

Ringdown tests of general relativity with spin-precession

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Abstract. The coalescence of black-hole binaries provides a unique arena for testing general relativity in the dynamical and strong-field regime. The ringdown phase, where the merger remnant relaxes to a stationary state, offers a direct probe of the theory. It is dominated by the oscillation frequencies of the remnant, the so-called quasinormal modes, which reveal the nature of the remnant. We present a parametrized test of general relativity that constrains deviations in the quasinormal modes for spin-precessing binaries. We show that neglecting spin precession can lead to false detections of GR deviations, even at current detector sensitivities, emphasizing the need for accurate modeling. Finally, we reanalyze the events of the third Gravitational-Wave Transient Catalog. Using a hierarchical combination of these events, we accurately constrain fractional deviations in the frequency and damping time of the dominant quasi-normal mode.

The ground-based detectors LIGO and Virgo have detected 90 gravitational wave (GW) candidates from the coalescence of compact binaries in the first three observing runs [1, 2, 3]. In 2023, the LIGO-Virgo-KAGRA (LVK) collaboration began its fourth observing run, reporting 218 candidates [4]. The final stage of a compact binary coalescence is the ringdown when the remnant settles down to a stationary configuration. The ringdown is dominated by the characteristic oscillation frequencies of the remnant, the so-called quasi-normal modes (QNMs),

$$\omega_{\ell mn} = \omega_{R,\ell mn} + i\omega_{I,\ell mn}. \quad (1)$$

It is modeled as a sum of exponentially damped sinusoids, with their frequencies and damping times corresponding to the QNMs of the remnant, as follows:

$$f_{\ell mn} = \frac{\omega_{R,\ell mn}}{2\pi}, \quad \tau_{\ell mn} = -\frac{1}{\omega_{I,\ell mn}}. \quad (2)$$

The least-damped dominant QNM ($\ell = m = 2, n = 0$) has been observed in the ringdown of $\mathcal{O}(10)$ GW events [5, 6]. The ringdown observations are compatible with Kerr black hole (BH) remnants with fractional deviations [6]:

$$\delta f_{220} = 0.02_{-0.07}^{+0.07}, \quad \delta \tau_{220} = 0.13_{-0.22}^{+0.21}, \quad (3)$$

where an agreement with general relativity (GR) corresponds to $\delta f_{220} = \delta \tau_{220} = 0$.



Parametrized ringdown test of GR

The pSEOBNR test introduces fractional deviations to the frequency and the decay time of the fundamental quasinormal modes in an inspiral-merger-ringdown waveform as [7, 8, 9, 10]:

$$f_{\ell m 0} = f_{\ell m 0}^{\text{GR}} (1 + \delta f_{\ell m 0}), \quad \tau_{\ell m 0} = \tau_{\ell m 0}^{\text{GR}} (1 + \delta \tau_{\ell m 0}), \quad (4)$$

where the GR values are predicted from estimates of the mass and spin of the remnant and fits with numerical relativity. The baseline waveform model is developed in the effective-one-body formalism and is constructed as [11, 12]:

$$h_{\ell m}(t) = h_{\ell m}^{\text{insp-plunge}}(t) \Theta(t_{\text{match}}^{\ell m} - t) + h_{\ell m}^{\text{merger-RD}}(t) \Theta(t - t_{\text{match}}^{\ell m}), \quad (5)$$

where $\Theta(t)$ is the Heaviside step function, the amplitude and phase of $h_{\ell m}(t)$ are continuously differentiable at the matching time and are obtained through fits with numerical relativity waveforms.

In the third observing run of the LVK collaboration, the baseline waveform model was constructed for quasi-circular binaries with spins aligned to the orbital angular momentum of the system and higher-order modes $(\ell, |m|) = (2, 2), (2, 1), (3, 3), (4, 4), (5, 5)$ [13, 14, 15, 16].

However, the presence of spin precession could bias us to find a false GR violation when the waveform model used to test GR does not include this effect [17, 8]. In the fourth observing run, the baseline waveform model is for spin-precessing binaries and with higher-order modes $(\ell, |m|) = (2, 2), (2, 1), (3, 3), (3, 2), (4, 4), (4, 3), (5, 5)$ [18, 19, 20, 21, 22]. In a spin-precessing binary, we can identify three reference frames:

1. the inertial frame of the observer (source frame);
2. an inertial frame where the z -axis is aligned with the final angular momentum of the system (J_f -frame);
3. a non-inertial frame where the z -axis is instantaneously aligned with the orbital angular momentum (co-precessing frame).

We develop a parametrized ringdown test of GR including spin precession [23], where the fractional deviations to the frequency and decay time of the fundamental quasinormal modes are introduced in the co-precessing frame [24]. In the source frame, the spin precession induces a mixing of modes with the same ℓ and different m .

Parameter estimation on synthetic signals

We inject a numerical relativity waveform, i.e., SXS:BBH:0165, with mass ratio $q = 0.167$, total mass $M = 95M_{\odot}$, precessing spin $\chi_p = 0.78$ and SNR = 18. Fig. 1 shows the posterior probability distributions for the fractional deviations in the frequency and damping time of the $(2, 2, 0)$ QNM when the parameter estimation is performed with pSEOBNR with aligned-spin and spin-precessing configurations [23]. The aligned-spin model finds a false GR deviation in the ringdown damping time, whereas the spin-precessing model recovers GR correctly.

We also inject the numerical-relativity waveform of a boson star merger with compactness $\mathcal{C} = 0.2$ and tidal deformability $\Lambda = 10$ [25]. The pSEOBNR test can correctly identify the signal that does not originate from a binary BH in GR [23]. The BH hypothesis is indeed excluded at 90% credible level, and the model finds biases in the recovery of the GR parameters, e.g., mass ratio and effective inspiral spin.

Parameter estimation on real data

We apply our model to real data by analyzing 12 GW events from the third GW transient catalog [3] with SNR > 8 in the inspiral and post-inspiral regime. This selection criteria mitigates the degeneracy between the fundamental ringdown frequency deviation parameter and the remnant mass.

The analysis of the events with pSEOBNR including spin-precession is compatible with previous analyses assuming aligned spin configurations [23, 6]. For GW200129.065458 [3], the spin-precessing model is favoured to the aligned-spin model with $\ln \mathcal{B} = 5.1$. Fig. 2 shows the one-dimensional combined constraints on δf_{220} and $\delta \tau_{220}$, obtained by multiplying the posteriors from individual events (in blue), and using hierarchical combination (in orange). The combined results with spin-precession (unfilled curves) are consistent with the aligned-spin ones (filled curves). By combining the events hierarchically, we obtain [23]

$$\delta f_{220} = 0.00_{-0.06}^{+0.06}, \quad \delta \tau_{220} = 0.15_{-0.24}^{+0.26}. \quad (6)$$

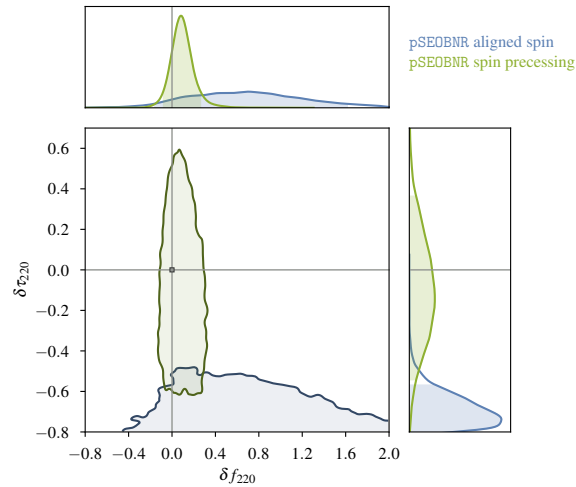


Figure 1: Posterior probability distributions for the fractional deviations in δf_{220} and $\delta \tau_{220}$ for a synthetic signal of a highly precessing waveform, SXS:BBH:0165. The vertical and horizontal lines mark the GR predictions, $\delta f_{220} = \delta \tau_{220} = 0$. The pSEOBNR model with aligned spins (blue) finds a false GR deviation in the damping time, whereas the pSEOBNR model including spin precession (green) recovers GR correctly [23].

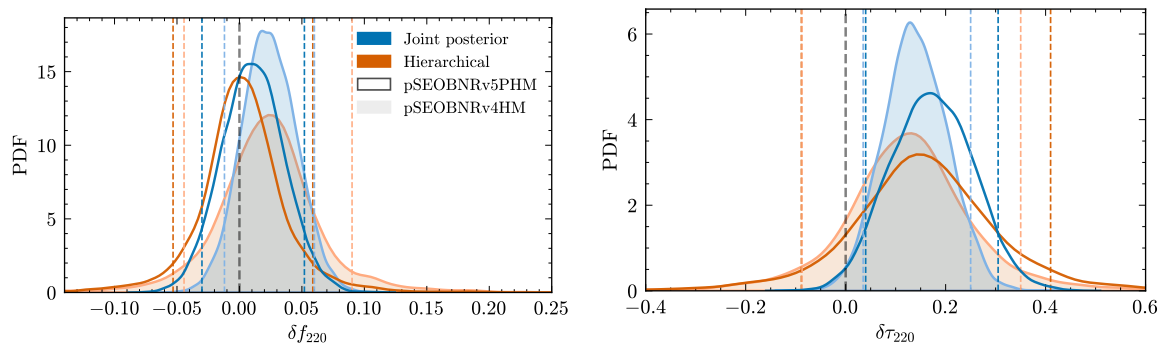


Figure 2: The combined constraints on the fractional deviations in the frequency and damping time of the (2, 2, 0) QNM, obtained by multiplying the posteriors from individual events (blue), and using a hierarchical combination (orange), from the GWTC-3 events. The Probability Density Function (PDF) with the pSEOBNR model including spin precession (pSEOBNRv5PHM, unfilled curves) agree with the one with aligned spins (pSEOBNRv4HM, filled curves) [23].

The joint posterior distribution seems in tension with the GR prediction for the damping time. This discrepancy can arise from a variety of factors, including parameter correlations, noise fluctuations, and intrinsic variance due to the limited number of events in the catalog [26].

Conclusions and future prospects

Parametrized tests allow us to constrain the degree to which the data agrees with GR. We presented a ringdown test of GR including spin-precession, which is applied to the analyses of the events observed in the forth observing run of the LVK collaboration. As future prospects, further improvements in waveform models are needed to perform accurate tests of GR, such as including eccentricity and higher-order modes. Finally, assessing the impact of waveform systematics and noise on testing GR is necessary for the next observing runs.

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