Correlations of Hawking radiation in acoustic black holes

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Abstract. I will review the main motivations and results regarding the proposal to use non-local correlations measurements to get evidence of the Hawking effect in acoustic black holes, focussing in particular on those built up with weakly interacting Bose Einstein condensates and rings of ions.

1. Introduction
Hawking's prediction of black holes evaporation is generally regarded as a milestone of modern theoretical physics. Combining Einstein's General Relativity and Quantum Mechanics, Hawking was able to show that black holes are not "black", but emit thermal radiation at a temperature inversely proportional to their mass [1]. This quantum mechanical process is triggered by the formation of a horizon and proceeds via the conversion of vacuum fluctuations into on-shell particles. Unfortunately, due to the extreme faintness of the effect, so far there is no experimental support for this amazing theoretical prediction.

In a remarkable work Unruh [2] showed that Hawking radiation is not peculiar to gravity, but is rather a purely kinematical effect of quantum field theory which only depends on field propagation on a black hole-type curved space-time background. This opens the concrete possibility to study the Hawking radiation process in completely different physical systems. As an example, the propagation of sound waves in Eulerian fluids can be described in terms of the same equation describing a massless scalar field on a curved spacetime characterized by an *acoustic metric* $G_{\mu\nu}$ which is function of the background flow: the curvature of the acoustic geometry is induced by the inhomogeneity of the fluid flow, while flat Minkowskii spacetime is recovered in the case of a homogeneous system. In particular, an *acoustic black hole* (or *dumb hole*) configuration is obtained whenever a subsonic flow turns supersonic: sound waves in the supersonic region are in fact dragged away by the flow and can not propagate back towards the *acoustic horizon* separating the supersonic and subsonic regions. Upon quantization, Hawking radiation is expected to appear as a flux of thermal phonons emitted from the horizon at a temperature proportional to its surface gravity. Even though a substantial effort has been spent on a variety of *analog models* [3] (e.g. superfluid Helium [4], phonons in atomic Bose-Einstein condensates [5], degenerate Fermi gases [6], slow light in moving media [7], or traveling refractive index interfaces in nonlinear media [8], ion rings [9]), the small Hawking temperature predicted even in these systems, and the actual impossibility to distinguish the Hawking contribution from the thermal background in the detection of the phonon content, has so far prevented any experimental verification of these predictions.

To overcome these difficulties, the detection of Hawking radiation via non-local correlation measurements has been first proposed in [10]. The idea is to get the quantum signature of the correlated Hawking quanta as a proof of the Hawking effect. Differently from the case of astrophysical black holes, both the external and the internal region of the acoustic black hole are in fact accessible to experiments: the quantum correlations between the Hawking radiation emitted inside and outside the acoustic black hole are responsible for a significant long-range correlation at points respectively inside and outside the acoustic black hole. This unique signature can be exploited to isolate Hawking radiation from the background of competing processes and experimental noise.

The main prediction contained in [10] has been numerically validated in [11]. Later correlations are calculated for black holes built with ion rings [9]. Then semi-analytical works followed [12, 13] explaining features that the first proposal paper didn't address.

2. Correlations of Hawking radiation in hydrodynamical systems

Eulerian fluids are described by the continuity and Bernoulli equations:

$$\dot{n} + \vec{\nabla} \cdot (n\vec{v}) = 0 \qquad \dot{\theta} + \frac{1}{2}\vec{v}^2 + \mu(n) = 0 \tag{1}$$

where n is the density (the mass of the fluid constituents is taken $m = 1$), $\vec{v} = \vec{\nabla}\theta$ the flow velocity, $\mu(n)$ is the specific enthalpy from which the state equation derives. The phonons propagation in these fluids can be obtained by the linearization of around the solution of Eqs. (1): $n + \hat{n}_1$ and $\theta + \hat{\theta}_1$. The core of the analogy is in the fact that it is possible to show that the fluctuation of the phase $\hat{\theta}_1$ satisfies an equation which is formally equivalent to a curved spacetime equation for a massless scalar field [2, 3]

$$\Box\hat{\theta}_1 = \frac{1}{\sqrt{-G}}\partial_\mu(\sqrt{-G}\, G^{\mu\nu}\partial_\nu)\,\hat{\theta}_1 = 0\,, \tag{2}$$

where $\Box$ is the curved d'Alembertian for the acoustic metric $G_{\mu\nu}$ of line element

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \frac{n}{c}[-c^2dt^2 + (d\vec{x} - \vec{v}dt)\cdot(d\vec{x} - \vec{v}dt)]\,, \tag{3}$$

G is the metric determinant, and c, defined by $c^2 = nd\mu/dn$, is the local sound speed. The density fluctuation $\hat{n}_1$ is algebraically related to the phase fluctuation $\hat{\theta}_1$:

$$\hat{n}_1 = -\frac{n}{c^2}(\partial_t\hat{\theta}_1 + \vec{v}\cdot\vec{\nabla}\hat{\theta}_1) \tag{4}$$

An acoustic horizon appears when the fluid turns supersonic, and, after its formation, Hawking radiation is expected as a thermal emission of phonons [2].

The Hawking process consists of the creation of correlated pairs of outgoing quanta triggered by the horizon formation [14, 15]: one member is emitted into an escaping mode, and corresponds to what is usually called Hawking particle; the other member, the so-called *partner*, is emitted into a trapped mode inside the horizon. Of course, this latter particle is definitively lost and can not be detected in the case of astrophysical black holes.

To get the correlations associated to the Hawking process the two-point function $H_2(x, x') = \langle\hat{\theta}_1(\vec{x}, t)\hat{\theta}_1(\vec{x}', t')\rangle$ is the relevant one, as it will be always possible to derive the density-density $\langle\hat{n}_1(\vec{x}, t)\hat{n}_1(\vec{x}', t')\rangle$ or the mixed one $\langle\hat{\theta}_1(\vec{x}, t)\hat{n}_1(\vec{x}', t')\rangle$ by simply acting the derivative operator

$\mathcal{D}_x$, defined as $\mathcal{D}_x = -\frac{n(\vec{x},t)}{c^2(\vec{x},t)}\left(\partial_t + v(\vec{x})\vec{\nabla}_{\vec{x}}\right)$ on H_2 once or twice, and then taking the $t' \to t$ limit if interested in the same-time correlations.

To obtain workable analytical expressions, the simplest case of an effective $1+1$ dimensional geometry is considered. This will be an excellent approximation whenever one can assume that the mode propagation in the transverse dimensions is negligible or actually frozen. Performing a dimensional reduction along the transverse direction [18], the two-point function for the field $\hat{\theta}_1$ can be approximated as:

$$\langle \hat{\theta}_1(x,t)\hat{\theta}_1(x',t')\rangle \simeq -\frac{\hbar}{4\pi\sqrt{C(x,t)C(x',t')}}\ln[\Delta\tilde{x}^-\Delta\tilde{x}^+] \tag{5}$$

where $C = \frac{n}{c}$ is the conformal factor for the metric (3), and $\tilde{x}^{\pm}$ are coordinates related to the light (sound) cone coordinates $x^{\pm} = t \pm \int \frac{dx}{(c\mp v)}$ by a conformal transformation, defining the proper quantum state. The expectation value entering the two-point function (5) is to be taken in the vacuum state of the positive frequency modes with respect to the proper $\tilde{x}^{\pm}$ coordinates [16]. After dimensional reduction, an effective potential term appears in the $1+1$ dimensional wave equation (2), which is responsible for backscattering of the modes. In what follows this term is neglected, and the final result is expected to be just slightly overestimated [17].

After the horizon formation, while the ingoing modes remain positive frequency modes with respect to the x^+ light cone coordinate, the outgoing modes are no longer positive frequency with respect to the x^- light cone coordinate. Indeed they emerge as positive frequency with respect to the generalized outgoing coordinate $\tilde{x}^- = \pm\frac{e^{-kx^-}}{k}$ [14, 16, 18], where the surface gravity at the horizon (located at $x^- = \infty$ or $\tilde{x}^- = 0$) is defined as

$$k = \frac{1}{2c}\frac{d}{dx}(c^2 - v^2)|_H \tag{6}$$

and fully determines the Hawking temperature $T_H = \hbar k/(2\pi\kappa_B)$ [16]. The $\pm$ signs in the definition of $\tilde{x}^-$ refer to outgoing modes which are respectively trapped inside the horizon or able to escape towards the asymptotic subsonic region.

To get the correlator associated to late times after the horizon has formed one must use $\ln(\Delta\tilde{x}^-\Delta x^+)$ as the master function defining the correct quantum state in Eq. (5).

3. Density-density correlations in weakly interacting Bose Einstein condensates

Let's move to the analog model of black hole based on a flowing atomic Bose-Einstein condensate (BEC) [5]. A weakly interacting BEC is described by an order parameter satisfying the Gross-Pitaevskii equations [19]. One can consider phonons by considering small fluctuations around a stationary and fully condensed state. One can write the Bose field operator as $\hat{\Psi} = e^{i\hat{\theta}}\sqrt{\hat{n}}$ in terms of the number density $\hat{n}$ and the phase $\hat{\theta}$ and then expand the density $\hat{n} = n + \hat{n}_1$ and phase $\hat{\theta} = \theta + \hat{\theta}_1$ operators around the classical background values n and θ fixed by the mean-field Gross-Pitaevskii equation.

We limit our attention to the so-called hydrodynamic limit, where perturbations are considered with wavelengths much longer than the healing length $\xi = \hbar/\sqrt{gn}$ (g the atom-atom interaction constant in the Gross-Pitaevskii equation). In this limit the Gross-Pitaevskii equations assume exactly the hydrodynamical form of Eqs. (1).

As the density operator $\hat{n}_1$ is algebraically related to $\hat{\theta}_1$ by $\hat{n}_1 = -\frac{1}{g}\left(\partial_t\hat{\theta}_1 + \vec{\nabla}\theta\cdot\vec{\nabla}\hat{\theta}_1\right)$, the one-time density-density correlation function

$$G_2(x,x') = \langle\hat{n}(x)\hat{n}(x')\rangle - \langle\hat{n}(x)\rangle\langle\hat{n}(x')\rangle = \langle\hat{n}_1(x)\hat{n}_1(x')\rangle \tag{7}$$

can be simply expressed in term of the two-points function for the field $\hat{\theta}_1$:

$$G_2(x,x') = \lim_{t' \to t} \mathcal{D}_x \mathcal{D}_{x'} \langle \hat{\theta}_1(x,t) \hat{\theta}_1(x',t') \rangle \,. \tag{8}$$

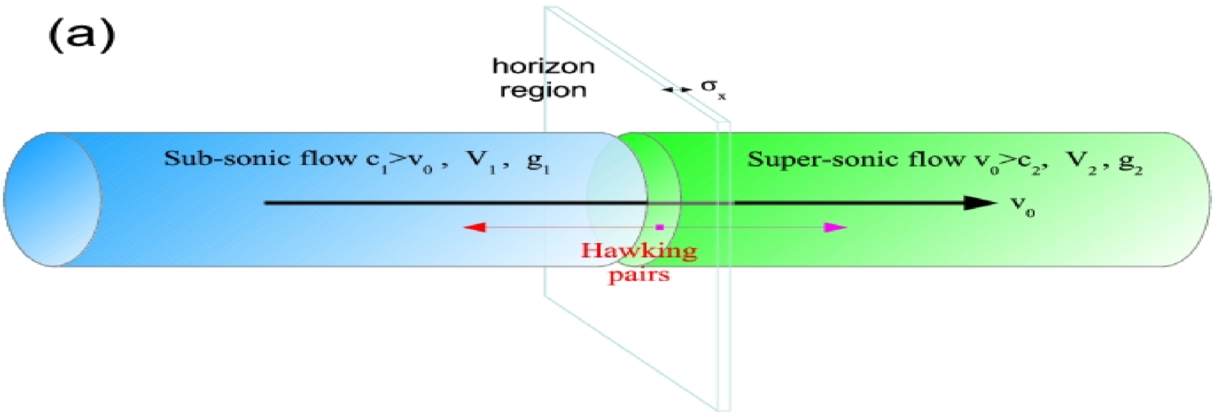

Figure 1. Sketch of the proposed set-up: the density n and the flow velocity v are kept constant, while the sound velocity c is linearly modulated in the region $[-x_0, x_0]$. The sonic horizon is at $x = 0$.

Even if the basic ingredients to get the general form of the G_2 corralator are all present, the calculation is done for a realistic experimental proposal to form an acoustic black hole. A one-dimensional BEC of uniform density n_{1D} is flowing with constant speed v along the x direction. For $t < 0$ the sound velocity $c(x,t)$ is constant in space and equal to $c(x,t) = c_1 > v$. At $t = 0$, the sound velocity in the $x < 0$ region is rapidly switched to a lower $c_2 < v$ value in order to create a horizon that separates a region of sub-sonic $v < c_1$ flow for $x > 0$ from a super-sonic $v > c_2$ one for $x < 0$. In the crossover region $x \in [-x_0, x_0]$, the sound velocity is assumed to vary linearly in x. In order for the hydrodynamic approximation to be valid x_0 has to be larger than ξ [1]. The sonic horizon (i.e. the locus where $c = v$) is taken at $x = 0$ (see Fig. 1).

A compact formula can then be obtained from Eqs.(5, 8) for the most interesting case when the x, x' points are located on opposite sides with respect to the horizon and lie well outside the modulation region i.e. $x > x_0$ and $x' < -x_0$:

$$\frac{G_2(x,x')}{n_{1D}^2} \simeq -\frac{k^2\,\xi_2\xi_1}{16\pi\, c_2 c_1} \frac{1}{\sqrt{(n_{1D}\xi_1)(n_{1D}\xi_2)}} \frac{c_1\, c_2}{(c_1 - v)(v - c_2)} \frac{1}{\cosh^2\left[\frac{k}{2}\left(\frac{x}{c_1 - v} + \frac{x'}{v - c_2}\right)\right]} \tag{9}$$

As shown in Fig. 2, $G_2(x,x')$ has a quite narrow, valley-shaped feature centered on the $x'/(v - c_2) = -x/(c_1 - v)$ straight line that describes correlations between pair of points located

[1] Such a modulation can be obtained by spatially modulating the transverse confinement of the waveguide and/or the nonlinear interaction constant $g(x)$ by means of a spatially varying magnetic field in the vicinity of a Feshbach resonance.

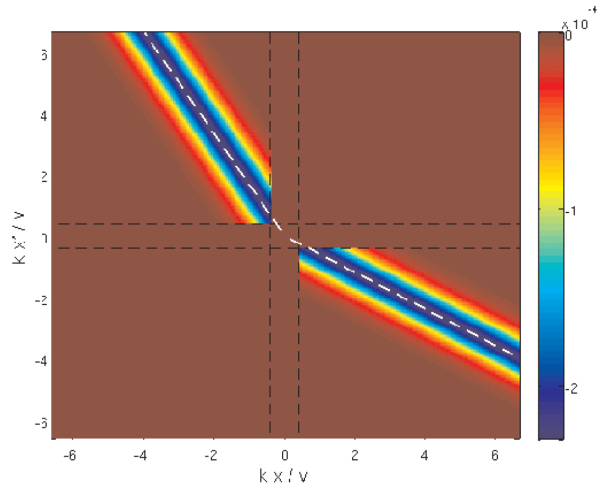

Figure 2. Density-density correlation pattern $|G_2(x,x')|/n_{1D}^2$. The valley-shaped feature indicated as a white dashed line is the signature of Hawking radiation. The black dashed lines indicate the $[-x_0, x_0]$ horizon region.

respectively inside and outside the black hole. The bottom value of the valley goes as the squared surface gravity and it is inversely proportional to the diluteness parameter $n_{1D}\xi$ of the gas. The neglected subleading term $O[|x-x'|^{-2}]$ contains the vacuum contribution of the ingoing modes and it is a consequence of the short-range repulsion of bosons [20]. On the other hand, no specific signal appears for pair of points on the same side of the horizon.

The position of the valley has a transparent physical interpretation: at all times, pairs of Hawking phonons almost simultaneously emerge from the horizon region and propagate into the sub- and super-sonic regions at speeds respectively $c_1 - v$ and $v - c_2$. After a propagation time Δt, the initial quantum correlation then reflects into a density correlation between $x = (c_1 - v)\Delta t$ and $x' = -(v - c_2)\Delta t$. The valley-shaped feature is immediately recovered once we integrate over all possible values of Δt. The width of the valley is proportional to the inverse of the surface gravity k^{-1} and it provides a simple way of estimating the Hawking temperature of the emission. Note that under the hydrodynamic assumption the width is much larger than the healing length ξ. The contribution of mode back-scattering, here neglected, quickly vanishes in the hydrodynamic limit: the corresponding terms are in fact of higher order in k. Although all the predictions have been obtained for the simplest configuration, the behavior of the correlation function at late ($t \gg k^{-1}$) times can be shown to be generic and independent on the details of the horizon formation process [16].

4. Numerical simulations

In [11] the analytical result of Eq. (9) has been validated by numerically simulating the dynamics of fluctuations around the mean field solution of the Gross-Pitaevskii equations during and after the formation of the horizon, by means of the so-called truncated Wigner method for the interacting Bose field [21].

At $t = 0$ well before the formation of the black-hole horizon, the condensate is assumed to be uniform and at thermal equilibrium in the moving frame at v_0 at a temperature T_0.

The numerical study of the Hawking radiation as appeared in [11] is based on a vast campaign of Wigner simulations of the condensate evolution after the formation of the black-hole horizon. Specific attention has been paid to the normal-ordered density-density correlation function

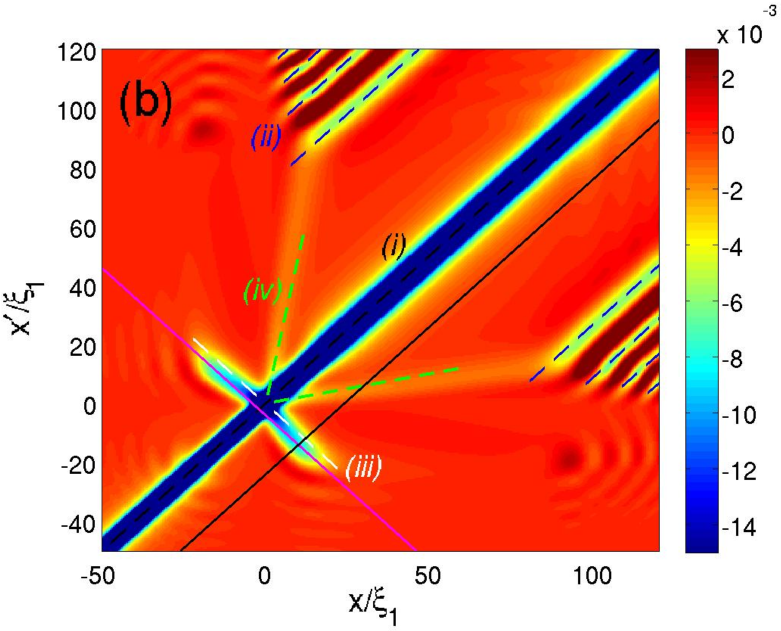

Figure 3. Density plot of the rescaled density correlation $(n\xi_1)\times[G_2(x,x')]$ well after the switch-on of the horizon. The dashed lines and the (i), (ii), (iii), (iv) labels identify the main features discussed in the text. The horizon has a spatial width $\sigma_x/\xi_1 = 0.5$. The initial temperature is $T_0 = 0$.

$G_2(x,x')$. One snapshots of $G_2(x,x')$ well after the horizon formation is shown in Fig.3.

The main features that are observed in the figure can be classified as follows:

- (i) A strong, negative correlation strip is always present along the main diagonal $x = x'$ and is almost unaffected by the horizon formation process.
- (ii) A system of fringes parallel to the main diagonal appears inside the black hole as soon as the horizon is formed. As time goes on, these fringes move away from the main diagonal at an approximately constant speed and eventually disappear from the region of sight.
- (iii) Symmetric pairs of negative correlation tongues extend from the horizon point almost orthogonally to the main diagonal. While their maximum height remains almost constant in time, their length linearly grows with time. These tongues involve pairs of points located on opposite sides of the horizon.
- (iv) A second pair of tongues appears for pairs of points located inside the black hole. Both their height and length scale in the same way as for feature (iii).

Apart from feature (i) which is the usual antibunching due to the repulsive atom-atom interactions [20], these observations illustrate a variety of effects of quantum field theory in a spatially and temporally varying background [16].

Feature (ii) is a transient effect that originates in the bulk of the internal $x > 0$ region and has no relation with the presence of a horizon. Detailed analysis is given in [22]: as a consequence of the time-modulation of the interaction constant g, correlated pairs of Bogoliubov phonons are generated during the short modulation time by a phonon analog of the dynamical Casimir effect [23]. As usual, the longer the switching time σ_t, the weaker the dynamical Casimir signal which eventually disappears in the limit of a very long switch.

On the other hand, features (*iii*) and (*iv*) do not depend on σ_t, but only on the possible presence of a horizon at long times. In particular, they completely disappear if the flow remains everywhere sub-sonic $v_0 < c_{1,2}$ [11]. This fact, together with their shape in the (x, x') plane and their persistence for indefinite times after the horizon formation suggests a strict link with the Hawking effect.

The predictions made in [10] are confirmed, by showing the quantum correlations within a pair of Hawking phonons emitted in respectively the inward and outward direction translate into a correlation between the density fluctuations at distant points located on opposite sides from the horizon. Once the horizon is formed, correlated pairs of Hawking phonons are continuously emitted at all times $t > t_0$. These phonons then propagate from the horizon in respectively the outward and inward direction at speeds $v_0 - c_1 < 0$ and $v_0 - c_2 > 0$. A generic time τ after their emission they are located at $x = (v_0 - c_1)\,\tau < 0$ and $x' = (v_0 - c_2)\,\tau > 0$, which defines a straight line of slope. As one can see in Fig.3, this line (indicated as a white dashed line) almost coincides with the axis of the numerically observed tongue (*iii*).

Feature (*iv*) originates from the (partial) elastic back-scattering of the Hawking particles in the horizon region: in this case both the Hawking and the partner phonons eventually propagate in the inward direction at speeds $v_0 \pm c_2$. Again, the analytically predicted slope $(v_0 - c_2)/(v_0 + c_2)$ (green dashed lines) well agrees with the axis of the numerically observed tongue (*iv*). Extended semi-analytically analysis of the process have appeared in [12, 13].

The identification of features (*iii*) and (*iv*) as signatures of Hawking radiation is further confirmed by a quantitative comparison of the numerical data with the predictions in [10]. The peak value G_2(peak) and the transverse full width at half maximum (FWHM) Δx_{FWHM} of the tongue (*iii*) confirms, as expected, that the Hawking emission is in fact stationary in both space and time, and that the agreement with the gravitational prediction is quantitatively excellent in the hydrodynamic limit $\sigma_x/\xi \gg 1$ where the physics of the many-body system is dominated by the hydrodynamic modes [11].

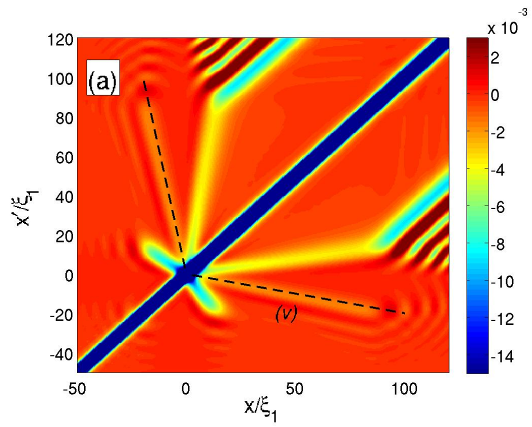

Figure 4. Panel (a): plot of the density correlation $(n\xi_1)G_2(x, x')$ for the same parameters as in Fig.3 but a finite initial temperature $k_B T_0/\mu_1 = 0.1$.

Even for an initial temperature $(k_B T/\mu_1 = 0.1)$ higher than the Hawking temperature $k_B T_H/\mu_1 \simeq 0.05$, the (*iii*) and (*iv*) Hawking tongues remain perfectly visible and are even a bit strengthened by stimulation effects [12, 24]. The new feature (*v*) that originates from thermal effects is located between the tongues (*iii*) and (*iv*) and is well distinct from them. Its attribution to partial reflection of thermal phonons is confirmed by the value of its slope, which is in very good agreement with the analytical prediction $(v_0 - c_1)/(v_0 + c_2)$ indicated as a dashed black line in Fig.4.

As the slope of the thermal tongue is completely different from the one $(v_0 - c_1)/(v_0 - c_2)$ of the Hawking tongue, the Hawking signal can be easily isolated in a correlation image from spurious thermal effects also at temperatures higher than T_H. This is even more remarkable as in this regime it appears difficult to separate the Hawking phonons from the thermal ones by looking just at the total phonon flux.

5. Acoustic black holes in ion rings

It has been recently proposed in [9] to get experimental evidence of Hawking radiation by using ion rings, and measuring the peculiar correlations associated to the Hawking process. This is a discrete analog of a hydrodynamical system that can show super- and subsonic regions, built up with ions trapped in a quadrupole ring trap [25] as schematically depicted in Fig. 5. Ions rotate on a ring with circumference L with an inhomogeneous, but stationary velocity profile $v(\theta)$. Thus the ions are inhomogeneously spaced. The necessary (de-)acceleration of the ions is guaranteed by additional electrodes exerting a force $F^e(\theta)$ on the ions. The harmonic oscillations of the ions around this equilibrium motion are phonons with velocities $c(\theta) \propto v(\theta)^{-1/2}$. Regions with large enough ion spacings and low enough velocities are supersonic; here phonons can only move into the direction of the ion flow and are trapped like light inside a black hole. The considered system shows a super- and a subsonic region. The border between these regions is analogous to a black hole horizon and is shown to emit Hawking radiation [2]. The details of the systems are given in [9].

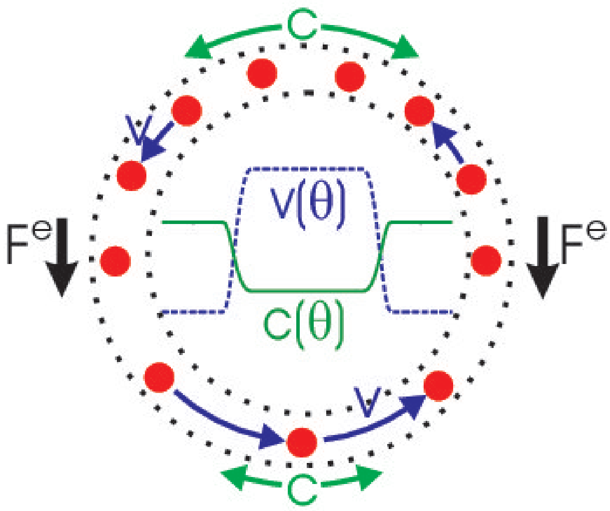

Figure 5. Schematic depiction of the ions rotating with velocity $v(\theta)$ and the phononic excitations with velocity $\pm c(\theta)$, that depend on the varying ion spacing. The external force $F^e(\theta)$ localized at the transition between the super- and subsonic region (de-)accelerates the ions. The inset shows a typical velocity profile with a black hole like region for $v(\theta) > c(\theta)$.

Small perturbations around the equilibrium motion positions $\theta_i(t) = \theta_i^0(t) + \delta\theta_i(t)$ are considered, and the canonical operators $\delta\theta_i$ and $-i\hbar\partial_{\delta\theta_i}$ describe the phononic oscillations of the ions. The quasi-free quantum dynamics of this harmonic system are governed by the classical linear equations of motion for the first and second moments. The stability of this equilibrium motion has been studied numerically. The limit of an infinite number of ions and only the Coulomb interaction between neighboring ions are considered, and an analogy with the standard Hawking effect is obtained. Then for a slowly varying $v(\theta)$ the system Lagrangian for the scalar

[2] Being a periodic system, it will show also the analog of the white hole horizon. It's presence will give constraints on the time-scale to perform the experiment.

field $\Phi(\theta_i^0(t), t) = \delta\theta_i(t)$ becomes

$$\mathcal{L} = \int d\theta \frac{\rho(\theta)}{2} \left[(\partial_t \Phi + v(\theta)\partial_\theta \Phi)^2 - [iD(\theta, -i\partial_\theta)\Phi]^2 \right] \tag{10}$$

with the dispersion relation $D(\theta, k) = c(\theta)k + \mathcal{O}(k^3)$, the conformal factor $\rho(\theta) = n(\theta) \cdot mL^2/(2\pi)^2$, and the density $n(\theta) = N/(v(\theta)T)$. This scalar field satisfying a linear dispersion relation at low wavenumbers, with sound velocity $c(\theta) = \sqrt{2(2\pi)^3 n(\theta) e^2/(4\pi\epsilon_0 \cdot mL^3)}$ is analogous to a massless scalar field in a black hole spacetime [2, 3]. Its quanta cannot escape a supersonic region with $v(\theta) > c(\theta)$, like photons trapped inside a black hole. The horizon of this analog model is located at $c(\theta_H) = v(\theta_H)$. Pairs of Hawking particles are emitted close to the black hole horizon with a black body distribution at the Hawking temperature given in Eq. (6).

For an experimental proof for Hawking radiation on the ion ring, correlations between the different regions on the ion ring after the creation of a black hole are studied and discussed. Starting from a thermal state of the excitations around homogeneously spaced rotating ions, a supersonic region is created in a small time interval τ. This is done by reducing the subsonic fluid velocity v_{min} in a Gaussian way, while leaving the average rotation velocity constant.

The normalized angle-angle correlations is analyzed

$$C_{ij} = \langle \delta\theta_i \delta\theta_j \rangle / \sqrt{\langle \delta\theta_i^2 \rangle \langle \delta\theta_j^2 \rangle}. \tag{11}$$

Fig. 6 shows the results at a fixed time starting from the ground state. Correlations between the super- and subsonic regions are created close to the black hole horizon and are moving away from it as expected. These correlations correspond to the pair creation of Hawking particles. In

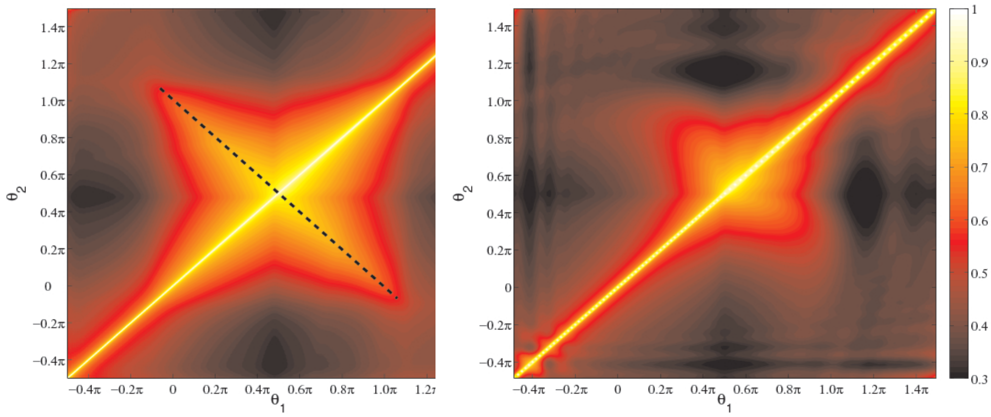

Figure 6. Correlations $C_{ij}(t)$ at time $t = T$ with real space lab frame positions. System parameters: $N = 500$ ions, $\sigma v_{min} T = 2\pi \cdot 0.25$, $e^2/4\pi\epsilon_0 = \frac{1.1}{2N}\frac{mL^3}{T^2}$, and $\tau = 0.05T$. Only nearest-neighbor interactions are considered (left panel). Full Coulomb interactions considered (right panel).

Fig. 6(left) the simulation is performed with interactions between neighboring ions only. The angle and the propagation velocity of the correlation features are in good agreement with the predictions based on the sound velocities and the ion velocities. For the long range Coulomb interactions the correlations behave similarly, and the slope agrees with the prediction one gets from the real dispersion relation [9]. This gives the first evidence of the robustness of Hawking radiation in a discrete system. The Hawking signature in the correlation pattern is estimated to be visible in a set of parameter usually used in everyday experiments with ions, and therefore the prospect to observe it in near future experiment is realistic.

6. Conclusions

In this paper the proposal to use correlation measurements to reveal the Hawking radiation in analog black holes is reviewed. The original proposal for weakly interacting BECs and the following application to ion rings, show that the presence of Hawking radiation is clear and peculiar in the correlation patterns.. It is therefore fair to conclude that so far correlation measurements are the most promising tool to get evidence of such an elusive effect in the near future experiments.

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References

[1] S.W. Hawking, Nature (London) **248**, 30 (1974); S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975)
[2] W.G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981)
[3] C. Barcelo, S. Liberati, and M. Visser, Living Rev. Rel. **8** (2005)
[4] T. A. Jacobson and G. E. Volovik, Phys. Rev. D **58**, 064021 (1998)
[5] L.J. Garay, J.R. Anglin, J.I. Cirac and P. Zoller, Phys. Rev. Lett. **85**, 4643 (2000); P. O. Fedichev and U. R. Fischer, Phys. Rev. Lett. **91**, 240407 (2003)
[6] S. Giovanazzi, Phys. Rev. Lett. **94**, 061302 (2005)
[7] U. Leonhardt and P. Piwnicki, Phys. Rev. Lett. **84**, 822 (2000); W. G. Unruh and R. Schützhold, Phys. Rev. D **68**, 024008 (2003).
[8] R. Schützhold and W. G. Unruh, Phys. Rev. Lett. **95**, 031301 (2005); T. G. Philbin, *et al.*, Science **319**, 1367 (2008)
[9] B. Horstmann, B. Reznik, S. Fagnocchi and J. I. Cirac, arXiv:0904.4801 [quant-ph]
[10] R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati and I. Carusotto, Phys. Rev. A **78**, 021603 (2008)
[11] I. Carusotto, S. Fagnocchi, A. Recati, R. Balbinot and A. Fabbri, New J. Phys. **10**, 103001 (2008)
[12] A. Recati, N. Pavloff and I. Carusotto, arXiv:0907.4305 [cond-mat.quant-gas]
[13] J. Macher and R. Parentani, arXiv:0905.3634 [cond-mat.quant-gas]
[14] R.D. Carlitz, R.S. Willey, Phys. Rev. D **36**, 2327 (1987)
[15] S. Massar, R. Parentani, Phys. Rev. D **54**, 7444 (1996); R. Brout *et al*, Phys. Rept. **260**, 328 (1995)
[16] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, Cambridge, UK, (1982)
[17] R. Balbinot *at al.*, Phys. Rev. **D** 63, 084029 (2001)
[18] R. Balbinot, A. Fabbri, S. Fagnocchi, and R. Parentani, Riv.Nuovo Cim.28, 1 (2005)
[19] L.P. Pitaevskii, S. Stringari, *Bose-Einstein condensation*, Clarendon Press, Oxford (2003)
[20] C. Mora, Y. Castin, Phys. Rev. **A 67**, 053615 (2003)
[21] A. Sinatra, C. Lobo, Y. Castin, J. Phys. **B 35**, 3599 (2002)
[22] I. Carusotto, R. Balbinot, A. Fabbri and A. Recati, arXiv:0907.2314 [cond-mat.quant-gas].
[23] A. Lambrecht, J. Opt. B: Quantum Semiclass. Opt. 7, S3 (2005); C. K. Law, Phys. Rev. A 49, 433 (1994)
[24] J. D. Bekenstein and A. Meisels, Phys. Rev. **D** 15, 2775 (1977)
[25] G. Birkl *et al.*, Nature 357, 310 (1992); T. Schätz *et al.*, Nature 412, 717 (2001)