On cosmic censor in high-energy particle collisions

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Abstract. In the context of large extra-dimension or TeV-scale gravity scenarios, miniature black holes might be produced in collider experiments. In many works the validity of the cosmic censorship hypothesis has been assumed, which means that there is no chance to observe trans-Planckian phenomena in the experiments since such phenomena are veiled behind the horizons. Here, we argue that "visible borders of spacetime" (as effective naked singularities) would be produced, even dominantly over the black holes, in the collider experiments. Such phenomena will provide us an arena of quantum gravity.

The higher-dimensional scenarios with large [1] and warped [2] extra dimensions were proposed to resolve or reformulate the hierarchy between the gravitational and electroweak interactions. In these scenarios the D-dimensional ($D > 4$) Planck energy E_P is related to that in 4-dimension $E_{P(4)}(\sim 10^{19}$ GeV) by $E_{P(4)}^2 \sim L^{D-4}E_P^{D-2}$, where L is the size of an extra dimension. Thus, if L is large enough, the fundamental Planck energy E_P can be as low as a few TeV. For instance, such a small Planck energy is realized if $L \sim 0.1$ cm when $D = 6$, $L \sim 10^{-7}$ cm when $D = 7$. If the Standard Model particles but the gravitons (and possibly other unobserved particles) are confined to our 3-brane these scenarios are consistent with all current observations.

One of the most striking predictions of such scenarios is the productions of a large number of mini black holes in high-energy particle collisions [3]. A simplified picture of the black hole production is put in the following way. The size of a black hole is characterized by Schwarzschild radius R_g, scaling with its energy as $R_g \propto E^{1/(D-3)}$. If colliding particles (partons) have a center-of-mass energy E above a threshold energy of order E_P and an impact parameter less than the Schwarzschild radius, a black hole of energy (mass) E is produced. The black holes so produced will decay thermally via the Hawking radiation and be detected in terrestrial collider experiments such as CERN Large Hadron Collider and in ultrahigh energy cosmic rays. Such possibilities have been extensively studied and known to give rise to rich phenomenology.

Quite recently, the present author and his collaborators pointed out another interesting possibility [4] of the high-energy particle collisions in the TeV-scale gravity. They argued that effective naked singularities called *the visible borders of spacetime* would be generated by high-energy particle collisions. A border of spacetime, proposed in [5] originally, is defined as a domain of spacetime where the curvature becomes trans-Planckian and acts as an border (or boundary) of classical sub-Planckian regimes. That is, the domain $\mathcal{A}$ of spacetime is called the border if the following inequality is satisfied:

$$\inf_{\mathcal{A}} F \geq \alpha E_P^2, \tag{1}$$

where α is a positive constant of order unity and F is given, for instance, by using the scalar polynomials of the Riemann tensor, or components of the Riemann tensor with respect to a parallelly transported orthonormal basis e^a_μ as

$$F := \max\left(|R^a_a|, |R^{ab}R_{ab}|^{1/2}, |R^{abcd}R_{abcd}|^{1/2}, |R_{\mu\nu\rho\sigma}|\right). \tag{2}$$

We denote the union of all borders in spacetime $\mathcal{M}$ by $\mathcal{U}_{\rm B}$. We call border $\mathcal{A}$ a visible border if $J^+(\mathcal{A}, \mathcal{M}) \cup (\mathcal{M} - \mathcal{U}_{\rm B})$ is not empty, where $J^+(\mathcal{A}, \mathcal{M})$ is the causal future of $\mathcal{A}$ in $\mathcal{M}$. We can also naturally introduce a globally visible border as an extension of a globally naked singularity [5].

In the rest of this article, we discuss whether the visible borders of spacetime, defined above as effective naked singularities, can be produced in high-energy particle collisions. We will see that the visible borders of spacetime would be generated in the collider experiments such as the Large Hadron Collider.

Let us consider a collision of two elementary particles. For simplicity, suppose that both particles are identical and structureless. The extra dimensions are assumed to be compactified into a length scale considerably larger than the fundamental Planck length E_P^{-1}, whereas the energy of the particles is assumed to be confined within the length scale E_P^{-1} in every direction of the extra dimensions; i.e., within a volume $V_{D-4}E_P^{-(D-4)}$ in the extra dimensions, where V_n is the volume of a unit n-ball $V_n = \pi^{\frac{n}{2}}/\Gamma\left(\frac{n}{2}+1\right)$, where $\Gamma(x)$ is the gamma function. Then, we consider the distribution of two particles in the remaining three-dimensional space.

We denote the center-of-mass (CM) energy of the two particles by E, which is much larger than their rest masses, and assume that the three momenta of these particles are parallel to each other. The minimum uncertainty in the position of each particle measured in the longitudinal direction of their three momenta is approximately E^{-1} due to the Lorentz contraction.

It is assumed that these two particles will form a black hole if they pass through a transverse disk of radius $R_{\rm g}$ at almost the same time, where $R_{\rm g}$ is the horizon radius of the Schwarzschild–Tangherlini black hole with gravitational mass E,

$$R_{\rm g} := E_P^{-1}\left(\frac{E}{E_P}\right)^{\frac{1}{D-3}}, \tag{3}$$

and where we have defined the fundamental Planck scale E_P so that the reduced Compton wavelength agrees with the gravitational radius if the particle mass is equal to E_P. In this normalization, D-dimensional Einstein's gravitational constant κ ($G_{ab} = \kappa T_{ab}$) is related to E_P in the form $\kappa = \frac{1}{2}(D-2)S_{D-2}E_P^{2-D}$, where $S_n = (n+1)V_{n+1}$ is the area of a unit n-sphere. Based on this picture, the total cross section of black hole formation is given by $\sigma_{\rm tot} = \pi R_{\rm g}^2$.

We consider the case in which these two particles pass through a transverse disk with a radius of $\nu R_{\rm g}$, where $\nu > 1$. In this case, a black hole does not form. When these two particles pass through a disk of radius $\nu R_{\rm g}$ at almost the same time, they can be momentarily confined within a $(D-1)$-volume $V \simeq \pi\nu^2 R_{\rm g}^2 \times E^{-1} \times V_{D-4}E_P^{-(D-4)}$, where the second factor E^{-1} is the uncertainty in the longitudinal position of each particle and $V_{D-4}E_P^{-(D-4)}$ is the volume in the extra dimensions. At this moment, the average energy density ρ in this volume is estimated to be

$$\rho \simeq \frac{E}{V} = \frac{E_P^D}{\pi\nu^2 V_{D-4}}\left(\frac{E}{E_P}\right)^{2\left(\frac{D-4}{D-3}\right)}. \tag{4}$$

If $\kappa\rho$ is larger than or equal to αE_P^2, then, through the Einstein equation, the average Ricci tensor satisfies the criterion for a border (1). The average density ρ is a monotonically increasing function of the CM energy E. Thus, a sufficiently large CM energy can produce a visible border,

Table 1. Values of $\alpha\nu_{\max}^2$ when $E = 14$ TeV.

E_p	$D = 5$	$D = 6$	$D = 7$	$D = 8$	$D = 9$	$D = 10$	$D = 11$
1 TeV	66.0	180	309	437	559	674	783
14 TeV	4.71	5.33	5.89	6.40	6.87	7.31	7.73

since no black hole forms in this situation. The condition for realizing a border, $\kappa\rho \geq \alpha E_P^2$, and Eq. (4) lead to

$$\nu^2 < \nu_{\max}^2 := \frac{(D-2)S_{D-2}}{2\pi\alpha V_{D-4}} \left(\frac{E}{E_P}\right)^{2\left(\frac{D-4}{D-3}\right)}. \tag{5}$$

If the upper bound $\nu_{\max}^2$ is less than or equal to unity, the border will never be visible. Since $\nu_{\max}^2$ will an increasing function of the CM energy, if $E > [2\pi\alpha(D-2)^{-1}S_{D-2}^{-1}V_{D-4}]^{\frac{1}{2}\left(\frac{D-3}{D-4}\right)}E_P$, $\nu_{\max}^2$ is larger than unity, then the border will be visible.

The value of $(\nu_{\max}^2 - 1)$ expresses the ratio of the production rate of visible borders to that of black holes. The production rate of visible borders increases with increasing CM energy. In the case of the LHC ($E = 14$ TeV), if the fundamental Planck scale E_P is equal to 1 TeV, the production rate of visible borders will be about $70\alpha^{-1}$ times larger than the production rate of black holes for $D = 5$, whereas it will about $800\alpha^{-1}$ times larger than that of black holes for $D = 11$ (see Table 1). Although there is an ambiguous factor α in the criterion for the border (1), the present result, which is based on dimensional analysis, implies that if black holes form at the LHC, more visible borders will be produced than black holes.

Note added

After completing this article, the present author and his collaborators published a paper, in which motivated by the observations in this paper, the particle creation by naked singularities in higher dimensions was investigated [6].

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1. References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D **59**, 086004 (1999).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).

[3] P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B **441**, 96 (1998); T. Banks and W. Fischler, arXiv:hep-th/9906038; R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. **85**, 499 (2000); S. B. Giddings and S. D. Thomas, physics," Phys. Rev. D **65**, 056010 (2002); S. Dimopoulos and G. L. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001).

[4] K. i. Nakao, T. Harada and U. Miyamoto, Phys. Rev. D **82**, 121501(R) (2010).

[5] T. Harada and K. i. Nakao, Phys. Rev. D **70**, 041501 (2004).

[6] U. Miyamoto, H. Nemoto and M. Shimano, "Particle Creation by Naked Singularities in Higher Dimensions," arXiv:1012.2374 [gr-qc].