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Tests of general relativity on astrophysical scales

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Abstract While tested to a high level of accuracy in the Solar system, general relativity is under the spotlight of both theoreticians and observers on larger scales, mainly because of the need to introduce dark matter and dark energy in the cosmological model. This text reviews the main tests of general relativity focusing on the large scale structure and more particularly weak lensing. The complementarity with other tests (including those on Solar system scales and the equivalence principle) is discussed.

Keywords General Relativity, Cosmology, Weak lensing

1 Introduction

Gravitational lensing is historically bound to the developments of general relativity (GR) and, more generally, of the theories of gravitation. Since the end of the eighteenth century, it was thought that light can be deflected by a gravitational field, in particular with the works of Georg von Soldner that postulated that light must behave as any other particle or of Robert Blair, John Michell and Pierre Simon de Laplace (see Ref. [1] for an historical discussion).

The deflection of light by any massive body is a central prediction of GR. In particular, the observations of the deflection of light emitted by distant stars by the Sun during the Solar eclipse on the 29 May 1919 by the expeditions led by Eddington and Cottingham on Principe island and by Davidson and Crommelin in the Nordeste region of Brasil is always considered as an experimental confirmation of the predictions of GR. Indeed, if such an observation was a test of GR, it is only because the mass of the Sun was supposed to be well-determined at the time. On

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the one hand, the light deflection angle predicted by GR is

$$\Delta\theta_{\text{GR}} = \frac{4GM_{\odot}}{bc^2}$$

where G is the Newton constant, M_{\odot} the Solar mass, b the impact parameter and c the speed of light, while, on the other hand, the dynamics of massive bodies, such as planets, are in an extremely good approximation still given by Kepler third law,

$$\frac{P}{2\pi} = \frac{a^3}{GM_{\odot}},$$

where P is the period of the orbit and a its semi-major axis. Measuring $\Delta\theta$ and b on one side and P and a on the other gives two estimates of GM_{\odot} ,

$$GM_{\odot}^{\text{lens}} = \frac{bc^2\Delta\theta}{4}, \quad GM_{\odot}^{\text{dyn}} = \frac{2\pi a^3}{P},$$

that needs, given the error bars, to be consistent. This illustrates that lensing *alone* does not allow to construct a test of the theory of gravity but that we need to check the consistency between various predictions such as lensing and dynamics of massive bodies.

Today, gravity, i.e. the only long range force that cannot be screened, is described by GR in which it is the consequence of the geometry of the spacetime. GR is consistent with all precision experimental tests available but most of these tests are restricted to the Solar system or to our Galaxy, so that they are only local. By considering astrophysical systems, we can extend the *domain of validity* of GR at large distance, low acceleration or low curvature. In particular, most attempts to construct a quantum theory of gravity or to unify it with other interactions predict the existence of partners to the graviton, i.e. extra fields contributing to a long range force, and thus to gravity (e.g. this the case in all extra-dimensional theories where some components of the extra-dimensional part of the metric behave as scalar fields from a 4-dimensional point of view; string theory also predicts the existence of a scalar field, the dilaton, in the graviton supermultiplet). It follows that deviation from GR [2] are expected but in many cases (such as scalar–tensor theories), the theory can be dynamically attracted toward GR so that cosmology can set sharper constraints than those obtained locally.

An other reason to test GR in these regimes is related to our current cosmological model. This model, which is consistent with almost all astrophysical data requires the addition of both dark matter (a fluid with negligible pressure that does not interact with standard matter) and dark energy (a fluid with a negative pressure), which represent, respectively 23 and 72% of the matter content of the universe. The need for these two components arises from the study of the dynamics of clusters, galaxies, large scale structures and of the cosmic expansion under the assumption that GR holds on astrophysical scales. This conclusion has been challenged by invoking possible modifications of GR either in a low acceleration regime to explain the galaxy rotation curves and cluster dynamics without introducing dark matter and at large distance to account for the late time acceleration of the cosmic expansion.

In conclusion, the possibility to sharpen our understanding of the validity of GR from astrophysical data and the need to understand the properties of dark

energy and dark matter, which are tied to the validity of GR, are our two driving motivations. The main difficulty is that, on astrophysical scales, most observations entangle the properties of gravity, matter, as well as other hypothesis such as the Copernican principle.

The bottom line of the construction of these tests is simple. Once GR is assumed valid, it describes the dynamics of the cosmic expansion, the growth of large scale structures, the propagation of light etc... so that many observables are not independent. Such observables can be used to construct consistency relations. Any sign of a violation of these relations will indicate the need to extend our description of gravity, but will not indicate us how. For instance, in the oversimplified Solar system example described above, we want the two estimates of GM_{\odot} to agree that is we must have

$$\frac{bc^2\Delta\theta}{4} - \frac{2\pi a^3}{P} = 0. \quad (1)$$

This is a relation between observable quantities ($b, \Delta\theta, P, a$) that has to be satisfied. Actually, it was first proposed in Ref. [3] to perform a similar test on cosmological scales using weak lensing and galaxy redshift suveys, followed by the analysis of Ref. [4].

The review is organized as follows. We start, in Sect. 2, by recalling the main hypothesis on which GR is based as well as the standard constraints obtained in the Solar system. We also discuss the use of alternative theories and draw the conclusions of what was learnt in the Solar system for constructing tests on astrophysical scales. In Sect. 3, we discuss briefly tests on galactic and cluster scales where the need for dark matter can be interpreted as the necessity to modify GR in a low acceleration regime. Larger scales are considered in Sect. 4 which focuses on the large scale structure of the universe.

2 Relativity and its Solar system tests

2.1 General relativity (in brief)

Let us recall that GR, Einstein's theory of gravity, relies on two independent hypothesis.

First, the theory rests on the *Einstein equivalence principle*, which includes the universality of free fall, the local position and local Lorentz invariances in its weak form (as other metric theories) and is conjectured to satisfy it in its strong form. We refer to Ref. [5] for a detailed description of these principles and their implications. The weak equivalence principle can be mathematically implemented by assuming that all matter fields, including gauge bosons, are minimally coupled to a single metric tensor $g_{\mu\nu}$. This metric defines the lengths and times measured by laboratory rods and clocks so that it can be called the *physical metric*. This implies that the action for any matter field, ψ say, is of the form

$$S_m[\psi, g_{\mu\nu}]. \quad (2)$$

This so-called *metric coupling* ensures in particular the validity of the universality of free-fall.

Then, the action for the gravitational sector is given by the Einstein–Hilbert action

$$S_{\text{gravity}} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g_*} R_*, \quad (3)$$

where $g_{\mu\nu}^*$ is a massless spin-2 field called the *Einstein metric*. The second hypothesis of GR states that both metrics coincide, i.e.

$$g_{\mu\nu} = g_{\mu\nu}^*.$$

2.2 Testing GR

It follows that one can aim at testing both the equivalence principle and the dynamical equations that derive from the Einstein–Hilbert action.

The assumption of metric coupling is well tested in the Solar system. First it implies that all non-gravitational constants are spacetime independent, which have been tested to a very high accuracy in many physical systems and for various fundamental constants [6; 7; 8; 9], e.g. at the 10^{-7} level for the fine structure constant on time scales ranging to 2–4 Gyears. Second, the isotropy has been tested from the constraint on the possible quadrupolar shift of nuclear energy levels [10; 11; 12] proving that matter couples to a unique metric tensor at the 10^{-27} level. Third, the universality of free fall of test bodies in an external gravitational field at the 10^{-13} level in the laboratory [13; 14]. The Lunar Laser ranging experiment [15], which compares the relative acceleration of the Earth and Moon in the gravitational field of the Sun, also probe the strong equivalence principle at the 10^{-4} level. Fourth, the Einstein effect (or gravitational redshift) states that two identical clocks located at two different positions in a static Newton potential U and compared by means of electromagnetic signals shall exhibit a difference in clock rates of $1 + [U_1 - U_2]/c^2$. This effect has been measured at the 2×10^{-4} level [16].

The parameterized post-Newtonian formalism (PPN) is a general formalism that introduces ten phenomenological parameters to describe any possible deviation from GR at the first post-Newtonian order [5]. The formalism assumes that gravity is described by a metric and that it does not involve any characteristic scale. In its simplest form, it reduces to the two Eddington parameters entering the metric of the Schwarzschild metric in isotropic coordinates

$$g_{00} = -1 + \frac{2Gm}{rc^2} - 2\beta^{\text{PPN}} \left(\frac{2Gm}{rc^2} \right)^2, \quad g_{ij} = \left(1 + 2\gamma^{\text{PPN}} \frac{2Gm}{rc^2} \right) \delta_{ij}.$$

Indeed, GR predicts $\beta^{\text{PPN}} = \gamma^{\text{PPN}} = 1$. These two phenomenological parameters are constrained (1) by the shift of the Mercury perihelion [17] which implies that $|2\gamma^{\text{PPN}} - \beta^{\text{PPN}} - 1| < 3 \times 10^{-3}$, (2) the Lunar laser ranging experiments [15] which implies $|4\beta^{\text{PPN}} - \gamma^{\text{PPN}} - 3| = (4.4 \pm 4.5) \times 10^{-4}$ and (3) by the deflection of electromagnetic signals which are all controlled by γ^{PPN} . For instance the very long baseline interferometry [18] implies that $|\gamma^{\text{PPN}} - 1| = 4 \times 10^{-4}$ while the measurement of the time delay variation to the Cassini spacecraft [19] sets $\gamma^{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$.

The PPN formalism does not allow to test finite range effects that could be caused e.g. by a massive degree of freedom. In that case one expects a Yukawa-type deviation from the Newton potential,

$$\Phi = -\frac{Gm}{r} \left[1 + \alpha \left(1 - e^{-r/\lambda} \right) \right],$$

that can be probed by ‘‘fifth force’’ experimental searches. λ characterizes the range of the Yukawa deviation while its strength α may also include a composition-dependence [6]. The constraints on (λ, α) are summarized in Ref. [20] which typically shows that $\alpha < 10^{-2}$ on scales ranging from the millimeter to the Solar system size.

GR is also tested with pulsars [21; 22] and in the strong field regime [23]. For more details we refer to Refs. [5; 24; 25]. Needless to say that any extension of GR has to pass these constraints. However, these deviations can be larger in the past, as we shall see, which makes cosmology an interesting field to extend these constraints.

2.3 Alternative theories of gravity

The ways of modifying GR are so various and in large number that we cannot review them here. We refer to Refs. [5; 26] for some examples.

Let us however introduce the simplest modification of GR, that is scalar–tensor theories of gravity in which gravity is mediated not only by a massless spin-2 graviton but also by a spin-0 scalar field that couples universally to matter fields (this ensures the universality of free fall). Their action can be written as, in the so-called Einstein frame,

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g_*} \left[R_* - 2g_*^{\mu\nu} \partial_\mu \varphi_* \partial_\nu \varphi_* - 4V(\varphi_*) \right] + S_{\text{matter}}[A^2(\varphi_*)g_{\mu\nu}^*; \Psi], \quad (4)$$

where G_* is the bare gravitational constant. The physical metric, to which matter is universally coupled, $g_{\mu\nu} = A^2(\varphi_*)g_{\mu\nu}^*$ is the product of the coupling function A , which characterizes the strength of the scalar interaction, and the Einstein frame metric $g_{\mu\nu}^*$. This theory involves a new degree of freedom coupled to matter.

It can be used to illustrate the effect of modification of GR on lensing (see Ref. [26] for more details). Consider the action for electromagnetism in d dimensions

$$S_{\text{Maxwell}} = \frac{1}{4} \int \sqrt{-g_*} g_*^{\mu\nu} g_*^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} d^d x$$

transforms to

$$S_{\text{Maxwell}} = \frac{1}{4} \int \sqrt{-g} g^{\mu\nu} g^{\rho\sigma} A^{d-4} F_{\mu\rho} F_{\nu\sigma} d^d x$$

under the conformal transformation $g_{\mu\nu} = A^2(\varphi_*) g_{\mu\nu}^*$. In the relevant case of a $d = 4$ dimensional spacetime, the Maxwell action is conformally invariant. Therefore light is only coupled to the spin-2 field $g_{\mu\nu}^*$ so that light deflection by a point mass M must be the same as in GR, i.e.

$$\Delta\theta = \frac{4G_*MA^2}{bc^2},$$

where A^2M is the deflecting mass in the Einstein frame. It thus seems that there is no effect on lensing, contrary to the standard lore that light deflection is smaller in scalar–tensor theories. Actually there is a crucial difference since in scalar–tensor theory massive bodies do feel the scalar field. It follows that the gravitational constant measured in a Cavendish experiment today is not G_* but $G_N = G_*A_0^2(1 + \alpha_0^2)$ with $\alpha \equiv d \ln A / d\varphi_*$ and where a subscript 0 indicates that the quantity is evaluated today. It follows that the dynamics of massive bodies, such as planetary orbits, determine $G_N M$ and not G_*MA^2 so that

$$\Delta\theta = \frac{4G_*MA_0^2}{bc^2} = \frac{4G_N M}{(1 + \alpha_0^2)bc^2} = \frac{\Delta\theta_{\text{GR}}}{1 + \alpha_0^2} \leq \Delta\theta_{\text{GR}},$$

as expected. Again, this shows that lensing *alone* cannot probe GR and that we need to compare different measurements. Note also that the gravitational constant (or more precisely the dimensionless number $Gm^2/\hbar c$) varies with time so that extending this argument to the cosmological context is not straightforward [27]. When the theoretical framework is specified then the post-Newtonian parameters can be computed (here $\gamma^{\text{PPN}} = -2\alpha^2/(1 + \alpha^2)$ and $\beta^{\text{PPN}} = \alpha^2/[2(1 + \alpha^2)^2]d\alpha/d\varphi_*$ as long as the potential is such that the field is light on Solar system scales) so that the PPN constraints can be translated to constraints on the parameters of the model.

In conclusion, this simple extension of GR illustrates that we always have to introduce new fields in the theory so that we have to specify their nature (here a scalar field) and the ways they couple to the matter fields (here universally with the strength α). The distinction between a modification of GR and dark matter (or energy) is thus slight since in *both cases we need to introduce new fields in our theory*. The main difference lies in the fact that the amount of dark matter or dark energy is set by initial conditions (e.g. the amount of dark matter is fixed initially and determines the properties of the potential wells in which baryonic matter falls to form the structure or the amount of dark energy is fixed by tuning some parameters and/or initial conditions so that it starts dominating today and the fact that $\rho_\Lambda : \rho_{\text{cdm}} : \rho_b \sim 14 : 5 : 1$ today calls for an explanation). In a modified GR model, the way standard matter generates potential wells or affect the dynamics of the universe is changed. Note however that the new degree of freedom are also gravitating so that in some models the distinction is even more subtle.

Among the most studied alternative theories of gravity, let us mention scalar–tensor theories discussed above, $f(R)$ theories which are, after a field redefinition, a sub-class of scalar–tensor theories, the DGP model [28] and the TeVeS [29] theory which is a relativistic version of the MOND [30] idea.

2.4 Lessons for extending the tests to astrophysical scales

GR is a well-defined theory of gravity with clear predictions so that the consistency of these predictions offers the possibility to test the theory in a model-independent way. This implies that we need various observables relating the same physical quantities (such as the mass in the example of the Sect. 1).

In cosmology, we can use almost the same observations as in the Solar system. Concerning light deflection, it cannot be measured (since the “undeflected” position of the sources cannot be determined; but for the particular case of microlensing) and we will have to use the distortion of light bundles, that is strong and weak lensing. Also, and contrary to the Solar system, we can have access of the evolution of the energy of the photons, related to the time variations of the gravitational potential in the case of the integrated Sachs–Wolfe effect. The dynamics of massive bodies can be obtained from the large scale structure of the universe, which give an information of the growth of the structures and their velocity. Among the tests of the equivalence principle, only the test on the constancy of fundamental constants can be generalized.

There are however limitations specific to cosmology. In particular, the cosmological structures evolve with time and this contains an information on gravity but also on the properties of matter which are difficult to disentangle (for instance, our prediction on the shape of the galaxy power spectrum are different whether there exist massive neutrinos or not). This also means that we may have to take evolution effects into account. Also, cosmological data have to be interpreted in a statistical way so that we always have a dependence of the initial conditions that cannot be forgotten. Then, the description of the dynamics of the universe involves the Copernican principle so that the interpretation of our tests will depend on such a hypothesis.

It follows that the tests that will be designed are indeed tests of GR but also depends on many other hypothesis so that they should probably be considered first as tests of the Λ CDM model.

3 Galaxy and cluster scales

The first interesting systems for testing GR in astrophysics are galaxies and clusters. It is now well-established that, as long as one assumes GR to hold, their dynamics can be understood by invoking the existence of dark matter.

The visible mass of spiral galaxies is rather concentrated so that Newtonian gravity predicts that the rotation curves should drop as $r^{-1/2}$ outside the bright part of these galaxies. But this has not been confirmed by more than a hundred rotation curve measurements [31; 32; 33].

Actually, in many spiral galaxies, the rotation curves at large distance from the center do not declined as the Keplerian model and $v_\infty \rightarrow \text{const}$. Moreover, this velocity is correlated to the luminosity of the galaxy. This correlation, known as the Tully-Fisher law, states that the luminosity of the galaxy scales as v_∞^4 , so that one expects that $v_\infty^4 \propto M$, for the total stellar mass. This has also supported the view of the dark matter explanation: if the velocity is constant in the outer region of the galaxy, this means that the centripetal acceleration scales as $a_r \propto r^{-1}$ and

Newton's law implies that the gravitational potential scales as $\ln r$. In the case of spherical symmetry, the Poisson equation implies that it should be sourced by a matter whose density profile scales as $\rho(r) \propto r^{-2}$, as for an isothermal sphere model. Thus, each spiral galaxy must contain a dark matter component with, if one assumes that the halo is spherical, a density profile scaling as r^{-2} at large distance. This reflects the discrepancy between two estimations of the mass: the luminous mass and the dynamical mass (see e.g. Ref. [34] for general exposition of the paradigm).

To avoid such an hypothesis, Milgrom [30] proposed a phenomenological modification, called MOND, that was able to account for the galaxy rotation curves [35], and more particularly to recover the Tully-Fisher law. MOND introduces a fundamental acceleration a_0 , of the order of $1.2 \times 10^{-10} \text{ m} \cdot \text{s}^{-2}$, such that the acceleration of any massive body is

$$a = a_N, \quad \text{if } a > a_0, \quad a = \sqrt{a_N a_0}, \quad \text{if } a < a_0$$

so that, at large distance, the Newtonian acceleration being GM/r^2 , the centripetal acceleration is $\sqrt{GMa_0}/r$. Since it is also given by v^2/r , one deduces that $v(r) \rightarrow (GMa_0)^{1/4}$. In this regime, the gravitational potential behaves as $\sqrt{GMa_0} \ln r$ instead of the standard Newtonian potential $-GM/r$. It follows that the deflection angle at large distance from the center of the galaxy is

$$\Delta \theta_{\text{MOND}} = \frac{2\pi\sqrt{GMa_0}}{c^2}.$$

This value is the same as the one expected from GR, as long as one is in the halo. Indeed, if interpreted within GR, the presence of dark matter suggested by the rotation curves is confirmed by lensing observations. Therefore in MOND, an in any modification of GR, one must predict that a given mass generates a larger potential and a larger deflection angle than in GR.

It follows that, one needs to estimate the mass of the galaxy, in a given theory of gravity, by different methods in order to check their compatibility. In particular, different notions of mass needs to be distinguished [26]: the *baryonic mass* M_b assumed to be proportional to the luminous mass, or stellar mass M_* ; the total *dynamical mass* $M_{\text{tot}}^{\text{dyn}}$, estimated from the rotation curves; and the total *lensing mass* $M_{\text{tot}}^{\text{lens}}$ determined by lensing observations. In the dark matter interpretation, and as well established by lensing observations, we have

$$\text{Dark Matter : } \quad M_b < M_{\text{tot}}^{\text{dyn}} \sim M_{\text{tot}}^{\text{lens}}.$$

In particular, such mass estimates were performed in Ref. [36] using six strong lensing galaxies from the CASTLES database. The total mass was estimated from lensing while the stellar mass was estimated from a comparison of photometry and stellar population synthesis. It demonstrates that dark matter is still needed (and that it is detected even in region where $a > a_0$). In particular this dark matter component cannot be explained by 2 eV neutrinos (see below).

If one assumes that the light deflection is given as in GR, then the previous equivalences told us that MOND predicts the same lensing as in GR within the dark matter halo. In particular the convergence at distance r is given by $\kappa(r) = r_E/2r$, where the Einstein radius is

$$r_E = 2\pi \left(\frac{v_\infty}{c}\right)^2 \frac{D_{ls}}{D_s} \quad (\text{MOND}) \quad r_E = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{ls}}{D_s} \quad (\text{DM}),$$

where the latter holds for a singular isothermal sphere with line-of-sight velocity dispersion σ_v . While formally similar, these expressions have however an interesting difference [37] since σ_v^2 scales as M_{tot} while v_∞^2 scales as $\sqrt{M_*}$ so that

$$r_E \propto \sqrt{M_*} \quad (\text{MOND}) \quad r_E \propto M \quad (\text{DM}).$$

The scaling of the Einstein radius with the stellar mass was measured [37] using the RCS and SDSS surveys to show that $r_E \sim M_*^{0.74 \pm 0.08}$. This seems in contradiction with the MOND prediction but the data used measurements of the shear at distances of some hundred of kpc, at which the environment effects can change the MOND prediction. It sets no constraint and the cold dark matter model since the fraction M_*/M is not known.

Indeed, MOND is a phenomenological description but not a field theory. As, we have seen earlier, the light deflection in scalar–tensor theories is smaller than in GR. This means that a MOND cannot derive from a simple scalar–tensor theory. It was realized (see Ref. [26] for more details) that this can be solved by coupling matter not to the metric $g_{\mu\nu}$ but rather to a “physical metric” involving both a scalar and a vector field

$$\tilde{g}_{\mu\nu} = e^{-2\phi} g_{\mu\nu} - 2U_\mu U_\nu \sinh 2\phi.$$

The first term is similar to what is performed in scalar–tensor theories and the second term, involving the vector field U_μ , allows to reconcile light deflection with the GR prediction. This theory is known as the TeVeS (Tensor–Vector–Scalar) theory [29]. Reference [38] compares the TeVeS predictions to a large sample of galaxy strong lenses from the CASTLES sample. Recently, Ref. [39] compares the predictions of TeVeS for both galaxy rotation curves and strong lensing (for high and low surface brightness galaxies) concluding that TeVeS, in its simplest form, cannot reproduce these data consistently. The analysis [40] of galaxy-scale strong lensing from the Sloan ACS (SLACS) survey indicates that $|\gamma^{\text{PPN}} - 1| < 5 \times 10^{-2}$. However this work emphasizes that setting constraints with such systems requires to know the properties of the lensing galaxies with a great accuracy, much greater than at present. The comparison of the stellar velocity dispersion to measurements of the Einstein radius allows to constrain γ^{PPN} , reaching [41] $\gamma^{\text{PPN}} = 0.88 \pm 0.05$ on kiloparsec scales at 68% C.L. while an early analysis based on 14 systems only gave [42] $\gamma^{\text{PPN}} = 0.93 \pm 0.1$.

On cluster scales, various estimates of the mass can be obtained by lensing (strong and weak), X-ray emission that characterizes the intracluster (baryonic) medium, and the SZ effect which gives an information of the electron distribution. By comparing these distributions, one can compare the location of the gas and the gravitational iso-potential. Earlier analysis used the comparison of X-ray and strong lensing [43] and then weak lensing [44] leading to the conclusion that the Poisson equation should be valid, within a factor 2, up to scales of 2 Mpc [45].

Interesting conclusions arise from the study of the colliding galaxy clusters 1E0657-56 ($z = 0.296$). In this system a smaller cluster, known as the “bullet cluster”, has crashed through a larger one and their intracluster gas has been stripped by the collision. On one hand, weak lensing shows that the lensing mass is concentrated in the two regions containing the galaxies rather than in the stripes containing the baryonic matter [46]. A similar observation [47] was made with the merging galaxy cluster MACS J0025.4-1222 ($z = 0.586$) for which the emitting gas, traced from its X-emission, is clearly displaced from the distribution of galaxies (from lensing). The rich cluster Abell 520 ($z = 0.201$) also exhibits the same properties [48] and contains a massive dark core, as deduced from lensing mass reconstruction, that coincides with the central X-ray emission peak. The analysis [49] of the cluster Abel 478 demonstrates that the X-ray, SZ and weak lensing data perfectly agree with a dark matter model (but does not prove they cannot be reproduced by a MOND model).

This seems to be a proof of the need of dark matter since, being collisionless, it continues to be located around the bullet, contrary to the baryonic gas. This was confirmed [50] by the reconstruction of the mass distribution from both strong and weak lensing. However, it seems that MOND could accommodate these observations in particular because the original TeVeS versions make different prediction when the system is not spherically symmetric. Reference [51] showed that it was possible to design a multi-centred baryonic system reproducing the weak-lensing signal of 1E0657-56 with a bullet-like light distribution. The same authors [52] then realized that a purely baryonic MOND model cannot accommodate the data and that the bullet cluster was dominated by dark matter whether one uses GR or MOND. In the latter case, it would require massive neutrinos with $m_\nu = 2$ eV.

The existence of such neutrinos seems however problematic. From the study weak lensing for 3 Abell clusters and 42 SDSS clusters, it was concluded that MOND cannot explain the data unless some dark matter is added and this dark matter cannot be accounted for by massive neutrinos [53]. This was confirmed [54] by the confrontation of strong and weak lensing from the HST Wide-Field Camera, excluding the dark matter to be neutrinos with mass in the range 2-7 eV.

In conclusion, it seems that MOND and TeVeS have difficulties to reproduce the observations of the distribution of dynamical, baryonic and lensing masses. Indeed none of the above mentioned results demonstrate that MOND is ruled out. They are analysis that show that the data can be consistently interpreted assuming GR and the existence and dark matter. One of the main difficulty to use these observations as a direct test of GR is the complex geometry of the systems that are used.

4 Cosmological scales

The construction of a cosmological model relies on the choice of the theory of gravity as well as on our understanding of the fundamental interactions of nature. However it also involves other hypothesis, such as the Copernican principle which states that we are not seating in a privileged place of space. Under such an hypothesis, and whatever the theory of gravity, the universe on large scales can be described by a Friedmann-Lemaître spacetime with metric

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j, \quad (5)$$

where t is the cosmic time, a the scale factor and γ_{ij} the spatial metric on constant t hypersurfaces. If one assumes that GR is a good description of gravity then the dynamical equations of such a model derives from the Einstein and the conservation equations

$$G_{\mu\nu} = 8\pi GT_{\mu\nu}, \quad \nabla_\mu T^{\mu\nu} = 0, \quad (6)$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ the total stress-energy tensor. Among this class of models our reference model is the Λ CDM model which includes a cosmological constant and cold dark matter and assume that initial conditions are consistent with the prediction of slow-roll inflation. Such a model is in very good agreement with most of the astrophysical data and it is self-consistent. But the fact that the dark sector represents 95% of the matter content of the universe and the cosmological constant problem drive us to test the hypothesis of our model, and in the first place the Copernican principle and GR. It is indeed difficult to anbandon these two hypothesis at the same time so that all the studies aiming at testing GR in that context assume that the spacetime metric remains of the form (5). Also most of them still include dark matter and aim at replacing dark energy by a modification of GR.

Two roads can be followed. Either one defines a class of gravity models that contains GR in some limit and then confronts it to cosmological data to see how close from GR, in this particular space of theories, the theory of gravity should seat. As an example, this was performed in depth for scalar–tensor theories Eq. (4) for which the implications of the background dynamics [55], of the cosmic microwave background [55], primordial nucleosynthesis [56], weak lensing [27], and local constraints [57], even allowing for extensions to a non-universal coupling of dark matter [58], were all studied. Or one tries to quantify the allowed deviations from the reference model while being as much as can be model-independent. The strategy is then to exhibit consistency relations, analogous to Eq. (1), between different observables, which must hold in our Λ CDM reference model, as first proposed on the particular case of the Poisson equation in Ref. [3]. As explained in Ref. [59], the modification of our reference framework can be classified in universality classes who have specific signatures and different tests can favour or disfavour some classes of modification.

4.1 Background dynamics

The dynamics of the background spacetime is dictated by the Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} \quad (7)$$

where $H = \dot{a}/a$ is the Hubble function and $K = 0, \pm 1$ is the curvature of the spatial sections. From the last equation, the recent acceleration of the cosmic expansion implies that $(\rho + 3P) < 0$ if GR is a good description of gravity.

At the background level, a modification of GR or the introduction of a dark energy component instead of the cosmological constant will change the Friedmann equation and can be taken into account in an effective way simply in terms of an effective fluid

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_{\text{de}}) - \frac{K}{a^2}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P + \rho_{\text{de}} + 3P_{\text{de}}), \quad (8)$$

where ρ_{de} and P_{de} can depend on H and its derivatives, as e.g. for scalar-tensor theories or DGP. This allows to define the equation of state of the dark energy from H (see Ref. [57]).

Indeed, without an explicit model, the extra-terms ρ_{de} and P_{de} are not known. Besides we know that, since they arise from the existence of a new degree of freedom, there must exist an associated equation of evolution [60]. The standard approach is to postulate that they are related by an equation of state, $\rho_{\text{de}} = w_{\text{de}}P_{\text{de}}$, and the most commonly used ansatz is [61; 62]

$$w_{\text{de}}(z) = w_0 + \frac{z}{1+z}w_a, \quad (9)$$

with w_0 and w_a constant. It is thus clear that the background dynamics cannot distinguish between a modification of GR and a properly tuned dark energy model. This lies in the fact that the only quantity at hand is $H(z)$ and most of the models of the literature can be tuned to reproduce the same function (see Ref. [59] for explicit examples). No null test of GR can be constructed with background data since they all are functions of $H(z)$.

4.2 Linear perturbation theory

4.2.1 Standard Λ CDM

As long as one assumes GR to hold and consider an almost Friedmann-Lemaître spacetime, the evolution of perturbations is well understood, see e.g. Ref. [63]. On sub-Hubble scales, focusing only on scalar perturbations which are dominant at late time, the space-time metric can be written as

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)a^2(t)\gamma_{ij}dx^i dx^j, \quad (10)$$

where Φ and Ψ are the two gravitational potentials.

The evolution equations on Hubble scales are given by the conservation of the matter stress-energy tensor (continuity and Euler equations)

$$\dot{\delta}_m = -\frac{\theta_m}{a}, \quad \dot{\theta}_m + H\theta_m = -\frac{1}{a}\Delta\Phi, \quad (11)$$

which leads to the standard second order evolution equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{1}{a^2}\Delta\Phi = 0. \quad (12)$$

Among the Einstein equations, we can keep only the Poisson equation

$$\Delta\Psi = 4\pi G\rho_m a^2 \delta_m \quad (13)$$

and

$$\Phi - \Psi = 0 \quad (14)$$

that arises from the fact that the matter anisotropic stress is negligible. This shows that the two gravitational potentials have to coincide, which is related to the fact that $\gamma^{\text{PPN}} = 0$ in GR, their spectrum has to be proportional to the matter power spectrum, as first pointed out in Ref. [3] and Eq. (11) implies that

$$\theta_m = -f\delta_m, \quad \text{with} \quad f = \frac{d\ln D}{d\ln a}, \quad (15)$$

where we have decomposed the density contrast as $\delta_m = D(t)\varepsilon(x)$ where ε encodes the initial conditions. It was shown [64] that, for a flat CDM model,

$$f \sim \Omega_m^{0.6} \quad (16)$$

was a good fit. The important feature here is that if GR holds, then the growth of structures is completely determined by $H(z)$. We foresee that one can check the compatibility of background data (such a distance-redshift relations, e.g. from SNIa) and large scale structure data, whatever the parameters entering the equation of state (9) or any other parameterisation (see below).

4.2.2 Extension

In order to construct tests of GR in this regime, one needs to construct the most general extension of this set of evolution equations, still assuming we are dealing with a metric theory of gravity.

In the case of dark energy alone, one needs to consider the effect of its stress-energy tensor, which can have non-vanishing anisotropic stress and density contrast, contrary to a pure cosmological constant but the equations of evolution of the other fluids are not modified, since otherwise this new component would be coupled to the standard matter non minimally. In GR modifications, there exists a new long range force and the evolution equation of matter will be of the form $\nabla_\mu T_i^{\mu\nu} = f_i^\nu$, where f^μ is a force term between the standard matter fields and the new degree of freedom. The way such force term appears in the equation and its relation to the Einstein equation is not obvious to describe in full generality

while being model-independent. As an example, consider scalar–tensor theories of gravity. In the Jordan frame, the equations of motion of the standard matter fields are not modified so that $f_i^\mu = 0$ but, performing a conformal transformation, the same theory, written in the Einstein frame, involves a force $f_\mu^i = \alpha(\varphi_*) T_i \partial_\mu \varphi_*$ that will appear even at the background level in the continuity equation. In this particular case, it is well understood that the modification of gravity appears as a time-dependent modification of the Newton constant in the Jordan frame while it is seen as a universal time-dependent modification of masses in the Einstein frame. Indeed, if the new force is not universal, it probably involves that mass ratios will be time-dependent, which can be tested [6]. Thus, we assume that we are working in the equivalent of the Jordan frame so that we assume that $f_i^0 = 0$ and that there is no creation of matter. The spatial component of the force can however be non vanishing and enters the Euler equation. Let us stress that while important from a physical point of view [65; 66], this is not dramatic from a phenomenological point of view since only the source term of Eq. (18) below will be changed. On the other hand, we shall have an equation of evolution for the new degree of freedom that shall also have a source term proportional to the matter stress-energy tensor [65]. Unfortunately, this equation remains unknown until we specify the model.

As long as we stick to linear perturbations, these extensions can be implemented by modifying the previous equations as

$$\dot{\delta}_m = -\frac{\theta_m}{a}, \quad \dot{\theta}_m + H\theta_m = -\frac{1}{a}\Delta\Phi + \mathcal{S}_{de}, \quad (17)$$

which leads to the standard second order evolution equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{1}{a^2}\Delta\Phi = \mathcal{S}_{de}. \quad (18)$$

The term $\mathcal{S}_{de}(k, a)$ encodes the new long-range force between the new degree of freedom and the standard matter (and dark matter!).

Then, we need to write down the Einstein equations. First, we can generalize the Poisson equation, written in Fourier space, as

$$-k^2\Psi = 4\pi GF(k, H)\rho_m a^2 \delta_m + \Delta_{\text{de}}. \quad (19)$$

The first term $F(k, H)$ accounts for a scale dependence of the gravitational interaction while Δ_{de} accounts for a possible clustering of the new degree of freedom, and in particular of dark energy if there is no modification of GR (this shows at this stage, that the Poisson equation can be modified without modification of GR if dark energy can cluster; also care needs to be taken in the case of massive neutrinos which can enter on the r.h.s. of this equation, see e.g. [67]; Ref. [68; 69] proposed an interesting example of a clustering dark energy model mimicking the DGP model). Then, there is the possibility to have an effective anisotropic stress so that

$$\Delta(\Phi - \Psi) = \Pi_{\text{de}}. \quad (20)$$

It follows that the deviation from GR is encoded in the four functions $(\mathcal{S}_{\text{de}}, F, \Delta_{\text{de}}, \Pi_{\text{de}})$ which, in the case of the Λ CDM model, reduces to $(0, 1, 0, 0)$ and, in the case of dark energy to $(0, 1, \Delta_{\text{de}}, \Pi_{\text{de}})$, even though in most cases Δ_{de} and Π_{de} are negligible. Their expression for quintessence, scalar–tensor and DGP models can be found in Ref. [59]. For the same reason that, in the case of a GR modification, at the background level P_{de} and ρ_{de} can depend on H , \mathcal{S}_{de} , Δ_{de} and Π_{de} can depend on Φ and Ψ , while F is a function of the background quantities only.

Equations (17–19) imply that the matter density evolves as

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi GF(k, a)\rho_m a^2 \delta_m = \mathcal{C}_{\text{de}} \quad (21)$$

with $\mathcal{C}_{\text{de}} = (\Delta_{\text{de}} + \Pi_{\text{de}})/a^2 - \mathcal{S}_{\text{de}}/a$ so that the matter power spectrum is expected to be deformed in shape, mainly because of the k -dependence arising from F and from the source term \mathcal{C}_{de} .

This idea to construct such a post- Λ CDM parameterisation on sub-Hubble scales was first proposed in Ref. [59], following the analysis of the particular case of scalar–tensor theory by Ref. [27]. Several similar approaches were then designed in Refs. [69; 70; 71; 74; 75; 76; 77; 78; 79; 80; 81] which went further in determining the relations with observational data (see discussion below). In particular Ref. [70] uses, instead of Π_{de} , a parameter η defined by

$$\Phi = (1 + \eta)\Psi,$$

so that it is a generalisation of the post-Newtonian parameter γ^{PPN} , as first proposed in the particular case of scalar–tensor theories in Ref. [27]. ($\Phi \neq \Psi$ in the Solar system is an indication of a modification of GR because one is dealing with vacuum solution of Einstein equation; again, this is not the case in cosmology since dark energy can have an anisotropic stress). Instead of using the the set (f, Δ_{de}) , most analysis, including Refs. [69; 70; 78; 79; 80], assume that the Poisson equation is modified to

$$-k^2\Psi = 4\pi GQ(k, a)\rho_m a^2 \delta_m$$

involving only one new function Q . In most cases at hand, this is a good approximation, but in full generality we should distinguish the large-scale modification of GR and the clustering of possible new degrees of freedom (see e.g. Ref. [68; 69]).

In particular Δ_{de} could be an independent random variable, not proportional to δ_{m} . The limit $\Delta_{\text{de}} = 0$ corresponds to pure modification of GR with negligible effect of the new degree to the total stress-energy tensor while $F = 1, \Delta_{\text{de}} \neq 0$ corresponds to models of clustering dark energy (and may also incorporate the effect of massive neutrinos; see e.g. [82] for a study of this degeneracy).

As such, this description is not complete since we have no equation to describe the evolution of the new degrees of freedom. At the background level, this gap is often filled by assuming a parameterisation of the dark energy equation of state. As we shall see, two roads may be followed from this point: either one parameterized the unknown functions that appear here or one construct null-tests.

4.3 Cosmological data

The previous analysis shows that in order to constrain the deviation from GR with the large scale structure of the universe we need to be able to extract information on the distribution of the four variables $(\Phi, \Psi, \delta_{\text{m}}, \theta_{\text{m}})$ which are not directly observable. Let us summarize briefly some of the observations that turn to be useful. We call $P_{XY}(k, z)$ the 3-dimensional power spectrum of the fields X and Y at redshift z (or equivalently time t or distance χ) defined by $\langle X(\mathbf{k}, z)Y(\mathbf{k}', z)^* \rangle = (2\pi)^3 P_{XY}(k, z) \delta^{(3)}(\mathbf{k} - \mathbf{k}')$ and $C_{XY}(\ell, z)$ their 2-dimensional (or angular) power spectra.

4.3.1 Background data

Background data usually include the luminosity distance-redshift relation probed by SNIa which provides a handle on $H(z)$ up to $z \sim 1.5$, the angular diameter distance mainly from the tangential component of the BAO measured by their imprint on galaxy distribution.

4.3.2 Large scale structure

The clustering of galaxies is one of the oldest measures of the properties of the large scale structure. The galaxy power spectrum P_{gg} is the simplest measure of the correlations in the galaxy number density n_g . In general the distribution of galaxies is biased with respect to the mass distribution and it is often assumed that they can be related by

$$\delta_g \equiv \frac{\delta n_g}{n_g} = b_1 \delta_{\text{m}} + \frac{b_2}{2} \delta_{\text{m}}^2, \quad (22)$$

where b_1 and b_2 are bias parameters. It is expected that the bias is a complicated function of time and of the masses of the halo hosting the galaxies [83], since it encodes in some way all the process of galaxy formation. We shall assume here, for simplicity, that we restrict to a linear bias so that $P_{gg}(k, z) = b_1^2(z) P_{\delta_{\text{m}} \delta_{\text{m}}}(k, z)$. Imaging data with photometric redshift provides a measurement of the angular

power spectrum of galaxies, which is a simple projection of the three-dimensional power spectrum

$$C_{gg}(\ell) = \int \frac{W_g^2(\chi)}{S_K(\chi)^2} P_{gg} \left(k = \frac{\ell}{S_K(\chi)}, \chi \right) d\chi, \quad (23)$$

S_K being the comoving angular distance and where W_g is the normalized redshift distribution of galaxies in the sample.

However, the redshift-space position of any galaxy differs from its real space position due to its peculiar velocity. The density contrast in redshift space, δ_g^s , and in real space, δ_g , can be related by imposing mass conservation [84]. In the linear regime, this leads to $\delta_g^s = \delta_g + \mu^2 \theta_g / aH$, where μ is the cosine of the line-of-sight angle so that the redshift-space power spectrum is

$$P_{gg}^s(k, \mu) = P_{gg}(k) + 2 \frac{\mu^2}{aH} P_{g\theta_g}(k) + \frac{\mu^4}{a^2 H^2} P_{\theta_g \theta_g}(k). \quad (24)$$

It is thus commonly modelled as [83; 84]

$$P_{gg}^s(k, \mu) = \left[P_{gg}(k) + 2 \frac{\mu^2}{aH} P_{g\theta_g}(k) + \frac{\mu^4}{a^2 H^2} P_{\theta_g \theta_g}(k) \right] F \left(\frac{k^2 \mu^2 \sigma_v^2}{H^2(z)} \right), \quad (25)$$

where F is a smoothing function and σ_v is the 1-dimensional velocity dispersion. The angular dependence enables to separate the different components [85] to get a measurement of the three spectra $P_{gg}(k, z)$, $P_{g\theta_g}(k, z)$ and $P_{\theta_g \theta_g}(k, z)$, in particular from the SDSS [86] and 2dF [87] galaxy redshift surveys. These low redshift analysis were extended to $z \sim 1$ in Ref. [88] using the VIMOS-VLT Deep Survey (VVDS) [89].

Indeed, in the standard Λ CDM, and in the linear regime, we have that $\theta_g = -a\dot{\delta}_m$ so that its growth rate is $D_{\theta_g} = -a\dot{D}$ so that $\theta_g = aHf\delta_m$. In that limit $P_{g\theta_g} = aH\beta P_{gg}$ and $P_{\theta_g \theta_g} = a^2 H^2 \beta^2 P_{gg}$ with $\beta = f/b$, and the three spectra are not independent.

4.3.3 Weak lensing

Gravitational lensing offers various possibilities. As previously, we restrict our analysis to metric theories of gravity.

First, either in the strong or weak regime, it can probe the sum of the two Bardeen potentials, $\Phi + \Psi$. Weak lensing surveys use the observed ellipticities of background galaxies (and more particularly the correlation of their shapes) to reconstruct a map of the cosmic shear, which can then be used to determine the convergence κ [90]. As long as photons travel on null geodesics and the geodesic deviation equation holds, the distortion of the shape of background galaxies can be computed from the Sachs equation [91] leading to

$$\kappa(\theta, \chi_i) = \frac{1}{2} \int W(\chi, \chi_i) \Delta_2(\Phi + \Psi), d\chi \quad (26)$$

for sources located in a bin centered round a redshift z_i and with $\chi_i = \chi(z_i)$ with

$$W(\chi, \chi_i) = \frac{S_K(\chi)S_K(\chi_i - \chi)}{S_K(\chi_i)}.$$

The convergence power spectrum for two sets of galaxies centered around z_i and z_j , as can be obtained by a tomographic survey, is thus

$$C_{\kappa\kappa}(\ell, z_i, z_j) = \frac{\ell^4}{4} \int W(\chi, \chi_i)W(\chi, \chi_j)P_{\Phi+\Psi}\left(k = \frac{\ell}{S_K(\chi)}, \chi\right) d\chi. \quad (27)$$

Until we have data allowing for the use of tomography, we have only access to the shear power spectrum averaged on the source redshift distribution, $P_{\kappa\kappa}(\ell)$ which is given by the same expression but with the window function

$$W(\chi) = S_K(\chi) \int_{\chi} W_g(\chi') \frac{S_K(\chi' - \chi)}{S_K(\chi')} d\chi'.$$

In conclusion, this allows to constrain the power spectrum $P_{\Phi+\Psi}(k, z)$. Note that the analysis of weak lensing requires in fact to know the non-linear power spectrum but the latest data from the CFHTLS [92] reach large angular scales ($\theta > 30$ arcmin.), which allows to work in the (quasi) linear regime, where theoretical predictions for the time evolution of the power spectrum are more reliable. Note also that the convergence power spectrum is often expressed in terms of the matter power spectrum, making use both of the Friedmann and Poisson equations. Indeed, the goal here is to relate the observables to their primary perturbation variables without using any equations.

Second, one can use galaxy–galaxy lensing, which arises when the deflecting and source galaxies are aligned, giving rise to a mean tangential shear around foreground galaxies. This will thus give an information on the correlation between the galaxy distribution and $\Phi + \Psi$ through the angular power spectrum

$$C_{g\kappa}(\ell, z_i, z_j) = \frac{\ell^2}{2} \int W_g(\chi, \chi_i)W(\chi, \chi_j)P_{g, \Phi+\Psi}\left(k = \frac{\ell}{S_K(\chi)}, \chi\right) d\chi. \quad (28)$$

This was for instance measured from the SDSS galaxy survey. Note that the magnification bias [83] can also help to extract some correlations since

$$\delta_g = b\delta_m + (5s - 2)\kappa$$

where $s = \text{dlog}N/\text{dm}$ is the logarithmic slope of the number count-magnitude function. This induces distortions [93; 94] that can also be used to test GR. Note that it was also proposed that the weak lensing of standard candles (SNIa, or GW sirens) can be used to measure the cross correlation between the magnification μ and δ_g [95; 96]. In particular $C^{\mu\mu}$ and $C^{g\mu}$ contain informations similar to $C^{\kappa\kappa}$ and $C^{g\kappa}$ respectively. This has not been investigated yet.

4.3.4 Integrated Sachs–Wolfe effect

The observation of the cosmic microwave background temperature anisotropies gives numerous important informations for our cosmological model, among which the initial power spectrum for the perturbations. While propagating from the last scattering surface to us, the energy of the photons changes due to the fact that they cross structures in formation, and thus propagate in a spacetime where Φ and Ψ are not constant. This induces a direction-dependent temperature change, known as the integrated Sachs–Wolfe effect [63]

$$\frac{\Delta T}{T} = \int (\dot{\Phi} + \dot{\Psi}) a(\chi) d\chi,$$

where the integral is performed along the photon geodesic. This ISW effect is correlated with the galaxy distribution with angular power spectrum

$$C_{g,ISW}(\ell) = \int P_{g,(\dot{\Phi}+\dot{\Psi})} \left(k = \frac{\ell}{S_K(\chi)}, \chi \right) \frac{a^2(\chi)}{\chi^2} d\chi. \quad (29)$$

This has been detected [97] by cross-correlating the CMB anisotropies to galaxy maps.

4.3.5 Conclusions

We see that astrophysical observations allow to measure many correlations between the perturbation variables. In order to construct tests of GR, one needs to relate δ_g to δ_m and thus understand the bias (and most importantly constrain its scale dependence). The example given above are the most promising to implement the tests of GR but, indeed, there exist many other ways to measure these quantities.

We also need to keep in mind that each of this method has its own systematics and limits. We cannot discuss this issue here, but it is central when actually deriving constraints.

4.4 Growth of matter perturbations

The first effect of a modification of GR is to change the growth of density perturbation. In the Λ CDM model, Eqs. (11–12) imply that the growth rate D evolves as

$$\ddot{D} + 2H\dot{D} - 4\pi G\rho_m D = 0. \quad (30)$$

This equation can be recast in terms of a as time variable [63] as

$$D'' + \left(\frac{d \ln H}{da} + \frac{3}{a} \right) D' = \frac{3}{2} \frac{\Omega_{m0}}{a^5} D, \quad (31)$$

from which it can be checked that $D = H$ is a solution so that the growing mode can be obtained as

$$D = \frac{5}{2} \frac{H}{H_0} \Omega_{m0} \int_0^a \frac{du}{[uH(u)/H_0]^3}, \quad (32)$$

which implies that if $H(z)$ is known from background observations, such as SNIa, then $D(z)$ is fixed. There is a rigidity between the expansion history and the growth rate.

The growth rate can be parameterized phenomenologically as [98; 99]

$$\frac{d \ln D}{d \ln a} = \Omega_m^\gamma. \quad (33)$$

Then, if GR is not modified, the index γ can be computed once $H(z)$, or equivalently the dark energy equation of state, is known and it was shown [100; 101] that

$$\gamma = 0.55 + 0.05[1 + w_{de}(z = 1)].$$

While being a good test of dark-energy model with a smooth energy distribution, it is not clear whether it can be considered as a test of GR. In particular, we can imagine that dark energy has an anisotropic stress or is clustering while GR is not modified (i.e. $F = 1$, $\mathcal{S}_{de} = 0$, $\Pi_{de} \neq 0$, $\Delta_{de} \neq 0$ so that $\mathcal{S}_{de} \neq 0$) so that one accomodate a value of γ by some properly designed model. To finish, such a parameterisation is too restrictive since it does not include the scale-dependence that is expected from the modification of GR (see however Ref. [81]). The extensions to include the super-Hubble regime were considered in Refs. [74; 81; 102]. It was recently proposed [103] that galaxy cluster velocities, measured from the kinetic SZ effect, may allow for a measurement of γ .

Several studies concentrate on a pure modification of the Poisson equation so that only the term $F(k, H)$ is modified. The effect of such a k -dependent term on the power spectrum was first studied in Ref. [3] where the function F was assumed to reproduce the effect of higher-dimensional gravity, as described at the time. In that particular case, where the only parameter is the length scale r_s , it was shown [105] that the cosmic shear [104] 3-point function implies that $r_s > 2h^{-1} \text{Mpc}$.

Similar analysis in the case of a Yukawa type modification of the gravitational potential,

$$\Phi(\mathbf{r}) = -G \int d^3 \mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \left[1 + \alpha \left(1 - e^{-|\mathbf{r} - \mathbf{r}'|/\lambda} \right) \right]$$

were then performed. With such a potential, the function $F(k, a)$ entering the Poisson equation is given by [106; 107]

$$F(k, a) = 1 + \alpha \frac{(a/k\lambda)^2}{1 + (a/k\lambda)^2}. \quad (34)$$

Such a modification causes the rate of growth to depend on k so that the scale λ shall have an imprint on the power spectrum (see also Ref. [108] for a general argument on the shape dependence). Interestingly, assuming an Einstein–de Sitter background cosmology, Eq. (21) can be solved analytically in terms of hypergeometric function [106; 107] to give the growing mode

$$\delta_+(k, a) = s_2 F_1 \left(\frac{5 - \sqrt{25 + 24\alpha}}{8}, \frac{5 + \sqrt{25 + 24\alpha}}{8}, \frac{9}{4}; -s^2 \right)$$

with $s \equiv a/k\lambda$.

Reference [106] considered the case of a Einstein–de Sitter background and analyzed the SDSS and 2dFGRS data up to $k \sim 0.15h/\text{Mpc}$, leading respectively to the constraints $\alpha = 0.025 \pm 1.7$ and $\alpha = -0.35 \pm 0.9$ at a 1σ level. A similar analysis was performed in Ref. [107] who used the Peacock and Dodds procedure [109] to describe the non-linear power spectrum. The analysis of the SDSS data sets the constraints $-0.5 < \alpha < 0.6$ (resp. $-0.8 < \alpha < 0.9$) for $\lambda = 5h^{-1}$ Mpc (resp. $\lambda = 10h^{-1}$ Mpc). The analysis was extended in Ref. [110] by performing both second order perturbations and N-body simulations to construct a mock galaxy catalog. Reference [111] extended these analysis by allowing a modified expansion rate, which should be the case if GR is modified. They also showed that the modification of the shape of the power spectrum is almost degenerate with the effect of massive neutrinos. Notice that a combined analysis [112] using CFHTLS weak lensing data and the SDSS matter power spectrum estimated from luminous red galaxies found no sign of deviation from GR on scales ranging between 0.04 and 10 Mpc. Even though this analysis used both matter distribution and weak lensing, it only constrained the shape of the power spectrum without implementing the consistency check proposed in Ref. [3].

N-body simulations with such a Yukawa modification of the gravitational potential were performed in Ref. [113] who concluded that the gravitational evolution is almost universal, at least for λ in the 1–20 Mpc range so that the Peacock and Dodds approach [109] can be adapted to get an analytical fit. It was extended by the simulations of Ref. [114] which include the possibility of an anisotropic stress and considered the case of DGP models with $r_s = (5, 10, 20)h^{-1}$ Mpc. To finish, the spherical collapse model and the estimate of the abundance of virialized objects was considered in Refs. [115; 116]. The scale dependence of the growth rate was proposed [117] to be studied in terms of

$$\varepsilon(k, a) = \Omega_m^{-\gamma}(a) \frac{d \ln D}{d \ln a} - 1,$$

which remains close to 0 for any smooth dark energy model. In particular it can be measured from future redshift surveys.

These studies allows to understand the effect of the modification of the Poisson equation, which is expected to be generic in any deviations from GR, and absent in all models of pure dark energy. They are thus very instructive but note that the background cosmology is in general not modified in a consistent way.

4.5 Testing GR on cosmological scales

There have been two main approaches to using cosmological data to constrain deviations from GR.

4.5.1 Parameterizing our ignorance

In the first approach, one tries to use the generalized set of perturbation equations (17–20) in order to compute various cosmological observables and compare them to astrophysical data. The main problem, as mentioned earlier, is that such a parameterisation cannot be complete unless the physics of the new degrees of freedom is known. We thus have two possibilities. Either one computes explicitly these terms in some classes of theories such as $f(R)$, DGP or scalar–tensor [59; 70; 71] or one specifies some ansätze for these functions, in the same spirit as we introduced the parameterisation (9) for the dark energy equation of state.

For instance, Refs. [70; 78] assume that the function $\Sigma \equiv Q(a, k)(1 + \eta(a, k)/2)$ can be expanded as $\Sigma = 1 + \Sigma_0 a$ with Σ_0 constant. The effect of the modification of GR is taken into account through a parameterisation of the form (33), with γ constant so that one ends up with 4 constant extra-parameters ($w_0, w_a, \gamma, \Sigma_0$) besides the standard cosmological parameters, the Λ CDM model corresponding to $(w_0, w_a, \gamma, \Sigma_0) = (-1, 0, 0.55, 0)$. Such a parameterisation was then used to discuss the sensitivity of various probes. Clearly, this choice misses a possible scale-dependence of F (or Q) which is generically expected if GR is modified [108]. This issue was recently addressed in Ref. [118] which proposes to expand the two unknown functions Q and η as

$$X(a, k) \simeq X_0(a) + X_1(a)aH/k.$$

Ref. [119] chooses the same two functions as functions of k and a and proposes different ansätze for their functional form in order to study the potential of upcoming and future tomographic surveys to constrain them. Reference [71] (see also Ref. [72]) proposed a parameterisation that depends on the scale and Ref. [73] a parameterisation of the redshift dependence of γ . On the other side Refs. [76; 77] focus only on the function $\eta(z)$, that they call $\varpi(z)$, in order to infer its influence on CMB anisotropy spectrum and weak lensing. ϖ was chosen to scale as $\varpi_0 \rho_{\text{de}}(z)/\rho_{\text{m}}(z)$, assuming a Λ CDM evolution for the background cosmology.

Note that in the standard Λ CDM, we must have that $\Psi(\mathbf{k}, a) = \Phi(\mathbf{k}, a) = -3\Omega_{\text{m}}(a)(Ha/k)^2 \delta_{\text{m}}(\mathbf{k}, a)/2$ and $\theta_{\text{m}}(\mathbf{k}, a) = -f \delta_{\text{m}}(\mathbf{k}, a)$. Thus, instead of parameterizing the unknown terms that enter the perturbation equations, we may think to parameterize directly their solution, for instance, as

$$\begin{aligned} \Phi(\mathbf{k}, a) &= -\frac{3}{2}\Omega_{\text{m}}(a)(Ha/k)^2 \delta_{\text{m}}(\mathbf{k}, a) [1 + c_{\Phi}(k, a)], \\ \Psi(\mathbf{k}, a) &= -\frac{3}{2}\Omega_{\text{m}}(a)(Ha/k)^2 \delta_{\text{m}}(\mathbf{k}, a) [1 + c_{\Psi}(k, a)], \end{aligned}$$

and

$$\theta_m(\mathbf{k}, a) = -f\delta_m(\mathbf{k}, a) [1 + c_\theta(k, a)],$$

where $\delta_m(\mathbf{k}, a)$ is supposed to scale as $\delta_m(\mathbf{k}, a) = \delta_m(\mathbf{k}, a_m)D(a) [1 + c_\delta(k, a)]$ and then finding physically motivated ansätze for the functions c_i . Such an alternative parameterisation was considered in Ref. [75] (see also Refs. [72; 73]).

In these approaches, the game is thus to replace free unknown functions by a set of parameters in order to be able to compute the different cosmological observable and then compare them to data. This allows in particular to understand the accuracy with which they can be constrained by forthcoming experiments, the main difficulty being to find the most relevant set of parameters that reproduce a large class of theories. Reference [78] utilised the large angular scales ($\theta > 30$ arcmin.) weak lensing data from the CFHTLS (in order to work in the linear regime), BAO and SNIa data to get constraints on $(\Omega_{m0}, \Sigma_0, \gamma)$ consistent (e.g. related to the parametrisation of the equation of state) with the standard Λ CDM. Note also that it is important that the background and perturbation dynamics be consistent since they derive from the same modification of GR, an issue often overlooked.

4.5.2 The art of correlating

A probably better idea to obtain constraints on deviation from GR is to construct null tests. Such tests are based on the simple fact that once the theory is completely specified, there must exist consistency relations, between different observables, in a similar way as the Solar system example of the introduction led to the consistency relation Eq. (1). Indeed, any departure from such a relation would indicate that some hypothesis of our model are not correct and that the theory needs to be extended, without telling how. Such tests are *null tests*, in the spirit of “traditional” physics in which a reference model is confronted to observations in order to determine the limits of its validity.

The use of cosmological data to perform such tests was first proposed in Ref. [3] who focused on the Poisson equation (13). If such an equation holds then the power spectra of the gravitational potential $P_\Phi(k)$ and of the matter distribution $P_{\delta_m}(k)$ must be related by

$$k^4 P_\Phi(k, a) = \frac{9}{4} \Omega_{m0} H_0^2 a^{-2} P_{\delta_m}(k, a),$$

whatever the cosmological scenario. This means that the scale dependence of the two spectra are related in a very specific way. In particular, if the Poisson equation is modified, the change of the shape of the matter power spectrum is, as we saw on the example of a Yukawa potential above, model dependent but the fact that the two spectra differ is a model independent conclusion. In particular such a relation can be tested by comparing weak lensing data to galaxy survey, if the scale dependence of the bias is mild, as expected from numerical simulation, since $C_{\kappa\kappa}$ and P_{gg} give access to P_Φ and P_{δ_m} . Note also that, it has a trivial generalisation if the fields are all proportional to the same stochastic variable (which is the case for adiabatic initial conditions) then $P_{\Phi\delta_m} = \sqrt{P_\Phi P_{\delta_m}}$, which again can be tested using galaxy–galaxy lensing.

Similar rigidities were exhibited between the background dynamics and the growth of the large scale structure. For instance, the growth equation (31), valid for a Λ CDM, and thus when GR hold, can be recast [120; 121] (see also Refs. [122; 123]) as a first order equation for H so that $H(z)$ can be inferred from background data and perturbation data independently, or equivalently the equation of state of dark energy (9) and the parameter γ defined in Eq. (33) are not independent when the dark energy is assumed to remain smoothly distributed. This was implemented in the analysis of Refs. [124; 125] who introduce two dark energy equations of state, one for the evolution of the background geometry and the other governing the growth. Using SNLS-SNIa, 2dF and SDSS galaxy redshift survey, CMB data and CTIO-lensing survey, they concluded that the two determinations of Ω_Λ were consistent and that the two constant dark energy equations of state have also to agree. These analysis consider only the effect of the growth factor and no other modification is considered. Another implementation performs a model-independent reconstruction of the growth rate from distance measurements and then compares to growth measurements [126; 127]. Reference [128] proposed a similar consistency test of the Λ CDM using low and high redshift SNIa survey by estimating Ω_m in three different ways (background geometry, growth, and shape of the power spectrum), all agreeing with the canonical value 0.25.

The original idea of Ref. [3] was extended to multiple cosmological probes. Reference [129] proposed to use the galaxy-velocity correlation and the galaxy-galaxy lensing, which give access to $\langle \delta_g \theta_m \rangle \propto bf \langle \delta_m^2 \rangle$ and $\langle \delta_g \kappa \rangle \propto b \langle \delta_m \Delta(\Phi + \Psi) \rangle$ so that the ratio of these two quantities is expected to be independent of the bias, at least in the regime of linear biasing. An estimator, \hat{E}_G based on the ratio of these two quantities, was constructed and it was demonstrated that it can distinguish a large class of models. Reference [69] worked out the relations between the various observables, including a discussion of quasilinear effects. It was also shown that the clustering of dark energy can mimic features of a modification of GR and investigated the way to combine data in order to distinguish the two effects. References [79; 80] designed consistency checks based on the redshift-space power spectrum and weak lensing in order to constraint the ratio Φ/Ψ and the Poisson equation. Reference [130] proposed an estimator to measure the ratio of the two gravitational potentials, again using weak lensing and redshift-space power spectrum. Reference [131] proposed a method to extract the effect of a modified Poisson equation and Ref. [118] analyzed the combination of imaging and spectroscopic surveys and Refs. [72; 132] discusses the gain of using tomography. Reference [133] proposed to use the ISW-structure correlation to constrain the growth rate of the density, and in particular its scale-dependence.

All these works are thus starting from the constitutive relations that exist in a Λ CDM, and thus assuming GR valid to construct from large scale structure survey some tests that will indicate the violation of one of these relations. They often construct estimators that are probed by using some extensions of GR (DGP, $f(R)$, scalar-tensor) and in order to forecast the power of coming surveys to distinguish between them.

5 Conclusions

This review has presented the tests of GR on astrophysical scales, but more generally of the Λ CDM model, based on the large scale structure and the global dynamics of the universe. In particular, it is important to make the distinction since some supposedly tests of GR proposed in the literature are in fact only tests of the Λ CDM model. Maybe the first answer these tests will give is whether there is a need for new physical degrees of freedom in our model and then start to characterize the nature and the couplings of this field with standard matter (and also dark matter).

As explained, future surveys will allow to map weak lensing, galaxy distribution and velocity on sub-Hubble scales with high accuracy. This will allow to construct many consistency checks of our cosmological model, and in particular of GR. A multi-probe approach will allow to have a better control of systematics which affect each probe.

Today, data shows no deviation from the Λ CDM model, and thus from GR on large scales. On galactic scales, the debate between dark matter and MOND-inspired models is yet unsettled even though the need of massive neutrinos to reconcile MOND with cluster data seem to disfavor this latter approach. Besides, all analysis on cosmological still assume the existence of dark matter.

It is important to keep in mind that these are not the only tests of the deviation from the standard Λ CDM that can be performed. Let us mention

- *Test of the weak equivalence principle.* They can be performed on a large band of redshifts, up to BBN time, by constraining the time variation of fundamental constants [6].
- *Test of the distance duality relation.* In standard cosmology the angular and luminosity distances are related by $D_L = (1+z)^2 D_A(z)$. This equation holds in any metric theory of gravity if the number of photons is conserved. By testing it [134], one can check the validity of Maxwell theory and constrain models such a photon-axion oscillation.
- *Test of the Copernican principle.* All the equations and solutions we have used, assumed the existence of a homogeneous and isotropic background spacetime. It is only an assumption based on the Copernican principle and recently many proposals to test it appear in the literature [135; 136; 137; 138].
- *Propagation of gravity waves.* In bi-metric theories of gravity, gravity waves and photons may not necessary follow the geodesics of the same metric so that there can exist a time delay between them [139]. Confronting their arrival times (as well as those of neutrinos, if massive) allows to set constraints on bimetric theories of gravity. If gravity propagates slower than light then some tight constraint can arise from the energy loss of cosmic rays by gravitational Čerenkov radiation [140] leading to $c_{\text{GW}}/c - 1 < 2 \times 10^{-19}$.

These tests will enable to check the robustness of the hypothesis on which our cosmological model rests. It will either confirm the need for the existence of dark matter and dark energy (thus extending drastically the domain of validity of GR) or offer new theoretical constructions to explain the late time acceleration of the cosmic expansion.

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