ABSTRACT

THE GRAVITATIONAL SCHWINGER EFFECT AND ATTENUATION OF GRAVITATIONAL WAVES

This paper will discuss the possible production of photons from gravitational waves. This process is shown to be possible by examining Feynman diagrams, the Schwinger Effect, and Hawking Radiation. The end goal of this project is to find the decay length of a gravitational wave and assert that this decay is due to photons being created at the expense of the gravitational wave. To do this, we first find the state function using the Klein Gordon equation, then find the current due to this state function. We then take the current to be directly proportional to the production rate per volume. This is then used to find the decay length that this kind of production would produce, gives a prediction of how this effect will change the distance an event creating a gravitational wave will be located, and shows that this effect is small but can be significant near the source of a gravitational wave.

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May 2017

THE GRAVITATIONAL SCHWINGER EFFECT AND ATTENUATION OF GRAVITATIONAL WAVES

by

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A thesis
submitted in partial
fulfillment of the requirements for the degree of
Master of Science in Physics
in the College of Science and Mathematics
California State University, Fresno

May 2017

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ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Singleton, for bringing me into this project at the beginning of my Masters program. I would also like to thank Dr. Preston Jones for collaborating with myself and Dr. Singleton throughout this process. I would like to thank the thesis committee members for agreeing to give me appropriate feedback to ensure that this work is of the highest standards.

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To my loving wife who believes in me more than I do.

INTRODUCTION

The search for a connection between gravity and electromagnetism has been going on for centuries. As early as 1855 Faraday speculated a relation between gravity and electricity [1], through his observations he noted "Such results, if possible, could only be exceedingly small; but, if possible, *i.e.* if true, no terms could exaggerate the value of the relation they would establish." A modern take on the subject was done by [2, 3, 4] where they examined the Feynman diagrams of two gravitons turning into two photons [5, 6, 7]. This effect, as speculated by Faraday, is exceptionally small. The cross section of this interaction is around 10^{-110}cm^2 for a frequency on the order the rest mass of an electron ($\omega \sim m_e$). However, with the recent detection of gravitational waves from LIGO [9] we know that the frequency of these waves is in fact much smaller, (on the order of 100Hz). This means that the cross section associated with the waves detected by LIGO are even smaller then the values found in [2, 3, 4]. The important thing to note here is that both of these cases give *very* small values, but they are both not zero. This shows that the process could happen over very large distances, like the scale of the observable universe. In order to analyze the process we will first describe a metric for a gravitational wave propagating through the vacuum a distance very far from the source. Using this far distance limit, we will be able to approximate the gravitational wave as a plane wave. After we have defined our metric we will use the Klein Gordon equation to find our state function for the gravitational wave as it interacts with the vacuum. A discussion on the justification for using the Klein Gordon equation for photons will be given in the Calculation Section. After finding the equation of state, we will find the current for the system which is directly proportional to the probability per unit volume per unit time. We will take this as the rate of photons produced by the gravitational wave as it propagates through the vacuum and examine the physical consequences of these results.

MOTIVATION

Graviton Graviton to Photon Photon ($g + g \to \gamma + \gamma$)

Our first motivation for pursuing the attenuation of the gravitational wave is to look at the cross section produced from the Feynman Diagram of two gravitons turning into photons. Feynman diagrams were first presented by Richard Feynman [10] as a way to visually represent complex quantum interactions. By examining the Feynman Diagrams we will come to a quick solution to the probability of this process actually occurring. When first looking at the process of $g + g \to \gamma + \gamma$, it is beneficial to start with a system similar to Compton scattering ($\gamma + e^- \to \gamma + e^-$). The main difference is in this case we will have a photon scattering off of a graviton instead of an electron. After we have found the scattering for the Compton like system, we will then rotate the particles in a clockwise manner in order to create the scattering system of $g + g \to \gamma + \gamma$.

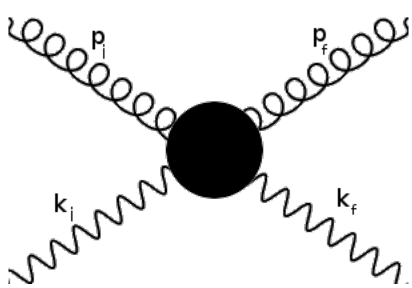


Figure 1. Compton Scattering like system of $g + \gamma \to g + \gamma$

Figure (1) is the process that is started in Ref. [3] section 5. Note that in earlier sections the authors of [3] examined many processes, including a

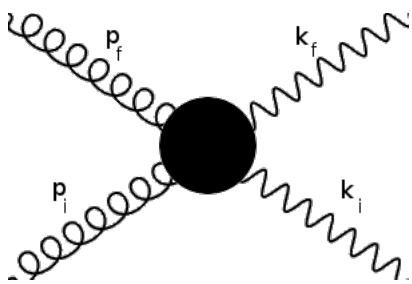


Figure 2. Rotated system from the compton like scattering

massive spin 1 particle interacting with a graviton. Thankfully, we can take this result as examined by [3] in section 5 and examine the limit as $m \to 0$ since the photon is a spin 1 massless particle. The result obtained is

$$\frac{d\sigma_{g\gamma}}{dt} = 2\pi G^2 \frac{s^4 + u^4}{s^2 t^2} \tag{1}$$

Where s,u,t are defined in the following way

$$s = (p_i + k_i)^2,\ t = (k_i - k_f)^2,\ u = (p_i - k_f)^2$$

These variables are found based on the two particles initial momenta and their final momenta. Refer to Fig. (3) for the definition of these variables known as Mandelstam variables. The Mandelstam variables will be useful when we wish to rotate our diagram and go from Fig. (1) to Fig. (2)

When examining Fig. (2), it is important to define our new *s, t, u* variables in the same way as before. For example, when looking at s' (our new s variable for Fig. (2)) we may want to simply state $s' = (p_f + p_i)^2$. However, s is originally defined in terms of the total incoming momenta. Since we have an

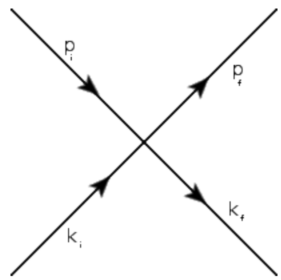


Figure 3. Define p_i, p_f, k_i, k_f where both p and k represent the momentum but for different particles.

outgoing momenta in the top of the figure we must take the sign into account, thus $s' = (p_i - p_f)^2$. We use this same process when finding t' and u' yielding

$$s' = (p_i - p_f)^2,\ t' = (p_i + k_i)^2,\ u' = (k_i - p_f)^2$$

It is easy to see from this that $s' = t$ and $t' = s$ and after examining conservation of momentum it is also easy to show that $u' = u$. After rotating the incoming and outgoing particles, we now have the differential cross section for $g + g \to \gamma + \gamma$

$$\frac{d\sigma_{gg}}{dt} = 2\pi G^2 \frac{t^4 + u^4}{s^4} \tag{2}$$

Currently, Equ. (2) gives us the differential cross section per momenta transfer of the k particle. In order to solve for the entire cross section, a change of variables is required to examine the system over the sold angle instead of the momenta. Using the change of variables given in [3][1] $\frac{dt}{d\Omega} = \frac{\omega_{CM}^2}{\pi}$

[1]It was noticed by the author that the units of the change of variables given in the reference was wrong. If the equation is examined closely it shows that the t is the variable of the difference in momenta squared, not time. This means that the units on the other side of

and using the angle relation of t and u with respect to s. This is to say, examine how the exchanges of momenta relate to the initial momenta. These substitutions lead to a differential equation for the cross section over the angle which can be integrated in order to calculate the full cross section.

$$\frac{d\sigma_{gg}}{d\Omega} = 2\pi G^2 \omega^2 \left[cos^8 \left(\frac{\theta}{2} \right) + sin^8 \left(\frac{\theta}{2} \right) \right] \tag{3}$$

Using this differential cross section we can calculate the full cross section for the process of $g + g \to \gamma + \gamma$, which gives

$$\sigma_{gg \to \gamma\gamma} = 2\pi \frac{G^2 (\hbar\omega)^2}{c^8} \tag{4}$$

It is important to note that the frequency associated with the recently observed gravitational waves is very small, $\sigma \approx 10^{-146}\text{cm}^2$. Even given a larger frequency, like one on the order of the mass of an electron as done in [2] gives a cross section of 10^{-110}cm^2, this is still an incredibly small cross section. However, it is not zero. This tells us that the effects of the attenuation of the gravitational wave will be small, but should still exist.

Schwinger Effect

Another motivation for examining the attenuation of gravitational waves is the work done by Schwinger. In this case we see that a strong field can produce particle pair production. in his paper [11], Schwinger uses the imaginary part of the electromagnetic field action integral in order to find the

the equation must be the units of energy squared. This means that we should have frequency squared given that we are taking $\hbar = 1$

probability of pair production from a constant electric field. The result that interests us is given as

$$\Gamma_{e^+e^-} = \frac{e^2 E_0^2}{4\pi^3 c\hbar^2} \exp\left[\frac{-\pi m_e^2 c^3}{eE_0\hbar}\right] \tag{5}$$

This result is the production of electron positron pairs per volume per time given a constant electric field. Due to the exponential, and the non zero mass of the electron, the electric field must be relatively large in order to get a reasonable production rate. This field strength can be found as the critical field by setting the terms inside the exponential to -1. Doing this gives an electric field of $E_{crit} = \frac{\pi m_e^2 c^3}{e\hbar} = 4 \times 10^{18} N/C$. It is important to note the major differences between the Schwinger system and our gravitational wave system. First, the Schwinger Effect as shown is for a constant electric field. With the gravitational wave, the amplitude varies in both space and time. The second major difference is the mass of the particles that are created in each case. With the Schwinger Effect, we are expecting to get positrons and electrons which have a non-zero mass. This causes a major exponential suppression for the system. In the case of gravitational waves, we are expecting photons to be produced which as far as experiments can tell have an effective mass of zero[2]. This means that the major factor requiring this process to occur only with an extremely high field strengths will not be present in our case. However, even given that there will be no exponential suppression it is important to note that an electric field is in general much stronger then a gravitational field. This can be seem by the fine structure constants of each field. For the electric

[2] The upper limit on the mass of a photon is $1.2 \times 10^{-51} g$ [12]

case, the fine structure constant is defined by $\alpha_{elec} = \frac{e^2}{\hbar c} \approx \frac{1}{137}$. Comparing this to the gravitational fine structure constant $\alpha_{grav} = \frac{Gm_e^2}{\hbar c} \approx 2 \times 10^{-45}$ it is clear to see that the electrical case is much bigger then the gravitational case. This implies that the amplitude of the gravitational wave will be much weaker in our case and could likely be the suppression of the production rate.

Zel'dovich radiation and Hawking Radiation

Finally, we want to discuss the implications of Zel'dovich radiation and Hawking radiation. Zel'dovich radiation originated from examining a rotating metal sphere. It was known that this rotating system would shoot of electromagnetic radiation tangentially due to the rotation. Zel'dovich simply asked the question, what would happen in the case of a very massive particle that is rotating? The system that was used during this analysis was a Kerr black hole. There are two main features of a black hole, the photosphere and the event horizon. The photosphere is the distance from that black hole where the space-time is so warped, the photons can no longer escape the space-time around the black hole. This is just like a ball spinning on the edge of a bowl. The event horizon is where a particle can no longer maintain a constant radius and is forced towards the singularity of the black hole. At the event horizon trying to avoid the singularity is like someone telling you to avoid the upcoming Monday, it cannot happen. When the black hole is rotating, these two characteristics of the black hole have a noticeable distance between them, as shown in Fig. (4)

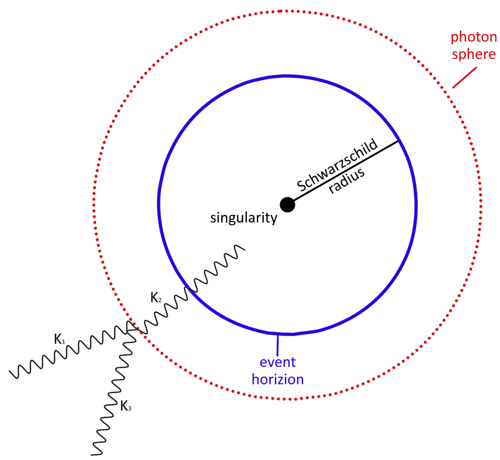


Figure 4. Kerr black hole with photosphere and event horizon shown

Zel'dovich radiation comes into play, when a photon of a characteristic energy interacts with the photosphere. Due to this interaction, the initial photon is destroyed and produces two other photons. We know from conservation of energy $k_1 = k_2 + k_3$. Where the k vectors represent the wave number of the photons. The photons produced by the radiation are dependent of the rotation of the black hole. For certain frequencies - $0 < \omega < m\Omega_H$ where ω is the frequency of the wave, m is the mass of the black hole, and Ω_H is the frequency of rotation of the black hole - it is actually possible for the photon leaving the black hole to have more energy then the photon that entered. Again, since energy must be conserved, this means that the photon leaving the black hole must have taken some of the energy from the black hole with it.

This idea was latter expanded upon by Steven Hawking who showed that it is possible that the initial photon is not actually required for this process to happen. Meaning the black hole will spontaneously generate photons at the expense of the black hole itself. Hawking's work is what we will be key in our examination of the decay of gravitational waves because it will give us the correct interpretation when we find the production rate.

DEFINING THE METRIC

In order to calculate the production rate of $g + g \rightarrow \gamma + \gamma$, we first need to discuss the space-time that is used. It is important to note at this point that we will be examining the gravitational wave far from the source. This is done for a few reasons, first, near the source the system is constantly changing. This makes it very hard to accurately discribe the system we are examining. The second reason we are examining the system far from the source is we will be able to approximate the gravitational wave as a plane wave. Finally, given our current technology we will only be able to observe gravitational waves far from their source. This greatly simplifies the system and allows for a clean analysis of what we would expect to detect with our current technology. To set this up we look at a flat space-time with the gravitational wave propagating in the z direction. The wave will be assumed to have a constant wave vector, k, and have a non-constant amplitude, h_+. It is important to note here that we are choosing the polarization of the gravitational wave. There are two orthogonal polarizations for gravitational waves which are cross polarization and plus polarization as shown in Fig. (5). The plus polarization streches the space-time in a straight up and down then side to side motion. Over time, this continued motion maps out a plus sign, hence the name plus polarization. The cross polarization shows the same behavior, just rotated 45 degrees. Normally we think of polarizations being 90 degrees apart like in the case of electric and magnetic fields. This is the case because the exchange particle of these fields is the photon which is a spin 1 particle. Since the graviton is a spin 2 particle it has a 45 degree rotation shown by the relation $\Theta_{rotation} = \frac{90°}{S}$, where S is the spin of the particle. Due to the 45 degree rotation the continued motion maps out a cross, which is why it is called cross polarization.

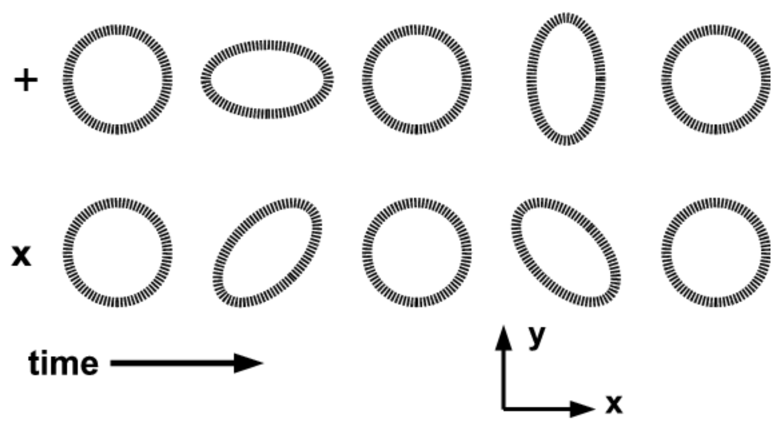


Figure 5. Demonstration of plus polarization vs cross polarization

The reason we are choosing the plus polarization is again for simplcity of the equations. If we were to use the cross polarization it would give us a metric that is not diagonal. Having a diagonal metric allows for a more straight forward equation of state without actually effecting the system. Given all of this, we can write the metric for our system by starting with a flat space-time and simply adding the gravitation wave. As shown in Fig. (5), the only part of the space-time that is effected is the x and y directions and due to using the plus polarization these are independent changes. To formally find the metric we start with the invariant distance equation that represents our system.

$$ds^2 = -dt^2 + f(u)^2 dx^2 + g(u)^2 dy^2 + dz^2 \tag{6}$$

Where $u = z - t$ and $v = z + t$ are known as light front coordinates. In order to use this form, there is a known condition that guarantees an exact solution for our yet to be determined functions $f(u)$ and $g(u)$.

$$\ddot{f}/f + \ddot{g}/g = 0 \tag{7}$$

Given Equ. (7), we can make some physical representations of how the gravitational waves would behave. It is also good to keep in mind some limiting cases when examining these equations. First, when the wave goes to zero (*i.e.* $h_+ \to 0$ or $k \to 0$) $f(u)$ and $g(u)$ should go to 1. Second, when we look at the extreme distance limits (*i.e.* taking $u \to \pm\infty$) the wave should not diverge to infinity. These are two physical limits that will guide how we look at the system.

From this equation we will be considering three examples. The first is an approximation in the amplitude of the gravitational wave that makes physical sense and the next two are examples of exact solutions to the system. The first approximate solution simply assumes

$$f(u) = 1 + h_+ e^{iku}, \;\; g(u) = 1 - h_+ e^{iku}$$

These ansatze represent the plane wave approximation that we are physically able to make due to the assumption we are examining the system far from the source. It is important to note here that this approximation only works to first order in h_+. Since we know gravitational waves have small amplitudes and we are far from the source it is reasonable to assume that h_+ is very small. This means that when looking at the functions that define our system we only need to use the first non vanishing order of h_+.[3]

The second ansatz function is purely a mathematical function that fits the conditions outlined for f and g to be an exact solution. Taking $f = g = 1 + ku$ is an easy solution to our conditions on f and g to be exact solutions. However, these functions describe a pulse of gravitons instead a wave of gravitons. This means we are only looking at a short burst instead of a full gravitational wave.[17]

The final ansatz function is an exact solution with explicit amplitude decay multiplied by our assumed plane wave. Taking $f(u) = e^{iku}e^{-ku}$ and $g(u) = e^{iku}e^{ku}$ gives us an exact solution but it is more explicit about how the amplitude decays.

In each case we should examine the limits of the ansatz to see if they make physical sense. First, if we were to get rid of the wave the ansatz should go back to the usual flat space-time (*i.e.* $f = g = 1$). This can be accomplished in two ways, first taking the wave amplitude $h_+ \to 0$ and second taking the wave number $k \to 0$. In each case, taking this limit creates a system where the gravitational wave would not exist[4]. In each of our ansatz functions taking these limits gives us the desired result of $f = g = 1$. Next, we

[3]This will be an important note to follow through the rest of the derivation because we will be seeing higher orders in h_+, but they are still in smallest non vanishing orders of h_+.

[4]It should be noted that for the case of taking the wave number to zero we may need to include a phase factor to guarantee this but that can be done without changing any results since its derivative is zero

need to check the limits when $u \to \pm\infty$. This is an important limit to check because we are examining the system over very large distances and these limits very well could be tested in a realistic way. Directly looking at these limits it is clear to see that the first ansatz functions keep behaving at $u = \pm\infty$ as any other value of u due to the wave property of the equation. The same cannot be said about the next two set of ansatz functions. Both of these equation diverge at both plus and minus infinity meaning they are not physical at these limits. With these tests on the limits of each of the ansatz functions, we will perform the main analysis on the first set of ansatz functions and comment on the significance of the other two.

CALCULATION

Normally when examining photons the Klein Gordon equation is not enough. This is because photons are massless spin 1 particles but the Klein Gordon equation is only valid for spin zero particles. Given this issue, we want to examine again the cross section $g + g \to \gamma + \gamma$. Referring back to [3], we will now examine the difference between $g + S \to g + S$ and $g + \gamma \to g + \gamma$ where S is a particle with variable spin. This comparison allows us to find the probabilty of each process happening [5] in order to see if we are justified in using the massless Klein Gordon equation. From these cross sections we see that the only difference between the two is a factor of order unity. We see this happening because the full spin state of the photon can be written as $A_\mu(x_\nu) = \epsilon_\mu \varphi(x_\nu)$. Where $A_\mu(x_\nu)$ is the vector potential, ϵ_μ is the polarization matrix, and $\varphi(x_\nu)$ is the spinless state function that obeys the Klein Gordon equation. This allows us to separate the full spin vector into a polarization matrix multiplied by the spinless state function. The result of doing this exchange is shown from the cross section, the only factor the polarization matrix brings into the system is a factor of order unity. With this justification we can remove the polarization matrix and focus only on the spinless state function which allows us to use the Klein Gordon equation. The metric for this generic system, and its covariant version, are described as

[5]Taking the limit $S \to 0$ to compare the photon with the spinless system

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & f(u)^2 & 0 & 0 \\ 0 & 0 & g(u)^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{f(u)^2} & 0 & 0 \\ 0 & 0 & \frac{1}{g(u)^2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

With the metric defined we can now start to setup the Klein Gordon equation which is given by

$$\frac{1}{\sqrt{-|g_{\mu\nu}|}} \left(\partial_\mu g^{\mu\nu} \sqrt{-|g_{\mu\nu}|} \partial_\nu \right) \varphi = 0. \tag{9}$$

Where $\sqrt{-|g_{\mu\nu}|}$ is the determinant of the matrix which is easy to show equals $f(u)g(u)$. Now that we have defined the metric we can solve the Klein Gordon equation in order to find the state function for the gravitational wave. It is more beneficial to start this derivation with the general equations $f(u)$ and $g(u)$ because we can find an almost general solution with this form before inserting a specific set of ansatz functions. Starting with the Klein Gordon equation and summing over its indices gives

$$\frac{1}{fg} \left(-\partial_t \{(fg)\partial_t\} + \frac{1}{f^2}\partial_x \{(fg)\partial_x\} + \frac{1}{g^2}\partial_y \{(fg)\partial_y\} + \partial_z \{(fg)\partial_z\} \right) \varphi = 0 \tag{10}$$

It is important to remember that $u = z - t$ and therefore is not dependent on x or y. Due to this the fg in the second and third terms can just pass through the derivative, leaving us with a second derivative in x and y. This expands Equ. (10) to

$$\frac{1}{fg}\Big(fg(-\partial_t^2+\partial_z^2)+\frac{g}{f}\partial_x^2+\frac{f}{g}\partial_y^2-\partial_t(fg)\partial_t+\partial_z(fg)\partial_z)\Big)\varphi=0 \tag{11}$$

At this point we want to make a change of variables to the light front coordinates. This is because we know whatever solution we end up with, we will never be able to fully separate the z and t variables. Therefore, the light front coordinates and it will be more representative of our actual system. A quick analysis will show that $\partial_t(fg)=-\partial_u(fg)$, $\partial_z(fg)=\partial_u(fg)$, $(\partial_z^2-\partial_t^2)=4\partial_u\partial_v$, $(\partial_t+\partial_z)=2\partial_v$, and $(\partial_t-\partial_z)=2\partial_u$ then substituting these expressions in and multiplying by f^2g^2 gives

$$\Big(4f^2g^2\partial_u\partial_v+2fg\partial_u(fg)\partial_v+g^2\partial_x^2+f^2\partial_y^2\Big)\varphi=0 \tag{12}$$

It is at this point that we need to put in some specifics for the system. We will use the physical system to perform our actual calculation of the production rate and will go back to the mathematical system to show that they will in fact yield similar results. Using the physical set of ansatz functions gives $f(u)=1+h_+e^{iku}$ and $g(u)=1-h_+e^{iku}$, where the equation is organized in terms of powers of h_+.

$$\Big[h_+^0\left(4\partial_u\partial_v+\partial_x^2+\partial_y^2\right)+2h_+e^{iku}\left(\partial_y^2-\partial_x^2\right)$$

$$+h_+^2 e^{2iku}\left(-8\partial_u\partial_v - 4ik\partial_v + \partial_x^2 + \partial_y^2\right) + h_+^4 e^{4iku}\left(4\partial_u\partial_v + 4ik\partial_v\right)\Big]\varphi = 0$$

It is important to point out a few aspects of this equation. If we look at the limit when the gravitational wave is not present, ($h_+ \to 0$), we expect to get the vacuum background. It turns out that this is exactly what happens when we take this limit in the light front coordinates[6]. Since we are looking at a gravitational wave propagating the in the z-direction, it becomes physically reasonable based on the isotropy of space-time and assuming a non thermal background that the x and y derivatives should act the same[6, 8, 15]. This leads us to $\partial_y^2\varphi - \partial_x^2\varphi = 0$. Applying separation of variables with our state function defines $\varphi = X(x)Y(y)U(u)V(v)$. Right away we have some insight on what can happen in the x and y directions. At most we expect to get constants from the second derivative of these functions since there is no direct dependence on x and y in our ansatz functions.

$$\begin{aligned} \partial_x^2 X(x) &= -k_x^2 \to X(x) = e^{ik_x x} \\ \partial_y^2 Y(y) &= -k_y^2 \to Y(y) = e^{ik_y y} \end{aligned} \tag{13}$$

Note the way we have set up this expression assumes that we also have waves traveling in the x and y directions. However, these are free waves since the gravitational wave is only traveling in the z direction. Also from our analysis of the x and y derivatives it becomes clear that $k_x^2 + k_y^2 = 2k_{xy}^2$. We

[6] It should be noted here that usually the light front coordinates are defined with a normalization factor of $1/\sqrt{2}$. This will change the result sightly making the vacuum state $2\partial_u\partial_v + \partial_x^2 + \partial_y^2$.

can also introduce three functions to compact the equation in terms of the derivatives ∂_u and ∂_v.

$$F\frac{\partial_u U}{U}\frac{\partial_v V}{V} + 4ikG\frac{\partial_v V}{V} - 2k_{xy}^2 H = 0 \tag{14}$$

Where $F(ku) \equiv (1 - 2h_+^2 e^{2iku} + h_+^4 e^{4iku}),\ G(ku) \equiv (h_+^4 e^{4iku} - h_+^2 e^{2iku}),\ H(ku) \equiv (1 + h_+^2 e^{2iku})$. Unfortunately we were unable to completely separate the variables. However, it is easy to complete our separation of variables by introducing the condition $\frac{\partial_v V}{V} = ik_v$. Again, this sets up the function in the non u direction to be a free wave. With this addition we are able to complete the separation of variables and find a solution for the function $U(u)$. Rearranging variables an defining $\lambda \equiv \frac{k_{xy}^2}{2k_v}$ gives us the differential equation for $U(u)$.

$$i\frac{\partial_u U}{U} = k\frac{G}{F} + \lambda\frac{H}{F} \tag{15}$$

Which when integrated gives

$$U = e^{\frac{\lambda}{k}} e^{\frac{-\lambda}{k\left(1-h_+^2 e^{2iku}\right)}} \left(1 - h_+^2 e^{2iku}\right)^{\frac{1}{2}\left(\frac{\lambda}{k}-1\right)} e^{-i\lambda u}\ . \tag{16}$$

Again, we should check to see what would happen if the gravitational wave was not in the system. Taking the limit as h_+ goes to zero gives $U = e^{-i\lambda u}$, which is just a free wave like the other directions. This shows that since the wave is only traveling in the u direction it makes sense for the other directions to have just plane waves. This gives us the final equation of state for both the system with the gravitational wave and the system for just the vacuum. Both of these states will be used in order to find the probability of particle production.

$$\varphi = Ae^{\frac{\lambda}{k}} e^{\frac{-\lambda}{k\left(1-h_+^2 e^{2iku}\right)}} \left(1 - h_+^2 e^{2iku}\right)^{\frac{1}{2}\left(\frac{\lambda}{k}-1\right)} e^{-i\lambda u} e^{ik_v v} e^{ik_x x} e^{ik_y y} \tag{17}$$

$$\varphi_0 = Ae^{-i\lambda u} e^{ik_v v} e^{ik_x x} e^{ik_y y} \tag{18}$$

From the state function we can now calculate the current of the system. The reason we want to find the current is because it is directly related to the production rate per volume that we are looking for, as shown by inspection in Ref. [14].

$$j_\mu = \Gamma_\mu \Delta t \tag{19}$$

This means that if we want to find the production rate per volume in the u direction, the direction of the gravitational wave, we can find it simply with this process. The current is defined by

$$j_u = -i(\varphi^* \partial_u \varphi - \varphi \partial_u \varphi^*) \tag{20}$$

It should be noted here that the derivatives are only on u since we are only interested in the current in the u direction. Looking at the state function φ, it is very clear that taking these derivatives will be tricky. Here is where we should remind ourselves what system we are trying to find and what system we have actually been analyzing. What we want to know is how does the gravitational wave interact with a flat space-time. That is to say, how does the gravitational wave interact only with the background and nothing else. What we have created is a state function where a wave of wavelength λ interacts with the gravitational wave. Recalling back to Hawkings work, this process should be spontaneous and therefore does not need the external photon to excite the gravitational wave for the decay to occur. In order to show this spontaneous process, we take the limit as λ goes to zero [7]. If we perform this limit the state function for the gravitational wave solely in the vacuum background reduces to

$$\varphi = A(1 - h_+^2 e^{2iku})^{-\frac{1}{2}} \tag{21}$$

Another step that we can do to make it more plain what our state function looks like is to approximate the square root. It is important to remember that h_+ represents the amplitude of the gravitational wave which means it is a very small number due to experimental measurements [9]. This means at most we only need to keep the first few orders of the expansion of the square root. In general $\frac{1}{\sqrt{1-\epsilon^2}} \approx 1 + \frac{\epsilon^2}{2} + \frac{3}{8}\epsilon^4$. By defining $\epsilon(u)^2 \equiv h_+^2 e^{2iku}$, Equ. (21) approximates to,

$$\varphi \approx A\left(1 + \frac{1}{2}\epsilon(u)^2 + \frac{3}{8}\epsilon(u)^4\right) \tag{22}$$

[7] Recall that $\lambda \equiv \frac{k_{xy}^2}{k_v}$. This means that in order to take $\lambda \to 0$, we also need to take the limits when $k_{xy} \to 0$ and $k_v \to 0$.

It is important at this point to remember the limits we put on our metric when we first started this analysis. The condition we used in order to make the metric work required the metric to be accurate to first order in h_+. This limit should extend throughout the rest of the analysis. Because of this we need to keep only the first non-vanishing term in h_+ when looking at our calculations.

$$|j_u| = |A|^2 k h_+^2 \left\{ e^{2iku} + e^{-2iku} + h_+^2 + \frac{3}{2} h_+^2 \left(e^{4iku} + e^{-4iku} \right) \right\} \tag{23}$$

In the expression above we have included the two lowest orders of h_+ in case one of them is a vanishing term. At this point we need to again examine the system that we are looking at. What we are trying to do is see what happens over a long period of time to our system. This means when examining the current of the system what we really want is the time average. In order to do this we should note the familiar definition $e^{ix} + e^{-ix} = 2\cos(x)$. Due to the symmetry in the $\cos(x)$ it is easy to see that this function time averages to zero. This leaves us with the time averaged current to the lowest non-vanishing power of h_+ to be

$$|j_u| = |A|^2 k h_+^4 \tag{24}$$

It is important to make note of the normalization constant that has just been floating around until this point. If we look at the normalization constant for a usual plane wave it is $|A|^2 = \frac{1}{2k}\frac{1}{V}$ where k is the wave number and V is some finite volume. We will be using this as our normalization constant because we are presenting the gravitational wave as a plane wave. This means when we finally apply this normalization constant the current simply becomes

$$|j_u| = \frac{1}{2V} h_+^4 \tag{25}$$

Now we can use Equ. (19) to find the probably rate per volume of this process actually occurring. The only question we need to ask ourselves is what Δt makes the most physical sense. Since we are looking at the system over a long period of time it makes the most sense to use a time unit that represents the system. Looking at the system in this sense there is a natural unit for time, $\Delta t \equiv \frac{1}{\omega}$, where ω represents the frequency of the wave. Using this as our natural unit of time means that the probability rate per volume comes out to be

$$\Gamma = \frac{\omega}{2V} h_+^4 \tag{26}$$

We can now examine what our current densities would be if we instead considered our other metrics shown earlier. First let us examine the case where $f(u) = g(u) = 1 + ku$. Starting from Eq. (12), and using this definition we find the state function to be

$$\left(4\left(1 + 2ku + k^2u^2\right)\partial_u\partial_v + 4k\left(1 + ku\right)\partial_v + \partial_x^2 + \partial_y^2\right)\varphi = 0 \tag{27}$$

As before, we will define $\varphi = U(u)V(v)X(x)Y(y)$, $\frac{\partial_x^2 X(x)}{X(x)} + \frac{\partial_y^2 Y(y)}{Y(y)} = 2k_{xy}^2$, and $\frac{\partial_v V(v)}{V(v)} = ik_v$. Subsituting these definitions into Equ. (27) simplifies our new state function to.

$$\left(1 + 2ku + k^2u^2\right)\frac{\partial_u U}{U} + k\left(1 + ku\right) = i\lambda \tag{28}$$

Where again, $\lambda \equiv \frac{k_{xy}^2}{k_v}$. Solving this differential equation for $U(u)$ and then multiplying it by the known $v(v)$, $X(x)$, and $Y(y)$ yields the state function for this ansatz function.

$$\varphi = A\frac{\exp\left[-i\frac{\lambda}{k(1+ku)}\right]}{1+ku}e^{ik_v v}e^{ik_x x}e^{ik_y y} \tag{29}$$

It is at point we want to take the limit as $\lambda \to 0$ in order to find the interaction of this gravitational system purely with the vacuum background. It is easy to see that applying this limit reduces the state function to something that is purely real. By definition this means that the current from this state will be zero since $\varphi^* = \varphi$, which results in no particle production. An explanation for this could be that this system is representing a gravitational pulse rather then sitting at one specific location and observing a wave passing by over a long period of time. This again is showing that the cross section of this event happening is very small and in order to observe its effects we need to observe the interatction of gravitons multiple times.

Finally we will examine the case where $f(u) = e^{iku}e^{-ku}$ and $g(u) = e^{iku}e^{ku}$. Again starting from Eq. (12), we can find the state function for this system.

$$\left(4e^{4iku}\partial_u\partial_v + 2e^{2iku}\partial_u\left(e^{2iku}\right)\partial_v + e^{2iku}e^{2ku}\partial_x^2 + e^{2iku}e^{-2ku}\partial_y^2\right)\varphi = 0, \tag{30}$$

and making the substitution

$\varphi = U(u)V(v)X(x)Y(y) = U(u)e^{ik_v v}e^{ik_x x}e^{ik_y y}$,

$$\left(i\frac{\partial_u U}{U} - k - e^{-2iku}e^{2ku}\frac{k_x^2}{4k_v} - e^{-2iku}e^{-2ku}\frac{k_y^2}{4k_v}\right) = 0. \tag{31}$$

In the limit when the gravitational wave is absent (*i.e.* $k \to 0$) the solution to (31) is again given by (18). When $k \neq 0$ the solution from (31) gives the state function,

$$\varphi = Ae^{\left(\frac{(1-i)}{4k}\lambda_x e^{-2iku}e^{2ku} + \frac{(1+i)}{4k}\lambda_y e^{-2iku}e^{-2ku}\right)}e^{-iku}e^{ik_v v}e^{ik_x x}e^{ik_y y} \tag{32}$$

Where $\lambda_x \equiv \frac{k_x^2}{4k_v}$ and $\lambda_y \equiv \frac{k_y^2}{4k_v}$. These definitions are very similar to our previous λ and the same limits should apply here. Recall that it was the limit on our original λ that gave us the condition to eliminate an interaction wave by taking the limits of k_x, k_y, and $k_v \to 0$. These limits will still give us just the gravitational wave and the background which is the situation we want to find the current for. Taking these limits we are left with $\varphi = Ae^{-iku}$.

From here, we can take this state function and run it through our current definition in order to find the production rate for this metric. Doing so we see $|j_u| = |A|^2 2k$ and as before we have defined the normalization constant as if it were for a plane wave to be $|A|^2 \equiv \frac{1}{V}\frac{1}{2k}$ which brings the current to $|j_u| = \frac{1}{V}$. This is very similar to the result from our first metric exepct there is no direct dependence on the amplitude. However, the main difference between these two metrics is that one amplitude is left as a variable and the other is explicitly defined in relation to the variable u. Given this explicit definition it makes sense that we would see the usual result except for the amplitude which is exactly what we got. It should be mentioned again that this metric is only valid for a limited range due to the diverging nature of the ansatz functions when $u \to \pm\infty$. Due to this, even though our third metric gives us a mathematical exact solution to f and g, it is more reasonable to use our first metric in order to gain physical understanding from this system.

RESULTS OF CALCULATIONS

Now that we have a production rate it is important to see the implications of these results. The most important aspects of these results are that it is dependent on the frequency of the wave and the amplitude of the wave. At this point we have only measured very small amplitudes at a very small frequency. Due to this we should be able to examine the decay length based on these measurements to get an idea of the characteristic length implied by this result. Normally, when looking at the decay length we use the form $\Lambda = \frac{c}{\Gamma}$. That is the say, the speed of the wave over the production rate. I stress that this is the usual thing to do because this process is used when you have a constant Γ. In our case the decay factor is not constant since it is dependent on the amplitude of the wave, which due to the nature of the decay will be changing. However, we can use this as a way to get a good first estimate of what we should expect to get out of the system. Based on this first estimate we expect the decay length of the system to be

$$\Lambda = \frac{c}{\Gamma} = \frac{2c}{\omega h_+^4} = \frac{2h_+^{-4}}{k} \tag{33}$$

Another method of finding the decay length is to do a more accurate calculation of what the distance as a function of amplitude is. To do this, we simply use the relation $\frac{dN}{dt} = -\Gamma N$. This is to say we are examining how the number of gravitons change as a function of time. We do need to make a change of variables first since we are more interested in the distance rather then time. To do this we simply use $z = ct$ since the wave moves at the speed of light. This changes the equation to

$$\frac{dN}{dz} = \frac{-\Gamma}{c}N = \frac{1}{2}kh_{+}^{4}N$$

With the usual wave definition that $\frac{\omega}{c} = k$. Since not much is yet known of the graviton, we have to make an assumption about what properties it has. The one we are making here is that the number of gravitons is proportional to the amplitude of the gravitational wave ($N \propto h_{+}^{n}$). At this point we will be the power general since it is not a know relationship. With this assumption it is easy to see that the differential equation reduces to $\frac{dh_{+}}{h_{+}^{5}} = -\frac{1}{2n}kdz$ and once we solve for the distance as a function of amplitude, $z(h_{+})$, it becomes

$$z(h_{+}) = \frac{n}{2}\frac{h_{+}^{4}}{k} + z_0 \tag{34}$$

At this point we will take inspiration from how the number operator for photons relate to the vector potential. In this case $N_{photon} \propto A_{\mu}^{2}$. Assuming a similar relationship with gravitons to their amplitude implies that $n = 2$. If this turns out to be incorrect, it is very easy to find the new equation based on out generalized model. [8]

[8]Another guess that we could have made was $n = 4$. If we examine the Schwinger Effect we see that the production rate is related to the Electric field magnitude squared, where in our case the production rate is related to the the fourth power of the gravitational wave amplitude. Either way, the results will be nearly identical since we will only be looking at order of magnitude arguments.

Using this we can find the decay length with the definition $\Lambda \equiv z(h_+/2) - z(h_+)$, this gives us

$$\Lambda = \frac{15h_+^{-4}}{k} \tag{35}$$

The good news is this is a very similar result to the one that we got from the assumption that Γ was constant. However, the new factor is close to a factor of ten so it is more useful to go with the full calculation of the decay length.

$h_+^{(0)}$	$\Lambda(m)$
10^{-21}	10^{91}
10^{-15}	10^{67}
10^{-9}	10^{43}
10^{-5}	10^{27}
10^{-1}	10^{11}

Table 1. Various values of the decay length Λ versus $h_+^{(0)}$ for $\omega \approx 3 \times 10^2 Hz$ and $k \approx 10^{-6}\ m^{-1}$. We begin with $h_+ \approx 10^{-21}$ which is roughly the measured strain reported for GW150914 [9].

Table 1 shows the various decay lengths based on a given initial amplitude. As is shown, even with a high initial amplitude 10^{-1} still requires a decay length of 10^{11}m. This again refers back to Equ. (2), showing us that the cross section for this process is incredibly small. However, this is still within the scope of the observable universe, meaning from a very distance source this effect could be significant. Unfortunately, as soon as the amplitude decreases slightly 10^{-5}, we see that the decay length based on this initial amplitude is on the order of the observable universe 10^{27}m. The most significant initial amplitudes are the first one presented in the table, 10^{-21} and the last 10^{-1}.

The first amplitude was measured by LIGO when the gravitational wave was detected. It is clear to see that with this as the initial amplitude, this process would be mostly impossible to detect. The last amplitude is what LIGO expects the amplitude of the wave to be as it was created from the black hole merger. This was found purely using the known $\frac{1}{r}$ fall off and using the Schwarzschild radius of the new black hole as a starting point.

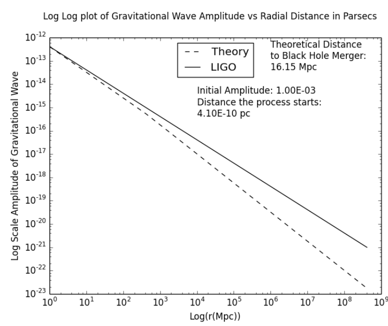
Now that we have a feel for the decay length of the system, a closer look can be taken. LIGO measured the distance based solely on the fact that the amplitude of the wave would decrease due to a $\frac{1}{r}$ fall off. Due to this assumption, we can find the amplitude of the gravitational wave at any distance with the following relationship.

$$h_+(r) = \frac{h_+(r_0)r_0}{r} \tag{36}$$

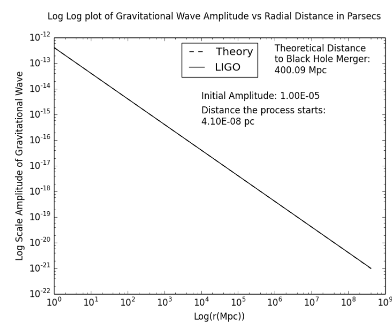
Normally this relationship implies that $h_+(r_0)$ is a constant value. A way that we can include the decay of the gravitational wave into photons is to invert Eq. (34) to get the wave amplitude as a function of distance.

$$h_+(r) = (k(r - r_0) + h_+(r_0)^{-4})^{-1/4} \tag{37}$$

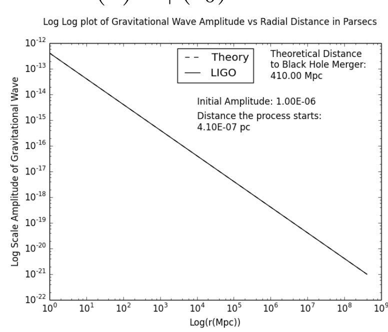
Where the constant was constrained such that when $r = r_0$, $h_+(r_0) = h_+(r_0)$. This expression for the wave amplitude will be used in Eq. (36) in order to include the decay from $g + g \to \gamma + \gamma$. It is important to note that the theory for this process assumed that we were “far from the source” in order to use the plane wave approximation. This means that we can examine how this system will behave, but we need to represent our initial distance (r_0) and initial amplitude ($h_+(r_0)$) around this assumption.



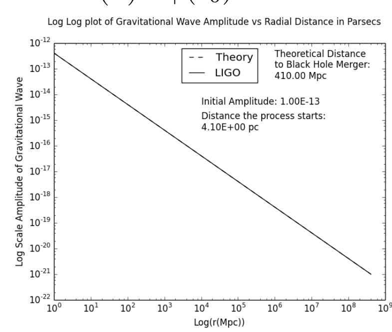
(a) $h_+(r_0) = 10^{-3}$



(b) $h_+(r_0) = 10^{-5}$



(c) $h_+(r_0) = 10^{-6}$



(d) $h_+(r_0) = 10^{-13}$

Figure 6. Log Log plot of gravitational wave amplitude vs distance in parsecs. This shows the difference between how the amplitude changes purely from the $\frac{1}{r}$ fall off assumed by LIGO and the decay in amplitude using the process $g + g \rightarrow \gamma + \gamma$. The distance from the event creating the gravitational wave is given in mega parsecs (Mpc) and should be compared with the value as measured by LIGO which is 410Mpc.

Figure (6) shows how this effect is huge when the amplitude of the wave is huge. For example, looking at Fig. (6a) with an initial amplitude of 10^{-3}, the distance from the event is 16.15 Mpc compared to the measured 410 Mpc. This is a huge difference, but again we need to ask the question, "is this far from the source?" This is why we are looking over a few initial amplitudes in order to get a feel of the system. Looking at Fig. (6c) which has an amplitude of 10^{-6}, we see that this is the largest amplitude where the distance from the event is the same as the distance measured by LIGO. This agrees with our earlier analysis of the decay lengths from Table (1), since the decay length for a wave of amplitude 10^{-5} gives a decay length on the order of the observable universe.

Another topic of interest is what the physical significance is when taking the limit of $\lambda \to 0$. This is an interesting thing to look at because if we examine our plane wave metric it leaves us with $\varphi = (1 - h_+^2 e^{2iku})^{-\frac{1}{2}}$. Again, what does this mean physically is the question. The reason we took the limit in the first place was to guarantee that the gravitational wave was interacting only with the vacuum and not another wave. A similar analysis was done when examining the Higgs Boson [16]. However, there is a clear difference between these two situations. In the case of the Higgs Boson there was a self interaction term (*i.e.* $\lambda\phi^4$) that causes the state function to reduced to $\varphi = \sqrt{\frac{m^2}{2\lambda}}$ when taking a similar limit. This is a constant and therefore does not create a current. In the case of the gravitational wave, we have a state function that is not independent of spatial and time coordinates, which is why we get particle production from the vacuum. This effect is also akin to the symmetry breaking in super conductors where it is the background lattice and phonons which provide the mechanism leading to a non-zero expectation value for Cooper pairs.

For the main part of this calculation, we looked at the Hawking case of gravitational wave decay be taking the external photon to zero. For completeness we should also examine the case of Zel'dolvich, where an external photon interaction does exist. In order to get the current of this system we refer back to Eq. (17) and do an expansion of powers in h_+ in order to get the first non-vanishing order in h_+. Doing this expansion we get.

$$|j_u| = -|A|^2 \left[2\lambda + \left(\frac{9}{2} \frac{\lambda^3}{k^2} - 12\frac{\lambda^2}{k} + \frac{13}{2}\lambda - k \right) h_+^4 \right]$$

At this point we want to make sure that we are refering to the current created soley from the interaction of an external photon with the gravitational wave. This current is found by taking the current and subtracting the current that would result in a vacuum background which is known to be $-|A|^2 2\lambda$. Δj_u defines this current and is shown to be

$$|\Delta j_u| = -|A|^2 \left(\frac{9}{2} \frac{\lambda^3}{k^2} - 12\frac{\lambda^2}{k} + \frac{13}{2}\lambda - k \right) h_+^4 \tag{38}$$

A major part of Zel'dovich radiation is the interesting interaction that occurs when $0 < \omega < m\Omega_h$, giving the photon leaving the black hole more energy than the photon that entered the black hole. In order to find a parallel between these two cases, we can simply plot the part of the current inside the parentheses vs a reduced external wavelegnth defined as $\frac{\lambda}{k}$. In order to find the point where the produced photon has more energy then the initial, we want this to be negative in order to make the overall current positive. This plot is shown in Fig (7) and we can see that there is only a small range where this occurs. If the incident photon wave number is between zero and two times the wave number of the gravitational wave (with an exception when

$\frac{\lambda}{k} = 0$), this will produce a photon with more energy then it came in with, again at the expense of the gravitational wave. Since gravitational waves so far have been observed to have very small frequencies, it is more likely that any photon interactions will result in the photon being absorbed by the gravitational wave. Thankfully any interactions the gravitational wave will have with photons will be rare events and corrections for this would likely show up as random error.

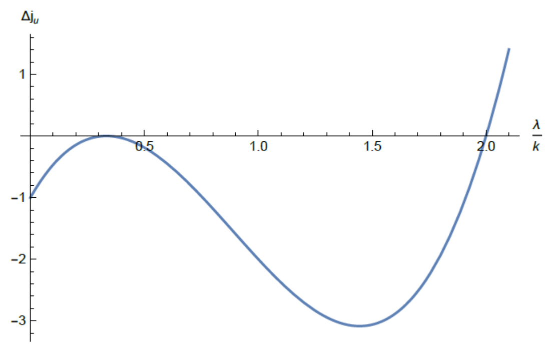


Figure 7. Gravitational wave current vs incoming photon reduced wave number

CONCLUSION

Over the course of this work, we have examined the possibility of attenuation of gravitational waves. This was modeled by examining the gravitational wave spontaneously producing photons based on previous work like the Schwinger effect with some slight differences. In our case, we were using massless scalar particles, which represented photons, as our produced particles. Since the biggest suppression from the Schwinger effect is from the exponential suppression of a non-zero particle mass, this process seemed much more likely to occur. However, we can examine the “fine structure constant” of gravity and electromagnetism to see that the electromagnetic effects will be much stronger. We were able to show this process having a non-zero result by taking the current from just the gravitational wave propagating through the vacuum. Since this result was non-zero, even in the limit with no external interaction, we were able to show that the decay of the gravitational wave holds under physical reasoning. However, the results we found also suggest that in order for this process to occur, the amplitude of the gravitational wave must be very large compared to the amplitudes measured so far by LIGO.

The main calculation that was done in this work showed that the current was non-zero even when taking the no interaction case ($\lambda \to 0$). This was done with guidance from Hawking radiation as proof that a gravitational wave can produce photons even without the system being agitated by an outside source (*i.e.* a particle interaction). This result was motivated by Hawking, but it is also surprisingly similar to the Higgs mechanism. In the case of the Higgs, there is a self interaction term that causes the state function to persist even when taking the limit of no interactions. The main difference between these two cases though is that when you take this similar limit with the Higgs mechanism, you get a constant state function as the result, which yields a current of zero. In the case of the gravitational wave, when you take this no interaction limit, the state function is still a function of space and time. This result gives us a non-zero current, which is directly proportional to the production rate of photons due to the gravitational wave.

From this work we can make two predictions. First, any gravitational wave that is measured on Earth would actually have a smaller amplitude then we currently expect. This would lead to the event causes the gravitational wave (*i.e.* two merging black holes) to be closer then we first expected. Finally, we were able to bring the plane wave approximation together with the expected $\frac{1}{r}$ fall off in order to see what the overall effects of this theory imply. This again shows that we expect this change to only matter close to the source, but it is still significant in order to accurately measure the event at large distances and implies that photons should be created as the wave propagates. In addition, since both of these waves travel at the speed of light, the photons should arrive at the same time as the gravitational wave. Surprisingly, it has already been observed [13] that a stream of photons accompanied the gravitational wave, and that these photons could be related to the gravitational wave. One of the aspects of these photons, however, is that they should have the same frequency as the gravitational wave. So far the observed frequency of gravitational waves is also small ($\sim$ 100Hz), which would lead to photons with wavelengths in the range of 100s of kilometers.

These wavelengths could easily have gone undetected. In fact we may need a space radar in order to actually observe these photons.

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