, c-number fields of the form:

$$\phi(r) = e^{in\theta} f(r) \qquad A_i = \epsilon^{ijz}\frac{x_j}{r}A(r) \tag{2.1}$$

Here, (r, θ, z) are cylindrical coordinates with z along the axis of the string, $(x_1, x_2) = (r\cos\theta, r\sin\theta)$. For definiteness, we will take $n > 0$. The asymptotic forms of the fields are

$$f(r) \to f' r^{|n|} \qquad A(r) \to 0 \qquad \text{for} \quad r \to 0 \tag{2.2}$$

$$f(r) \to f_o \qquad A(r) \to -\frac{n}{qr} \qquad \text{for} \quad r \to \infty. \tag{2.3}$$

Witten considers a $U(1) \times U(1)'$ gauge theory where the first $U(1)$ factor is the unbroken gauge symmetry Q of electromagnetism and the second $U(1)'$ is the unbroken gauge symmetry R which gives rise to the string. The Lagrangian for the fermions in this theory is taken to be:

$$\begin{aligned} L &= i\psi^\dagger \not{D}\psi + i\chi^\dagger \not{D}\chi + i\lambda^\dagger \not{D}\lambda + i\delta^\dagger \not{D}\delta \\ &\quad + ig\phi(r,\theta)\psi^T\epsilon\chi - ig\phi^*(r,\theta)\lambda^T\epsilon\delta \end{aligned} \tag{2.4}$$

ψ, χ, λ and δ are 2-component (left–handed Weyl) spinors, $\not{D} = \sigma^\mu D_\mu$ where σ^μ are the unit and Pauli matrices and $D_\mu = \partial_\mu + ieA_\mu + iqA'_\mu$ where e (q) is the $U(1)$ $(U(1)')$ charge of the fermion. Spinor indices have been suppressed (we are using Van der Waerden conventions). The charge assignments are summarized in Table I.

Table I. Model charge assignments:

field	e	q
ϕ	0	2
ψ	1	-1
χ	-1	-1
λ	1	1
δ	-1	1

Far from the flux tube the Higgs field may be regarded as having a constant VEV. Then we see that the pairs of Weyl spinors, $(\psi,\ \chi)$ and $(\lambda,\ \delta)$ form two four–component massive Dirac fields. If we consider one such field by itself, say the pair $(\psi,\ \chi)$, we see that it has a gauged electric current with charge e and a gauged axial current of charge q. Such a situation is not allowed by itself because of the axial vector anomaly, so we must introduce the pair $(\lambda,\ \delta)$ with gauged axial charge $-q$ to cancel the anomaly. Thus the Lagrangian of eq.(2.4) is the minimal anomaly free model we can consider.

There remain a number of currents which are not coupled to gauge fields but which do possess anomalies. For example, there is the axial current:

$$j_\mu = \psi^\dagger \sigma_\mu \psi + \chi^\dagger \sigma_\mu \chi + \lambda^\dagger \sigma_\mu \lambda + \delta^\dagger \sigma_\mu \delta \tag{2.5}$$

with the usual axial vector anomaly:

$$\partial^\mu j_\mu = \frac{e^2}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{q^2}{8\pi^2} F'_{\mu\nu} \tilde{F}'^{\mu\nu}. \tag{2.6}$$

The interesting current for the present discussion, however, is the vector current (which may be viewed as the difference between the electric currents of the two corresponding Dirac spinors):

$$j_\mu = \psi^\dagger \sigma_\mu \psi - \chi^\dagger \sigma_\mu \chi - \lambda^\dagger \sigma_\mu \lambda + \delta^\dagger \sigma_\mu \delta \tag{2.7}$$

It is readily verified that j_μ has a mixed anomaly of the form discussed in the introduction:

$$\partial^\mu j_\mu = \frac{eq}{4\pi^2} F_{\mu\nu} \tilde{F}'^{\mu\nu}. \tag{2.8}$$

Also, it is readily verified that the matrix element of j_0 in a state consisting of $N/2$ right–moving zero–modes and $N/2$ left–moving anti–zero–modes is N/L and is equivalent to the matrix element of the z component of the electric current in such a state. This is because the pair $(\psi,\ \chi)$ produces chiral left–moving zero modes and the pair $(\lambda,\ \delta)$ produces chiral right–moving zero modes, as we now show.

We may demonstrate the existence of the zero–modes by solving the equations of motion for the spinor fields in the vortex. Consider the fields ψ and χ for which the equations of motion,

$$\sigma^\mu D_\mu \psi + g\phi^* \epsilon \chi^* = 0 \qquad \sigma^\mu D_\mu \chi + g\phi^* \epsilon \psi^* = 0 \tag{2.9}$$

are solved by the separation of variables $\psi = \alpha(z,t)\beta(r,\theta)s^+,\ \chi = \alpha^*(z,t)\tilde{\beta}(r,\theta)s^+$ where s^+ is the constant spinor $\begin{pmatrix}1\\0\end{pmatrix}$. One has:

$$(\partial_t - \partial_z)\alpha = 0 \tag{2.10}$$

$$e^{-i\theta}\left(\partial_r - \frac{i}{r}\partial_\theta + qA'\right)\beta - ge^{-in\theta} f(r)\tilde{\beta}^* = 0 \tag{2.11}$$

$$e^{-i\theta}\left(\partial_r - \frac{i}{r}\partial_\theta + qA'\right)\tilde{\beta} - ge^{-in\theta} f(r)\beta^* = 0. \tag{2.12}$$

Modes move in the $-z$ direction at the speed of light. The equations for β and $\tilde{\beta}$ have been studied in detail by Jackiw and Rossi [2]. They find n regular solutions trapped at the origin of the (r,θ) plane. We note that the equations for the lower components of ψ and χ do not admit regular solutions which explains our taking the solutions to be proportional to s^+. For $n < 0$, one finds $|n|$ regular solutions proportional to $s^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ which move in the $+z$ direction. Note that for fixed $n > 0$ we will thus have from the Lagrangian of eq.(2.4) n left-moving zero-modes built of the pair $(\psi,\ \chi)$ and n right-movers from the pair $(\lambda,\ \delta)$. Incidently, the antiparticle of a left (right) mover is also a left (right) mover and we see that the left and right moving modes must be viewed as completely independent (e.g., the right-movers are chiral "electrons" while the left-movers are chiral "muons"). Thus, as described in the introduction this model admits superconductivity and is the simplest anomaly free example of the class of models considered in ref. [3].

Note that the Lagrangian of eq.(2.4) possesses discrete symmetries of the form $(\psi,\ \chi) \to -(\psi,\ \chi)$; $(\lambda,\ \delta) \to (\lambda,\ \delta)$. This leads to a chiral symmetry in the $1+1$ effective theory and thus the masslessness of the zero-modes. However, this symmetry in the parent theory need not be present. With a slight modification this model leads to *massive* charged states trapped on the string. Additional mass terms in the Lagrangian must be neutral in both e and q charge as well as Lorentz invariant. One cannot construct any such quantities out of the pair $(\psi,\ \chi)$ or $(\lambda,\ \delta)$, but we may consider cross-terms of the form $\psi\delta$ and $\chi\lambda$. These are simply the analogues of flavor mixing Dirac mass terms.

The Lagrangian is taken to be:

$$\begin{aligned} L \;=\; & i\psi^\dagger \not{D}\psi + i\chi^\dagger \not{D}\chi + i\lambda^\dagger \not{D}\lambda + i\delta^\dagger \not{D}\delta \\ & + ig\phi(r,\theta)\psi^T\epsilon\chi - ig\phi^*(r,\theta)\lambda^T\epsilon\delta \\ & + \mu(\psi^T\epsilon\delta + \chi^T\epsilon\lambda) + h.c \end{aligned} \tag{2.13}$$

Note that the μ terms are a consequence of the abandonment of the discrete flavor symmetries in the parent theory; such terms will generally occur whether through explicit flavor mixing, or through induced effects so long as the flavor symmetries are broken somewhere in the parent theory. The equation of motion for, e.g., ψ is:

$$i\sigma^\mu D_\mu\psi + ig\phi^*\epsilon\chi^* - \mu\epsilon\delta^* = 0. \tag{2.14}$$

The equations of motion are solved in much the same manner as was done in the previous case. The separation of variables is taken as:

$$\psi = c_1 \alpha(z,t)\beta(r,\theta)s^+ \tag{2.15}$$

$$\chi = c_2 \alpha^*(z,t)\tilde{\beta}(r,\theta)s^+ \tag{2.16}$$

$$\lambda = c_3 \alpha(z,t)\tilde{\beta}^*(r,\theta)s^- \tag{2.17}$$

$$\delta = c_4 \alpha^*(z,t)\beta^*(r,\theta)s^- \tag{2.18}$$

where c_i are constants which depend on w and p, the energy and z-momentum of the trapped states and β and $\tilde{\beta}$ satisfy the same equations as those studied above. The equation for α is now

$$\left(-\partial_t^2 + \partial_z^2 - \mu^2\right)\alpha = 0. \tag{2.19}$$

These are modes trapped on the string which behave as particles with mass μ, having energy w and z-momentum p such that $w^2 = p^2 + \mu^2$. The equations in the (r,θ) plane are identical to those studied by Jackiw and Rossi and the same counting of solutions applies. The analysis outlined here will be discussed in more detail in a future paper [8].

The effective field theory for the fermions trapped on the string reduces to the problem of 1+1 dimensional fermions interacting with a 3+1 dimension electromagnetic field. The effective Lagrangian is:

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\Psi}\gamma^i D_i \Psi + \mu\bar{\Psi}\Psi \tag{2.20}$$

Here, μ,ν run from 0 to 3 and i,j are 0 or 3. The theory can be most easily studied using the technique of bosonization. The connection between the fermion theory and the equivalent bose theory is made through the identifications:

$$j^i \equiv :\bar{\Psi}\gamma^i\Psi: \;=\; \frac{1}{\sqrt{\pi}}\epsilon^{ij}\partial_j\phi \tag{2.21}$$

$$:\bar{\Psi}\Psi: \;=\; -c\mu\cos\sqrt{4\pi}\phi \tag{2.22}$$

where c is a numerical constant related to the Euler constant (we will not be specific about the value of c in the present discussion). The Lagrangian for the Bose theory is:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_i\phi\partial^i\phi - \frac{e}{\sqrt{\pi}}A_i\epsilon^{ij}\partial_j\phi + c\mu^2\cos\sqrt{4\pi}\phi. \tag{2.23}$$

Our model somewhat resembles the massive Schwinger model [9, 10], but has an important distinction: in the massive Schwinger model, both electromagnetic and fermionic fields live in 1+1 dimensions and one can integrate out the vector boson to obtain essentially the effective Lagrangian of eq.(2.23) but with the addition of a mass term, $\phi^2e^2/2\pi$ and an additional parameter θ associated with the presence of an external electric field and entering the Lagrangian through the argument of the cosine "mass" term. Since we do not integrate out the photon field we have no such mass term and the θ parameter becomes irrelevant.

III. Elementary Applications

The equation of motion for ϕ is (*assuming a string of fixed length*):

$$\partial_i\partial^i\phi + \frac{e}{\sqrt{\pi}}\epsilon^{ij}\partial_iA_j + \sqrt{4\pi}c\mu^2\sin\sqrt{4\pi}\phi \;=\; 0 \tag{3.1}$$

consider first the simple case of a constant electric field applied along the direction of the string, $E = \epsilon^{ijz}\partial_iA_j = constant$. ϕ will be constant in z and eq.(3.1) becomes:

$$\ddot{\phi} \;+\; \frac{e}{\sqrt{\pi}}E \;+\; \sqrt{4\pi}c\mu^2\sin\sqrt{4\pi}\phi \;=\; 0 \tag{3.2}$$

The equation can be integrated once to give:

$$\dot{\phi}^2 \;+\; \frac{2eE}{\sqrt{\pi}}\phi \;+\; 4c\mu^2\sin^2\sqrt{4\pi}\phi/2 \;=\; 0 \tag{3.3}$$

where the integration constant is chosen so that $\phi(t=0)=0$. In the model studied by Witten, $\mu = 0$ and $J \equiv ej^3 = e^2Et/\pi$. The current builds up until the energy of the trapped fermions is equal to the mass of the fermions in the vacuum $m = gf_0$ and the particles are ejected from the string into the normal vacuum in which they are supermassive. The maximum current in the string is then $J_{max} = em/\pi$. If the electric field is turned off, then $\dot{J} = 0$ and the current runs without dissipation.

In the case at hand, the μ term acts as a capacitance. We note that the equation for ϕ is equivalent to that of a pendulum in a gravitational field where ϕ is the angle made by the pendulum with respect to the vertical and E is the applied torque. Clearly, for $eE \leq 2\pi c\mu^2$, $\dot{J} = -e/\sqrt{\pi}\ddot{\phi} = 0$ has a solution and the current reaches the maximum value:

$$J_{max} = e\mu\left(\frac{2c}{\pi}\right)^{1/2}\left(-\sqrt{4\pi}\Lambda\phi - 2\sin^2\sqrt{4\pi}\phi/2\right)^{1/2}\Big|_{\sin\sqrt{4\pi}\phi=-\Lambda} \tag{3.4}$$

where $\Lambda \equiv eE/2\pi c\mu^2$. For $\Lambda \ll 1$ one has $\sqrt{4\pi}\phi \simeq -\Lambda$ and $J_{max} = (4\pi^3 c)^{-1/2}e^2E/\mu$. ϕ now executes simple harmonic motion with frequency of oscillation $\omega^2 = 4\pi c\mu^2$ and the current will produce radiation at this characteristic frequency.

In astrophysical settings, currents are generated by motion through magnetic fields where charges on the string see an electric field βB (β is v/c where c is the speed of light, not to be confused with the constant c used above). With this in mind, we write the condition that the current reach a maximum as:

$$\left(\frac{B}{1\ Gauss}\right)(\beta)\left(\frac{1\ eV}{\mu}\right)^2 \leq 1.69\times10^2\times2\pi c \tag{3.5}$$

and the maximum current achieved as:

$$J_{max} = 1.44\times10^{-6}(4\pi c)^{-1/2}\left(\frac{B}{1\ Gauss}\right)(\beta)\left(\frac{1\ eV}{\mu}\right)\ \text{Amperes.} \tag{3.6}$$

For comparison, the currents achieved in the pure superconducting case studied by Witten can be of the order 10^{20} amperes. For $eE > 2\pi c\mu^2$ the current grows with time and $\phi \to eEt^2/2\sqrt{\pi}$, $J \to e^2Et/\pi$ in agreement with the result obtained in the superconducting case.

We may also consider the case where a large current, J_0, has been generated in the string and external fields have been turned off. Integrating eq.(3.1) gives:

$$\dot{\phi}^2 = \frac{\pi}{e^2}J_o^2 - 4c\mu^2\sin^2\sqrt{4\pi}\phi/2 \tag{3.7}$$

and the solution for ϕ is given by $F\left(k, \sqrt{4\pi}\phi/2\right) = -\pi/et$ where $k^2 \equiv 4c\mu^2e^2/\pi J_o^2$ and F is as elliptic integral of the first kind. where F is an elliptic integral of the first kind. Expanding in powers of $c\left(2\mu/J_o\right)^2$ we find:

$$J = -J_o(1-\frac{k^2}{4}) + \frac{J_0 k^2}{4}\cos\frac{2\pi J_0 t}{q} \tag{3.8}$$

The current has a large constant component, modulated by small oscillations. Because of these oscillations the string will radiate and lose power at a rate $\sim \omega A^2$ where A is the amplitude of the oscillations.

Finally we consider the scattering of light by the string. Following the analysis of ref. [3], we consider an electromagnetic wave with incident direction perpendicular to the string and with the electric field parallel to the string. Choosing the gauge $A_t = A_x = A_y = 0$ and noting that there is no z-dependence in the problem, we have the coupled equations for $A_z = A(x,y,t)$ and $\phi = \phi(t)$:

$$\ddot{A} - \nabla^2 A - \frac{e}{\sqrt{\pi}}\dot{\phi}\delta^2(x) = 0 \tag{3.9}$$

$$\ddot{\phi} + \frac{e}{\sqrt{\pi}}\dot{A}_z(x=y=0) + \sqrt{4\pi}c\mu^2\sin\sqrt{4\pi}\phi = 0 \tag{3.10}$$

These equations differ from those for the superconducting case only in the addition of the μ^2 term in the second equation.

Taking $A(x,y,t) = e^{-i\omega t}A(x,y)$, $\phi = e^{-i\omega t}\phi_0$ and considering small oscillations, we find:

$$(-\omega^2 + 4\pi c\mu^2)\phi = \frac{i\omega e}{\sqrt{\pi}}A(x=y=0) \tag{3.11}$$

$$\left(-\nabla^2 + \frac{e^2}{\pi}f(\omega)\delta^2(x)\right)A(x,y) = \omega^2 A(x,y) \tag{3.12}$$

$$f(\omega) = \frac{\omega^2}{\omega^2 - 4\pi c\mu^2} \tag{3.13}$$

The scattering solution is

$$A(x) = e^{ik\cdot x} - \frac{e^2}{\pi}f(\omega)G(x,0)A(0) \tag{3.14}$$

where:

$$A(0) = \frac{1}{1 + \frac{e^2}{\pi} f(\omega) G(0,0)}. \tag{3.15}$$

G(0,0) though divergent if the string is taken as a true δ function, is finite for a string of finite size and is equal to $\sim 1/2\pi \ln(\Lambda/\omega)$ where Λ depends on the core radius of the string.

The mass term in our theory enters the analysis through $f(\omega)$. For $\omega \sim \sqrt{4\pi}\mu$, $f(\omega) \to \infty$ and the small ϕ assumption breaks down. Clearly, this corresponds to a resonance in scattering of waves. For large scale magnetic fields of, for example, the size of a galaxy, $\omega \sim 10^{-11} sec^{-1}$, and $\mu >> \omega$ for masses in excess of a few electron volts, the amplitude of the scattered radiation is reduced by a factor ω^2/μ^2 as compared with the superconducting case. This reduction is quite severe and indicates that such strings could not be detected from scattered radiation.

Throughout the preceding discussion we held the length scale of the string fixed; this is generally not the case in reality since the strings are contracting due to gravitational and electromagnetic energy losses. The effect of this is to add a term $-\dot{L}\dot{\phi}/L$ to the lhs of eq.(3.2) and subsequent modifications to the following equations. This offers the interesting possibility that strings with arbitrarily small initial seed currents can develop exponentially large final currents by shrinking. The resonant behavior of these strings may allow excitation by thermal and other unconventional mechanisms. Thus, although passage of a present epoch string through a galaxy may not lead to large currents for large μ, relic strings with large currents may still be possible.

Thus these considerations significantly alter the standard superconducting string scenario. Perhaps the most striking behavior in the present situation is the resonant LC circuit behavior of the string and its attendant narrow band radiation, which may be detectable and would be an impressive signature for the existence of such objects. In a future publication we intend to amplify these considerations [8].

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References

1. H. Nielsen and P Olesen, Nucl. Phys. B61 (1973) 45.
2. R. Jackiw and P. Rossi, Nucl. Phys. B190 (1981) 681.
3. E. Witten, Nucl. Phys. B249 (1985) 557.
4. J. P. Ostriker, C. Thompson, and E. Witten, Princeton University preprint (to be published in Phys. Lett. B), C.T. Hill, D.N. Schramm, T.P. Walker, FERMILAB-Pub-86/146-T.
5. E. Weinberg, Phys. Rev., D24, 2669 (1981)
6. C. Callan, J. Harvey, Nucl. Phys. B250, 427 (1985)
7. C. Nohl, Phys. Rev. D12, 1840 (1975); H. de Vega, Phys. Rev. D18, 2932 (1978)
8. C.T. Hill and L.M. Widrow, in preparation.
9. S. Coleman, Phys. Rev. D 11, 2088. S. Mandelstam, Phys. Rev. D11 (1975), 3026.
10. S. Coleman, R. Jackiw, and L. Susskind, Ann. Phys. 93 (1975), 267. S. Coleman, Annals of Phys, 101, (1976) 239.