The Application of Gravitational Lensing on Observation of Dark Matter Candidates

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Abstract. Basic properties of gravitational lensing could help scientists in detecting the structure of the universe. For instance, the non-luminous dark matter, cannot detected by normal astronomical telescope. The large population low-mass halo which is the cosmological mode of cold dark matter, CDM, locating along the line-of-sight will perturb the strong lensing. In that way, alternation of dark matter from CDM to WDM will be detected by analysis the halo perturbed image. The main proposes of this review is to introduce the history of gravitational lensing, describe the principle of gravitational lensing and state the application of gravitational lensing in modern astrophysics.

Keywords: Strong lensing, Weak lensing, Dark matter.

1. Introduction

In real universe, not only stars like the sun, both space and time can also be curved by gravity caused by huge amount of matter, for instance, cluster of galaxies. When the light from distance galaxy behind the matter but in the same line of sight is observed, it will be magnified and distorted to rings, crosses, arcs and etc. This phenomenon is called the gravitational lensing.

200 years ago, Sir Isaac Newton explained how gravity acted on light in his book Optics. He believed that gravity was a force that could pull light toward a massive object. According to Newton's theory, the deflection angle of the light emitted by a star and passed by the surface of the sun is 0.87" while it was calculated as 1.75" by using Einstein's general theory of relativity which claimed that the gravity of an object would distort the space and time around it, and the light would travel along the distorted path [1]. So, which result was correct?

2. Theory of Gravitational Lensing

In order to understand the application of gravitational lensing better, we need to understand how gravity bends the light path. In the introduction section, we have mentioned that deflection angles derived from the Newtonian theory and Einstein's theory are inconsistent. The angle calculated based on Einstein's theory is twice that of Newtonian theory. Therefore, in this section, these two methods will be introduced separately, and the reason for the inconformity will be discussed.

2.1. Newton's Theory

By the late 18th century, the speed of light was well calculated. However, the constitution of light was still unknown [4]. Therefore, before using Newton's theory to calculate the deflection, there is an assumption need to be made. The light emitting from a star is a pulse of light which could be considered as a particle of matter moving at the speed of light.

Firstly, establishing a rectilinear coordinates which the sun is located at the origin of xy coordinate. And a light particle moving with speed $c = a$ along a nearly straight path $y = r_0$ with acceleration m/r^2. The r is the distance between the light particle and the sun which could be written as $r^2 = r_0^2 + x^2$. The component of acceleration transverse to the ray is $a = (\frac{m}{r^2})(\frac{r_0}{r}) = \frac{mr_0}{r^3}$. And the small segment $y = \frac{1}{2}ax^2$. The angle of the ray is $\frac{dy}{dx} = ax = \tan(\theta) \approx \theta$. Then we could get $\frac{d\theta}{dx} = a = \frac{mr_0}{r^3}$. Integrating the result from $x = -\infty$ to $x = +\infty$, to the lowest order of approximation, the total Newtonian angular deflection will be: $\frac{2m}{r_0}$.

However, this method still does not take into account the speed of the light corpuscle and simplify the light path to a virtually straight line. Therefore, this process should be modified.

In 1804, Soldner published a more rigorous calculation [5]. Instead of using rectilinear coordinates, the polar coordinates is more suitable for analysis.

The component of velocity now is: $\dot{x} = -r\omega\sin(\theta) + \dot{r}\cos(\theta)and\dot{y} = r\omega\cos(\theta) + \dot{r}\sin(\theta)$

The acceleration would be: $\ddot{x} = \cos(\theta)(\ddot{r} - r\omega^2) - \sin(\theta)(r\dot{\omega} + 2\dot{r}\omega)$

$$\ddot{y} = \sin(\theta)(\ddot{r} - r\omega^2) + \cos(\theta)(r\dot{\omega} + 2\dot{r}\omega)$$

For vectors whose x and y components are proportional to cos(θ) and sin(θ) respectively are parallel to r, whereas the vectors whose x and y components are proportional to -sin(θ)and cos(θ)respectively are perpendicular to r.

Therefore, we could get: $a_{radial} = \ddot{r} - r\omega^2anda_{tangential} = r\dot{\omega} + 2\dot{r}\omega$

According to Newton's theory, the force acting on a test particle is purely a radial force with magnitude $-m/r^2$, where m is the mass of a large gravitating body and $G = c = 1$ in geometrical units. So for a test particle: $\ddot{r} - r\omega^2 = -\frac{m}{r^2}andr\dot{\omega} + 2\dot{r}\omega = 0$

From the equation above, $r\dot{\omega} + 2\dot{r}\omega = 0$ could be written as $\frac{dr^2\omega}{dt} = r\dot{\omega} + 2\dot{r}\omega = 0$

Let $h = r^2\omega$, as$\frac{dr^2\omega}{dt} = 0$, it is a constant. And let $u = r^{-1}$, as $h = r^2\omega$, we can get $h/\omega = u^{-2}$Then it follows that: $\ddot{r} = -h^2u^2\frac{d^2u}{d\theta^2}and\dot{r} = -h\frac{du}{d\theta}$

Substituting these equations to a_{radial} can obtain: $-h^2u^2\frac{d^2u}{d\theta^2} - u^{-1}(hu^2)^2 = -mu^2$

The general solution to the equation above is: $u(\theta) = A\cos(\theta) + \frac{m}{h^2}$

In order to obtain the constant, A, let us consider such a situation which the pulse of light at its closest approach to the gravitating body. This closest distance is r_0. At this point, $\frac{du}{d\theta} = 0$. Therefore, $A = \frac{1}{r_0} - \frac{m}{h^2}$. And the solution should be: $u(\theta) = \frac{m}{h^2}[1 + (\frac{h^2}{mr_0} - 1)\cos(\theta)]$

As u is reciprocal of r, we can convert the solution to the form

$$r(\theta) = \frac{\frac{h^2}{m}}{1+(h^2/mr_0-1)\cos(\theta)} \quad (1)$$

which is the path of a particle in a stationary gravitational field.

Since r_0 is much larger than m, the path is hyperbola which slightly differs from a straight line. And the asymptotes of the hyperbola is: $\theta_{asymp} = \arccos(-\frac{m}{r_0-m}) = \pm(\frac{\pi}{2} + \frac{m}{r_0} + \ldots)$

The total deflection angle σ is:

$$\sigma_{Newtonian} = \theta_{asymp} - \pi = (\frac{\pi}{2} + \frac{m}{r_0} + \ldots) - \pi = 2\frac{m}{r_0} \tag{2}$$

This result is consistent with the previous simplified method.

Now we could calculated the deflection angle. As the mass of sun is about m=1475 meters in gravitational units and closest distance the light particle passing by is the radius of the sun, r = 6.95 · 10^8m [4]. Plug these parameters in the equation 2, according to the Newtonian prediction, the result angle would be 0.000004245 radians which is equivalent to 0.875 seconds of arc.

However, there are still some problems the Newtonian theory could not explain. If the light is a test particle, it could be accelerate or decelerate as an ordinary matter. But, the light is observed travelling at a constant single speed in non-relativistic absolute space and time.

2.2. Einstein's Theory

If a pulse of light is emitted from the bottom of an elevator which moving upward at a constant acceleration a, and is absorbed by its top. The speed of light would be:

$$c = \frac{x_2 - x_1}{t_2 - t_1} = \frac{L + \frac{a}{2}(t_2^2 - t_1^2)}{t_2 - t_1}$$

Where x_2 and x_1 are the bottom and top position of the elevator respectively, t_1 and t_2 are emission and absorption time and L is the distance travelled by the light pulse.

Rearrange this formula to the form:

$$c(t_2 - t_1) = L + \frac{a}{2}(t_2^2 - t_1^2) \tag{3}$$

If the emission time varies, in order to evaluate the change in reception time, we need to dfferentiate the equation (3) and assume $t_1 = 0$ for t_2 in (3), we can get:

$$\frac{dt_2}{dt_1} = \frac{1}{\sqrt{1-\frac{2aL}{c^2}}} \tag{4}$$

The frequencies of light is inversely proportional to the incremental time intervals, according to (4), therefore:

$$\frac{v_2}{v_1} = \sqrt{1 - \frac{2aL}{c^2}} \approx 1 - \frac{2aL}{c^2} \tag{5}$$

The equation (5) indicate the fraction of reduced frequency of the light traveling at the end point.

Now the acceleration of the elevator could be considered as the gravity of a stationary gravitational field which equals to a. As the acceleration varies with the distance from gravitating body with mass m, we need to determine the total frequency shift. For a large change in position, the acceleration is approximately constant for each divided small step. The unit of gravitational constant and the speed of light are both equal to 1 and each unit of radial distance L=1.

$$\ln(\frac{v_2}{v_1}) = \int_{r_1}^{r_2} \ln(1 - \frac{m}{r^2})dr \approx -\int_{r_1}^{r_2} \frac{m}{r^2}dr \approx m(\frac{1}{r_2} - \frac{1}{r_1})$$

Hence, in 1919, Einstein predicted that if a light emitting at radial distance r_1 with frequency v_1, at the point with radial distance r_2, the frequency v_2 will have a relationship with v_1:

$$\frac{v_2}{v_1} = 1 + m(\frac{1}{r_2} - \frac{1}{r_1})$$

This can also be seen as a consequence of different rates of proper time at two radial distances. For stationary positions a in gravitational field, the only non-zero term on the right side of the line element $(d\tau)^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ is the term involving coordinate time differentials $\frac{d\tau}{dt} = \sqrt{g_{tt}}$ [4]. The rate of proper time to the first order would be $1 - m/r$ and g_{tt} is $1 - 2m/r$.

$$(d\tau)^2 = (1 - \frac{2m}{r})(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \tag{6}$$

If the gravitating body is at the origin of a xy coordinate and the light ray moving in y=R direction, as the light has dτ = 0, its speed in x direction is:

$$c(r) = \frac{dx}{dt} = \sqrt{1 - \frac{2m}{r}} \approx 1 - \frac{m}{r} \text{ and } \frac{\partial c}{\partial y} = \frac{\partial}{\partial y}(1 - \frac{m}{\sqrt{x^2+y^2}}) = \frac{m}{r^3}y$$

To calculate the total deflection, simplify y=R and integrate from $x = -\infty$ to $x = +\infty$.

$$\text{deflection} = mR\int_{-\infty}^{\infty} \frac{1}{(x^2+R^2)^{3/2}}dx = 2\frac{m}{R} \tag{7}$$

However, the result from equation (7) is not complete. Not only the variation of the time-time component of the metric but also the variation of the spatial components should be taken into account. And the equation (6) should be written as:

$$(d\tau)^2 = (1 - \frac{2m}{r})(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 - \frac{1}{r^2}(\frac{2m}{r-2m})(xdx + ydy + zdz)^2 \tag{8}$$

Take$d\tau = 0$ and $dy = dz = 0$, the speed of light is: $c(r) = \frac{dx}{dt} = \sqrt{-\frac{g_{tt}}{g_{xx}}}$

And instead of g_{xx} = 1, it should be: $g_{xx} = -1 - \frac{x^2}{r^2}(\frac{2m}{r-2m})$

Therefore, the speed of light, c should be written as:

$$c(r) = \sqrt{-\frac{g_{tt}}{g_{xx}}} \approx 1 - \frac{m}{r}\left(1 + \frac{x^2}{r^2}\right)$$

And

$$\frac{\partial c}{\partial y} = \frac{\partial}{\partial y}(1 - \frac{m}{\sqrt{x^2+y^2}}(1 + \frac{x^2}{x^2+y^2})) = \frac{4x^2+y^2}{r^5}ym$$

Using the same condition, y=R and integrate the entire path gives:

$$\text{deflection} = mR\int_{-\infty}^{\infty} \frac{4x^2+R^2}{(x^2+R^2)^{5/2}}dx = 4\frac{m}{R} \tag{9}$$

As the equation (9) shows, it is twice of the equation (7).

3. Category

3.1. Strong Lensing

Gravitational lensing, the most easily observable lensing is cause by concentrated dense of mass like galaxies. It may provide a multi-location source, which is the result of complex shape or offset of background sources [6]. Properties of strong lensing and its observables connect it with the detection of universe structure. Observes like flux ratio or relative position of strong lensing can reversal the gravitational potential and derivative of its lens (or called reflector). Geometrical relationship among observers, lens and light sources also can vary the observables of strong lensing. [7].

3.2. Weak Lensing
It deflects light slightly when the light far from the center of massive object like core of clusters. Weak lensing affects all photons in the universe. However, weak lensing can only be approximated with matrix which includes magnification, shear and rotation due to the slight defection. [6-7].Weak lensing will magnify and distorted light emitted by sources, which gives people statistical properties of matter between observer and ensemble of sources and also help astronomers make assumptions of properties of sources [8].

3.3. Lensing in Different Scale

3.3.1. Macrolensing. It is largest scale of lensing. Galaxies with different total mass will have different feature in separate image carried by it. Its nature can seem as combination of the lensing proprieties of dark matter halo, bulge and the dish. Image of it can be described in SIE model which use two curve to indicate two regions of different multiplicity in sources. The outer curve only produce one image, the inner caustic curve produce four images, and sources in between produce two images [7].

3.3.2. Millilensing. It is intermediate angular scale gravitational lensing caused by substructure of the universe. These substructures can be luminous or dark. The magnitude of millilensing can be changed according to different mass of substructure introduce it [7].

3.3.3. Microlensing. Its name from characteristic 1 microarcsecond size of a star's Einstein radius. It is caused by the smallest scale objects such as stars or planets. Only physically small object will get its influence. Only small fraction of light of massive object like clusters will be magnified, others will remain unchanged [6-7].

4. Dark Matter

4.1. Introduction
The idea of this non-luminous matter is proposed when scientists realised that all parts of a galaxy moves in a similar speed while the edge of a galaxy was considered have lower speed. And this matter are thought not to involve in electromagnetic interaction.[9]

4.2. Dark Matter Candidates

4.2.1. Axion-like Particles. Axion and axion-like particles are created from spontaneous breaking of a global U(1) symmetry and are classified as pseud-Nambu-Goldstone. The key characteristics of ALPs are small mass and small coupling, which makes them cosmologically stable. In early universe, cold relics will be produced through misalignment mechanism of ALPs. And energy density of axion field is in zero-mode homogeneous anxion field. Hubble friction will frozen the field at first. After the expansion rate of the universe became lower than axion mass, the oscillation of field will start. Today, the relationship between dark matter density and ALPs is heavily depended on the initial displacements of axion field relative to the minimum of axion potential [10].

4.2.2. Primordial Black Holes. Primordial black holes are black holes that formed in the early stage of universe. Due to the relationship between cosmological density at different time after Big Bang and object's mass that is required to fall into its Schwarzschild radius, PBHs would have mass around cosmological horizon and have a large mass range. Although all back hole can catch the light, most of them are affected by BBN constraint, which make them to be formed by baryons. However, PBHs are formed in the stage that before BBN. Thus PBHs are classified as non-baryon and behave like cold dark matter [12].

5. Application: Searching Dark Matter

5.1. Searching ALPs with Strong Gravitational Lensing

The birefringence phenomenon caused by parity-violating interaction between ALPs and photons through coupling gaγ is the main method used to detect the properities of ALPs. Since the period of oscillation of ALPs field is depended on ALP mass. Measurement on amount of rotation of polarization plane of polarized light relative to plane of emission Δ θ (which depends on gaγ) can be used to determine ALP mass [14].

In detection of ALPs, there are mainly two difficulties. Firstly, Faraday rotation creates complex variation in Stocks Q and U. Secondly, the sentitivity of instruments cannot allow scientists accurately determine the coupling gaγ and ALP mass simultaneously [14].

Because of Farady rotation, any measurement of Δ θ required knowledge of intrinstic polarisation angle of linear polarized sources θ_α and offset angle created by proper calibration θ_γ since Faraday rotation-corrected polarisation angle that people can get is sum of all three angle above (Δ θ, θ_α and θ_γ). Strong gravitational lensing can easily solve this problem. Strong lensing produces time- separated images that can be observed in same time. Polarisation plane of each lensed image is created by different amount of birefringence. So different Faraday rotation-corrected polarisation angles can be get from different lensed images as all these angles share the same θ_α and θ_γ but different Δ θ. Effects of θ_α and θ_γ can be eliminated by the subtraction between these angles [14].

ALP field and birefringence angle oscillate in same time. When the spatial variations of ALP field is negligible relative to the frequency of photon in that field. Amount of rotation Δ θ can be represented by an equation involving coupling gaγ. In addition, Δ θ is measured through amount of rotation difference between two lensed image, which is created by strong lensing. In that case, properties of ALP field in emitting region is the only factor affecting amount of rotation Δ θ. Thus, the relation between Δ θ and ALP mass can be formulated and all factors can be measure accurately. Ultimately, with muti-measurement on lensing system, the coupling gaγ and ALP mass can be determine simultaneously [14].

5.2. Weak Gravitational Lensing and Dark Energy.

Based on the theory of weak gravitational lensing, recently, the Subaru Telescope is equipped with the most impressive Hyper- SuprimeCam - a 870 megapixel CCD camera. It will undertake a 1200 deg2 survey2. The high technology used make it the deepest weak lensing survey. This survey contrast the difference between grow rate of structure and the expansion rate of the universe, which detection need utilize time delay of gravitational lensing, which proves whether the dark energy exist. In addition, date get from the survey will also be used to determine whether the dark energy's energy density is a constant property of the space or it changes over time [7].

5.3. Microlensing and PBHs

PBHs are classified based on their mass. For different mass rage, different methods are used for observation. Microlensing is mainly used for detection of PBHs larger than 1012g [15]. Since the light source is too far from lensing object (PBHs in this case), so lensed images can only form a single image. But microlensing has other detection way. The luminosity of image is different for which only formed by single beam of light and those formed by muti-images. In addition, because the source of light and lensing object are always moving respect to observers, lensing events are instaneous. Thus, increase in brightness which caused by gravitational lensing can be measured [12]. Microlensing is especial useful in study of PBHs, since scientists can utilize knowledge of angle between source respected to observer and Einstein angle with mass of PBHs and the likely place and velocity distribution of these PBHs to calculate the magnification of PBHs [15].

6. Conclusion

The theory of gravitational lensing has undergone a period of development from Newtonian version to the Einstein's general relativity which has been proved in 20-century. Based on Einstein's theory and observation, gravitational lensing is divided to three categories which are strong lensing, weak lensing and microlensing.

As an essential tool in astrophysics, it is necessary for research like detection of dark matter, observing radio-quit quasar, analysis of properties of distant planets. As the most general application of gravitational lensing, dark matter detection already gains several achievement. These achievements are like dark matter's small electroweak and self- interaction cross section are essential resources we can use to analyze the distribution of universe.

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