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DISPERSION RELATIONS FOR A CYLINDRICAL WAVEGUIDE WITH MULTILAYER WALLS *

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Abstract

An algorithm to determine the eigenvalues of the eigenfunctions of a multilayer cylindrical waveguide is constructed. A relationship of dispersion relations and impedances is found and a method to calculate the resonant frequencies of the wake field for a linear and a helical motion of a particle is described. The damping coefficients of the eigenmodes at the resonant frequencies are determined.

INTRODUCTION

The bulkiness of the system of equations determining the eigen numbers of the eigenmodes of a multilayer waveguide with a large number of layers, Fig.1, prevents its effective use. Relatively simple equations can be compiled for single-layer [1] and two-layer [2, 3] walls. A simple algorithm for the efficient calculation of impedances and wake fields for a waveguide with an arbitrary number of layers was developed in [4]. A similar algorithm can be constructed for the eigenwave numbers. The two algorithms can be combined into a single system.

BUILDING A SOLUTION

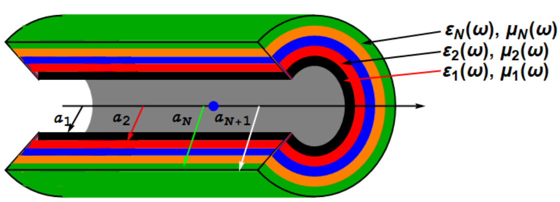

Figure 1: Multilayer cylindrical waveguide.

In the case of cylindrical symmetry an arbitrary electrical $\vec{E}^i$ and magnetic $\vec{H}^i$ field component, generated in each layer $(i = 0,\ 1,\ 2\ldots, N+1)$ of the pipe (including inner $i = 0$ and external $i = N+1$ vacuum regions) can be presented by the superposition of the fundamental solutions of the homogeneous Maxwell equations $\vec{E}_m^{(i,Z)}, \vec{H}_m^{(i,Z)}$:

$$\vec{E}^{(i)} = \begin{cases} \sum_{m=-\infty}^{\infty} \vec{E}_m^{(i,J)}, & i = 0 \\ \sum_{m=-\infty}^{\infty} \left\{\vec{E}_m^{(i,J)} + \vec{E}_m^{(i,H)}\right\}, & i = 1,2\ldots N \\ \sum_{m=-\infty}^{\infty} \vec{E}_m^{(i,H)}, & i = N+1 \end{cases}$$

$$\vec{H}^{(i)} = \begin{cases} \sum_{m=-\infty}^{\infty} \vec{H}_m^{(i,J)}, & i = 0 \\ \sum_{m=-\infty}^{\infty} \left\{\vec{H}_m^{(i,J)} + \vec{H}_m^{(i,H)}\right\}, & i = 1,2\ldots N, \\ \sum_{m=-\infty}^{\infty} \vec{H}_m^{(i,H)}, & i = N+1 \end{cases}$$

where $(Z = J, H)$

$$\vec{E}_m^{(i,Z)} = A_m^{(i,Z)} \vec{E}_{m,TM}^{(i,Z)} + B_m^{(i,Z)} \vec{E}_{m,TE}^{(i,Z)},$$

$$\vec{H}_m^{(i,Z)} = A_m^{(i,Z)} \vec{H}_{m,TM}^{(i,Z)} + B_m^{(i,Z)} \vec{H}_{m,TE}^{(i,Z)} \quad (1)$$

Here $A_m^{(i,Z)}$ and $B_m^{(i,Z)}$ are arbitrary weight coefficients;

$$\vec{E}_{m,TM}^{(i,Z)} = -\nu_m^{(i)^{-2}} \mathrm{rot}\, \vec{R},\quad Z_0 \vec{H}_{m,TM}^{(i,Z)} = j\varepsilon_i k \nu_m^{(i)^{-2}} \vec{R},$$
$$Z_0 \vec{H}_{m,TE}^{(i,Z)} = -\nu_m^{(i)^{-2}} \mathrm{rot}\, \vec{R},\quad \vec{E}_{m,TE}^{(i,Z)} = -j\mu_i k \nu_m^{(i)^{-2}} \vec{R},$$
$$\vec{R} = \vec{e}_z \times \vec{\nabla} P^Z,\ P^z = Z_m e^{j(n\varphi + pz - \omega t)}. \quad (2)$$

If $Z = J$, $Z_m = J_m\left(\nu_m^{(i)} r\right)$ and if $Z = H$, $Z_m = H_m^{(1)}\left(\nu_m^{(i)} r\right)$; $Z_0 = 120\pi\ \Omega$ is the impedance of free space; ε_i, μ_i are the electric and magnetic permeability of i^{th} layer. In (2) p is the longitudinal eigenvalue of the m^{th} mode. For a given mode it is the same for all layers; $\nu_m^{(i)}$ is the transverse eigenvalue of the m^{th} mode in the i^{th} layer. Its relationship with the transverse eigenvalue of the same mode in the internal vacuum cavity ($k = \omega/c$, ω is the frequency, c is the speed of light in vacuum, $\nu_i = \nu_m^{(i)}$, $i = 0,\ 1,\ 2\ldots N+1$) is:

$$\nu_i^2 = k^2 \varepsilon_i \mu_i - p^2 = k^2(\varepsilon_i \mu_i - 1) + \nu_0^2. \quad (3)$$

The homogeneous system of equations (in the absence of charges and currents in the cavity of the waveguide), obtained by matching the tangential field components at the boundaries, reduces to a system of four equations (see [4]):

$$\hat{D}(k, \nu_0) \cdot \hat{X} = 0 \quad (4)$$

where $\hat{X} = \left\{A_m^{(0,J)},\ A_m^{(N+1,H)},\ B_m^{(0,J)},\ B_m^{(N+1,H)}\right\}$ is a four-element single-column and $\hat{D}\left(k, \nu_m^{(0)}\right)$ is a four-column square matrix:

$$\hat{D}(k, \nu_0) = \hat{Q} \widehat{W}_H(\nu_{N+1} a_{N+1}) - \widehat{W}_J(\nu_0 a_1) \quad (5)$$
$$\hat{Q} = \prod_{i=1}^{N} \{W(\nu_i a_i) W^{-1}(\nu_i a_{i+1})\},$$
$$W(\nu_i r) = W_J(\nu_i r) + W_H(\nu_i r)$$

$$W_J(\nu_i r) = \begin{pmatrix} -\frac{mp}{\nu_i^2 r} J_m(\nu_i r) & 0 & -\frac{j\mu_i k}{\nu_i} J'_m(\nu_i r) & 0 \\ J_m(\nu_i r) & 0 & 0 & 0 \\ \frac{j\varepsilon_i k}{\nu_i} J'_m(\nu_i r) & 0 & -\frac{mp}{\nu_i^2 r} J_m(\nu_i r) & 0 \\ 0 & 0 & J_m(\nu_i r) & 0 \end{pmatrix}$$

$$W_H(\nu_i r) = \begin{pmatrix} 0 & -\frac{mp}{\nu_i^2 r} H_m(\nu_i r) & 0 & -\frac{j\mu_i k}{\nu_i} H'_m(\nu_i r) \\ 0 & H_m(\nu_i r) & 0 & 0 \\ 0 & \frac{j\varepsilon_i k}{\nu_i} H'_m(\nu_i r) & 0 & -\frac{mp}{\nu_i^2 r} H_m(\nu_i r) \\ 0 & 0 & 0 & H_m(\nu_i r) \end{pmatrix}$$

The equation for ν_0 follows from the requirement of the existence of a nonzero solution of equation (4):

$$det\left\{\hat{D}(k, \nu_0)\right\} = 0 \quad (6)$$

The uniqueness of the solution of eq. (6) is provided by the additional initial conditions. In the presence of a highly conductive outer wall, they look like this: $\nu_0 \to j_{m,i}$ $\left(J_m(j_{m,i}) = 0\right)$ at $\omega \to 0$ for TM modes and $\nu_0 \to \nu_{m,i}$ $\left(J'_m(\nu_{m,i}) = 0\right)$ at $\omega \to 0$ for TE modes [2].

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RELATIONSHIP WITH RADIATION

The frequency distributions of the wake fields (impedances) in the presence of a particle are also calculated using the partial area method [4]. In contrast to the calculations of the eigenvalues, here, in addition to the general solutions of the homogeneous Maxwell equations containing indefinite weight factors, particular solutions of the non-uniform Maxwell equations are also required. The result is a system of linear equations with non-zero right-hand sides, which in this case is reduced to a system of four inhomogeneous equations [4] relative to the weighting factors to be determined. As a partial solution, as a rule, charge fields in free space [5] of the solution for the ideal waveguide [4] are used. The phase factors of the particular and general solutions must be consistent.

The phase factor of a particular solution of the inhomogeneous Maxwell's equations is determined by the shape of the current density due to a moving point particle. In the case of a particle moving parallel to the waveguide axis with a speed V, its phase factor has the form $\omega/\mathrm{V}\,\mathrm{z}-\omega\mathrm{t}$, and the phase factor of the eigenmode [4] is equal to $\sqrt{\omega^2/c^2-\nu_0^2}\mathrm{z}-\omega\mathrm{t}$. Equating them to each other, we obtain for the transverse eigenvalue $\nu_0=\mathrm{f}(\omega)=\mathrm{j}\,\omega/(\mathrm{V}\gamma)$, where $\gamma=1/\sqrt{1-\mathrm{V}^2/c^2}$ the Lorentz factor and j imaginary unit.

In the case of a helical motion of a particle, the phase factor of its charge density has the form $(\omega-\mathrm{m}\omega_0)/\mathrm{V}\,\mathrm{z}-\omega\mathrm{t}$ [6, 7], where now V is the longitudinal velocity of the particle, and ω_0 is the revolution frequency. The transverse wave number now looks like:

$$\nu_0=\mathrm{f}(\omega)=\sqrt{\omega^2/c^2-(\omega-\mathrm{m}\omega_0)^2/\mathrm{V}^2}. \tag{7}$$

As a result, the equation for the amplitudes $\hat{X}$ of the m^{th} term of the multipole expansion of the radiation field for both linear and helical motion of a particle in a multilayer waveguide is written as:

$$\hat{D}\big(k,\mathrm{f}(\omega)\big)\cdot\hat{X}+\hat{G}=0 \tag{8}$$

with the solution

$$\hat{X}=-\hat{D}^{-1}\big(k,\mathrm{f}(\omega)\big)\cdot\hat{G} \tag{9}$$

Here $\hat{G}=\{E_\varphi^0,\ E_z^0,\ Z_0H_\varphi^0,\ Z_0H_z^0\}$ is a 4-element single-column matrix with the tangential electric and magnetic field components of the partial solution on the inner surface of the waveguide $r=a_1$. As a particular solution, the solution for the motion of a particle in an ideal waveguide [4] can be used in case of a linear motion of a particle. In the case of a helical motion of a particle, the solution for the radiation field of a particle moving along a helical trajectory in free space, obtained in [7], where it was used as a particular solution in constructing a complete solution for the radiation field of particle moving along a helical trajectory in a resistive waveguide, is applicable. The transition to the case of a single-layer resistive waveguide [7] is carried out at $N=2$ and $a_2=a_1$ with $a_3\to\infty$ in (1) - (5).

Impedances are determined by means of the Lorentz force acting on the test particle located at the arbitrary point r, θ, z (cylindrical coordinate system associated with the waveguide axis, Fig. 1) in the inner cavity of waveguide:

$$\vec{\mathrm{F}}=\vec{\mathrm{E}}_{\mathrm{m}}^{(0,\mathrm{J})}+\vec{\mathrm{V}}/\mathrm{c}\times\mathrm{Z}_0\vec{\mathrm{H}}_{\mathrm{m}}^{(0,\mathrm{J})} \tag{10}$$

Substitution of (1) for i = 0 and Z = J into (10) gives the cylindrical components of the Lorentz-force:

$$\vec{\mathrm{F}}_{\mathrm{m}}=\mathrm{A}_{\mathrm{m}}^{(0,\mathrm{J})}\left\{\frac{1}{c^2\gamma}\mathrm{J}'_{\mathrm{m}}(\mathrm{f}(\omega)\mathrm{r}),\ \frac{1}{c^2}\frac{\mathrm{mV}}{\mathrm{r}\omega}\mathrm{J}_{\mathrm{m}}(\mathrm{f}(\omega)\mathrm{r}),\mathrm{J}_{\mathrm{m}}(\mathrm{f}(\omega)\mathrm{r})\right\}e^{\mathrm{j}\mathrm{m}\theta}.$$

Here $\mathrm{F}_{\mathrm{m,r}}/\mathrm{qr}_{\mathrm{q}}$, $\mathrm{F}_{\mathrm{m},\theta}/\mathrm{qr}_{\mathrm{q}}$ are transverse impedance components, $\mathrm{F}_{\mathrm{m,z}}/\mathrm{q}$ is the longitudinal impedance and r_{q} is an offset. The amplitude $\mathrm{A}_{\mathrm{m}}^{(0)}$ can be directly expressed in terms of the elements of the matrix $\hat{\mathrm{D}}\big(\mathrm{k},\mathrm{f}(\omega)\big)$:

$$\mathrm{A}_{\mathrm{m}}^{(0,\mathrm{J})}=\mathrm{B}_\theta^0\det\{\hat{\mathrm{D}}^{(3,1)}\big(\mathrm{k},\mathrm{f}(\omega)\big)\}/\mathrm{Det}\{\hat{\mathrm{D}}\big(\mathrm{k},\mathrm{f}(\omega)\big)\},$$

where $\hat{\mathrm{D}}^{(3,1)}(\omega)$ is the algebraic complement of the element {3,1} of the matrix $\hat{\mathrm{D}}\big(\mathrm{k},\mathrm{f}(\omega)\big)$, B_φ^0 is the single non-zero term in the matrix $\hat{G}$ [4]:

$$\mathrm{B}_\theta^0=-\mathrm{j}\frac{\mathrm{qZ_0h_m}}{\pi\beta\mathrm{a}_1}\frac{\mathrm{J_m}(\mathrm{f}(\omega)\mathrm{r_q})}{\mathrm{J_m}(\mathrm{f}(\omega)\mathrm{a}_1)},\quad \beta=\frac{\mathrm{V}}{\mathrm{c}},\ \mathrm{h}_0=1,\ \mathrm{h}_{\mathrm{m}>0}=2$$

The roots of equation $\det\{\hat{\mathrm{D}}\big(\mathrm{k},\mathrm{f}(\omega)\big)\}=0$ give the complex resonant frequencies of the wake field. The real components determine the real resonant frequencies of the eigenmodes, and the imaginary parts characterize their attenuation at the same frequencies, Fig. 2.

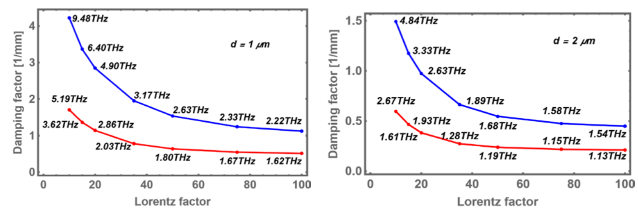

Figure 2: Damping factors of the dipole mode ($m=1$) at the resonant frequencies (for the marked γ values) of the radiation due to the linear motion of a particle in a two-layer copper-dielectric waveguide; $a_1=2$ mm, $d_1=1\ \mu$m (left), $d=2\ \mu$m (right); $\varepsilon=2$ (blue), $\varepsilon=10$ (red).

Damping factors and resonance frequencies decrease with increasing γ and dielectric constant. In each case shown in Fig. 2, we have a single resonance (characteristic of a thin dielectric layer [2, 3]) and, accordingly, a single resonant frequency due to the synchronization of the particle velocity with the phase velocity of the TM_{11} mode [2,3].

In the case of a helical motion, Fig. 3, the dipole term of the multipole expansion contains the modes that form the radiation in front of $(\mathrm{z}-\mathrm{Vt}>0)$ and behind $(\mathrm{z}-\mathrm{Vt}<0)$ the particle. The TE modes, which make the main contribution to the radiation [6], have much lower damping factors than the TM modes for both direct and backward radiation. Increasing the permittivity leads, for a sufficiently thin dielectric layer ($\mathrm{d}_1=1\ \mu\mathrm{m}$, Fig. 3, left), to a general decrease of the damping of the modes and to a convergence of the frequencies of the neighbouring TE and TM modes. Vice versa, the backward-directed radiation almost does not feel the presence of a dielectric. With a thick inner layer ($\mathrm{d}_1=10\ \mu\mathrm{m}$, Fig. 3, right), the optimal radiation conditions are created at $\varepsilon_1=2$. In this case, there is a significant decrease in attenuation both for TE and TM modes.

Thus, a unified approach has been developed for calculating the radiation field of a particle in a multilayer pipe both for its linear motion and in its helical trajectory.

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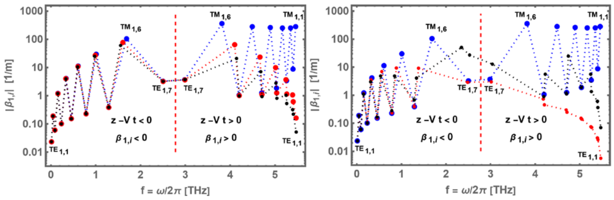

Figure 3: Damping factors of dipole ($m = 1$) modes contributing to the radiation of a particle moving along a helical trajectory in a copper-dielectric waveguide, installed in a helical undulator. Damping coefficients at ε = 1 (blue), ε = 2 (red), ε = 10 (black). Undulator parameters: K = 0.42 (undulator number), undulator period 8 cm, particle energy 15 MeV, $a_1 = 1$ cm; $d_1 = 1\ \mu m$ (left) $d_1 = 10\ \mu m$ (right).

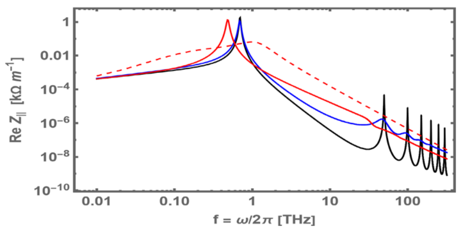

Figure 4: Longitudinal impedance of copper-lossles dielectric ($\varepsilon = 10$) waveguide with geometric resonances (black); smoothing effect of dielectric losses ($\varepsilon = 10 + 3j$, blue); smoothing effect of an additional inner metal NEG coating (taken from [8]): $\sigma = 1.4 \cdot 10^4\ \Omega^{-1}\mathrm{m}^{-1}$ (red, solid), $\sigma = 8 \cdot 10^5\ \Omega^{-1}\mathrm{m}^{-1}$ (red, dashed).

GEOMETRIC RESONANCE

The longitudinal impedance of a copper-dielectric pipe with a thin dielectric layer (Fig. 4, black), has a single narrow-band resonance generated by the TM_{01} mode [2]. However, at frequencies much higher than the fundamental resonant frequency, a periodic series of resonances occurs. For the given parameters of the waveguide ($a_1 = 1\ cm$, $d_1 = 1\ \mu m$, $\varepsilon_1 = 10$), the resonant frequencies are found at 50, 100, 150 ... THz. Fig. 5 shows the distributions of transverse eigenvalues for the TM_{01} and TM_{02} modes. The first of them does not contain any features at the specified frequencies, while the second contains perturbations both in the real and imaginary parts at the frequencies equal to additional resonances of the impedance (cf. Fig. 4 and 5). The observed features can be revealed by considering the dispersion equation for a two-layer pipe with a perfectly conducting outer wall:

$$\mathrm{Det}\{\widehat{D}\} = \left(\frac{J'_m(\nu_0 a_1)}{\nu_0 J_m(\nu_0 a_1)} - \frac{\mu'_1 U_{11}}{\nu_1 U_{01}}\right)\left(\frac{J'_m(\nu_0 a_1)}{\nu_0 J_m(\nu_0 a_1)} - \frac{\varepsilon'_1 U_{10}}{\nu_1 U_{00}}\right) - \frac{m^2 p^2}{a_1^2 k^2}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_0^2}\right)^2 = 0, \quad (11)$$

$$U_{fl} = \frac{1}{\nu_1^2}\left\{\frac{\partial^l H_m(\nu_1 a_1)}{\partial a_1^l}\frac{\partial^f J_m(\nu_1 a_2)}{\partial a_2^f} - \frac{\partial^l J_m(\nu_1 a_1)}{\partial a_1^l}\frac{\partial^f H_m(\nu_1 a_2)}{\partial a_2^f}\right\},$$

where $f, l = 0, 1$. After substituting the high-frequency asymptotes of functions U_{fl} [4] in (11), it is transformed to:

$$\mathrm{Det}\{\widehat{D}\} = \left(\frac{J'_m(\nu_0 a_1)}{\nu_0 J_m(\nu_0 a_1)} - \frac{\mu_1 \mathrm{tg}(\nu_1 d_1)}{\nu_1}\right)\left(\frac{J'_m(\nu_0 a_1)}{\nu_0 J_m(\nu_0 a_1)} + \frac{\varepsilon_1}{\nu_1 \mathrm{tg}(\nu_1 d_1)}\right) - \frac{m^2 p^2}{a_1^2 k^2}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_0^2}\right)^2 = 0, \quad d_1 = a_2 - a_1 \quad (12)$$

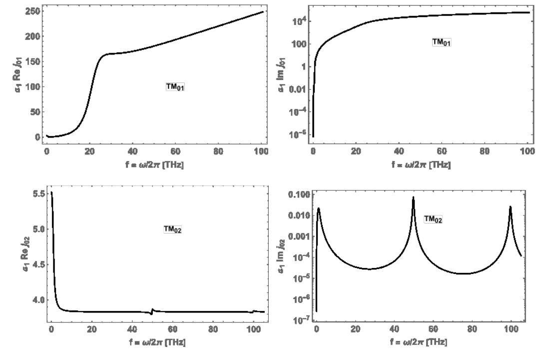

Figure 5: Real (left) and imaginary (right) parts of the mode TM_{01} (top) and TM_{02} (bottom) transverse eigenvalues.

The parameter ν_1 (transverse wavenumber of the mode in the inner layer), as follows from (3), at $\varepsilon_1\mu_1 > 1$ in the high-frequency region ($k \gg 1$) in the case of

$$k\left|\sqrt{\varepsilon_1\mu_1 - 1}\right| \gg |\nu_0| \quad (13)$$

is independent from the parameter ν_0:

$$\nu_1 \approx k\left|\sqrt{\varepsilon_1\mu_1 - 1}\right| \quad (14)$$

and, at $\mathrm{Im}(\varepsilon_1\mu_1) = 0$, equation (12) has singularities at the real frequencies:

$$f = \omega/2\pi = c\,j/\{4d_1\sqrt{\varepsilon_1\mu_1 - 1}\} \quad (15)$$

($j = 1, 2, 3$ ... for $m > 0$, and, at $m = 0$: $j = 2, 4, 6$ ... for TM modes, $j = 1, 3, 5$ ... for TE modes), which are displayed as periodic bursts (Fig. 5, bottom) at high, but finite, conductivity of the outer copper wall. The frequency (15) does not depend on the waveguide radius or the mode number: it depends on the effective (optical) thickness of the inner layer $d_1\sqrt{\varepsilon_1\mu_1 - 1}$ and thus, the resonance can be defined as "geometric". A condition for its occurrence is inequality (13) which is not always satisfied. It does not hold for the TE_{01} mode where no perturbations are observed (Fig. 5, top). With the chosen parameters ($d_1 = 1\ \mu m, \varepsilon_1 = 10$, $\mu_1 = 1$), the resonant frequencies have the values $25j$ THz ($j = 1, 2, 3 \ldots$) for $m > 0$, $25j$ THz ($j = 1, 3, 5 \ldots$) for TE_{0j} modes and $25j$ THz ($j = 2, 4, 6 \ldots$) for TM_{0j} modes, therefore, the geometric resonances in Fig. 4 are due to the last ones. Comparison of the geometric and the fundamental resonance frequencies ($f_r = c/2\pi g$ with $g = \sqrt{2\varepsilon_1(\varepsilon_1 - 1)^{-1}(a_1 d_1)^{-1}}$ [2]) give: $f/f_r = \pi/\sqrt{2\varepsilon_1}\ \sqrt{a_1/d_1}$. Equality $f = f_r$ can be achieved with $a_1/d_1 = 2\varepsilon_1/\pi^2$.

CONCLUSION

A versatile consideration of issues related to eigenmodes and wakefields in multilayer cylindrical waveguides has been undertaken. A unified approach to the determination of mode eigenvalues and frequency distributions of wakefields has been developed. The transformed dispersion matrix is a basis for the transition from the eigenvalue equation to the radiation field of the particle. A relationship between geometric resonance and periodic perturbations of mode transverse eigenvalues has been established.

Due to their weakness, the presence of geometric resonances doesn't change the single-mode nature of the structure.

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REFERENCES

[1] B. W. Zotter and S. A. Kheifetz, *Impedances and wakes in high energy particle accelerators*, Singapore, World Scientific, 1997.

[2] M.I. Ivanyan, L.V. Aslyan, K. Floettmann, F. Lemery, and V. M. Tsakanov, "Wakefields in conducting waveguides with a lossy dielectric channel", *Phys. Rev. Accel. Beams*, vol. 23, p. 041301, Apr. 2020. doi:10.1103/PhysRevAccelBeams.23.041301

[3] M. I. Ivanyan, F. Lemery, K. Floettmann, and L.V. Aslyan, "*Transverse impedance of lossy circular metal-dielectric waveguides*", in *Proc. 12th Int. Particle Accelerator Conf. (IPAC'21)*, Campinas, SP, Brazil, May 2021, pp. 2093-2096. doi:10.18429/JACoW-IPAC2021-TUPAB269

[4] M.I. Ivanyan, E. Laziev, A. Tsakanian, V. Tsakanov, A. Vardanyan and S. Heifets, "Multi-layer tube impedance and external radiation", *Phys. Rev. ST Accel. Beams*, vol. 11, p. 084001, Aug. 2008. doi:10.1103/PhysRevSTAB.11.084001

[5] A. Piwinski, "Wake fields and ohmic lossesin round vacuum chambers", DESY, Hamburg, Germany, Rep. DESY HERA 92-11, May 1992.

[6] M.I. Ivanyan, A.H. Grigoryan, B. K Sargsyan, T. L. Vardanyan, A. V. Tsakanian and V. M. Tsakanov, "Mode enhancement of THz helical undulator radiation in waveguide", *Laser Phys*, vol. 30, p. 115002, Oct. 2020. doi:10.1088/1555-6611/abb1bd

[7] M. Ivanyan, B. Grigoryan, B. Sargsyan, A. Grigoryan, K. Floettmann and F. Lemery, "*Radiation of a particle moving along a helical trajectory in a resistive-wall cylindrical waveguide*", in *Proc. IPAC'22*, Bangkok, Thailand, June 2022, pp. 2150-2153. doi: 10.18429/JACoW-IPAC2022-WEPOTK037

[8] M. Dehler *et al.*, "*Characterization of NEG Coatings for SLS 2.0*", in *Proc. IPAC'19*, Melbourne, Australia, May 2019, pp. 1662-1665. doi:10.18429/JACoW-IPAC2019-TUPGW108