The primordial power spectra in modified loop quantum cosmology

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We review recent developments in the primordial power spectra of two modified loop quantum cosmological models (mLQCs) which originate from the quantization ambiguities while loop quantizing the spatially-flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe. The properties of the background dynamics and the primordial scalar power spectra in two modified models, namely mLQC-I and mLQC-II, are reported. In both models, the inflationary scenario can be naturally extended to the Planck regime when a single scale field is minimally coupled to gravity with an inflationary potential and the big bang singularity is replaced with a quantum bounce. The qualitative difference lies in the behavior of the contracting phase where a quasi de Sitter phase emerges in mLQC-I. When applying the dressed metric approach and the hybrid approach to mLQCs, we find the most distinguishable differences between these models and the standard loop quantum cosmology (LQC) occur in the infrared and intermediate regimes of the power spectra.

Keywords: Loop quantum cosmology; bouncing cosmology; primordial power spectrum.

1. Introduction

Loop quantum cosmology (LQC)[1,2] provides an elegant resolution to the big bang singularity in the standard big bang cosmology by replacing it with a quantum bounce and also naturally extends the inflationary paradigm into the Planck regime. The loop quantization of the spatially flat FLRW spacetime in LQC is based on the homogeneity and isotropy of the spacetime. It has been found that the dynamics of the resulting quantum theory can be well described by the effective dynamics for the states which are sharply peaked around the classical trajectories at late times.[3] However, as is common in any quantum theory, different regularizations of the classical Hamiltonian may give rise to different quantum theory and finally result in distinct physical consequences. So it is important to check the robustness of the theoretical predictions from standard LQC by studying its variants originating from quantization ambiguities. The modified LQC models initially proposed in[4] provide a good platform to investigate the impacts of the quantization prescriptions on the dynamics and the observations.

In standard LQC, when quantizing a spatially flat FLRW universe, the Lorentz term was initially treated in the same way as the Euclidean term since they are proportional to each other at the classical level due to the homogeneity and isotropy of the background spacetime.[5] A separate treatment of these two terms was first implemented in[4] where two modified LQC models, namely mLQC-I and mLQC-II, were proposed due to different regularizations of the Lorentz term. Later, mLQC-I was rediscovered by computing the expectation value of the Hamiltonian constraint in LQG with complexifier coherent state.[6] The right Friedmann and Raychauduri equations of these two models were first reported in,[7,8] and it was found that after taking into account an inflaton field in mLQCs, the inflationary phase turns out to be an attractor in both models.[8] As a result, the inflationary scenario can also be naturally extended to the Planck regime in both models. In the following, assuming the validity of the effective dynamics in mLQCs, we first summarize the main features of the background dynamics of these two mLQCs and their similarities and differences with/from standard LQC. Then we report the results on the numerical simulations of the power spectra in both mLQCs in the dressed metric and the hybrid approaches.

2. The background dynamics in the modified LQC

Although the big bang singularity is resolved and replaced with a quantum bounce in both mLQCs, the differences in the background dynamics between these models are still distinct. Similar to standard LQC, the evolution of the background dynamics in mLQC-II is symmetric with respect to the bounce when gravity is coupled to a massless scalar field. The maximum energy density in mLQC-II is about $1.73m_{\text{pl}}^4$ which is larger than the maximum energy density ($\approx 0.41m_{\text{pl}}^4$) in LQC. It turns out that the evolution of the universe in mLQC-II is similar to that in LQC when gravity is coupled to the same matter content. In contrast, in mLQC-I, the contracting and the expanding phases are no longer symmetric with respect to the bounce point which takes place at the maximum energy density ($\approx 0.097m_{\text{pl}}^4$). A characteristic quasi de Sitter phase quickly emerges in the contracting phase when the universe is evolved backwards from the bounce point. This de Sitter phase is dominated by a Planck-sized effective cosmological constant and the asymptotic forms of the Friedmann and Raychauduri equations reveal a rescaled Newton's constant in the contracting phase as well.[7] As a result, when filled with an inflaton field, the classical universe can only be recovered in the expanding phase of mLQC-I while in the contracting phase, the universe remains quantum with a Planck-scale Hubble parameter.

Meanwhile, the numerical simulations of the background dynamics shows qualitatively similar behavior of the universe in the expanding phase of mLQCs and standard LQC.[9] The initial conditions of the background dynamics which are relevant to CMB data are those dominated by the kinetic energy of the inflaton field at the bounce. For these initial conditions, three distinct phases can be observed in the expanding phase: the bouncing phase with $w \approx 1$ which lasts only a few number of e-foldings, the transient transition phase where the equation of state quickly decreases from positive unity to near negative unity and the slow-roll inflationary phase. These generic features of the background dynamics shared by mLQCs and LQC imply the robustness of the preinflationary dynamics in loop quantizations of the spatially flat FLRW universe and the different regularizations of the Lorentz term in the classical Hamiltonian constraint leave imprints only in the contracting phase.

3. The primordial power spectra in the modified LQC

To compute the primordial power spectra in the mLQCs, one can appeal to the techniques developed in standard LQC where several distinct approaches are widely studied. Among them the most popular ones are the dressed metric approach[10–12]and the hybrid approach.[13–16] We have applied both these approaches to mLQCs and numerically obtained the resulting scalar power spectrum.[17,18] Irrespective of specific model and approach, the power spectrum can be generally divided into three distinctive regimes: the infrared (IR) regime, the intermediate regime and the ultraviolet (UV) regime.

The behavior of the IR regime sensitively depends on the initial states, the specific model and the applied approach. For example, when the fourth-order adiabatic states are employed as the initial states in the contracting phase, the magnitude of the power spectra remains almost constant in LQC and mLQC-II in both dressed metric and hybrid approaches. In contrast, when the second-order adiabatic states are used as the initial states, the magnitude of the power spectra keeps increasing with the comoving wavenumber in LQC and mLQC-II in both approaches. In mLQC-I, the power spectrum in the IR regime strongly relies on the approach we use. In the dressed metric approach, due to the emergent de Sitter phase, the preferred choice of the Bunch-Davies (BD) vacuum state yields a Planck-sized magnitude of the power spectrum in the IR regime. While in the hybrid approach, the effective mass changes so drastically as compared with the one in the dressed metric approach that the adiabatic states can be again chosen as the initial states of the scalar perturbations, leading to a power spectrum whose magnitude is comparable to that of LQC and mLQC-II in the IR regime. From our numerical results, we find the relative difference between LQC and mLQC-II in the IR regime turns out to be less than 40% in the dressed metric approach and less than 50% in the hybrid approach. Due to the similar background dynamics between the LQC and mLQC-II, the relative difference between LQC and mLQC-II is much less than that between LQC(mLQC-II) and mLQC-I.

The power spectrum keeps oscillating throughout the intermediate regime in all of the three models with the dressed metric and the hybrid approaches. This oscillatory behavior is believed to be associated with the bouncing regime where adiabatic conditions are violated for the modes whose comoving wavenumbers are close to the characteristic wavenumber in each model and thus particle creations become appreciable.[19] For the adiabatic initial states and in particular the BD vacuum state in the dressed metric approach of mLQC-I, the magnitude of the power spectrum in this regime is amplified as compared with that in the UV regime. Moreover, the relative difference of the power spectra between different models turns out to be larger than that in the IR regime due to the oscillatory behavior of the power spectra. In particular, the relative difference of the power spectra between LQC and mLQC-II in the intermediate regime can be up to 100% in both approaches and the relative difference between LQC and mLQC-II is also smaller than that between LQC(mLQC-II) and mLQC-I.

Finally, regardless of the initial states, the specific model and approach used to compute the power spectra, we always observe a scale-invariant UV regime whose magnitude agrees with the observations. The relative difference between the magnitudes of the power spectra from three models turns out to be less than 1%. As a result, only the IR and intermediate regimes are mostly affected by the different regularizations of the classical Hamiltonian constraints, there is little impact on the UV regime due to the quantization ambiguities as it should be.

4. Summary

We have studied the background dynamics and the primordial power spectra in mLQCs and compare the results with those from LQC. The background dynamics in mLQC-II is qualitatively similar to that in LQC with a symmetric evolution with respect to the bounce when gravity is minimally coupled to a massless scalar field. An asymmetric evolution is observed only in mLQC-I where the classical limit can only be recovered in the expanding phase and there shows up an emergent de Sitter spacetime in the contracting phase. The difference between the background dynamics also affects the power spectrum in the IR and intermediate regime. From our numerical results, we find the relative difference of the power spectra between LQC and mLQC-II in these two regimes is much smaller than the relative difference between LQC(mLQC-II) and mLQC-I in the dressed metric and hybrid approaches. Moreover, mLQC-I also serves as a good example to explicitly show the differences between the dressed metric and hybrid approaches. In particular, a Planck scale magnitude of the power spectrum in the IR regime of mLQC-I with the dressed metric approach is observed while in the hybrid approach the magnitude of the power spectrum is suppressed in the IR regime of mLQC-I. This difference is completely due to the distinct construction of the Mukhanov-Sasaki equation in each approach which leads to different effective masses. Finally, the power spectrum in the UV regime is found to be consistent with the CMB data in all three models regardless of the approach and the initial states.

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