

Tunneling of massless and massive particles from a quantum deformed Schwarzschild black hole surrounded by quintessence

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Abstract

We study radiation of a quantum deformed Schwarzschild black hole surrounded by a quintessence field, through a tunneling process. When the background spacetime of a black hole is covered by a scalar field, such as a quintessence field, metric of the black hole changes for a particular range of the mass. Then, the geometry of the Schwarzschild black hole with one black hole horizon converts to geometry with two horizons: a black hole horizon and a cosmological horizon. In the presence of the quintessence as a background field, we study the tunneling process for massless and massive particles and we obtain the temperature of the black hole. We calculate and compare particles' tunneling rate from both of the black hole horizon and the cosmological horizon. The obtained temperature is regular, radiation modes encompass correlations and there is the Planck scale remnant with the quintessence contents. As an important result, thermodynamics of the Schwarzschild black hole surrounded by the quintessence field has a significant difference in the range $-\frac{2}{3} < w_q < -\frac{1}{3}$ relative to the ordinary case. Also, the behavior of temperatures in our case becomes similar to the Schwarzschild de-Sitter black hole temperatures. Furthermore, we show that quantum correction of the black hole in an embrace of the quintessence field changes the location of horizons but the quintessence field is more effective in this change. Eventually, while the quantum correction prevents to reach the singularity at $r = 0$ in the final stage of the evaporation of the quantum deformed Schwarzschild black hole surrounded by quintessence, there is a Planck scale remnant with quintessence content.

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1. Introduction

There are so many enigmatic objects in the world that scientists have been working on them for a century, nowadays we know that they really exist by a close-up image of the center of galaxy M87; they are the black holes. After releasing the general relativity by Einstein in 1915, the Schwarzschild solution has been accepted as the most important spherically symmetric solution which describes the space-time outside of the gravitational mass. This solution is known as a mathematically specified surface, the so-called “event horizon”; The radius of this surface is named the Schwarzschild radius. A black hole forms if a gravitational mass collapses to itself until it reaches the Schwarzschild radius. Until 1974, it was believed that nothing can escape from the event horizon and then Hawking announced his exciting theoretical discovery that black hole can emit radiation which is called “Hawking radiation” [1]. Hawking’s opinion opened the major ways to new investigations in the field. As an important progress, Parikh and Wilczek explained [2,3] the tunneling process. Particle and antiparticle pairs be created in the neighborhood of horizon; one of these created objects can tunnel through the horizon and go away from the black hole as radiation. However, there are many unsolved problems in this picture such as the temperature divergence and information loss. Also, they considered the quantum process without applying the quantum effects on the black hole itself. We apply Parikh-Wilczek method and solve the mentioned problems by considering quantum effects in a background of quintessence field. Besides the Parikh and Wilczek method, mentioning the Refs. [4–6] is useful for a comprehensive view of the tunneling process. In Ref. [4], authors explained the tunneling process while

the particle treated as a spherical shell and so in their method a shell of matter interacts with a black hole instead of particles moving along the geodesics. Also, authors in Refs. [5,6] applied the method based on the complex paths.

Another issue is that the Schwarzschild metric suffers from the singularity problem. The coordinate singularities could be put down with the coordinate transformation, but the intrinsic singularities stay and prevent a perfect global space-time structure. In this condition, it's clear that in the strong gravitational field (for instance, near the Planck scale), the Schwarzschild solution is not applicable and we should consider the quantum effects in these scales. We don't know exactly what happens inside the black hole and near the horizon, but we hope that the ultimate quantum gravity theory has an answer for these issues finally. As we know, it is possible that quantum corrections propound on the right-hand side or left-hand side of the Einstein field equations. This issue has been discussed in details in several literature. We benefit from the proposed method in Ref. [7] and use a deformed metric due to quantum corrections by considering spherically symmetric excitations. In this picture, space-time is depicted in asymptotically flat sheets paste on the hypersurface of constant radial parameter $r = r_{min}$, so if an observer stays on the sheet out of the horizon, another sheet will be behind of the horizon. As a result, the space-time structure has been regular because of the singularity at $r = 0$ transfers to $r_{min} \equiv r_{Pl}$. Also, they showed that the metric for out of the gravitational mass at scales larger than the Planck scale is not Ricci flat and the scalar curvature tends to zero; So we don't observe it in the gravitational experiment. This deformed metric is pervasive whatever gravitational ghosts and matter be present or absent. More discussion about horizons in this picture is given in Refs. [8,9]. We are going to probe the thermodynamics of such a black hole in the presence of a quintessence field. We expect that linkage between thermodynamics and gravity would be useful for the formulation of the final quantum gravity theory.

Observational data of COBE, WMAP, Planck and other spacecrafts affirms that the universe is not only expanding, but also it has a positively accelerating expansion. These observations mutate our thoughts about the universe. So in late decades, scientists tried to understand this accelerating expansion. One of their suggestions is that the universe is full of the obscure stuff with negative pressure, the so-called "Dark Energy". Dark energy forms the nearly 70 percent of the universe content and we don't know what is it truly yet. One of the candidates for dark energy is the "cosmological constant". However, for accepting the cosmological constant as a candidate for dark energy, there are several unsolved problems yet such as non-evolutionary nature, fine-tuning and coincidence problem. This issue is an accompaniment with some predictions of particle physics and gets proposed the presence of particles with negative pressure such as the quintessence [10,11]. In other words, the quintessence is a scalar field that arises in particle physics and this is one of the important candidates of the dark energy [12–14]. Quintessence action is written by an ordinary scalar field minimally coupled to gravity. The equation of state parameter w_q for quintessence is in the range $-1 \leq w_q \leq -\frac{1}{3}$. What matters to us is the Schwarzschild black hole in a quintessence background and especially its thermodynamics. In Ref. [15], Kiselev solved the Einstein field equation in quintessence background and presented the form of the density for quintessence matter as $\rho_q = -\frac{c}{2} \frac{3w_q}{r^{3(w_q+1)}}$ and then he rendered the metric of this Schwarzschild black hole. In several references, the thermodynamics of the Schwarzschild black hole surrounded by quintessence has been inspected. For example Ref. [15] has ascertained that the temperature of the quintessence Schwarzschild black hole is less than the ordinary Schwarzschild black hole one. Several references have worked on heat capacity and it has been indicated that in the presence of the quintessence field, there is a second-

order phase transition. As a result, for the quintessence Schwarzschild black hole there exists a stable phase against the ordinary Schwarzschild black hole that has a negative heat capacity and is unstable [16,17]. In Ref. [16] it has been proved that the phase transition doesn't happen for a sharp w_q , but it occurs for a range $-\frac{2}{3} < w_q < -\frac{1}{3}$; Moreover, they studied tunneling process in a perturbative way that differs from our work. Also, Reissner-Nordsröm black hole surrounded by quintessence has been studied in Ref. [18] and the phase transition for a charged black hole in the quintessence field has been found out. They have obtained similar results in the presence of the quintessence field [19,20]. In Ref. [19] it has been pointed out that for Reissner-Nordström black hole surrounded by quintessence, by increasing the density of quintessence matter, the phase transition point is shifted toward a lower entropy and temperature of the black hole is decreased too. Mass, temperature, heat capacity and entropy of Schwarzschild and Reissner-Nordsröm black hole surrounded by quintessence have been glanced in Ref. [21].

Along these studies, we consider the thermodynamics of the Schwarzschild black hole in the quintessence field through particle tunneling from the horizon in the framework of semiclassical quantum tunneling that has been proposed by Parikh and Wilczek [2]. We show how changes in different values of w_q influence on the horizons and tunneling processes. Moreover, we consider the quantum corrections and quintessence background together and we probe thermodynamics of such a black hole structure for the first time. It should be noted that in Ref. [22], thermodynamics of quantum corrected Schwarzschild black hole surrounded by quintessence has been studied by using the standard thermodynamic equations but our method is different from the mentioned study.

First, we consider a Schwarzschild black hole in the background of quintessence field and we achieve the temperature of Hawking radiation with a tunneling process for different w_q and for massive and massless particles separately. In section 3, we look into the tunneling process from quantum corrected Schwarzschild black hole's horizon surrounded by quintessence. Finally, we summarize our results in the conclusion.

2. Tunneling of massless and massive particles from Schwarzschild black hole surrounded by quintessence

Since the tunneling process happens on the horizons, before starting of studying tunneling process, we need a precise review of the Schwarzschild black hole metric and horizons in the presence of the minimally coupled scalar field such as the quintessence. The metric is introduced in Ref. [15] as follows

$$ds^2 = -g_{00}(r)dt^2 + g_{00}^{-1}(r)dr^2 + r^2 d\Omega^2, \quad (2.1)$$

$$g_{00}(r) = 1 - \frac{2M}{r} - \frac{c}{r^{3w_q+1}}. \quad (2.2)$$

Where w_q is the equation of state parameter of the quintessence field and c is a normalization factor related to the energy density, $\rho_q = -\frac{c}{2r^{3(w_q+1)}}$. We show the behavior of this metric for various M , c and w_q in Figs. 1 and 2.

Considering various values of w_q , we have different horizon(s). If $w_q = -\frac{1}{3}$, there is one horizon for any values of other parameters. If $w_q = -\frac{2}{3}$, we have two horizons for a Schwarzschild black hole surrounded by quintessence that these are coincident when $M = \frac{1}{8c}$; This value for w_q is very remarkable for us because it is an appropriate limit in confrontation with observation (95% CL) for today's equation of state parameter of cosmic fluid [23]. We have said before, there

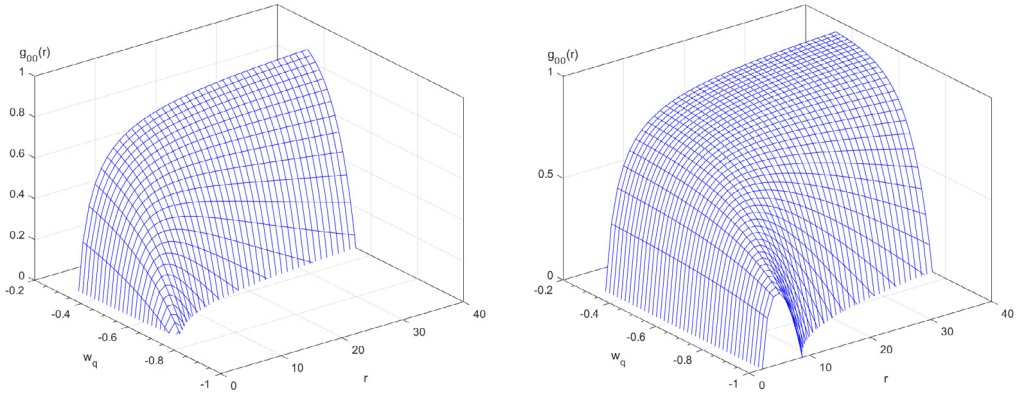


Fig. 1. Behavior of $g_{00}(r)$ versus r and w_q . The right panel is depicted with $c = 0.1$. The left panel is with $c = 0.01$. By decreasing c , the possibility of having two horizons is more probable.

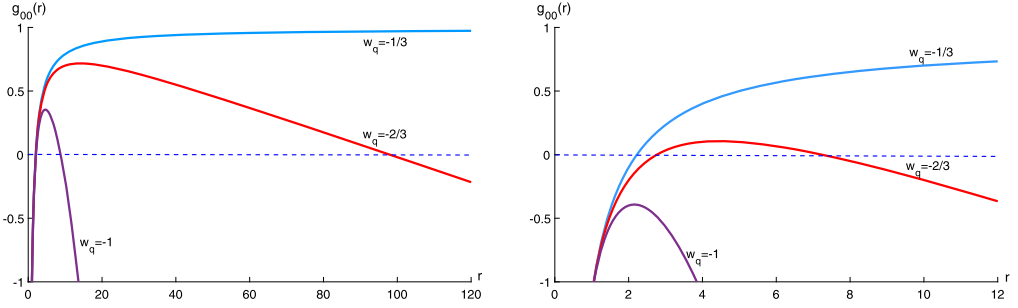


Fig. 2. Behavior of $g_{00}(r)$ versus r with $M = 1$. The curves are depicted from top to down for $w_q = -\frac{1}{3}, -\frac{2}{3}, -1$. The right panel is illustrated with $c = 0.1$ and the left one is with $c = 0.01$.

is a phase transition in the range between $w_q = -\frac{1}{3}$ and $w_q = -\frac{2}{3}$ so it is valuable that compare our result for these two values of w_q . Finally, if $w_q = -1$ it is possible to have two real horizons or no horizon at all. This case corresponds to the Schwarzschild de-Sitter metric [24] and implies that $c \sim \frac{\Lambda}{3}$.

Parikh and Wilczek described Hawking radiation as a particles' tunneling from the horizon. Particle and antiparticle pairs create near the horizon; Since there is no classical path for crossing the horizon, virtual particle tunnels to outside the horizon semi-classically. In this picture, the height of the tunneling wall is determined by tunneling particle itself. In other words, particle tunnels from the barrier that has been created by itself. On the other hand, the hypothetical observer will see that the black hole radius reduces by emitting the particle of energy ω . For this tunneling process, the existence of the metric which is nonsingular on the horizon is necessary but the given metric (2.1) is singular on the horizon and we should resolve this obstacle. To construct the nonsingular line element on the horizon, a new time coordinate has been defined, t_p by $t_p = t - f(r)$ [25,26] named the Painlevé coordinate transformation. Using this transformation on the metric (2.1), we have

$$ds^2 = -g_{00}(r)dt_p^2 + 2f'(r)g_{00}(r)dt_p dr + \left(\frac{1}{g_{00}(r)} - g_{00}(r)(f'(r))^2\right)dr^2 + r^2 d\Omega^2. \quad (2.3)$$

In view of the fact that the constant-time slice should be flat, we obtain the condition $\frac{1}{g_{00}(r)} - g_{00}(r)(f'(r))^2 = 1$. Applying this restriction on the Eq. (2.3), the metric (2.1) changes to the following form

$$ds^2 = -g_{00}(r)dt_p^2 + dr^2 + 2\sqrt{1 - g_{00}(r)} dt_p dr + r^2 d\Omega^2. \quad (2.4)$$

To continue, we discuss the thermodynamics of a quintessence Schwarzschild black hole with different w_q .

2.1. Tunneling of massless particles

2.1.1. The case with $w_q = -\frac{1}{3}$

Substituting $w_q = -\frac{1}{3}$ in Eq. (2.2), we have

$$g_{00}(r) = 1 - \frac{2M}{r} - c, \quad (2.5)$$

where there is one horizon in this case

$$r = \frac{2M}{1-c}. \quad (2.6)$$

First, we calculate imaginary part of the action for a particle that is moving from an initial state in r_{in} to the final state in r_{out}

$$\text{Im}S \equiv \text{Im} \int E dt = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{Pr} d\tilde{p}_r dr, \quad (2.7)$$

where $r_{in} = \frac{2M}{1-c} - \epsilon$ and $r_{out} = \frac{2(M-\tilde{\omega})}{1-c} + \epsilon$, $\tilde{\omega}$ is energy of the particle and we suppose this as a self interaction. With Hamilton equation, $dp_r = \frac{dH}{\dot{r}}$, (2.7) changes to the following form

$$\text{Im}S = \text{Im} \int_{r_{in}}^{r_{out}} \int_M^{M-\tilde{\omega}} \frac{dH}{\dot{r}} dr = -\text{Im} \int_0^{\tilde{\omega}} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} d\tilde{\omega}. \quad (2.8)$$

We consider the light-like geodesics for massless particles' tunneling. Regarded to (2.4), we have

$$\dot{r}_p^2 + 2\sqrt{1 - g_{00}(r)} \dot{r}_p - g_{00}(r) = 0. \quad (2.9)$$

As a result, we find

$$\dot{r}_p = \pm 1 - \sqrt{1 - g_{00}(r)}. \quad (2.10)$$

Where $+$ and $-$ signs indicate the outgoing and ingoing trajectories respectively. By substituting (2.10) in (2.8), the imaginary part of the action is given by

$$\text{Im}S = -\text{Im} \int_0^{\tilde{\omega}} \int_{r_{in}}^{r_{out}} \frac{dr d\tilde{\omega}}{1 - \sqrt{1 - g_{00}(r)}}. \quad (2.11)$$

Given the presence of a pole in the range of integral, we utilize the residue calculus to find

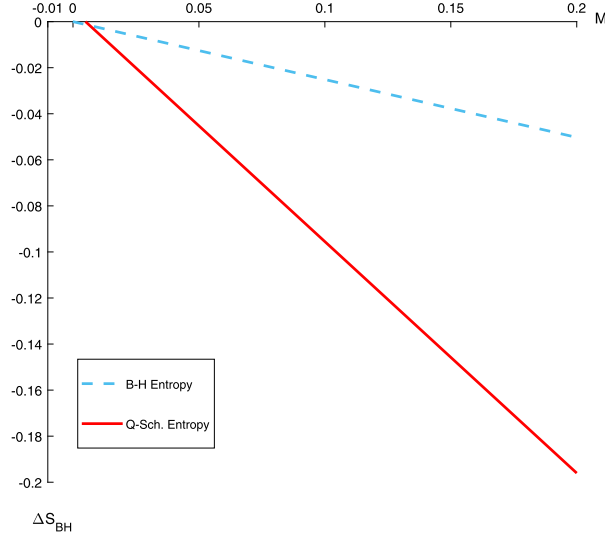


Fig. 3. Behavior of ΔS_{BH} versus M with $c = 0.5$ and $\omega = 0.01$. Ordinary Bekenstein-Hawking Entropy (dashed line) and Quintessence Schwarzschild Entropy with $w_q = -\frac{1}{3}$ (solid line).

$$\text{Im}S = \int_0^\omega \frac{8\pi(M - \tilde{\omega})}{(c-1)^2} d\tilde{\omega}. \quad (2.12)$$

Therefore, the imaginary part of the action is calculated as the following form

$$\text{Im}S = \frac{2\pi\omega(2M - \omega)}{(c-1)^2}. \quad (2.13)$$

Emission rate can be represented in terms of imaginary part of the action as follows

$$\Gamma \simeq e^{-2\text{Im}S} = e^{-\beta\omega} = e^{+\Delta S_{BH}}. \quad (2.14)$$

And finally we get the emission rate as follows

$$\Gamma \simeq \exp\left(-\frac{4\pi\omega(2M - \omega)}{(c-1)^2}\right). \quad (2.15)$$

According to Fig. 3 we see that with particle tunneling, black hole's entropy is reducing ($\Delta S_{BH} < 0$) which is obvious, because with tunneling of particles to out of the black hole horizon, number of surface quanta is reducing and so entropy decreases. Furthermore, in the presence of quintessence with $w_q = -\frac{1}{3}$, this reduction of entropy is stronger. Another issue is that quintessence prevents more evaporation of the black hole. Moreover, for a remnant mass in the presence of quintessence there is no further entropy change for the remnant.

For calculation of the correlation between radiated modes, we apply the method that introduced in Ref. [27] and suppose a massless particle of energy ω_1 and another massless particle of energy ω_2 that are tunneling from the horizon and compute the correlation function as follows

$$\chi(\omega_1 + \omega_2; \omega_1, \omega_2) = \ln[\Gamma(\omega_1 + \omega_2)] - \ln[\Gamma(\omega_1)\Gamma(\omega_2)], \quad (2.16)$$

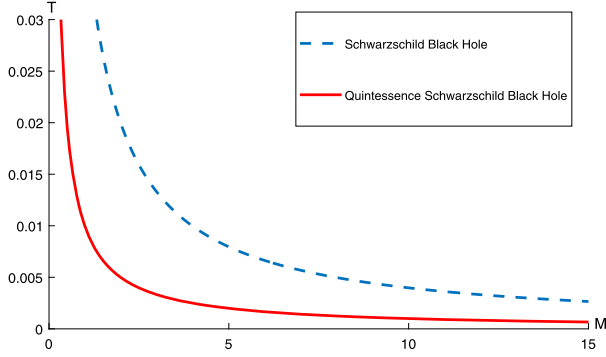


Fig. 4. Temperature versus Mass for ordinary Schwarzschild black hole (dashed line) and Schwarzschild black hole surrounded by a quintessence (solid line). Existence of the quintessence field predicts the lower temperature for black hole, but it can't prevent temperature divergency. The figures are plotted with $c = 0.5$.

$$\begin{aligned}
 \chi(\omega_1 + \omega_2; \omega_1, \omega_2) &= \frac{8\pi(\omega_1 + \omega_2)(2M - (\omega_1 + \omega_2))}{(c-1)^2} \\
 &\quad - \frac{8\pi\omega_1(2M - \omega_1)}{(c-1)^2} - \frac{8\pi\omega_2(2M - \omega_2)}{(c-1)^2} \\
 &= \frac{16\pi\omega_1\omega_2}{(c-1)^2}.
 \end{aligned} \tag{2.17}$$

Correlation function is not zero here; this means that the probability of tunneling of two particles of energy ω_1 and ω_2 is not the same as probability of tunneling of one particle with energy $\omega_1 + \omega_2$ and so there is correlation between the emitted modes. Also, existence of ω^2 in Eq. (2.13) demonstrates existence of correlation between radiate modes and we see this correlation in Eq. (2.17). As a result, the quintessence field causes the information to be emerged from black hole as correlation between emitted modes and this potentially resolves the information loss problem.

For large M , we neglect ω^2 in (2.15) and finally we achieve the temperature of Schwarzschild black hole surrounded by quintessence with $w_q = -\frac{1}{3}$ in the following form

$$T = \frac{1}{\beta} = \frac{(c-1)^2}{8\pi M}. \tag{2.18}$$

By plotting T versus M for ordinary Hawking temperature and Schwarzschild quintessence one, we conclude that quintessence with $w_q = -\frac{1}{3}$ can not prevent temperature divergency in this setup but as will be seen in the next section, with the choice $w_q = -\frac{2}{3}$, the behavior of temperature will become so different. Eq. (2.18) shows that with neglecting c , we have the Hawking temperature, see Fig. 4.

So, as an important result in this subsection, the correlations between the emitted modes are not zero in our study. This means that the probability of tunneling of two particles of energy ω_1 and ω_2 is not the same as probability of tunneling of one particle with energy $\omega_1 + \omega_2$. This means that there are correlations between the emitted modes and these correlations are capable of explaining the issue of information loss in essence.

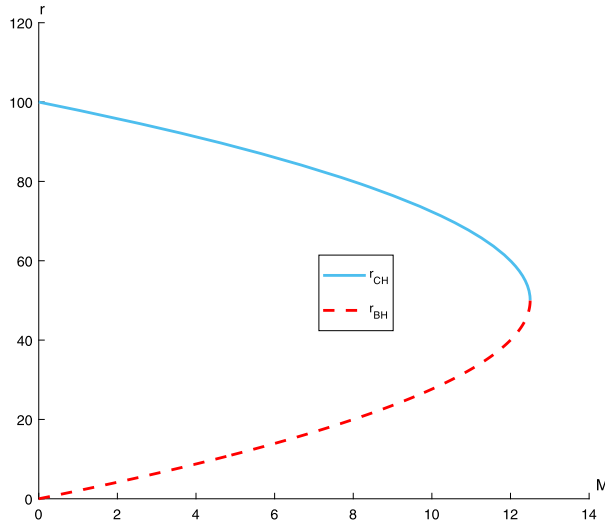


Fig. 5. r versus M with $c = 0.01$, r_{BH} and r_{CH} are coincident for $M = \frac{1}{8c}$.

2.1.2. The case with $w_q = -\frac{2}{3}$

Substituting $w_q = -\frac{2}{3}$ in Eq. (2.2), we have

$$g_{00}(r) = 1 - \frac{2M}{r} - cr. \quad (2.19)$$

In this case there are two horizons located at

$$r_{BH} = \frac{1 - \sqrt{1 - 8cM}}{2c}, \quad r_{CH} = \frac{1 + \sqrt{1 - 8cM}}{2c}. \quad (2.20)$$

Note that r_{BH} is the black hole horizon and r_{CH} is the cosmological horizon similar to the Schwarzschild-de Sitter black hole. As we see from Fig. 5, r_{BH} and r_{CH} are coincided for $M = \frac{1}{8c}$. When $M = \frac{1}{8c}$, there can be something like that the Schwarzschild-de Sitter extreme black hole named Nariai black holes [28,29]. Nariai solution describes such a situation when the event horizon coincides with the cosmological horizon. Besides, compared with the next section one should pay attention that without applying the quantum effect, r_{BH} gets the condition $r = 0$ and so there is an intrinsic singularity yet. In other words, the existence of the quintessence cannot prevent the intrinsic singularity.

Regarded to the fact that there are two horizons, the black hole event horizon and the cosmological horizon, there are two temperatures defined locally with particles' tunneling from each of the horizons. Gibbons and Hawking have investigated the properties of the black hole event horizon and the cosmological horizon in details [30]. They have shown that there are thermodynamics similarities between the black hole event horizon and the cosmological horizon. This issue for black holes in de Sitter background has been regarded in several papers [31–34,36]. In Ref. [31], particles' tunneling from cosmological horizon in de Sitter space has been investigated. In Ref. [32], both of cosmological event horizon temperature and black hole temperature has been calculated and compared. In Ref. [33], the relation between the cosmological event horizon and Hawking temperature has been shown. In Ref. [34], thermodynamics of the black hole in anti-de Sitter space has been investigated. In Ref. [36], it has been pointed out that in

such situations when there are geometry with two horizons, there are two temperatures too, so that there is the non-equilibrium thermodynamics. Therefore, authors in Ref. [36], investigated evolution of such black holes. In the following, we calculate the local temperature of the cosmological horizon and the black hole horizon respectively. First, we consider the outgoing particles' tunneling from the cosmological horizon. Similar to the previous section, we calculate the imaginary part of the action for particle which is going from the initial state at $r_{in} = \frac{1+\sqrt{1-8cM}}{2c} - \epsilon$ to the final state at $r_{out} = \frac{1+\sqrt{1-8c(M-\tilde{\omega})}}{2c} + \epsilon$ as follows

$$\text{Im}S = -\text{Im} \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{dr d\tilde{\omega}}{1 - \sqrt{1 - g_{00}(r)}}. \quad (2.21)$$

Substituting (2.19) in (2.21) we have two poles in the range of integral. So, to deduce the poles we expand the denominator in terms of r_{out}

$$\dot{r}_p = 1 - \sqrt{1 - [g_{00}(r_{out}) + g'_{00}(r_{out})(r - r_{out}) - \dots]}, \quad (2.22)$$

where a prime indicates derivative with respect to r . Therefore, we have

$$\text{Im}S = -\text{Im} \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{dr d\tilde{\omega}}{1 - \sqrt{1 + \left(c - \frac{8c^2(M-\tilde{\omega})}{(1+\sqrt{1-8c(M-\tilde{\omega})})^2}\right)\left(r - \frac{1+\sqrt{1-8c(M-\tilde{\omega})}}{2c}\right)}}. \quad (2.23)$$

Using the residue calculus (see [35]), we find

$$\text{Im}S = - \int_0^\omega \frac{(1 + \sqrt{1 - 8c(M - \tilde{\omega})})^2 d\tilde{\omega}}{c \left[1 - 8c(M - \tilde{\omega}) + \sqrt{1 - 8c(M - \tilde{\omega})}\right]}. \quad (2.24)$$

The imaginary part of the action can be calculated to find

$$\text{Im}S = \frac{\pi}{2c^2} \left[4\omega c - \sqrt{1 - 8cM} + \sqrt{1 - 8c(M - \omega)} \right]. \quad (2.25)$$

According to Eq. (2.14), finally we get the emission rate as follows

$$\Gamma \simeq \exp \left[- \frac{\pi}{c^2} \left(4\omega c - \sqrt{1 - 8cM} + \sqrt{1 - 8c(M - \omega)} \right) \right]. \quad (2.26)$$

By plotting ΔS_{BH} versus M , we see that ΔS_{BH} is rapidly changing for a specific range of the mass and then it changes in the same way as the ordinary Schwarzschild black hole. As might be expected, the final entropic state, in other words, the entropy of state with the quintessence field content is related to the stable and ultraviolet fixed point state, see Fig. 6.

Existence of correlation between radiated modes is clear from Eq. (2.27):

$$\begin{aligned} \chi(\omega_1 + \omega_2; \omega_1, \omega_2) = & \frac{\pi}{c^2} \left[4(\omega_1 + \omega_2)c - \sqrt{1 - 8cM} + \sqrt{1 - 8c(M - \omega_1 + \omega_2)} \right] \\ & - \frac{\pi}{c^2} \left[4(\omega_1)c - \sqrt{1 - 8cM} - \sqrt{1 - 8c(M - \omega_1)} \right] \\ & - \frac{\pi}{c^2} \left[4(\omega_2)c - \sqrt{1 - 8cM} - \sqrt{1 - 8c(M - \omega_2)} \right]. \end{aligned} \quad (2.27)$$

For large M , with Taylor expansion of (2.25) in terms of ω , we obtain

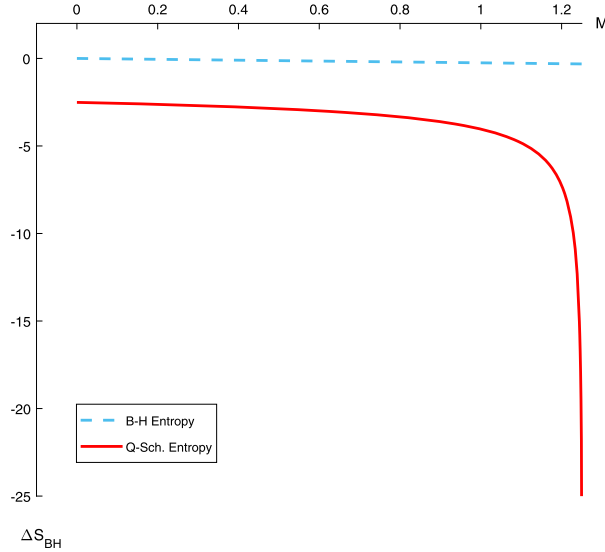


Fig. 6. ΔS_{BH} versus M with $c = 0.1$ and $\omega = 0.01$. Ordinary Bekenstein-Hawking Entropy (dashed line) and Quintessence Schwarzschild Entropy with $w_q = -\frac{2}{3}$ (solid line).

$$\text{Im}S = \frac{2\pi(1 + \sqrt{1 - 8cM})}{c(\sqrt{1 - 8cM})}\omega. \quad (2.28)$$

And finally, we derive the temperature of Schwarzschild black hole surrounded by quintessence with equation of state parameter $w_q = -\frac{2}{3}$ as follows

$$T_{CH} = \frac{1}{\beta} = \frac{c(\sqrt{1 - 8cM})}{4\pi(1 + \sqrt{1 - 8cM})}. \quad (2.29)$$

With the same calculation process, we can set the local temperature due to particles' tunneling from black hole event horizon r_{BH} as follows

$$T_{BH} = \frac{c(\sqrt{1 - 8cM})}{4\pi(1 - \sqrt{1 - 8cM})}. \quad (2.30)$$

Note that both of these temperatures in the absence of the quintessence field recover the Hawking temperature if we rewrite Eqs. (2.29) and (2.30) (according to Eq. (2.20)) in the form $T = \frac{(\sqrt{1-8cM})}{4\pi r}$ and set $c = 0$. As mentioned previously, when the mass of the Schwarzschild black hole reaches a special mass, then the quintessence field (with $w_q = -\frac{2}{3}$) causes the one horizon Schwarzschild black hole to become two horizons Schwarzschild black hole surrounded by the quintessence. Fig. 7 shows that when the mass of black hole reaches to $M = \frac{1}{8c}$, correspond to the coincident horizons, both temperatures are vanishing. This situation may seem stable at the classical level but it's unstable situation obviously under the small perturbations in a quantum or semiclassical viewpoint (similar to Nariai solution for de Sitter space [31]). After that, the temperature of black hole horizon is more than the temperature of cosmological horizon, $T_{BH} \geq T_{CH}$ (see also [15]). Therefore, we can infer that there is a pure current from the black hole horizon towards the cosmological horizon. Eventually, radiation continues to reach

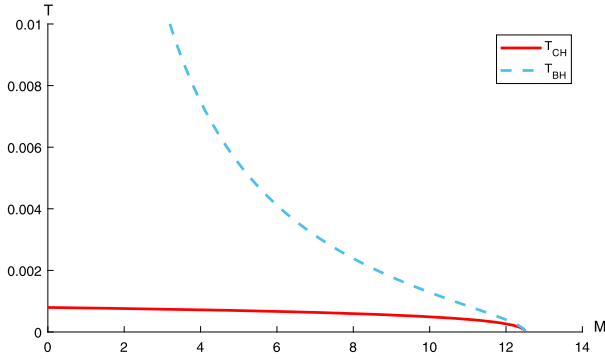


Fig. 7. Temperature of black hole horizon (dashed line) and Temperature of cosmological horizon (solid line) versus Mass. When the Schwarzschild black hole is surrounded by the quintessence with $w_q = -\frac{2}{3}$, the temperature becomes regular. With $M = \frac{1}{8c}$, both temperatures are vanishing. Then, because of $T_{BH} \geq T_{CH}$, there is a pure flow from the black hole horizon towards the cosmological horizon. In the final stage of evaporation, there is a remnant temperature with quintessence content when the mass of the black hole is over. The figure is depicted with $c = 0.01$.

the finite cosmological temperature with the quintessence content. As a more explanation, Fig. 7 demonstrates that the quintessence field with $w_q = -\frac{2}{3}$ prevents the divergence of temperature in the final stage of evaporation of the Schwarzschild black hole and there is a temperature remnant with the quintessence content. Our result for the behavior of the black hole temperature (Eq. (2.30)) and cosmological temperature (Eq. (2.29)) is coincident with the result of several research papers about the temperature of the quintessence Schwarzschild black hole (see for instance [15,29]) and the Schwarzschild in de Sitter Space [25,26,31,36]. Furthermore, to more discussion on the temperature of the black hole and cosmological horizons and related local and non-local thermodynamics, one can refer to Ref. [36]. On the other hand, it should be noted that the main difference between our work and the results of the author in Ref. [22] is that we predicted a remnant temperature for the final stage of the Hawking radiation of the quintessence Schwarzschild black hole. There is no trace of such an important result in Ref. [22].

2.2. Massive particles

As a generalization, now we consider a massive particles' tunneling from the Schwarzschild black hole surrounded by quintessence. In some research papers, authors have considered outgoing massive particles' tunneling in the form of the massive shell and so they have applied the wave equation for that particle and moved forward based on formulation of the WKB approximation [37]. Our work in this section is based on Parikh and Wilczek method [2] and the techniques that have been used in Ref. [38] for massive particles' tunneling. In this method, massive particles cross the horizon on the timelike geodesics. We describe tunneling of this particle from the cosmological horizon of quintessence Schwarzschild black hole and we use the result for the forthcoming section. To read more and discussion about the massive particles' tunneling from the cosmological horizon, one can be referred to Refs. [37,39–44].

To start, we need to derive equation of motion from the following Lagrangian

$$\mathcal{L} = \frac{m}{2} g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}. \quad (2.31)$$

In Ref. [10], authors discussed advantage and disadvantage of exponential potential for quintessence field. Also in Refs. [45,46] the authors introduced different potentials and discussed the corresponding outcomes. However, we continue our calculation without any potential. Using the Painlevé metric for time-like geodesics and equation of motion that is derived from Euler-Lagrange equation with concern on t_p , we have the following two equations

$$-g_{00}(r) \dot{t}_p^2 + \dot{r}_p^2 + 2\sqrt{1 - g_{00}(r)} \dot{t}_p \dot{r}_p = -1, \quad (2.32)$$

$$p_{t_p} \equiv -\frac{\partial \mathcal{L}}{\partial \dot{t}} = m(-g_{00}(r) \dot{t}_p + \sqrt{1 - g_{00}(r)} \dot{r}_p) = cte. \equiv \omega. \quad (2.33)$$

With these two equations, we obtain

$$\dot{r}_p = \sqrt{\omega^2 - m^2 g_{00}(r)}, \quad (2.34)$$

$$\dot{t}_p = \frac{1}{g_{00}(r)} \left(\omega + \sqrt{1 - g_{00}(r)} \sqrt{(\omega^2 - m^2 g_{00}(r))} \right). \quad (2.35)$$

Finally, the radial motion of particle has the following form

$$\dot{r} = \frac{g_{00}(r) \sqrt{\omega^2 - m^2 g_{00}(r)}}{\omega + \sqrt{(1 - g_{00}(r))(\omega^2 - m^2 g_{00}(r))}}. \quad (2.36)$$

We use this equation in next subsection for the quintessence black hole and the quantum quintessence black hole with different w_q .

2.2.1. The case with $w_q = -\frac{1}{3}$

Substituting (2.5) into (2.36) and replacing $\dot{r}$ in imaginary part of the action (Eq. (2.8)) leads to

$$\text{Im}S = -\text{Im} \int_m^\omega \int_{r_{in}}^{r_{out}} \frac{\tilde{\omega} + \sqrt{(c + 2 \frac{M-\tilde{\omega}}{r})(-m^2(1-c-2 \frac{M-\tilde{\omega}}{r}) + \tilde{\omega}^2)}}{(1-c-2 \frac{M-\tilde{\omega}}{r}) \sqrt{[-m^2(1-c-2 \frac{M-\tilde{\omega}}{r}) + \tilde{\omega}^2]}} dr d\tilde{\omega}, \quad (2.37)$$

where r_{in} and r_{out} are defined in section 2.1.1. One notices that considering a massive particle leads to the lower limit of integral in Eq. (2.8) changes from m to ω in Eq. (2.37) [38]. Imaginary part of the action is calculable with the following integral

$$\text{Im}S = -\int_m^\omega 4\pi \frac{M - \tilde{\omega}}{(c-1)^2} d\tilde{\omega}. \quad (2.38)$$

Finally, we have

$$\text{Im}S = -\frac{2\pi}{(c-1)^2} \left[2M(m-\omega) - (m^2 - \omega^2) \right]. \quad (2.39)$$

Also, the emission rate is calculated as follows

$$\Gamma \simeq \exp \left[\frac{4\pi}{(c-1)^2} \left(2M(m-\omega) + (m^2 - \omega^2) \right) \right]. \quad (2.40)$$

The result of comparing (2.13) and (2.39) is that the mass of the particle in the tunneling process causes the change of entropy to be increased. Using Eqs. (2.14) and (2.40), we obtain the

temperature of the Schwarzschild black hole surrounded by quintessence with $w_q = -\frac{1}{3}$, with tunneling of a massive particle from the horizon as follows

$$T = \frac{1}{\beta} = \frac{(c-1)^2}{8\pi M}. \quad (2.41)$$

As we see, based on comparing (2.41) and (2.18), temperature of the black hole has no dependence on the mass of the tunneling particle. In other words, the mass of the particle has no effect on temperature, see the same result in Refs. [37–40,47].

2.2.2. The case with $w_q = -\frac{2}{3}$

Substituting (2.19) into (2.36) and replacing $\dot{r}$ in imaginary part of the action, we obtain

$$\text{Im}S = -\text{Im} \int_m^{\omega} \int_{r_{in}}^{r_{out}} \frac{\tilde{\omega} + \sqrt{(cr + \frac{2(M-\tilde{\omega})}{r})(\tilde{\omega}^2 - m^2(1 - cr - \frac{2(M-\tilde{\omega})}{2}))}}{(1 - cr - \frac{2(M-\tilde{\omega})}{2})\sqrt{(cr + \frac{2(M-\tilde{\omega})}{r})(\tilde{\omega}^2 - m^2(1 - cr - \frac{2(M-\tilde{\omega})}{2}))}} dr d\tilde{\omega}. \quad (2.42)$$

With residue calculus, we achieve

$$\text{Im}S = - \int_m^{\omega} \frac{\pi (1 + \sqrt{1 - 8c(M - \tilde{\omega})})}{c \sqrt{1 - 8c(M - \tilde{\omega})}} d\tilde{\omega}. \quad (2.43)$$

Therefore, the imaginary part of action is calculated as the following form

$$\text{Im}S = -\frac{\pi}{4c^2} \left[4c(m - \omega) + \sqrt{1 - 8c(M - m)} - \sqrt{1 - 8c(M - \omega)} \right]. \quad (2.44)$$

Finally we derive the emission rate as follows

$$\Gamma \simeq \exp \frac{\pi}{2c^2} \left(4c(m - \omega) + \sqrt{1 - 8c(M - m)} - \sqrt{1 - 8c(M - \omega)} \right). \quad (2.45)$$

As has been explained previously, in comparison with the emission rate of massless particle tunneling (2.26), the emission rate of massive particle tunneling is larger. If we calculate temperature, as expected, results would be the same as equations (2.29) and (2.30). To summarize, trace of the particles' mass just can be sensed in the surface of the black hole and related entropy and emission rate but there is no trace of the mass in temperature of radiation that has received at infinity.

3. Temperature of quantum deformed Schwarzschild black hole surrounded by quintessence through the tunneling process

In this section, we investigate particle tunneling from a Schwarzschild black hole that is deformed with quantum correction and surrounded by the quintessence field. To start the discussion, firstly we should determine the metric and horizons of this black hole as has been introduced in Ref. [22] as follows

$$ds^2 = -g_{00}(r)dt^2 + g_{00}^{-1}(r)dr^2 + r^2 d\Omega^2, \quad (3.1)$$

$$g_{00}(r) = -\frac{2M}{r} + \frac{1}{r}\sqrt{r^2 - a^2} - \frac{c}{r^{3\omega_q+1}}. \quad (3.2)$$

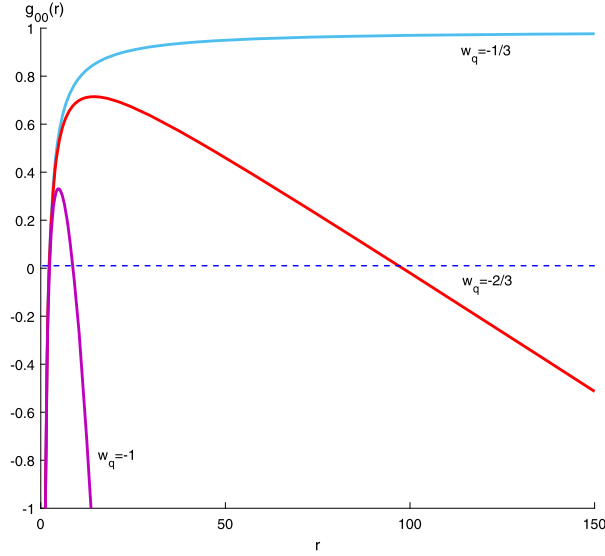


Fig. 8. Behavior of $g_{00}(r)$ versus r with $c = 0.01$ and $a = 0.1$. Diagrams are depicted from top to down for $w_q = -\frac{1}{3}, -\frac{2}{3}, -1$.

Where a is a constant related to the gravitational constant and it has the dimension of length [7]. Fig. 9 demonstrates the behavior of $g_{00}(r)$ for different values of equation of state parameter and the quintessence normalization factor (which is connected with the energy density of the quintessence field). Also, comparing Fig. 8 with Fig. 2 or equations (3.2) and (2.2), we see that the effect of quintessence is more effective than the quantum effect which is an obvious result, because the quantum effect is usually more important for $r \sim r_{Pl}$ regime and it prevents formation of an intrinsic singularity.

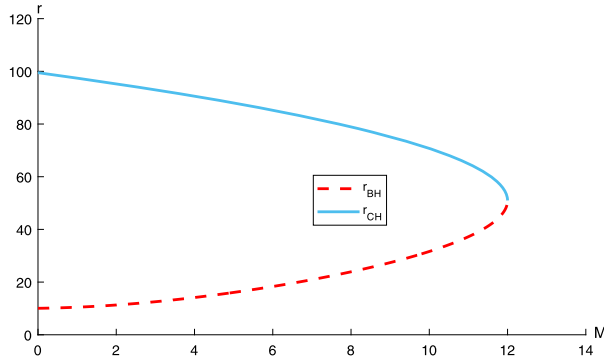
3.1. Massless particles

To continue our study of tunneling process with massless particles, we set $w_q = -\frac{2}{3}$ and find the location of horizons analytically as follows

$$g_{00}(r) = -\frac{2M}{r} + \frac{1}{r}\sqrt{r^2 - a^2} - cr = 0, \quad (3.3)$$

$$\begin{cases} r_{BH} = \sqrt{\frac{1-4Mc-\sqrt{1-4a^2c^2-8cM}}{2c^2}}, \\ r_{CH} = \sqrt{\frac{1-4Mc+\sqrt{1-4a^2c^2-8cM}}{2c^2}}. \end{cases} \quad (3.4)$$

As in the previous section, there are two horizons: the black hole horizon (r_{BH}) and the cosmological horizon (r_{CH}). It should be noted that for $M = \frac{1-4a^2c^2}{8c}$ two horizons will be coincident; in comparison with a black hole without quantum correction (section 2), we see that this mass is smaller. Actually, when the mass of the black hole reaches this special value, quantum effects and quintessence field form two new horizons and with decreasing of mass, the difference

Fig. 9. r versus M with $c = 0.01$ and $a = 10$.

between these two increases. As an important result in this section, when quantum correction is considerable, it prevents to reach the singularity at $r = 0$, as it is illustrated in Fig. 9.

Within the standard method of tunneling process, we apply the Painlevé coordinates and pay attention to the lightlike geodesics for tunneling of a massless particle to arrive at the imaginary part of the action as follows

$$\text{Im}S = -\text{Im} \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{dr d\tilde{\omega}}{1 - \sqrt{1 + \frac{2M}{r} - \frac{\sqrt{r^2 - a^2}}{r}} + cr}. \quad (3.5)$$

Using the residue theorem and extension of the metric around the cosmological horizon, we have

$$\text{Im}S = - \int_0^\omega \frac{4\pi[1 - 4c(M - \tilde{\omega}) + \zeta] \sqrt{1 - 2a^2c^2 - 4c(M - \tilde{\omega}) + \zeta}}{c \left[-2\sqrt{2}a^2c^2 + (1 - 8c(M - \tilde{\omega}) + \zeta) \sqrt{1 - 2a^2c^2 - 4c(M - \tilde{\omega}) + \zeta} \right]} d\tilde{\omega}, \quad (3.6)$$

where ζ is defined as

$$\zeta \equiv \sqrt{1 - 4a^2c^2 - 8c(M - \tilde{\omega})}. \quad (3.7)$$

Finally, we find the imaginary part of the action as follows

$$\text{Im}S = \frac{\pi}{2c^2} \left[4\omega c - \sqrt{1 - 4a^2c^2 - 8Mc} + \sqrt{1 - 4a^2c^2 - 8c(M - \omega)} \right]. \quad (3.8)$$

According to (2.14) and Taylor expansion of (3.8), we find the local temperature of quantum corrected Schwarzschild black hole surrounded by quintessence with $w_q = -\frac{2}{3}$ through the tunneling of massless particle from the cosmological horizon in the following form

$$T_{CH} = \frac{1}{\beta} = \frac{c\sqrt{1 - 4a^2c^2 - 8cM}}{4\pi(1 + \sqrt{1 - 4a^2c^2 - 8cM})}. \quad (3.9)$$

Without repeating the calculations, the local temperature of the black hole horizon is obtained as follows

$$T_{BH} = \frac{1}{\beta} = \frac{c\sqrt{1 - 4a^2c^2 - 8cM}}{4\pi(1 - \sqrt{1 - 4a^2c^2 - 8cM})}. \quad (3.10)$$

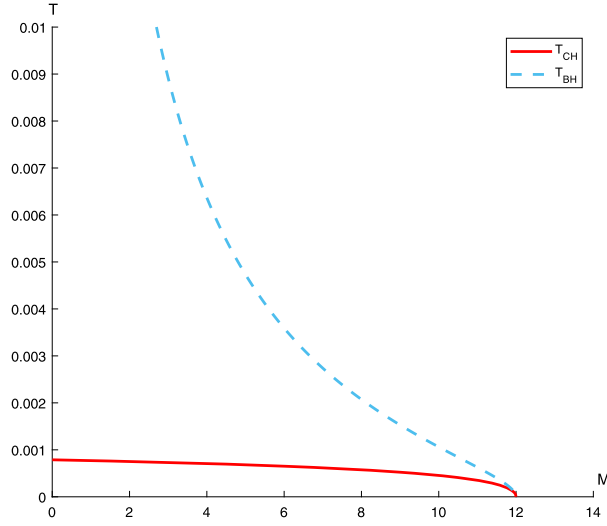


Fig. 10. Temperature of black hole horizon (dashed line) and Temperature of cosmological horizon (solid line) versus Mass. With $M = \frac{1-4a^2c^2}{8c}$, both temperatures are vanishing. Considering quantum corrections in the presence of the quintessence field, by decreasing M , one gains the remnant with the Planck scale and quintessence content. The figure is depicted with $c = 0.01$ and $a = 10$.

In comparison with the result of the local temperature of the black hole and the cosmological horizons without considering quantum correction, the general behavior of temperature doesn't change significantly. In other words, the behavior of temperature is similar to the last result in section 2.1.2, but the important point is that the effect of the quintessence field on temperature is more significant than the quantum correction term. Therefore, even though the quantum corrections prevent divergence of temperature [47], but the quintessence field has an even more important role in this respect. Eventually, while the quantum correction prevents to reach the singularity at $r = 0$, in the final stage of the evaporation of Schwarzschild black hole surrounded by quintessence, there is a Planck scale remnant with quintessence content, see Fig. 10.

3.2. Massive particles

To completion, regarding Eq. (2.8) and substituting (3.3) into (2.36) we have

$$\text{Im}S = -\text{Im} \int_m^{r_{out}} \int_{r_{in}} \frac{\tilde{\omega} + \sqrt{(1 + \frac{2M}{r} - \frac{1}{r}\sqrt{r^2 - a^2} + cr)(\tilde{\omega}^2 - m^2(-\frac{2(M-\tilde{\omega})}{r} + \frac{1}{r}\sqrt{r^2 - a^2} - cr))}}{(-\frac{2(M-\tilde{\omega})}{r} + \frac{1}{r}\sqrt{r^2 - a^2} - cr)\sqrt{\tilde{\omega}^2 - m^2(-\frac{2(M-\tilde{\omega})}{r} + \frac{1}{r}\sqrt{r^2 - a^2} - cr)}} dr d\tilde{\omega}. \quad (3.11)$$

By solving this integral and expanding in terms of ω , the imaginary part of the action is in the following form

$$\text{Im}S = \frac{2\pi(1 + \sqrt{1 - 4a^2c^2 - 8cM})}{c\sqrt{1 - 4a^2c^2 - 8cM}}(\omega - m) - \frac{4\pi}{(1 - 4a^2c^2 - 8cM)^{\frac{3}{2}}}(\omega^2 - m^2) + \dots \quad (3.12)$$

Existence of m and ω^2 and higher order terms of m and ω in the imaginary part of the action, displays that the emission rate is not purely thermal and there is a correlation between radiation modes ($\chi \neq 0$).

By considering Eqs. (2.14) and (3.12), finally we find the temperature of quantum corrected Schwarzschild black hole surrounded by quintessence with tunneling of a massive particle from the cosmological horizon as follows

$$T_{CH} = \frac{1}{\beta} = \frac{c\sqrt{1-4a^2c^2-8cM}}{\pi(1+\sqrt{1-4a^2c^2-8cM})}. \quad (3.13)$$

As we have explained previously, this is the temperature of cosmological horizon due to massless particle tunneling because the mass alone cannot change the temperature.

4. Summary and conclusion

We probed the tunneling process of a Schwarzschild black hole in the background of a minimally coupled scalar field (such as a minimally coupled quintessence field).

Firstly we checked the geometry of the quintessence Schwarzschild black hole and demonstrated that c , as the normalization factor (which is related to the energy density) and w_q are effective on the number of horizons. By increasing c , the probability of formation of two horizons for smaller w_q is decreasing. Moreover, due to the observational evidence for w_q , we regarded the special case $w_q = -\frac{2}{3}$.

We have shown that if $w_q = -\frac{1}{3}$, the temperature of the quintessence Schwarzschild black hole is less than the ordinary Schwarzschild black hole temperature but the general behavior of the temperature doesn't change significantly. But, if $w_q = -\frac{2}{3}$, there is the considerable difference. After the mass of the black hole reaches a special value, $M = \frac{1}{8c}$, the geometry changes to a black hole with two horizons: a black hole horizon and a cosmological horizon. By considering the tunneling process from both horizons, we conclude that the temperature of the black hole horizon is larger than the cosmological horizon. So, there is the net flow from the black hole horizon towards the cosmological horizon. Eventually, radiation continues to reach the finite cosmological temperature with the quintessence content. We have demonstrated that the final stage of evaporation of the Schwarzschild black hole surrounded by quintessence is a remnant with no gravitational mass content, but with the quintessence field. As a comparison, one can see that the behavior of temperature versus the mass matches with the behavior of temperature of Schwarzschild black hole in de Sitter space. The other significant result is a major change in the temperature of the Schwarzschild black hole surrounded by quintessence in the range of $-\frac{2}{3} < w_q < -\frac{1}{3}$. In Ref. [16], it has been indicated that a phase transition occurs in this range and as we have shown, the behavior of temperature is changing significantly in this special range for w_q .

We have studied the change of entropy in the tunneling process. In the case $w_q = -\frac{2}{3}$, an interesting effect has been observed for the special mass range; variation of entropy is rapidly and after that, the behavior of entropy variation is the same as in ordinary Schwarzschild black hole. Adding a mass for the tunneling particle in the tunneling process from a quintessence Schwarzschild black hole's horizons, results in that the mass of particle causes more reduction of the entropy of the black hole. In summary, the trace of the particles' mass just can be sensed in the surface of the black hole and related entropy and emission rate, but there is no trace of the mass in temperature of radiation that has received at infinity.

After all, in section 3, by adding quantum effect on the Schwarzschild black hole, we have considered the tunneling process from the cosmological horizon and the black hole horizon of quantum deformed Schwarzschild black hole surrounded by a minimally coupled scalar field such as a quintessence field. Considering quantum effects caused by smaller c , there are two horizons for Schwarzschild black hole in the quintessence field with $w_q \leq -\frac{2}{3}$. Also, quantum effects form a regular space-time in $r \sim r_{PI}$ and prevent from the singularity at $r = 0$. The incorporation of quantum effects in our case as studied here leads to a reduction of the special mass of the black hole, $M = \frac{1-4a^2c^2}{8c}$, which after it, the black hole is mainly affected by the quintessence field. Also, the quantum correction leads to less temperature for both the black hole and the cosmological temperatures. Eventually, while the quantum corrected prevent to reach the singularity at $r = 0$, in the final stage of the evaporation of quantum deformed Schwarzschild black hole surrounded by quintessence, there is the remnant with the Planck scale and the quintessence content. Finally, we note that although we have started with the same metric as in Ref. [22], but the author of Ref. [22] has applied the standard thermodynamics equations while we have worked in the Parikh-Wilczek tunneling framework which is essentially a quantum tunneling process. The main difference between our work and Ref. [22] is that while in Ref. [22] the black hole completely evaporates and there will be no trace of black hole remnant, in our tunneling mechanism black hole cannot evaporate totally and there would be a black hole remnant (a Planck size remnant) that could be interpreted as a candidate for the Dark Matter.

CRediT authorship contribution statement

Kourosh Nozari: Supervision, Conceptualization, Methodology, Checking all calculations, Writing - reviewing and editing. **Sareh Eslamzadeh:** Investigation, Calculations, Software, Data curation, Writing - original draft preparation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.nuclphysb.2020.115136>.

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