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Resumen

En el siguiente trabajo, se realiza la construcción general de la teoría supersimétrica dándose por conocidos los fundamentos en teoría de grupos en relación a grupo de Poincaré y por lo tanto se inicia desde las propiedades más generales de los espinores. Se desarrolla la teoría a partir de su formalismo de álgebras graduadas, construyéndose el grupo de Super-Poincaré y a partir del manejo de variables de Grassmann se construye la teoría supersimétrica desde el formalismo de Supercampos y generándose los Lagrangianos escalares, vectoriales (abelianos y no abelianos) y mixtos más generales permitidos por la condición de renormalizabilidad. Finalmente los desarrollos de esta teoría se implementan en el modelo estándar, donde se encuentran todas las matrices de masa en relación a las Super-partículas, los escalares y fermiones del modelo estándar. Adicionalmente, se construye un modelo supersimétrico al incluir una simetría $U(1)_X$ adicional al MSSM de modo que sea libre de anomalias quirales e incluyendo las tres familias de fermiones y algunos fermiones exóticos. A partir de la asignación de cargas X se construye el superpotencial más general permitido por el criterio de renormalización, se contruye el potencial escalar asociado y se obtienen las condiciones que permiten recrear la masa del boson de Higgs observado, el cual en este escenario puede explicar de forma natural la masa de $125GeV$. Finalmente, se obtienen expresiones analíticas para la masa de los fermiones y en particular se calcula la masa de los más ligeros (e, u, d y s) a nivel de un loop teniendo en cuenta las contribuciones debido a partículas y sus respectivos supercompañeros.

Abstract

In the following work, it is realized the general construction of the supersymmetric theory where the fundamentals in group theory is considered as known in relation to Poincaré group. Thus it begins from the most general properties of spinors. The theory is developed from its graded algebras formalism constructing then the super-Poincaré group, and with the use of Grassmann variables the supersymmetric theory is built from the superfield formalism generating then the most general scalar, vectorial (abelian and non abelian) and mixed Lagrangians allowed by the renormalization condition. Finally the developments of this theory are applied to the standard model, where it has been found all the mass matrices related with superparticles, scalars and standard model fermions. Additionally, it is build a supersymemtric model by including an additional $U(1)_X$ symmetry to the MSSM in such a way that it is chiral anomaly free and including all three fermions families and some exotic fermions. From the X charge assignation the most general renormalizable superpotential is written, the associated scalar potential is consequently obtained and the condition for reproducing the SM Higgs boson , which can explain naturally ira $125GeV$ mass. Finally, analytic expression for scalars and fermions are given where all 1-loop contributions due to particles and superparticles are considered for the lightests fermions masses (e, u, d y s).

Keywords: Supersymmetry, Standard Model, Superfields, Graded Algebras, Sparticles masses, Higgs boson, neutrino masses, CKM matrix, PMNS matrix.

Publications

Some figures have appeared previously in the following publication and pre-print:

- Alvarado, J. S., Diaz, C. E., & Martinez, R. (2019). Nonuniversal U (1) X extension to the MSSM with three families. Physical Review D, 100(5), 055037.

- Alvarado, J. S., & Martinez, R. (2020). PMNS matrix in a non-universal $U(1)_X$ extension to the MSSM with one massless neutrino. arXiv preprint arXiv:2007.14519.

Additionally, a conference presentation, a document in a conference proceedings can be found and an additional paper result of this work can be found:

- Alvarado, J. S., Bulla, M. A., Martinez, D. G., & Martinez, R. (2020). Explaining muon $g-2$ anomaly in a non-universal $U(1)_X$ extended SUSY theory. arXiv preprint arXiv:2010.02373.

- Alvarado, J. S., Diaz, C. E., & Martinez, R. (2019). A non-universal U (1) X gauge extension to the MSSM. `https://indico.cern.ch/event/801886/contributions/3615207/attachments/1954324/3245698/COMHEP_4.pdf`

- Alvarado, J. S., Diaz, C. E., & Martinez, R. (2019). A $U(1)_X$ extension to the MSSM with three families. arXiv preprint arXiv:1902.08566. `https://inspirehep.net/conferences/1724194`

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Contents

List of Tables

List of Figures

1 Introduction

Despite in the ancient Greece there was a notion of *atoms* given by Democritus as the fundamental component of the universe, none of our ancestors could have imagined how mysterious, huge and non predictable our universe turns out to be. We have discovered so many interesting phenomena against the common sense that new physics has become a problem of discovering rather than predict or modeling. Even when the notion of an atom was confirmed in experiments and ideas from Jhon Dalton we can in a certain way affirm that there is something even more fundamental. Now that atom looks like a giant object from the eyes of particle physics and its components nature transcends the understanding of matter composition and behaviour to reach the most fundamental and philosophical questions about the origin of the universe.

It all starts from the atom nuclei, where particles with the same electric charge remain very close with a stability that has lived on a time close to the universe age. Nowadays, we understand that there is more interactions rather than the electromagnetic forces being the first one a result of a $SU(3)$ symmetry among particles that we can not see easily but makes our universe possible. Furthermore, the notion of a force as the consequence of a certain field that fills the entire space has been broken by the discovery of gauge bosons; quantum couriers that *communicate* particles how to behave under the presence of other particles. But not everything is as stable and perfect as we want idealize, sometimes it happens that an atom expels and electron that it was not there before, technically known as β decay. It is neither an electrical effect nor due to the nuclei interactions. It is a new interaction among particles that respects a $SU(2) \times U(1)$ symmetry and teach us that every particle has two counterparts, *chirality*, because it only affects to one of them.

The standard model of particles put together the previously mentioned interactions. The first one is called the *strong interaction* and the second one is *electroweak interaction* summed up in a $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant theory. It exposes a theory of interacting massless particles until the universe temperature decreased enough to leave a certain particle field, the Higgs boson, in a minimum non zero value, whose interaction with other particles provides mass and breaks the original symmetry into a lower one. To date, this model describes very accurately most of the subatomic processes that we observe in colliders at CERN but still there are many unresolved questions indicating more physics beyond the standard model.

For instance, how can fermions like the electron and the top quark to have a mass with a difference of 6 order of magnitude if they interact a priori with the same Higgs bosons?. That seems unnatural and it is called the Fermion Mass Hierarchy problem (FMH), which opens the possibility of having more particles than observed and that's the starting point of several proposals. Theories with additional Higgs bosons can be found such as the two Higgs doublet model (2HDM) or more elaborate extension with three, four or even more scalar particles that can be doublets or singlets under $SU(2)$. In this way, fermions can have different masses by interacting with different scalar particles acquiring a vacuum expectation value. Even more, higher symmetry groups can be considered in such a way that eventually breaks into the standard model one such as $U(1)$ extensions, the 331 models, the $left-right$ symmetric models among others in order to explain the existence of additional particles by the proposal of a new interaction that generates processes yet unobserved.

Another possibility comes by the assumption of heavy fermion states, that explain the small mass values thanks to the seesaw mechanism [37], a mathematical property of matrices with a sector much bigger than others. However, this mechanism has more relevance in the case of neutrinos. According to the standard model, they should be massless but the discovery of neutrino oscillations points to the opposite, and if it was not enough, they do not acquire mass with the same mechanism of the rest of particles so both the seesaw and heavy particles are being considered until nature shows its reality in the experiments.

Supersymmetry was one of the most promising scenarios because it provides an explanations for a lot of unresolved questions such as the theoretical Higgs boson mass, the FMH, grand unification among other. It comes from our idealized view of a symmetric universe that the idea of having equal number of fermions and bosons related by pairs, fermion-boson supersymmetric partners, that brings to live the SUSY theory. Unfortunately, it has not been observed in experiments but it is still alive under the theoretical promise of string theories which locates the exotic particles at an unreachable energy scale at the same time that makes a proposal for quantum gravity.

The present work, exposes a proposal for solving the FHM problem by considering a $U(1)$ extension to the Minimal Supersymmetric Standard Model (MSSM) that can recreate all the standard model blackground without implying unobserved particles at energies already explored. It is based on the particle and charge assignation of its non-SUSY counterpart found in [44] and it is developed under the requirement of reproducing a $125.3GeV$ scalar particle as the lightest one, the correct gauge boson masses and the correct values for the Cabibbo-Kobayashi-Maskawa (CKM) and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices that parametrizes the flavor changing interactions between fermion mass eigenstates. Additionally, the extension has a non-universal charge assignations which opens the possibility of flavor changing neutral currents that in the SM are highly suppressed by the GIM mechanism but in the SM extensions represents hidden physics that may occur at higher energies.

Thus, the first chapter is devoted to the fundamentals of supersymmetry and some basic of particle physics like Lorentz invariance, the action functional, spontaneous symmetry breaking and chiral anomalies based taken from the cited bibliographic material. The second chapter describes the general features of the MSSM prior to present the $U(1)_X$ extension in chapter 3 as an extension of the MSSM. The last chapter closed this work with a short discussion and conclusions about the model and possible future prospects.

2 Supersymmetry

Supersymmetry (SUSY) rather than an extension of a theory, is a generalization via a higher symmetry among Fermions and Bosons. Unfortunately, such a symmetry can not be described with a Lie algebra so Graded Lie algebras might be introduced as well as new Grassmaniann coordinates that makes the supersymmetric theory consistent with Lorentz invariance and guarantees a supersymmetric invariance. In this chapter, the foundations of supersymmetry are described from the definition of spinors [53] and the action functional to the development of gauge-invariant supersymmetric lagrangians [53][36][61].

2.1 Spinors and the Poincaré Group

Spinors can be defined as two-component objects that transform in the fundamental representation of the $SL(2,C)$ group, which is a double cover of the Lorentz group. Moreover, spinors are considered anticommuting objects which operate ina very specific way. In this section, a brief introduction of spinor algebra is done on the basis of the $SL(2,C)$ group.

2.1.1 The fundamental representation

In the fundamental representation, $SL(2,C)$ transformations are 2×2 matrices M of unit determinant acting on a complex-valued 2-dimensional object ψ_α better known as *left-handed Weyl* spinor. Then the transformation rule is defined by:

$$\psi_\alpha \rightarrow \psi'_\alpha = M_\alpha^{\ \beta}\psi_\beta \tag{2-1}$$

where the position of indices is of big importance. In this case, left-handed spinor indices are contracted from upper-left to lower-right.

Now, we must define the *dual representation* given by the transformations M^{-1T} where T represents a transpose operation. However, there exist a similarity transformation connecting M and M^{-1T} which makes the dual representation equivalent the fundamental one. [16][10] This representation acts on the dual spinor ψ^α which consequently transform as:

$$\psi^\alpha \rightarrow \psi^\alpha = (M^{-1T})^\alpha_{\ \beta}\psi^\beta = \psi^\beta (M^{-1})_\beta^{\ \alpha} \tag{2-2}$$

Moreover, it allows to define a $SL(2,C)$ invariant inner product by:

$$< \phi, \psi > \equiv \phi^\alpha \psi_\alpha \tag{2-3}$$

Since the fundamental and its dual representation are equivalent, spinors are related as well by a $SU(2,C)$ invariant tensor ϵ which lowers or raises indices. such tensor is fixed by the relation [55]:

$$\epsilon^{\beta\gamma}\epsilon_{\gamma\alpha} = \delta^\beta_\alpha \quad \epsilon^{\alpha\beta} = -i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \epsilon_{\alpha\beta} = (\epsilon^{\alpha\beta})^T = (\epsilon^{\alpha\beta})^{-1} = i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{2-4}$$

where σ_2 is a Pauli matrix and $\delta^\beta_\alpha = diag(+,+)$ is a trivial invariant tensor such that $\delta_\alpha^{\ \beta}\psi_\beta = \psi_\alpha$.
Now, the contraction $\psi^\beta\epsilon_{\beta\alpha}$ transforms as:

$$\psi^\beta\epsilon_{\beta\alpha} \to (M^{-1T})^\beta_{\ \eta}\psi^\eta M_\beta^{\ \gamma} M_\alpha^{\ \delta}\epsilon_{\gamma\delta} = \psi^\eta\delta_\epsilon^{\ \gamma} M_\alpha^{\ \delta}\epsilon_{\gamma\delta} = M_\alpha^{\ \delta}\psi^\gamma\epsilon_{\gamma\delta} \tag{2-5}$$

so it transforms as a spinor, so the relationship between spinors and dual spinors is given by:

$$\psi_\alpha \equiv \psi^\beta\epsilon_{\beta\alpha} \qquad \psi^\alpha \equiv \epsilon^{\alpha\beta}\psi_\beta \tag{2-6}$$

2.1.2 The antifundamental representation

The fundamental and its dual representation were equivalent because there is a similarity transformation that connects both transformations. Nevertheless, the complex conjugate of the transformations cannot be justified by a similarity transformation. Thus, the antifundamental representation of the $SL(2,C)$ group are the complex conjugate matrices:

$$\bar{M}_{\dot\alpha}^{\ \dot\beta} := (M_\alpha^{\ \beta})^* \tag{2-7}$$

where the index contraction goes from lower left to upper right. The dotted indices are nothing but a mnemonic way of distinguish the fundamental and antifundamental representations and avoid index contraction between the both. Likewise, its dual is defined by $\bar{M}^{-1T}$ and such transformations act on *right-handed* Weyl spinors as:

$$\bar\psi_{\dot\alpha} \to \bar\psi'_{\dot\alpha} = \bar\psi_{\dot\beta}(M^\dagger)^{\dot\beta}_{\ \dot\alpha} \qquad \bar\psi^{\dot\alpha} \to \bar\psi'^{\dot\alpha} = (M^{\dagger -1})^{\dot\alpha}_{\ \dot\beta}\bar\psi^{\dot\alpha} \tag{2-8}$$

By comparing with the fundamental representation and its dual we can identify then:

$$\bar\psi_{\dot\alpha} = (\psi_\alpha)^* \tag{2-9}$$

and the antifundamental and its dual representation are connected by the epsilon tensor[55]:

$$(\epsilon_{\alpha\beta})^* = \bar{\epsilon}_{\dot{\alpha}\dot{\beta}} = \epsilon_{\dot{\alpha}\dot{\beta}} \tag{2-10}$$

However, we must take into account how indices are contracted in this representation. Then we can write:

$$\bar{\psi}_{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}} \qquad \bar{\psi}^{\dot{\alpha}} = -\bar{\psi}_{\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}} \qquad \bar{\psi}_{\dot{\alpha}} = \bar{\psi}_{\dot{\beta}}\delta^{\dot{\beta}}_{\ \dot{\alpha}} \tag{2-11}$$

2.1.3 The Lorentz and Poincaré groups

The Lorentz group $SO(1,3)$ can be understood as the subgroup of matrices Λ of the general linear group $GL(4,R)$ with unit determinant that leave invariant the Minkowski metric:

$$\Lambda^T\eta\Lambda = \eta \tag{2-12}$$

being $\eta_{\mu\nu} = diag(+,-,-,-)$. Such group has six generators: three hemitian generators associated to space rotations, J_i; and three antihermitian related to Boost ,K_i, for $i = 1,2,3$ that satisfy:

$$[J_i, J_j] = i\epsilon_{ijk}J_k \qquad [J_i, K_j] = i\epsilon_{ijk}K_k \qquad [K_i, K_j] = -i\epsilon_{ijk}J_k \tag{2-13}$$

though the linear combination $J_i^{\pm} = \frac{1}{2}(J_i \pm iK_i)$ is introduced to identify the equivalence to $SU(2)$ algebras by satisfying:

$$[J_i^{\pm}, J_j^{\pm}] = i\epsilon J_{ijk}J_k^{\pm} \qquad [J_i^{\pm}, J_j^{\mp}] = 0. \tag{2-14}$$

However, for a more compact notation, the generators are written in terms of an antisymmetric tensor $M_{\mu\nu}$ with four-vector indices defined as:

$$M_{0i} = K_i \qquad M_{ij} = \epsilon_{ijk}J_k \tag{2-15}$$

and can be rewrittena as[53]:

$$M_{\mu\nu} = i(x_\mu\partial_\nu - x_\nu\partial_\mu) + S_{\mu\nu} \tag{2-16}$$

$$= L_{\mu\nu} + S_{\mu\nu} \tag{2-17}$$

where $L_{\mu\nu}$ are the angualr momentum generators, $S_{\mu\nu}$ the spin generators and $\mu,\nu = 0,1,2,3$. It can be proven that Lorentz group is homeomorphic to $SL(2,C)$ which means that for any matrix $A \in SL(2,C)$ there exists an associated Lorentz matrix Λ such that:

$$\Lambda(A)\Lambda(B) = \Lambda(AB) \tag{2-18}$$

being A and B $SL(2,C)$ matrices, the relationship between transformations can be proven to be [10]:

$$\Lambda^{\mu}_{\nu}(A) = \frac{1}{2}Tr[\sigma^{\mu}A\sigma_{\nu}A^{\dagger}] \tag{2-19}$$

where $\sigma_{\mu} = (\mathcal{I}, \vec{\sigma})$ leading to the relationship between spinors and four-vectors and A acts on the structure $X = x^{\mu}\sigma_{\mu}$ in the adjoint representation. However, we would like to write the vector as a direct product of spinors $X = \xi\xi^{\dagger}$ so $X' = AXA' = A\xi\xi^{\dagger} = (A\xi)(A\xi)^{\dagger} = \xi'\xi'^{\dagger}$ so a Lorentz transform *Lambda* on a vector corresponds with the matrix A on a spinor [64]. The importance of this results lies in the fact that there exists an Homeomorphism between the Lorentz group and the $SL(2,\mathbf{C})$ group [52], where the latter can be decomposed into two subgroups. On the one hand there are unitary matrices (Boost) while on the other hand there are Hermitian ones (Rotations).
Finally, the Poincaré Group is the same Lorentz group augmented by translations, so its algebra is given satisfy:

$$[P_{\mu}, P_{\nu}] = 0 \tag{2-20}$$
$$[M_{\mu\nu}, M_{\rho,\sigma}] = -i\eta_{\mu\rho}M_{\nu\sigma} - i\eta_{\nu\sigma}M_{\mu\rho} + i\eta_{\mu\sigma}M_{\nu\rho} + i\eta_{\nu\rho}M_{\mu\sigma} \tag{2-21}$$
$$[M_{\mu\nu}, P_{\rho}] = -i\eta_{\rho\mu}P_{\nu} + i\eta_{\rho\nu}P_{\mu} \tag{2-22}$$

2.2 The action functional

In classical mechanics a detailed description of the action, the equations of motion and the Noether theorem is performed [34]. Nevertheless, in a field theory they arise more properly by considering Lorentz invariance. This action functional contains all the information of the physical system under considerations and that is where its importance lies. The functional depends on the trajectory over the phase space being the classical path the case where the functional is an extremal. Despite we are interested in a Quantum Field Theory (QFT), in practice is needed first to perform a classical treatment prior to quantize and look for the new implications. In this section the conditions required to the construction of a suitable action are discussed.

2.2.1 Behavior of the fields under the Poincare Group

The Scalar Field

If we consider a general infinitesimal Lorentz transformation $\Lambda^{\mu}_{\nu} = \delta^{\mu}_{\nu} + \epsilon^{\mu}_{\nu}$ we find ϵ^{μ}_{ν} to be antisymmetric due to the invariance of the metric tensor:

$$(\Lambda^T)^{\mu}_{\ \nu}\eta^{\nu}_{\ \rho}\Lambda^{\rho}_{\ \sigma} = \eta^{\mu}_{\sigma} \tag{2-23}$$

$$\Lambda_{\nu}^{\ \mu}\eta^{\nu}_{\ \rho}\Lambda^{\rho}_{\ \sigma} = \eta^{\mu}_{\sigma} \tag{2-24}$$

$$(\delta_{\nu}^{\ \mu} + \epsilon_{\nu}^{\ \mu})\eta^{\nu}_{\ \rho}(\delta^{\rho}_{\ \sigma} + \epsilon^{\rho}_{\ \sigma}) = \eta^{\mu}_{\sigma} \tag{2-25}$$

$$\eta^{\mu}_{\ \rho}\epsilon^{\rho}_{\ \sigma} + \epsilon_{\nu}^{\ \mu}\eta^{\nu}_{\ \sigma} \approx 0 \tag{2-26}$$

$$\epsilon^{\mu}_{\ \sigma} = -\epsilon_{\sigma}^{\ \mu} \tag{2-27}$$

where only linear terms in ϵ are considered. Then, for the case of a scalar field which is a scalar function of the coordinates that must acquire the same value on different reference frames related by a Lorentz transformation we have the relation $\delta\phi = \phi'(x') - \phi(x) = \delta_0\phi + \delta x^{\mu}\partial_{\mu}\phi = 0$, being $\delta_0\phi = \phi'(x) - \phi(x)$ a transformation that does not change the coordinates and it is assumed in an approximate form that $\partial_{\mu}\phi' = \partial_{\mu}\phi$. As a consequence, the scalar field transforms as:

$$\begin{aligned}\delta_0\phi &= -\delta x^{\mu}\partial_{\mu}\phi \\ &= -\epsilon^{\mu\rho}x_{\rho}\partial_{\mu}\phi \\ &= -\frac{i}{2}\epsilon^{\mu\rho}L_{\mu\rho}\phi\end{aligned} \tag{2-28}$$

which depends only on the orbital angular momentum generators $L_{\mu\rho} = i(x_{\mu}\partial_{\rho} - x_{\rho}\partial_{\mu})$ and $\delta x^{\mu} = \Lambda^{\mu\rho}x_{\rho} - x^{\mu} \approx \epsilon^{\mu\rho}x_{\rho}$, if we apply the general Lorentz transformation $\delta_0\phi = -\frac{i}{2}\epsilon^{\rho\sigma}M_{\rho\sigma}\phi$ being M the generator of Lorentz group introduced in Eq. 2-16, we conclude that the Spin operator vanishes which means that the scalar field describes a spin zero particle.

The Vector Field

A non trivial $S_{\rho\sigma}$ spin operator can be built by considering a construction involving $\partial_{\mu}\phi$ which is invariant under translations, representing in fact a vector field, whose transformations are given by applying the infinitesimal transformation and 2-16:

$$\delta\partial_{\mu}\phi = -\epsilon^{\nu}_{\mu}\partial_{\nu}\phi \qquad \delta_0(\partial_{\mu}\phi) = -\frac{i}{2}(\epsilon^{\rho\sigma}S_{\rho\sigma})^{\nu}_{\mu}\partial_{\nu}\phi - \frac{i}{2}\epsilon^{\rho\sigma}L_{\rho\sigma}\partial_{\mu}\phi \tag{2-29}$$

with the spin operator defined as $(S_{\rho\sigma})^{\nu}_{\mu} = g_{\rho\mu}g^{\nu}_{\sigma} - g_{\sigma\mu}g^{\nu}_{\rho}$. In a similar fashion a tensor field transforms as (2-29) and the spin operator acting on the field would be a sum of the defined $(S_{\rho\sigma})^{\nu}_{\mu}$ for each index. For instance, for a rank 2 tensor it is [53]:

$$(S_{\rho\sigma}B)_{\mu\nu} = -i(g_{\sigma\mu}B_{\rho\nu} + g_{\rho\nu}B_{\sigma\mu} + g_{\sigma\nu}B_{\rho\mu} - g_{\rho\mu}B_{\sigma\nu}) \tag{2-30}$$

It is worth to notice that the contraction between indices μ and ν lead to the scalar case $S_{\rho\sigma} = 0$.

However, there is another representation of vector fields as invariant determinant hermitian matrices since transform according to the representation $(\frac{1}{2},\frac{1}{2})$ of the Lorentz group, such matrix generate invariants as:

$$A = \begin{pmatrix} A_0 + A_3 & A_1 + iA_2 \\ A_1 - iA_2 & A_0 - A_3 \end{pmatrix} \qquad A_\mu A^\mu \qquad \partial_\mu A_\nu \partial^\mu A^\nu \qquad \partial_\mu A_\nu \partial^\nu A^\mu \qquad \partial^\mu A_\mu$$

The spinor field

Spinor fields are going to be described in the Weyl representation, where a 4-component field is described by two chiral 2-component fields $\psi_L \in (\frac{1}{2},0)$ and $\psi_R \in (0,\frac{1}{2})$ being the spinor representations $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$. In this case, the Lorentz group is isomorphic to $SU(2)$ so rotation generators are given by Pauli matrices. On the contrary, Boost transformations are not hermitian so they cannot be represented unitarily. Nevertheless, $\vec{k} = -\frac{i}{2}\vec{\sigma}$ can be considered as Boost generators since they satisfy the commutation relations. As a consequence, the most general Lorentz transformation for left-handed Weyl spinors is given by:

$$\Lambda_L = exp\left[\frac{\vec{\sigma}}{2}\cdot(\vec{\omega} - i\vec{\nu})\right]$$

where $\vec{\omega}$ and $\vec{\nu}$ are the transformation parameters for rotations and Boost respectively and $\vec{\sigma}$ Pauli matrices. On the other hand, since left and right representations are conjugate, they are related by a parity transformation that only changes the Boost generators sign in the Lorentz transformation:

$$\Lambda_R = exp\left[\frac{\vec{\sigma}}{2}\cdot(\vec{\omega} + i\vec{\nu})\right]$$

As a result, these two kind of transformations fulfill properties as:

$$\Lambda_L^{-1} = \Lambda_R^\dagger \qquad \Lambda_L^T = \sigma^2\Lambda_L^{-1}\sigma^2 \qquad \sigma^2\Lambda_L^T\sigma^2\Lambda_L = 1 \qquad \Lambda_L^T\sigma^2\Lambda_L = \sigma^2$$

It is from those properties that it can be proven that a spinor ψ_L transforming as $(\frac{1}{2},0)$ can build a spinor $\sigma^2\psi_L^*$ transforming as $(0,\frac{1}{2})$ and vice versa.

In addition, recalling the anticommuting nature of spinor fields, we can build invariant scalars as $\chi_L^T\sigma^2\psi_L = -i\chi_{L1}\psi_{L2} + i\chi_{L2}\psi_{L1}$ and 4-vectors such as $i\psi_L^\dagger\sigma^\mu\psi_L$ and $i\psi_R^\dagger\bar{\sigma}^\mu\psi_R$ which together with the operator ∂_μ that provides translation invariance we get in the easiest case complex Lorentz invariants, or real ones if we consider linear combinations:

$$\frac{1}{2}\psi_L^\dagger\sigma^\mu\partial_\mu\psi_L - \frac{1}{2}\partial_\mu\psi_L^\dagger\sigma^\mu\psi_L \equiv \frac{1}{2}\psi_L^\dagger\sigma^\mu\overleftrightarrow{\partial}_\mu\psi_L \tag{2-31}$$

In a similar fashion it is done for ψ_R and $\bar{\sigma}^\mu$. Since a trivial solution is found by equaling ψ_L and $\sigma^2\psi_L^*$ a four-component spinor is built in the Dirac basis by introducing the Pauli adjoint $\bar{\psi}\gamma^0$.

$$\psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L = \psi^\dagger\gamma^0\psi \equiv \bar{\psi}\psi \qquad \frac{1}{2}\left(\psi_L^\dagger\sigma^\mu\overleftrightarrow{\partial}_\mu\psi_L + \frac{1}{2}\psi_R^\dagger\bar{\sigma}^\mu\overleftrightarrow{\partial}_\mu\psi_R\right) = \frac{1}{2}\bar{\psi}\gamma^\mu\overleftrightarrow{\partial}_\mu\psi$$

Besides, a charge conjugation operation can be defined leading to the definition of a Majorana spinor

$$\psi^c \equiv \begin{pmatrix} \sigma^2\psi_R^* \\ -\sigma^2\psi_L^* \end{pmatrix} \qquad (\psi^c)^c = \psi \qquad \psi^M \equiv \begin{pmatrix} \psi_L \\ -\sigma^2\psi_L^* \end{pmatrix} \Rightarrow \textit{Majorana Spinor}$$

2.2.2 Lorentz Invariant actions for the fields

General properties

Despite we have built some Poincare invariant expressions with well defined transformations, we have to turn them into actions describing reasonable physical theories. The main features of a correct action, $\mathcal{S}$, are:

1. **$\mathcal{S}$ must be real:** In classical mechanics, a complex potential lead to absorption, implying that matter suddenly disappears.

2. **$\mathcal{S}$ must involve second derivatives:** S$\mathcal{S}$ must lead to the correct equations of motion which can be achieved only with second order derivatites. Besides, higher order derivatides usually present problems with causality such as the Abraham-Lorentz equation [40]

3. **Poincaré group invariance:** when the equations of motion are a eigenvalue condition of the operator $\partial_\mu\partial^\mu$ it is said taht we are dealing with a free field since it can be identified as a group casimir with the equations of motion restricted to the free particle representation.

4. **Units:** The action has units of angular momentum, equivalently in natural units it is dimensionless so the lagrangian density must have units of $[L^{-4}]$

5. **Canonical transformations invariance:** In general, the action in invariant under transformations like $\mathcal{L}' = \mathcal{L} + \partial_\mu\Lambda^\mu$ being Λ^μ an arbitrary function. In classical mechanics it represents a canonical transformation leaving the action invariant since the surface term does not change the equations of motion.

Scalar and spinor field action

On the one hand, the most general lagrangian density for a free real scalar field is given by:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - V[\phi(x)] \Rightarrow \qquad \partial_\mu\partial^\mu\phi(x) + \frac{\partial V[\phi(x)]}{\partial\phi(x)} = 0 \tag{2-32}$$

on the other hand, for a weyl spinor field the kinetic term takes the form:

$$\mathcal{L}_L = \frac{1}{2}\psi_L^\dagger\sigma^\mu\overleftrightarrow{\partial}_\mu\psi_L \qquad \mathcal{L}_R = \frac{1}{2}\psi_R^\dagger\bar{\sigma}^\mu\overleftrightarrow{\partial}_\mu\psi_R \qquad \mathcal{L}_{Dirac} = \mathcal{L}_L + \mathcal{L}_R = \frac{1}{2}\psi^\dagger\gamma^\mu\overleftrightarrow{\partial}_\mu\psi \tag{2-33}$$

where the Dirac lagrangian arises if parity is of interest. In the case when $\psi_R = -\sigma^2\psi_L$, $\mathcal{L}_R$ is equivalent to $\mathcal{L}_L$ up to a total divergence, making of ψ a Majorana spinor whose lagrangian is the same $\mathcal{L}_L$. It is common to find the Dirac kinetic term with the operator ∂_μ acting only to the right and without the $\frac{1}{2}$ factor. However, this distinction is irrelevant as long as gravity coupling is not studied. Additionally, these kinetic terms are invariant under conformal transformations, global phase shift and chiral transformations.

$$\psi \to e^{i\delta}\psi \qquad\qquad \psi \to e^{i\beta\gamma_5}\psi$$

where the respective conserved currents are:

$$\begin{aligned} j^\mu &= i\bar{\psi}\gamma^\mu\psi = i\psi_L^\dagger\sigma^\mu\psi_L + i\psi_R^\dagger\bar{\sigma}^\mu\psi_R \\ j_5^\mu &= i\bar{\psi}\gamma^\mu\gamma^5\psi = i\psi_L^\dagger\sigma^\mu\psi_L - i\psi_R^\dagger\bar{\sigma}^\mu\psi_R \end{aligned}$$

Another invariant quadratic non kinetic terms can be build for Majorana and Dirac spinors:

$$\begin{aligned} \mathcal{L}_L^m &= \frac{im}{2}(\psi_L^T\sigma^2\psi_L + \psi_L^\dagger\bar{\sigma}^2\psi_L^*) & \mathcal{L}_{L5}^m &= \frac{m}{2}(\psi_L^T\sigma^2\psi_L - \psi_L^\dagger\bar{\sigma}^2\psi_L^*) \\ &= -\frac{im}{2}\bar{\psi}_M\psi_M & &= -\frac{m}{2}\bar{\psi}_M\gamma_5\psi_M \\ \mathcal{L}_D^m &= im\bar{\psi}_M\psi_M & \mathcal{L}_{D5}^m &= im\bar{\psi}_M\gamma^5\psi_M \\ &= im(\psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L) & &= -m(\psi_L^\dagger\psi_R - \psi_R^\dagger\psi_L) \end{aligned}$$

where m is a mass dimentions parameter. These terms are not chiral invariant but we can still build a chiral invariant from a linear combination:

$$\sigma(x)\mathcal{L}_D^m + i\pi(x)\mathcal{L}_{D5}^m = im\bar{\psi}_M[\sigma(x) + i\gamma_5\pi(x)]\psi_M$$

where σ y π must transform as $\delta\sigma = 2\beta\pi(x)$ and $\delta\pi = -2\beta\sigma(x)$ leaving $\sigma^2 + \pi^2$ invariant. As a result, if we include kinetic terms for σ and π and a potential, the chiral invariant lagrangian is:

$$\mathcal{L}_f = \frac{1}{2}\psi^\dagger\gamma^\mu\overleftrightarrow{\partial}_\mu\psi + i\hbar\bar{\psi}_M[\sigma(x) + i\gamma_5\pi(x)]\psi_M + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - V(\sigma^2 + \pi^2)$$

It is also invariant under global phase shifts and a parity transformation, being $\sigma(x)$ a scalar field and $\pi(x)$ a pseudo-scalar field.

2.2.3 Spontaneous symmetry breaking

To introduce the systematics of Spontaneous Symmmetry Breaking (SSB) we can consider the non-linear sigma model [50] involving a set of N real scalar fields $\phi^i(x)$ with lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^i(x))^2 + \frac{1}{2}\mu^2(\phi^i(x))^2 - \frac{\lambda}{4}(\phi^i)^4 \tag{2-34}$$

This lagrangian is invariant under a rotation of the fields $\phi^i \rightarrow R^{ij}\phi^j$ represented by the orthogonal group $O(N)$. In general, the potential $V(\phi) = -\frac{1}{2}\mu^2(\phi^i(x))^2 + \frac{\lambda}{4}(\phi^i)^4$ is bounded from bellow since $\lambda > 0$ and it has a minimum depending on the value of μ^2. If $\mu^2 < 0$ there is a trivial minimum at $(\phi^i)^2 = 0$ contrary to the case where $\mu^2 > 0$ where the minimum of the potential is reached for any field that satisfies $(\phi^i)^2 = \frac{\mu^2}{\lambda} \equiv (\phi^i_0)^2$. This condition only give us the magnitude of the vector ϕ^i_o but not its direction. So, it is convenient to choose coordinates such that ϕ^i_0 points to the $N-th$ direction $\phi^i_0 = (0, 0, ..., 0, v)$ $(v = \frac{\mu^2}{\lambda})$ so now we define a set of shifted fields as perturbations from the minimum as $\phi^i = (\pi_1, \pi_2, ..., \pi_{N-1}, v+\sigma)$ so the lagrangian takes the form:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \pi^k)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\lambda}\mu\sigma^3 - \sqrt{\lambda}\mu(\pi^k)^2\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}(\pi^k)^2\sigma^2 - \frac{\lambda}{4}(\pi^k)^4 \tag{2-35}$$

which describe a massive σ field and a set of $N-1$ massless π^k fields which are invariant under rotations of the $O(N-1)$ group rather than $O(N)$ which is no longer a symmetry of the lagrangian. All in all, this means that when a field ϕ^i acquires a Vaccuum Expectation Value (VEV) v, the original symmetry $O(N)$ is broken into $O(N-1)$ and $N-1$ massless bosons arise. This results is not a particularity of the model, it comes out from the so called *Goldstone Theorem* [35].

2.2.4 Goldstone Theorem

It is a general result that massless states arise when a continuous symmetry has been broken, this massless particles are known as *Goldstone Bosons*. Particularly, the $O(N)$ group represent rotations on any of the $\frac{N(N-1)}{2}$ planes (which equally represents the number of symmetries), while $O(N-1)$ has only $\frac{(N-1)(N-2)}{2}$ symmetries so the number of broken symmetries (planes where there is no invariance) is $N-1$.Goldstone theorem states that for every spontaneously broken symmetry there must be a massless particle.

Refering to the previous potential, we can see that the massless π^k fields represents perturbations along the tangential directions of the potential while σ is a perturbation in the transversal direction. In the general case, the massless states represent perturbations along the tangential direction of the potential and are associated with the broken generators of the symmetry group which do not left the vacuum invariant. On the contrary, massive states are perturbations on the transversal directions and the unbroken generators do left the vaccum invariant [50].

It is worth to notice that there will not be any Goldstone bosons if the broken symmetry is discrete, but in the case of continuous symmetries it can be global or local. In the latter, there is an additional result. Local symmetries involves gauge bosons which enter into the lagrangian with the covariant derivative. When the symmetry has been broken, N massless Goldstone bosons appear but at the same time N gauge bosons acquire a finite mass value. So for each Goldstone boson there is a massive gauge boson and for each massive particle there is a massless gauge boson. However, the systematics of the gauge boson mass generation will be presented in later.

2.2.5 Chiral anomalies

Let's consider a gauge theory with massless fermions, the lagrangian reads:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar{\psi}\not{D}\psi \tag{2-36}$$

$$= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \mathcal{L}_{matter} \tag{2-37}$$

where the lagrangian is invariant under global chiral transformations and local transformations of a certain gauge group represented by a gauge boson field V_μ that enters into the covariant derivative D_μ. The classical equations of motions imply:

$$(D_\mu F^{\mu\nu})^a = -J^{a\nu}_{matter} \tag{2-38}$$

where $J^{a\nu}_{matter} = \frac{\partial\mathcal{L}_{matter}}{\partial V^a_\mu}$ is the classical matter current which is conserved if the field equations of motion are satisfied i.e. $D_\nu J^\nu_{matter} = 0$ being a consequence of gauge invariance. However, it is not true at the quantum level, $D_\nu < J^\nu_{matter} > \neq 0$. Let's consider a chiral transformation:

$$\psi \to U\psi = e^{i\epsilon^a(x)T_a\gamma^5} \qquad \bar{\psi} \to \bar{\psi}\bar{U} = \bar{\psi}i\gamma^0 e^{-i\epsilon^a(x)T_a\gamma^5}i\gamma^0 \tag{2-39}$$

since $\bar{U} \neq \bar{U}^{-1}$ due to the anticommutation of gamma matrices, it makes a contribution in the path integral measure given by:

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{iS_{matter}[\psi,\bar{\psi},V_\mu]} \\
&= \int \mathcal{D}\psi'\mathcal{D}\bar{\psi}'e^{iS_{matter}[\psi',\bar{\psi}',V_\mu]} \\
&= \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{i\int d^4x\epsilon(x)a(x)}e^{iS_{matter}[\psi,\bar{\psi},V_\mu]+i\int d^4x\epsilon(x)\partial_\mu J_5^\mu(x)} \qquad (2\text{-}40) \\
&= \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{iS_{matter}[\psi,\bar{\psi},V_\mu]}\left[1+i\int d^4x\epsilon(x)(a(x)+\partial_\mu J_5^\mu(x)))+\mathcal{O}(\epsilon^2)\right] \qquad (2\text{-}41)
\end{aligned}
$$

where

$$
a_\alpha(x) = -\frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}tr[t_\alpha F_{\mu\nu}F_{\rho\sigma}] \qquad (2\text{-}42)
$$

where $a(x)$ is the anomaly arising from the integration measure and J_5^μ is the classical conserved current arising from the transormation of the matter lagrangian [9][12] and the trace is over the representation of the group generators. This result from manipulating the ill defined expression for $Det(U)^{-2}$ which need to be regulated[9].
if we perform a local chiral transformations, we can associate a conserved current when the fields satisfy equations of motion by:

$$
\delta S = \int d^4x(-J_a^\mu(x))\partial_\mu\epsilon^a(x) = \int d^4x(\partial_\mu J_a^\mu(x))\epsilon^a(x) \qquad (2\text{-}43)
$$

In this case the variation must be zero for every ϵ leading to a conserved current $\partial_\mu J_a^\mu(x) = 0$. If we compare this with equation (2-41) we conclude that at a quantum level the current is not conserved. It satisfies:

$$
-\partial_\mu < J_a^\mu(x) >_A = -\frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}tr[t_\alpha F_{\mu\nu}F_{\rho\sigma}] \qquad (2\text{-}44)
$$

where the vacuum expectation value is done in a fixed A_μ background, and $a(x)$ is known as the *anomaly*. The latter result comes from the consideration of an abelian group and it means that at a quantum level we can not have always gauge invariance and chiral invariance at the same time. Its presence changes the ward identities of the model and induces new phenomena. In the case of the $SU(3)$ quark model, it explain the decay of the π^0 meson into two photons by generating an anomalos Ward identity for the 3-point vertex function $< \mathcal{T} J_\alpha^\mu(x)J_\beta^\nu(y)J_\gamma^\rho(z) >= \Gamma_{\alpha\beta\gamma}^{\mu\nu\rho}(x,y,z)$ depicted in the triangle diagrams shown in figure **2-1**

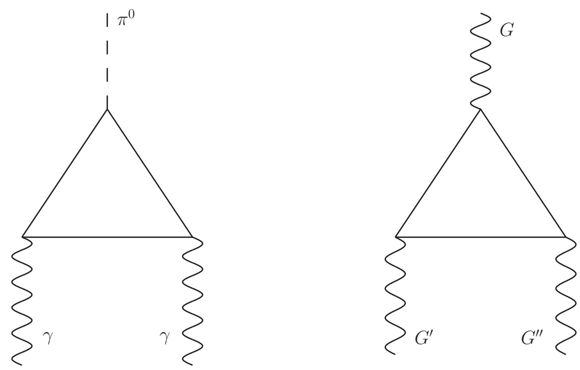


Figure **2-1**: anomalous triangle diagram that generates the decay $\pi^0 \rightarrow \gamma\gamma$ in a) and the general triangle diagram coupling to different gauge bosons G, G' an G'' belonging in general to different symmetry groups in b)

In the general case of a non-abelian group with chiral fermions in the triangle it becomes:

$$-(D_\mu < J^\mu(x) >)_\alpha = \mathcal{A}_\alpha^{L/R}(x) \tag{2-45}$$

$$\mathcal{A}_\alpha^{L/R} = \mp\frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\partial_\mu(V_\nu^\beta\partial_\rho V_\sigma^\gamma - \frac{i}{4}V_\nu^\beta[V_\rho, V_\sigma]^\gamma)C_{\alpha\beta\gamma} \qquad C_{\alpha\beta\gamma} = tr[t_\alpha\{t_\beta, t_\gamma\}] \tag{2-46}$$

where $\{...\}$ represents the anticomutator and T_α is the generator of the symmetry group and the trace is over the representations i.e. over the particles under considerations, implying that left and right handed fermions contribute with different signs to the anomaly but bosons do not. Additionally, for a theory invariant under several symmetry groups, the generators in $C_{\alpha\beta\gamma}$ mix among them so the gauge boson in each triangle corner can be different as shown in figure **2-1**-b. The anomaly caused by the triangular diagrams with fermions inside is what we call the chiral anomaly but it is worth to mention that there are some *safe groups* where $C_{\alpha\beta\gamma}$ vanishes like $SU(2)$, $SO(2N+1)$, $SO(4N)$, $E(6)$ etc. Nevertheless, it might happen another kind of anomaly, the *gravitational anomaly* generated by local Lorentz transformations when we have a $U(1)$ factor, presenting a $SO(4) - SO(4) - U(1)$ anomaly. They correspond to a triangle diagram with fermions coupling to one $U(1)$ gauge field and two gravitons as shown in figure **2-2**.

Since all particles are expected to couple universally to gravity, the coefficient $C_{\alpha\beta\gamma}$ reduces to $C_{\alpha\beta\gamma} = tr[tt_A^{SO(4)}t_B^{SO(4)}] \sim \delta_{AB}tr[t]$ which is nothing but the sum of all $U(1)$ charges. This anomaly must cancel in order to consistently couple gravity to matter under the $U(1)$. Furthermore, it is important to mention that there are no pure gravitational anomalies $(SO(4) - SO(4) - SO(4))$ in four dimensions.

When we build a theory we want to avoid anomalies, and one way of achieving it is by having a correct charge assignations for a $U(1)_X$ symmetry. In the case of the standard model in we might have anomalies for:

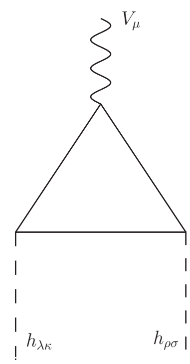


Figure **2-2**: $U(1) - SO(4) - SO(4)$ triangle diagram responsible of the $U(1)$-gravitational anomaly.

- $SU(3) \times SU(3) \times SU(3)$
- $SU(3) \times SU(3) \times U(1)$
- $SU(2) \times SU(2) \times SU(2)$
- $U(1) \times U(1) \times U(1)$
- $U(1)$-Gravitational

but the hypercharge assignation for particles makes that all posibilities vanish [12].

2.3 Super Poincaré algebra

Graded Lie algebras

In general, the symmetries of a field theory are represented by the Lie algebra of a symmetry group. However, we need a generalization since a symmetry between bosons and fermions via spinorial parameters can not be represented with a Lie algebra. The Coleman-Mandula [43] and Haag [24] theorems demand this generalized group as a direct sum of Poincaré group with the additional symmetries. Therefore, it is needed to generalize this Lie group concept, which is done by generalizing its algebra.

Lie algebra

A Lie algebra consist in a vector space defined over a field, where a composition rule denoted by "$\circ : L \times L \to L$" is defined and must accomplish the following properties:

1. **Closure**: $v_1 \circ v_2 \in L$

2. **Linearity:** $v_1 \circ (v_2 + v_3) = (v_1 \circ v_2) + (v_1 \circ v_3)$

3. **Antisymmetry:** $v_1 \circ v_2 = -v_2 \circ v_1$

4. **Jacobi identity:** $v_1 \circ (v_2 \circ v_3) + v_3 \circ (v_1 \circ v_2) + v_2 \circ (v_3 \circ v_1) = 0$

Graded Lie algebra

A graded algebra Z_{N+1} [20] consist in a total subspace $L = \bigoplus_{k=0}^{N} L_k$ as the direct sum of $N+1$ vector subspaces L_k with a composition rule $\circ : L \times L \to L$ that fulfill the following properties:

1. **Closure**

2. **Linearity**

3. **Grading:** $\forall x_i \in L_i \to x_i \circ x_j \in L_{i+jmod(2)}$

4. **Supersymetrization:** $x_i \circ x_j = -(-1)^{ij} x_j \circ_i$

5. **Generalized Jacobi identity:**
$(x_k \circ (x_l \circ x_m))(-1)^{km} + (x_m \circ (x_k \circ x_l))(-1)^{lk} + (x_l \circ (x_m \circ x_k))(-1)^{ml} = 0$

A general graded algebra only need the first 3 conditions, while the remaining two makes of it a graded Lie algebra. We are interested in a Z_2 ($L = L_0 + L_1$), being L_0 associated with Poincare algebra and L_1 with the supersymmetric transformations. Let $E_i \in L_0$ y $Q_a \in L_1$, then the product among elements in the different subspaces must obey supersymmetrization and is given by:

Product	*Supersimetrization*	*Definition*
$E_i \circ E_j \in L_0$	*Antisymmetric*	$[E_i, E_j]$
$E_i \circ Q_a \in L_1$	*Antisymmetric*	$[E_i, Q_a]$
$Q_a \circ Q_b \in L_0$	*Symmetric*	$\{Q_a, Q_b\}$

Table **2-1**: Z_2 graded Lie algebra products

In analogy to the spin-statistic theorem, we define the space L_0 as the bosonic sector and L_1 as the fermionic sector. It can be seen that the subspace L_0 satisfy by itself the conditions for a Lie algebra.

Super-Poincaré Algebra

The Poincaré algebra is given by [41]:

$$[P_\mu, P_\nu] = 0 \tag{2-47}$$
$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu) \tag{2-48}$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}) \tag{2-49}$$

where the generators $M_{\mu\nu}$ y P_μ generate a 10 dimensional vector space denoted by L_0. Then, the $L_0 \times L_1$ product shown in table **2-1** must have a matrix representation in L_0 but with dimensions of L_1. Therefore, let's consider a 4-dimensional L_1, so there are four base elements (Q_a, $a = 1, 2, 3, 4.$). A grading must be defined and it is done in the most trivial way possible, done first by Wess and Zumino, the result is the super-Poincaré algebra which is given by [66]:

$$[P_\mu, P_\nu] = 0 \tag{2-50}$$
$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda}P_\mu - g_{\mu\lambda}P_\nu) \tag{2-51}$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}) \tag{2-52}$$
$$[P_\mu, Q_a] = 0 \tag{2-53}$$
$$[M_{\mu\nu}, Q_a] = -(\sigma_{\mu\nu})_{ab}Q^b \tag{2-54}$$
$$\{Q_a, Q_b\} = a(\gamma^\mu C)_{ab}P_\mu \tag{2-55}$$
$$\{Q_a, \bar{Q}_b\} = -a(\gamma^\mu C)_{ab}P_\mu \tag{2-56}$$
$$\{\bar{Q}_a, \bar{Q}_b\} = -a(C^{-1}\gamma^\mu)_{ab}P_\mu \tag{2-57}$$

With this choice, most terms in the Jacobi identities vanish trivially. Besides, Q_a are Majorana spinors that must commute with momentum so it becomes a symmetry of the system since it commutes with the Hamiltonian as well.

Haag theorem allows the presence of an internal symmetry with generators B_l and the Coleman-Mandula theorem implies that they must commute with P_μ and $M_{\mu\nu}$ so it becomes necessary to introduce a set of N spinorial (central) charges Q_a^α in such a way that the algebra closes in subspace L_1 with the products:

$$[Q_a^\alpha, B_l] = iS_{l\beta}^\alpha Q_a^\beta$$
$$[B_l, B_m] = ic_{lm}^k B_k$$

where $S_{l\beta}^\alpha$ is a representation of the internal symmetry group algebra, so it is done the extension:

$$\{Q_a^\alpha, Q_b^\beta\} = 2\delta^{\alpha\beta}(\gamma^\mu)_{ab}P_\mu.$$

However,we are interested in $N = 1$ supersymmetry and the states are labeled through the Casimir operators denoting its mass and super-spin, a generalization of the spin since the Pauli-Lubansky vector does not make a Casimir operator now.

2.4 Superspace and Superfields

In general, the more symmetries a theory have the bigger dimensional space is. For instance, in classical mechanics, a 3D space is invariant under Galilean transformations, but it requires to be extended to four dimensions if a Lorentz invariant theory wants to be constructed. Likewise, to construct a SUSY invariant theory let's consider a 4-dimensional space plus another 4 grassmannian (anticomuting) directions $\{\theta_A\}_{A=1,2}$, $\{\dot{\theta}_{\dot{B}}\}_{\dot{B}=\dot{1},\dot{2}}$ [36] which together make up what we call the *superspace*. In the Weyl formalism, θ and $\dot{\theta}$ are $SL(2,\mathcal{C})$ transforming Weyl spinors in the adjoint representation and allow to turn the graded algebra into a Lie algebra as follows:

$$[\theta Q, \bar{\theta}\bar{Q}] = 2(\theta\sigma^{\mu}\bar{\theta})P_{\mu} \tag{2-58}$$

$$[\theta Q, \theta Q] = [\theta^A Q_A, \theta^B Q_B] = 0 \tag{2-59}$$

$$[\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] = [\bar{\theta}_{\dot{A}}\bar{Q}^{\dot{A}}, \bar{\theta}_{\dot{B}}\bar{Q}^{\dot{B}}] = 0 \tag{2-60}$$

Furthermore, the supersymmetry generators can be represented through a translation in these anticommuting coordinates plus a traslation in normal coordinates via a spinorial parameter [36]:

$$\boxed{Q_A = -i(\partial_A - i\sigma^{\mu}_{A\dot{B}}\bar{\theta}^{\dot{B}}\partial_{\mu})} \tag{2-61}$$

$$\boxed{\bar{Q}^{\dot{A}} = -i(\bar{\partial}^{\dot{A}} - i(\bar{\sigma}^{\mu})^{\dot{A}B}\theta_B\partial_{\mu})} \tag{2-62}$$

A superfield Φ is an operator defined over the superspace that must be understood in terms of its power series, which turns out to be finite thanks to the anticommuting coordinates which allow up to quadratic terms in θ or $\bar{\theta}$, it can be written as:

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) = f(x) &+ \theta^A\phi_A(x) + \bar{\theta}_{\dot{A}}\bar{\chi}^{\dot{A}} + (\theta\theta)m(x) + (\bar{\theta}\bar{\theta})n(x) + (\theta\sigma^{\mu}\bar{\theta})V_{\mu}(x) \\ &+ (\theta\theta)\bar{\theta}_{\dot{A}}\bar{\lambda}^{\bar{A}}(x) + (\bar{\theta}\bar{\theta})\theta^A\psi_A(x) + (\theta\theta)(\bar{\theta}\bar{\theta})d(x).\end{aligned} \tag{2-63}$$

The properties of the component fields are given by their lorentz transformations so:

- $f(x)$, $m(x)$, $n(x)$ are complex scalar or pseudoscalar fields.
- $\psi(x)$, $\phi(x)$ are left handed Weyl spinors.
- $\bar{\chi}(x)$, $\bar{\lambda}(x)$ are right handed Weyl spinors.

- $V_\mu(x)$ is a four-vector.
- $d(x)$ is a scalar field.

Under a supersymmetric transformation given by $\delta_s\Phi = [i\alpha^A Q_A + i\bar{\alpha}_{\dot{A}}\bar{Q}^{\dot{A}}]\Phi(x,\theta,\bar{\theta})$, the component fields transform as shown bellow being of big importance the $\theta\theta\bar{\theta}\bar{\theta}$ component which transform as a total derivative, so inside an action it is invariant under SUSY transformations [36].

$$\delta_s f(x) = \alpha(x)\phi(x) + \bar{\alpha}\bar{\chi}(x) \tag{2-64}$$
$$\delta_s \phi_A(x) = 2\alpha_A(x)m(x) + (\sigma^\mu\bar{\alpha})_A(i\partial_\mu f(x) + V_\mu(x)) \tag{2-65}$$
$$\delta_s \bar{\chi}^{\dot{A}}(x) = 2\bar{\alpha}^{\dot{A}}n(x) + (\alpha\sigma^\mu\epsilon)^{\dot{A}}(i\partial_\mu f(x) - V_\mu(x)) \tag{2-66}$$
$$\delta_s m(x) = \bar{\alpha}\bar{\lambda}(x) - \frac{i}{2}\partial_\mu\phi(x)\sigma^\mu\bar{\alpha} \tag{2-67}$$
$$\delta_s n(x) = \alpha\psi(x) + \frac{i}{2}\alpha\sigma^\mu\partial_\mu\bar{\chi}(x) \tag{2-68}$$
$$\delta_s V_\mu(x) = \alpha\sigma_\mu\bar{\lambda}(x) + \psi(x)\sigma_\mu\bar{\alpha} + \frac{i}{2}\alpha\partial_\mu\phi(x) - \frac{i}{2}\partial_\mu\bar{\chi}(x)\bar{\alpha} \tag{2-69}$$
$$\delta_s \bar{\lambda}^{\dot{A}}(x) = 2\bar{\alpha}^{\dot{A}}d(x) + \frac{i}{2}\bar{\alpha}^{\dot{A}}\partial^\mu V_\mu(x) + i(\alpha\sigma^\mu\epsilon)^{\dot{A}}\partial_\mu m(x) \tag{2-70}$$
$$\delta_s \psi_A(x) = 2\alpha_A d(x) - \frac{i}{2}\alpha_A\partial^\mu V_\mu(x) + i(\sigma^\mu\bar{\alpha})_A\partial_\mu n(x) \tag{2-71}$$
$$\delta_s d(x) = \frac{i}{2}\partial_\mu\psi(x)\sigma^\mu\bar{\alpha} - \frac{i}{2}\partial_\mu\bar{\lambda}(x)\bar{\sigma}^\mu\alpha \tag{2-72}$$

Nevertheless, we can reduce the number of fields by imposing covariant restrictions by considering the covariant derivatives $\bar{D}_{\dot{A}} = -\bar{\partial}_{\dot{A}} - i(\theta\sigma^\mu)_{\dot{A}}\partial_\mu$ and $D_A = \partial_A + i(\sigma^\mu\bar{\theta})_A\partial_\mu$ with lead to the definition of chiral superfields:

$$\bar{D}_{\dot{A}}\Phi(x,\theta,\bar{\theta}) = 0 \qquad \textit{Right-handed superfield} \tag{2-73}$$
$$D_A\Phi^\dagger(x,\theta,\bar{\theta}) = 0 \qquad \textit{Left-handed superfield} \tag{2-74}$$

whose field content now is restricted to:

$$\Phi_L(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)F(y) \qquad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \tag{2-75}$$
$$\Phi_R(z,\theta) = A^*(z) + \sqrt{2}\bar{\theta}\bar{\psi}(z) + (\bar{\theta}\bar{\theta})F^*(z) \qquad z^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta} \tag{2-76}$$

where $A(x)$ and $F(x)$ are complex scalar fields with no defined parity but the first one represents a scalar field whose superpartner is $\psi(x)$ while the second is an auxiliary field that closes the algebra *off-shell* called the *F-term*. It is however also important to consider superfield products which are easy to calculate since there cannot be powers of θ and $\bar{\theta}$ greater than 2. The product of two and three superfields becomes:

$$\Phi_i\Phi_k = A_iA_k + \sqrt{2}\theta[A_i\psi_k + \psi_iA_k] + (\theta\theta)[A_iF_k + F_iA_K - \psi_i\psi_k] \tag{2-77}$$

$$\begin{aligned}\Phi_l\Phi_i\Phi_k &= A_lA_iA_k + \sqrt{2}\theta[A_lA_i\psi_k + A_l\psi_iA_k + \psi_lA_iA_k] + ...\\ &... + (\theta\theta)[A_lA_iF_k + A_lF_iA_k + F_lA_iA_k - \psi_l\psi_kA_i - \psi_l\psi_iA_k - A_l\psi_i\psi_k]\end{aligned} \tag{2-78}$$

So the product of superfields is also a superfield. However, when considering the product of a left handed with a right-hanede superfield one gets:

$$\begin{aligned}\Phi_i^\dagger\Phi_j &= [A_i^* + \sqrt{2}\bar\theta\psi_i^* + (\bar\theta\bar\theta)F_i^*][A_j + \sqrt{2}\theta\psi_j + (\theta\theta)F_j]\\ &= A_i^*(x)A_j(x) + 2\bar\theta\bar\psi_i(x)\theta\psi_j(x)\\ &+ \sqrt{2}\theta[\psi_j(x)A_i^*(x)]\\ &+ \sqrt{2}\bar\theta[\bar\psi_i(x)A_j(x)]\\ &+ (\theta\theta)[A_i^*(x)F_j(x)]\\ &+ (\bar\theta\bar\theta)[F_i^*(x)A_j(x)]\\ &+ i(\theta\sigma^\mu\bar\theta)[(\partial_\mu A_j(x))A_i^*(x) - (\partial_\mu A_i^*(x))A_j(x)]\\ &- \sqrt{2}(\theta\theta)\bar\theta_{\dot A}\left[\frac{i}{2}\sigma^\mu_{A\dot B}\epsilon^{\dot B\dot A}(\psi_j^A(x)\partial_\mu A_i^*(x) - A_i^*\partial_\mu\psi_j^A(x)) + \bar\psi_i^{\dot A}F_j(x)\right]\\ &+ \sqrt{2}(\bar\theta\bar\theta)\theta^A\left[-\frac{i}{2}\sigma^\mu_{A\dot A}(\bar\psi_i^{\dot A}(x)\partial_\mu A_j(x) - A_j(x)\partial_\mu\bar\psi_i^{\dot A}(x)) + \psi_{jA}(x)F_i^*(x)\right]\\ &+ (\theta\theta)(\bar\theta\bar\theta)\Bigg[- A_i^*\Box A_i + i(\partial_\mu\bar\psi_i)\bar\sigma^\mu\psi_i + F_i^*(x)F_i(x) + \textit{total derivatives}\Bigg]\end{aligned} \tag{2-79}$$

As a result, the highest component of the product contains the kinetic terms for the scalar field and a Majorana spinor field, plus a term for the non-propagating auxiliary field which in addition to the invariance under SUSY transformations represents the simplest SUSY lagrangian.

Scalar superfields contain the information about scalar and fermion fields. However, to introduce gauge fields we need to introduce vector superfields, defined by the reality condition $V(x,\theta,\bar\theta) = V^\dagger(x,\theta,\bar\theta)$ leading to the general field expansion:

$$V(x,\theta,\bar\theta) = C(x) + \theta\phi + \bar\theta\bar\phi + (\theta\theta)M + (\bar\theta\bar\theta)M^* + \theta\sigma^\mu\bar\theta V_\mu + (\theta\theta)\bar\theta\bar\lambda + (\bar\theta\bar\theta)\theta\lambda + (\theta\theta)(\bar\theta\bar\theta)D \tag{2-80}$$

with $C(x)$, $M(x)$ and $D(x)$ scalar fields, ϕ and λ spinor fields and V_μ a vector field. There are also any other possible vector superfields, in fact, $\Phi^\dagger\Phi$ satisfies the reality condition as well as $\Phi + \Phi^\dagger$, being Φ a chiral field. Expressing the latter in x-coordinates we get:

$$\Phi + \Phi^\dagger = A + A^* + \sqrt{2}\theta\psi + \sqrt{2}\bar{\theta}\bar{\psi} + (\theta\theta)F + (\bar{\theta}\bar{\theta})F^* + i(\theta\sigma^\mu\bar{\theta})\partial_\mu[A - A^*] - ...$$
$$... - \frac{i}{\sqrt{2}}(\theta\theta)\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi - \frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\theta\sigma^\mu\partial_\mu\bar{\psi} - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box[A + A^*] \tag{2-81}$$

Now then, we can do the substitution:

$$\lambda(x) \to \lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\phi}(x)$$
$$D(x) \to D(x) - \frac{1}{4}\Box C(x)$$

$$V(x,\theta,\bar{\theta}) = C(x) + \theta\phi + \bar{\theta}\bar{\phi} + (\theta\theta)M + (\bar{\theta}\bar{\theta})M^* + \theta\sigma^\mu\bar{\theta}V_\mu + ...$$
$$... + (\theta\theta)\bar{\theta}\left(\bar{\lambda} - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\phi(x)\right) + (\bar{\theta}\bar{\theta})\theta\left(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\phi}(x)\right) + (\theta\theta)(\bar{\theta}\bar{\theta})D \tag{2-82}$$

so now the vector superfield is invariant under the transformation:

$$\begin{aligned} V(x,\theta,\bar{\theta}) \to V'(x,\theta,\bar{\theta}) &= V(x,\theta,\bar{\theta}) + \Phi(x,\theta,\bar{\theta}) + \Phi^\dagger(x,\theta,\bar{\theta}) \\ &= C(x) + A(x) + A^*(x) \\ &+ \theta[\phi(x) + \sqrt{2}\psi(x)] \\ &+ \bar{\theta}[\bar{\phi}(x) + \sqrt{2}\bar{\psi}(x)] \\ &+ (\theta\theta)[M(x) + F(x)] \\ &+ (\bar{\theta}\bar{\theta})[M^*(x) + F^*(x)] \\ &+ \theta\sigma^\mu\bar{\theta}[V_\mu(x) + i\partial_\mu(A(x) - A^*(x))] \\ &+ (\theta\theta)\bar{\theta}\left(\bar{\lambda}(x) - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu(\phi(x) + \sqrt{2}\psi(x))\right) \\ &+ (\bar{\theta}\bar{\theta})\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu(\bar{\phi}(x) + \sqrt{2}\bar{\psi}(x))\right) \\ &+ (\theta\theta)(\bar{\theta}\bar{\theta})\left(D(x) - \frac{1}{4}\Box(C(x) + A(x) + A^*(x))\right) \end{aligned} \tag{2-83}$$

which generalizes gauge invariance through the transformation of component fields:

$$\begin{aligned} C(x) &\to C'(x) = C(x) + A(x) + A^*(x) & \phi(x) &\to \phi'(x) = \phi(x) + \sqrt{2}\psi(x) \\ M(x) &\to M'(x) = M(x) + F(x) & V_\mu(x) &\to V'_\mu(x) = V_\mu(x) + i\partial_\mu(A(x) - A^*(x)) \\ \lambda(x) &\to \lambda'(x) = \lambda(x) & D(x) &\to D'(x) = D(x) \end{aligned}$$

Under this choice, λ and D are invariant as well as V_μ which shows the well known abelian transformation leading to a super-gauge invariant $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. This generalized gauge freedom let us chose a gauge where C', ϕ' and M' vanish, better known as the Wess-Zumino gauge where vector superfields reduces to:

$$V_{WZ}(x,\theta,\bar{\theta}) = \theta\sigma^{\mu}\bar{\theta}[V_{\mu}(x) + i\partial_{\mu}(A(x) - A^{*}(x))] + (\theta\theta)\bar{\theta}\bar{\lambda} + (\bar{\theta}\bar{\theta})\theta\lambda + (\theta\theta)(\bar{\theta}\bar{\theta})D(x) \quad (2\text{-}84)$$

where $V_{\mu}(x)$ is a gauge field, λ its fermionic superpartner, a gaugino, and $D(x)$ an auxiliary field. Unfortunately, the Wess-Zumino gauge breaks supersymmetry in the sence that a SUSY variation of $\phi_A(x)$ and $M(x)$ violates the gauge condition $C(x) = \phi_A(x) = M(x) = 0$. The latter means that the Wess-Zumino gauge is not covariant but it reduces the degrees of freedom from 16 to just 8. Anyway, together with vector superfields a supersymmetric field strenght is defined as:

$$W_A = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_A e^{V} \qquad \bar{W}_A = -\frac{1}{4}DDe^{-V}\bar{D}_{\dot{A}}e^{V} \quad (2\text{-}85)$$

$$V_{ij} = V^{a}(T_a)_{ij} \qquad \textit{in the adjoint representation}$$

$$W_A = \lambda_A(y) + 2\theta_A D(y) + (\sigma^{\mu\nu}\theta)_A F_{\mu\nu} - i(\theta\theta)\sigma^{\mu}_{A\dot{B}}D_{\mu}\bar{\lambda}^{\dot{B}}(y) \quad (2\text{-}86)$$

$$\bar{W}_{\dot{A}} = \bar{\lambda}_{\dot{A}}(z) + 2D(z)\bar{\theta}_{\dot{A}} - \epsilon_{\dot{A}\dot{B}}(\bar{\sigma}^{\mu\nu}\bar{\theta})^{\dot{B}}F_{\mu\nu} + i(\bar{\theta}\bar{\theta})(D_{\mu}\lambda(z)\sigma^{\mu})_{\dot{A}} \quad (2\text{-}87)$$

being $F_{\mu\nu}$ the general non-abelian field strenght and D_{μ} the covariant derivative both defined as follows:

$$\begin{aligned} F_{\mu\nu} &= F^{abelian}_{\mu\nu} - \frac{i}{2}[V_{\mu}, V_{\nu}] \qquad D_{\mu}\bar{\lambda} = \partial_{\mu}\bar{\lambda} - \frac{i}{2}[V_{\mu}, \bar{\lambda}] \quad (2\text{-}88) \\ &= \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu} - \frac{i}{2}[V_{\mu}, V_{\nu}] \end{aligned}$$

2.5 Supersymmetric Lagrangians

Since now we are considering an 8-dimensional space, the action functional must integrate over all of them. However, Grassmann number have interesting properties and one of them is that integration works identically as differentiation [14], so $\int d^2\theta\theta^A = \int d^2\theta = 0$ and the highest products of θ and $\bar{\theta}$ work as delta functions:

$$\int d^2\theta f(\theta)\delta^2(\theta) = f(0) \qquad \delta^2(\theta) = \theta\theta \quad (2\text{-}89)$$

In this way, the integration over a general superfield projects to the component $(\theta\theta)(\bar{\theta}\bar{\theta})$ which we already know is invariant under supersymmetric transformations and in the case of $\Phi^{\dagger}\Phi$ generates the kinetic terms for a lagrangian. We still however need to introduce gauge invariant kinetic terms. It is done by introducing the Käller potential, which is a function of the vector fields entering in the kinetic part of the lagrangian as:

$$\mathcal{L} = \Phi^\dagger K(V)\Phi$$

it has interesting mathematical properties but we are interested in the canonical Käller potential $K(V) = e^V$ with $V = V^a T_a$, which after integration gives the kinetic terms for the scalar and fermion fields with covariant derivatives $D_\mu \phi = \partial_\mu \phi - \frac{i}{2} V_\mu \phi$, two terms for the auxiliary fields and the allowed interactions between the scalar field, the fermion field and gauginos:

$$\int d^4\theta \Phi^\dagger e^V \Phi = D_\mu A^* D^\mu A - i\bar{\psi}\bar{\sigma}^\mu D_\mu \psi + |F|^2 + |A|^2 D + \frac{1}{\sqrt{2}} A\bar{\psi}\bar{\lambda} + \frac{1}{\sqrt{2}} A^* \psi\lambda \tag{2-90}$$

it is still missing the Yang-Mills term for gauge fields, which arises from the super-field strength tensor. After integration one obtains the Yang-Mills term for the gauge field, a kinetic term for the gaugino and a term for the non-propagating auxiliary field $D(x)$:

$$\begin{aligned} A &= \int d^4x \int d^4\theta [W^A W_A \delta^2(\bar{\theta}) + \bar{W}_{\dot{A}} \bar{W}^{\dot{A}} \delta^2(\theta)] \\ &= \int d^4 [8D^2(x) - F_{\mu\nu} F^{\mu\nu} - 4i\lambda\sigma^\mu \partial_\mu \bar{\lambda}] \end{aligned} \tag{2-91}$$

so the most general gauge invariant action is:

$$S = \int d^4x \left[\int d^4\theta \frac{1}{2} \Phi^\dagger e^V \Phi + \int d^2\theta Tr[W^A W_A] + \int d^2\theta Tr[W[\Phi]] + H.C. \right] \tag{2-92}$$

$$\begin{aligned} = \int d^4x \Big[& D_\mu A^* D^\mu A - i\bar{\psi}\bar{\sigma}^\mu D_\mu \psi + |F|^2 + |A|^2 D + \frac{1}{\sqrt{2}} A\bar{\psi}\bar{\lambda} + \frac{1}{\sqrt{2}} A^* \psi\lambda \\ & + 8D^2(x) - F_{\mu\nu} F^{\mu\nu} - 4i\lambda\sigma^\mu \partial_\mu \bar{\lambda} + V(A, \psi, F) \Big] \end{aligned} \tag{2-93}$$

where we have included a superpotential $W[\Phi] = \left(g_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \right) \delta^2(\bar{\theta}) + H.C.$ which in the most general renormalizable case may include the product of up to 3 superfields which after integration only survives the $\theta\theta$ component of equations in (2-77).
In order to be consistent with the literature convention, we are going to rescale the fields as:

$$V_\mu \to 2gV_\mu \qquad \lambda \to 2ig\lambda \qquad D \to gd, \tag{2-94}$$

leading to our final version of the lagrangian:

$$\begin{aligned} \mathcal{L} &= D_\mu A^* D^\mu A - i\bar{\psi}\bar{\sigma}^\mu D_\mu \psi + ... \\ & ... + F^* F + \frac{1}{2} d^2 + gA^* A d - ig\sqrt{2} A\bar{\psi}\bar{\lambda} + ig\sqrt{2} A^* \psi\lambda + ... \\ & ... - \frac{1}{4} Tr[F_{\mu\nu} F^{\mu\nu}] - i\lambda\sigma^\mu D_\mu \bar{\lambda} + V \end{aligned} \tag{2-95}$$

3 Minimal Supersymmetric Standard Model (MSSM)

3.1 Gauge Symmetries and Lagrangian

In general, any theory can be extended to have supersymmetry invariance just by promoting the fields to superfields. As shown before, it produces the right kinetic terms, the corresponding interactions involving gauginos and the auxiliary field terms. In principle, it represents a free theory but the equations of motion for D and F become non-trivial when we include a superpotential, which in fact is what characterises the theory.

The immediate supersymmetric generalization of the SM is the collection of the lagrangian of 16 superfields which leads to the particle content of 32 particles shown in figure **3-1** being all of the right hand side expected to be found in accelerators in the near future.

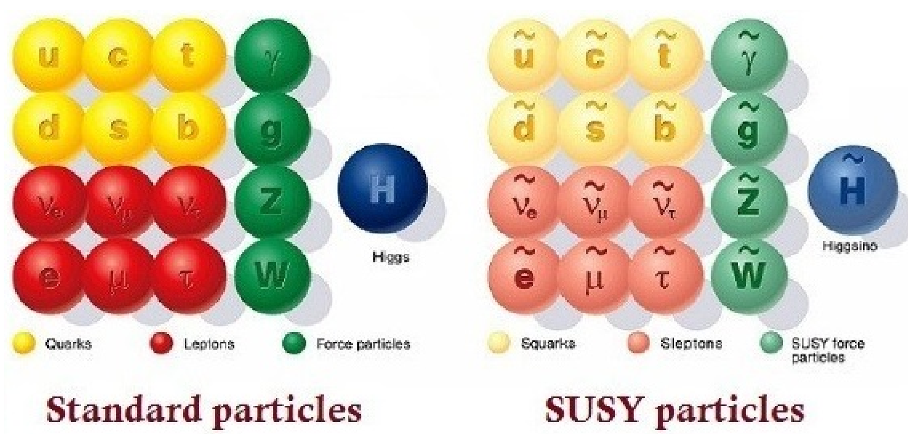


Figure **3-1**: SM SUSY extension particle content [1]

This particle content turns out to be invariant under certain symmetry groups. On the one hand, Quarks may interact among them by exchanging gluons, this follows from a $SU(3)_C$ invariance where the subscript C stands for a Color invariance though in the sense of Quantum Chromodynamics (QCD). On the other hand, fermions have both chiral counterparts, left and right handed (with the exception of neutrinos which only are left handed), where the left ones organizes in three $SU(2)$ doublets while the right handed fermions are singlets under the same group leading to a $SU(2)_L$ invariance of the lagrangian responsible of part of the electroweak interactions [33]. Additionally, an additional quantum number called *hypercharge* (Y) responds to a $U(1)_Y$ invariance of the lagrangian [65]. The inclusion of such a quantum number forbids non-observed interaction and provides the correct electrical charge of particles according to the Hell-Mann-Nishihima relation [18] besides completing the electroweak theory. All in all, the theory is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariant [56], as a result of the work of Glashow, Salam and Weinberg [33][56][65]. The hypercharge assignation of superfields can be seen in table **3-1**.

Consequently, the most general superpotential that can be considered is:

$$\begin{aligned} W_{SM} = \bar{U} y_u Q H_u - \bar{d} y_d Q H_d - \bar{e} y_e L H_d + \mu H_u H_d \\ + \epsilon_{ij}[\lambda_L \hat{L}^i \hat{L}^j \hat{E} + \lambda_L \hat{L}^i \hat{Q}^j \hat{D} - \mu' \hat{L}^i H_2^j + \lambda_B \hat{U} \hat{D} \hat{D}] \end{aligned} \tag{3-1}$$

where the first line respects R-Symmetry [45] and the second does not. It can be seen that the interaction of the model are family universal i.e. the interaction is the same for all three families and the second line may lead to Lepton/Baryon number violating terms which in principle are relevant for leptogenesis or Baryogenesis theories [39][58] but they are forbidden if R-Symmetry is included. Furthermore, since the product of left handed and right handed superfields produce terms involving derivatives, the superpotential must be a holomorphic function of superfields i.e. there are not conjugate superfields. However, this impass can be surpassed by considering the Majorana notation, where right handed fields are just conjugate left handed ones. As a result, when imposing supersymmetry to any theory, only holomorphic interactions should be considered before promoting fields into superfields.

The particle content shown in figure **3-1** correspond to just extending the standard model to a SUSY invariance. Although the SM is anomaly free this extension is not, chiral $U(1)$ anomalies are induced by Higgsinos. To avoid this problem, a second Higgs doublet with opposite hypercharge is considered making the final piece of the Minimal Supersymmetric Standard Model (MSSM).

Table **3-1**: Particle content of the MSSM with hypercharge assignation, the indices k, p run over family memebers.

Left-Handed Fermions	Y	Right-Handed Fermions	Y
SM Quarks			
$\hat{Q}^k = \begin{pmatrix} \hat{u}^k \\ \hat{d}^k \end{pmatrix}_L \equiv \begin{pmatrix} \hat{u} \\ \hat{d} \end{pmatrix}_L, \begin{pmatrix} \hat{c} \\ \hat{s} \end{pmatrix}_L, \begin{pmatrix} \hat{t} \\ \hat{b} \end{pmatrix}_L$	$+1/3$	$\hat{U}^k \equiv \hat{u}_L^{k\,c}$ $\hat{D}^k \equiv \hat{d}_L^{k\,c}$	$4/3$ $-2/3$
SM Leptons			
$\hat{L}^p = \begin{pmatrix} \hat{\nu}^p \\ \hat{e}^p \end{pmatrix}_L \equiv \begin{pmatrix} \hat{\nu}^e \\ \hat{e} \end{pmatrix}_L, \begin{pmatrix} \hat{\nu}^\mu \\ \hat{\mu} \end{pmatrix}_L, \begin{pmatrix} \hat{\nu}^\tau \\ \hat{\tau} \end{pmatrix}_L$	-1	$\hat{E}^p \equiv \hat{e}_L^{p\,c}$	-2
Higgs doublets			
$\hat{H}^1 = \begin{pmatrix} \hat{H}_1^0 \\ \hat{H}_1^- \end{pmatrix}$	-1	$\hat{H}^2 = \begin{pmatrix} \hat{H}_2^+ \\ \hat{H}_2^0 \end{pmatrix}$	$+1$

3.2 Interactions

3.2.1 Self-Interactions

From the standard theory we know that covariant derivatives include the interactions with gauge bosons. Now, it also will contain the interactions involving gauginos which become relevant for the construction of gaugino-Higgsino mixing matrix. Let's consider a fermion field ψ with its scalar superpartner A with a gauge interaction via a V_μ gauge boson and its respective gaugino λ^a. The covariant derivatives take the following form:

$$D_\mu A = \partial_\mu A + igV_\mu^a T^a A \tag{3-2}$$

$$D_\mu \psi = \partial_\mu \psi + igV_\mu^a T^a \psi \tag{3-3}$$

$$D_\mu \lambda^a = \partial_\mu \lambda^a + gf^{abc} V_\mu^b \lambda^c \tag{3-4}$$

From the kinetic term for the field λ, $\mathcal{L}_{kin-\lambda} = -i\bar{\lambda}\bar{\sigma}^\mu D_\mu \lambda$, a Yukawa-like interaction between gauge boson and gaugino arises, $\mathcal{L}_{int} = \frac{ig}{2}\bar{\lambda}^a \gamma_\mu \lambda^b V_\mu^c f^{abc}$, as shown in figure **3-2**-e. The latter implies that a high energy photon migh decay into a fotino-antifotino pair. However, if we consider the kinetic term for the scalar and fermion field and preserve only the interaction terms we get:

$$\mathcal{L}_{int-A\psi} = igV_\mu^a T_{ij}^a A_j^* \overset{\leftrightarrow\mu}{\partial} A_i - g^2 V_\mu^a (V^\mu)^b (T^a T^b)_{ij} A_j^* A_k + gV_\mu^a T_{ij}^a \bar{\psi}_i \gamma_\mu P_L \psi_j + \sqrt{2} g f_{aji} (A_i^* \lambda^a P_L \psi_j + A_i \bar{\psi}_j P_R \lambda^a) \tag{3-5}$$

These are nothing but the interactions of the scalar field with the gauge boson via a three and four leg vertices, the Yukawa interaction between the fermion and the gauge boson and a fermion-boson-gaugino interaction, all of them shown in figure **3-2**. The last of those interactions imply that boson/fermion can decay into its respective fermion/boson superpartner by emmitting a gaugino. This process imply that atoms might decay, making them unstable. In order to this proccess to be unreachable by the atomic energy scale, we conclude that supersymmetry must be broken. This will be further discussed in the next sections.

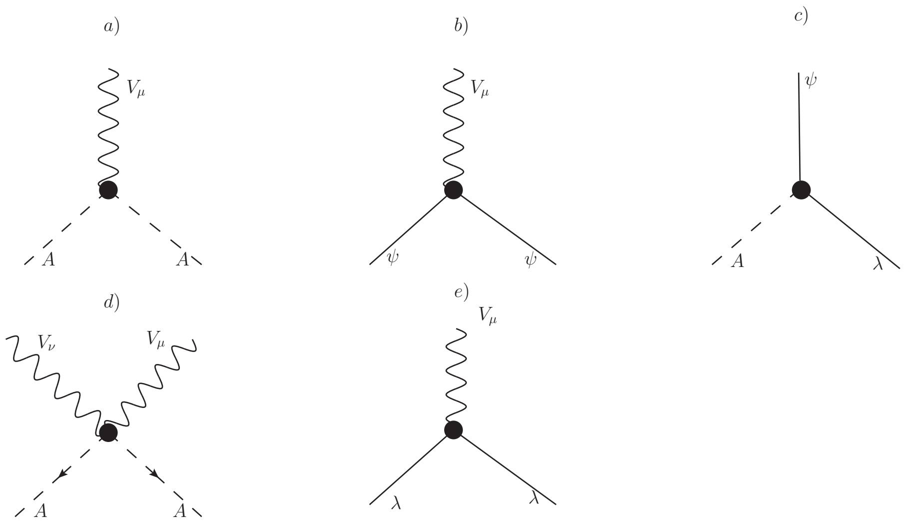


Figure **3-2**: Interactions between scalar-fermion superpartners and gauge bosons/fermions

The latter interaction would apply to particles who are not singlets under the symemtry group under consideration, so all doublets in table **3-1** may interaction with Winos, Binos and Gluinos in the case of quarks.

3.2.2 Superpotential: Fermions and F-Term potential

In equation 3-1 we stated the most general superpotential as the product of up to three superfields. However, we know that becase the product of three superfields leads to up to quartic interaction which is the limit for renormalizability. If we impose lepton/baryon number conservation the superpotential reduces to:

$$W_{SM} = \hat{U}^i y_u^{ij} \hat{Q}^j \hat{H}_2 - \hat{D}^i y_d^{ij} \hat{Q}^j \hat{H}_1 - \hat{E}^i y_e^{ij} L^j H_1 + \mu H_1 H_2 + H.C. \tag{3-6}$$

where the indices $i, j = 1, 2, 3$ run over family members while the product between superfield doublets is mediated by the Levi-Civita pseudotensor ϵ_{ij} as will be shown next. By using Eq. 2-77 for the superfield product and projecting to the $\theta\theta\bar{\theta}\bar{\theta}$ component we get (omitting family indices):

$$
\begin{aligned}
\epsilon_{mn}\hat{U}\hat{Q}^m\hat{H}_2^n\Big|_{\theta\theta} &= \hat{U}(\hat{u}_L\hat{H}_2^0 - \hat{d}_L\hat{H}_2^+)\Big|_{\theta\theta} \\
&= H_2^0\tilde{u}_R^*F_{u_L} + \tilde{u}_L\tilde{u}_R^*F_{H20} + H_2^0\tilde{u}_LF_{u_R} - \tilde{H}_2^0u_L\tilde{u}_R^* - \tilde{H}_2^0u_L^C\tilde{u}_L - u_Lu_L^CH_2^0 \\
&\quad - H_2^+\tilde{u}_R^*F_{d_L} - \tilde{d}_L\tilde{u}_R^*F_{H2+} - H_2^+\tilde{d}_LF_{u_R} + \tilde{H}_2^+d_L\tilde{u}_R^* + \tilde{H}_2^+u_L^C\tilde{d}_L + d_Lu_L^CH_2^+ \\
\epsilon_{mn}\hat{D}\hat{Q}^m\hat{H}_1^n\Big|_{\theta\theta} &= H_1^-\tilde{d}_R^*F_{u_L} + \tilde{u}_L\tilde{d}_R^*F_{H1-} + H_1^-\tilde{u}_LF_{d_R} - \tilde{H}_1^-u_L\tilde{d}_R^* - \tilde{H}_1^-d_L^C\tilde{u}_L - u_Ld_L^CH_1^- \\
&\quad - H_1^0\tilde{d}_R^*F_{d_L} - \tilde{d}_L\tilde{d}_R^*F_{H10} - H_1^0\tilde{d}_LF_{d_R} + \tilde{H}_1^0d_L\tilde{d}_R^* + \tilde{H}_1^0d_L^C\tilde{d}_L + d_Ld_L^CH_1^0 \\
\epsilon_{mn}\hat{E}\hat{L}^m\hat{H}_1^n\Big|_{\theta\theta} &= H_1^-\tilde{e}_R^+F_{\nu_L} + \tilde{\nu}_L\tilde{e}_R^+F_{H1-} + H_1^-\tilde{\nu}_LF_{e_R} - \tilde{H}_{1L}^-\nu_L\tilde{e}_R^+ - H_{1L}^-e_L^C\tilde{\nu}_L - \nu_Le_L^CH_1^- \\
&\quad - H_1^0\tilde{e}_R^+F_{e_L} - \tilde{e}_L^-\tilde{e}_R^+F_{H10} - H_1^0\tilde{e}_L^-F_{e_R} + \tilde{H}_{1L}^0e_L^-\tilde{e}_R^+ + \tilde{H}_{1L}e_L^C\tilde{e}_L^- + e_l^-e_L^CH_1^0 \\
\epsilon_{mn}\hat{H}_1^m\hat{H}_2^n\Big|_{\theta\theta} &= H_1^0F_{H20} + F_{H10}H_2^0 - \tilde{H}_{1L}^0\tilde{H}_{2L}^0 - H_1^-F_{H2+} - F_{H1-}H_2^+ + \tilde{H}_{1L}^-\tilde{H}_{2L}^+
\end{aligned}
$$

Since this interaction terms now are F-term dependent, we can see that their equation of motion is no longer trivial. They enter into the lagrangian as a F-term potential V(F) given by:

$$
V(F) = -\sum_k F_k^*F_k \qquad\qquad F_k^* = -\frac{\partial\hat{W}_{SM}|_{\theta\theta\bar{\theta}\bar{\theta}}}{\partial F_k} \tag{3-7}
$$

and in our case the F-terms acquire the form:

$$
\begin{aligned}
-F_{H10}^* &= y_\tau\tilde{e}_L^-\tilde{e}_R^+ + y_b\tilde{d}_L\tilde{d}_R^* - \mu H_2^0 & -F_{H1-}^* &= -y_\tau\tilde{\nu}_L\tilde{e}_R^+ - y_b\tilde{u}_L\tilde{d}_R^* + \mu H_2^+ && \text{(3-8)}\\
-F_{H20}^* &= y_t\tilde{u}_L\tilde{u}_R^* - \mu H_1^0 & -F_{H2+}^* &= -y_t\tilde{d}_L\tilde{u}_R^* + \mu H_1^- && \text{(3-9)}\\
-F_{e_L}^* &= y_\tau H_1^0\tilde{e}_R^+ & -F_{e_R}^* &= y_\tau(H_1^0\tilde{e}_L^- - H_1^-\tilde{\nu}_L) && \text{(3-10)}\\
-F_{u_L}^* &= -y_bH_1^-\tilde{d}_R^* + y_tH_2^0\tilde{u}_R^* & -F_{u_R}^* &= -y_t(H_2^+\tilde{d}_L - H_2^0\tilde{u}_L) && \text{(3-11)}\\
-F_{d_L}^* &= y_bH_1^0\tilde{d}_R^* - y_tH_2^+\tilde{u}_R^* & -F_{d_R}^* &= y_b(H_1^0\tilde{d}_L - H_1^-\tilde{u}_L) && \text{(3-12)}\\
-F_{\nu_L}^* &= -y_\tau H_1^-\tilde{e}_R^+ &&&& \text{(3-13)}
\end{aligned}
$$

we can see that this potential only involves scalar interactions among the scalar superpartners yet unobserved. Besides containing all possible interactions allowed by symmetries, they can contribute to the sparticles mass matrices after implementing the spontaneous symmetry breaking. All in all, the superpotential contribution is given by:

$$
\begin{aligned}
W|_{\theta\theta} = V(F) &- y_u \tilde{H}_2^0 u_L \tilde{u}_R^* - y_u \tilde{H}_2^0 u_L^C \tilde{u}_L - y_d \tilde{H}_1^0 d_L \tilde{d}_R^* - y_d \tilde{H}_1^0 d_L^C \tilde{d}_L - y_e \tilde{H}_{1L}^0 e_L^- \tilde{e}_R^+ \\
&- y_e \tilde{H}_{1L} e_L^C \tilde{e}_L^- + y_u \tilde{H}_2^+ d_L \tilde{u}_R^* + y_u \tilde{H}_2^+ u_L^C \tilde{d}_L + y_u d_L u_L^C H_2^+ + y_d \tilde{H}_1^- u_L \tilde{d}_R^* \\
&+ y_d \tilde{H}_1^- d_L^C \tilde{u}_L + y_d u_L d_L^C H_1^- + y_e \tilde{H}_{1L}^- \nu_L \tilde{e}_R^+ + y_e H_{1L}^- e_L^C \tilde{\nu}_L + y_e \nu_L e_L^C H_1^- \\
&- y_u u_L u_L^C H_2^0 - y_d d_L d_L^C H_1^0 - y_e e_l^- e_L^C H_1^0 - \mu \tilde{H}_{1L}^0 \tilde{H}_{2L}^0 + \mu \tilde{H}_{1L}^- \tilde{H}_{2L}^+
\end{aligned} \tag{3-14}
$$

3.2.3 D-Term Potential

From equation 2-95 we see that the part of the lagrangian involving the auxiliary D-term is:

$$
\mathcal{L}_D = \frac{1}{2} d^a d^a + g d^a A_i^* T_{ij}^a A_j \tag{3-15}
$$

being T_{ij}^a the generators of the symmetry group. By applying their equation of motion a D-term potential arises as $V_D = -\frac{1}{2}\sum_k d_k^* d_k$ with $d^a = -g T_{ij}^a A_i^* A_j$. In our case, we have two contributions, one from the $SU(2)$ symmetry with interaction g and another from the $U(1)_Y$ with interaction g' leading to:

$$
\begin{aligned}
D^a &= \frac{g}{2}\left[H_1^{m*}\tau_{mn}^a H_1^n + H_2^{m*}\tau_{mn}^a H_2^n + \tilde{Q}_m^{k*}\tau_{mn}^a \tilde{Q}_n^k + \tilde{L}_m^{p*}\tau_{mn}^a \tilde{L}_n^p\right] && \text{(3-16)} \\
D' &= \frac{g'}{2}\left[H_2^{m*}H_2^m - H_1^{i*}H_1^m + \frac{1}{3}\tilde{Q}^{mk*}\tilde{Q}^{mk} - \frac{4}{3}\tilde{u}_R^{k*}\tilde{u}_R^k + \frac{2}{3}\tilde{d}_R^{k*}\tilde{d}_R^k - \tilde{L}^{mp*}\tilde{L}^{mp} + 2\tilde{e}_L^{p\,c}\tilde{e}_R^+\right]
\end{aligned}
$$

$$
\begin{aligned}
V_D = -\frac{g^2}{8}\Big[&(H_1^{m*}H_1^m)^2 + (H_2^{m*}H_2^m)^2 + (\tilde{Q}^{mk*}\tilde{Q}^{mk})^2 + (\tilde{L}^{pk*}\tilde{L}^{pk})^2 \\
&- 2(H_1^{m*}H_1^m)(\tilde{Q}^{nk*}\tilde{Q}^{nk}) + 4|H_2^{m*}\tilde{Q}^{mk}|^2 - 2(H_1^{m*}H_1^m)(\tilde{L}^{np*}\tilde{L}^{np}) + 4|H_1^{m*}\tilde{L}^{mp}|^2 \\
&- 2(H_2^{m*}H_2^m)(\tilde{Q}^{nk*}\tilde{Q}^{nk}) + 4|\tilde{Q}^{mk*}\tilde{L}^{mk}|^2 - 2(H_2^{m*}H_2^m)(\tilde{L}^{np*}\tilde{L}^{np}) + 4|H_2^{m*}\tilde{L}^{mp}|^2 \\
&- 2(\tilde{Q}^{mk*}\tilde{Q}^{mk})(\tilde{L}^{np*}\tilde{L}^{np}) + 4|H_1^{m*}\tilde{Q}^{mk}|^2 - 2(H_1^{m*}H_1^m)(H_2^{n*}H_2^n) + 4|H_1^{m*}H_2^m|^2\Big] \\
&- \frac{g'^2}{8}\left[H_2^{m*}H_2^m - H_1^{m*}H_1^m + \frac{1}{3}\tilde{Q}^{mk*}\tilde{Q}^{mk} - \frac{4}{3}\tilde{u}_R^{k*}\tilde{u}_R^k + \frac{2}{3}\tilde{d}_R^{k*}\tilde{d}_R^k - \tilde{L}^{mp*}\tilde{L}^{mp} + 2\tilde{e}_L^{pC*}\tilde{e}_L^{pC}\right]^2 .
\end{aligned} \tag{3-17}
$$

where the indices $m, n = 1, 2$ are $SU(2)$ indices and $k, p = 1, 2, 3$ are family indices. Similar to the $F - Term$ potential, it only contains interactions among scalar particles. In this way, sparticles does not need a superpotential to interact among them or to acquire a mass value after symmetry breaking. However, the resulting mass matrix will be consistent with the fact that particles and sparticles have the same mass, and since no supersymmetric particle has been observed we need to consider supersymmetry breaking.

3.2.4 Soft-Breaking potential

As mentioned before, if supersymmetry is a symmetry of nature it must be broken because sparticles with the same mass of their partners have not been observed and because some interaction have not been observed neither. Now, we do not really know how must be broken. Spontaneous supersymmetry breaking is not an option because it would produce a Goldstino (massless goldstone fermion)[7] and we know that such a fermion does not exist. As a consequence, it must be explicitly broken. We add a soft breaking potential [45] to the lagrangian, where the soft recalls that these new terms do not introduce quadratic divergences so it is restricted to bilinear and trilinear couplings allowed by gauge symmetries as long as it does not involve any gauge singlet [31]. These soft-breaking interactions affect the mass matrices indeed, so they must involve only the unobserved particles to provide a mass value far beyond the electroweak scale, being a scalar potential too. The potential is written without writing the family index explicitly as:

$$
\begin{aligned}
V_{Soft} =& (m^2_{\tilde{Q}u})^{kl}\tilde{u}^{*k}_L\tilde{u}^l_L + (m^2_{\tilde{U}})^{kl}\tilde{u}^{C*k}_L\tilde{u}^{Cl}_L - \frac{\sqrt{2}(m_u A_t)^{kl}}{v_2}\epsilon_{ij}H^i_2\tilde{u}^{k*}_L\tilde{u}^{Cl}_L \\
&+ (m^2_{\tilde{Q}d})^{kl}\tilde{d}^{k*}_L\tilde{d}^l_L + (m^2_{\tilde{D}})^{kl}\tilde{d}^{C*k}_L\tilde{d}^{Cl}_L - \frac{\sqrt{2}(m_d A_b)^{kl}}{v_1}\epsilon_{ij}H^i_1\tilde{d}^{*k}_L\tilde{d}^{Cl}_L \\
&+ (m^2_{\tilde{l}e})^{kl}\tilde{e}^{*k}_L\tilde{e}^l_L + (m^2_{\tilde{e}})^{kl}\tilde{e}^{C*k}_L\tilde{e}^{Cl}_L - \frac{\sqrt{2}(m_\ell A_\tau)^{kl}}{v_1}\epsilon_{ij}H^i_1\tilde{e}^{k*}_L\tilde{e}^{Cl}_L \\
&+ (m^2_{\tilde{l}\nu})^{kl}\tilde{\nu}^{*k}_L\tilde{\nu}^l_L + M_2\tilde{W}^-\tilde{W}^+ + \frac{M_1}{2}\bar{\tilde{B}}\tilde{B} + \frac{M_2}{2}\bar{\tilde{W}}_3\tilde{W}_3 \\
&+ m^2_1|H_1|^2 + m^2_2|H_2|^2 - m^2_{12}\epsilon_{ij}(H^{i*}_1 H^j_2) + H.C.
\end{aligned}
\tag{3-18}
$$

where m_α, $\alpha = u, d, \ell$ correspond to the up-like quarks, down-like quarks and charged lepton mass matrices respectively. The potential is made of diagonal terms for all sparticles, mass terms for gauginos and left-right mixing terms. Since this potential break supersymmetry, we expect that all m's have a value in the SUSY breaking scale, hopefully in the TeV scale.

3.3 Mass matrices

Now that we have written the most general $SU(2)_L \otimes U(1)_Y$ potential without lepton/baryon number violation. A spontanous symmetry breaking is implemented by the replacement:

$$
H^1 = \begin{pmatrix} H^0_1 \\ H^-_1 \end{pmatrix} = \begin{pmatrix} \frac{v_1 + h_1 + i\eta_1}{\sqrt{2}} \\ H^-_1 \end{pmatrix} \qquad H^2 = \begin{pmatrix} H^+_2 \\ H^0_2 \end{pmatrix} = \begin{pmatrix} H^+_2 \\ \frac{v_2 + h_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}
\tag{3-19}
$$

with v_1, v_2 the vacuum expectation value (VEV) of the respective scalar fields. Additionally, h_1, h_2 are CP-even fields while η_1, η_2 are CP-odd. Instead of working with v_1 and v_2, we define a general VEV v and a mixing angle β all of them related by the triangle in figure (**3-3**).

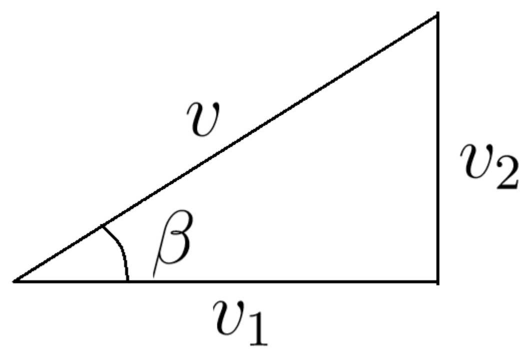


Figure **3-3**: Relationship among VEV's and mixing angle

First, when considering the kinetic terms for the scalar doublets, the covariant derivative takes the form $D_\mu = \partial_\mu - igW_\mu^a T_a - ig'\frac{Y}{2}B_\mu$, being $T_a = \frac{\sigma_a}{2}$ the $SU(2)$ generators and Y the hypercharge, so the symmetry breaking results in mass terms for the gauge bosons coming from the, kinetic terms $D_\mu \Phi_1 D^\mu \Phi_1$ and $D_\mu \Phi_2 D^\mu \Phi_2$ similar to the second term in Eq. (3-5). They are made out of a mixing between W_μ^3 and B_μ gauge bosons given by M_o^2 and a mass term for the charged W bosons.

$$M_0^2 = \frac{1}{4}\begin{pmatrix} g^2v^2 & -gg'v^2 \\ -gg'v^2 & g'^2v^2 \end{pmatrix}, \qquad m_\omega = \frac{gv}{2} \tag{3-20}$$

Since the mass matrix has null determinant the two mass eigenvalues are $m_\gamma = 0$ (photon) and $m_z = \frac{(g^2+g'^2)}{2}v^2 = \frac{m_\omega^2}{cos^2(\theta_w)}$ (Z-boson) being θ_w the Weinberg angle [65] which relates the coupling constants g and g' through $\tan\theta_w = \frac{g'}{g}$.
In the case of fermions, the last line on Eq. (3-14) generates bilinear mass terms for fermions identical to the SM mass matrices but with two different VEV for different isospin states.

$$m_u = \frac{v_2}{\sqrt{2}}\begin{pmatrix} y_u^{11} & y_u^{12} & y_u^{13} \\ y_u^{21} & y_u^{22} & y_u^{23} \\ y_u^{31} & y_u^{32} & y_u^{33} \end{pmatrix} \quad m_d = \frac{v_1}{\sqrt{2}}\begin{pmatrix} y_d^{11} & y_d^{12} & y_d^{13} \\ y_d^{21} & y_d^{22} & y_d^{23} \\ y_d^{31} & y_d^{32} & y_d^{33} \end{pmatrix} \quad m_\ell = \frac{v_1}{\sqrt{2}}\begin{pmatrix} y_e^{11} & y_e^{12} & y_e^{13} \\ y_e^{21} & y_e^{22} & y_e^{23} \\ y_e^{31} & y_e^{32} & y_e^{33} \end{pmatrix} \tag{3-21}$$

On the one hand, fermion mass eigenvalues are determined exclusively by a fine-tunning problem, which is what we want to avoid for generaing a Fermion Mass Hiearchy (FMH). On the other hand, since no right-handed neutrino is included in the SM, no right-handed neutrino superfield is included as well so in the MSSM neutrinos remain massless too. There

is however another kind of fermions on this theory, the higgsinos and gauginos.
If we consider the interaction terms in the Higgs kinetic terms indicated in Eq. (3-5) for the SM symmetry groups, a gaugino-higgsino mixing arises afer SSB, which together with the two last terms in Eq. (3-14) constitutes the charged-Higgsinos mass matrix and the gaugino-higgsino mass matrix in the basis $(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ as given by:

$$\begin{pmatrix} \tilde{W}^- & \tilde{H}_1^- \end{pmatrix} \begin{pmatrix} m_2 & \sqrt{2}m_\omega sin(\beta) \\ \sqrt{2}m_\omega cos(\beta) & \mu^2 \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \end{pmatrix} \tag{3-22}$$

$$M_{Char} = \begin{pmatrix} m_2 & \sqrt{2}m_\omega sin(\beta) \\ \sqrt{2}m_\omega cos(\beta) & \mu^2 \end{pmatrix} \tag{3-23}$$

$$\mathbf{M'}_{Neut} = \begin{pmatrix} m_1 & 0 & -m_z cos(\beta) sin(\theta_w) & m_z sin(\beta) sin(\theta_w) \\ 0 & m_2 & m_z cos(\beta) cos(\theta_w) & -m_z sin(\beta) cos(\theta_w) \\ -m_z cos(\beta) sin(\theta_w) & m_z cos(\beta) cos(\theta_w) & 0 & -\mu \\ m_z sin(\beta) sin(\theta_w) & -m_z sin(\beta) cos(\theta_w) & -\mu & 0 \end{pmatrix} \tag{3-24}$$

The mass eigenstates of the charged Higgsino matrix are known as *Charginos* while on the neutral Higgsino matrix are called *Neutralinos*, being the lightest of then the ideal Dark Matter candidate [27]. On the side of superpartners the following general mass matrices are obtained in 3×3 blocks:

$Up - Squarks$

$$M_t = \begin{pmatrix} m_{\tilde{Q}u}^2 + m_u^2 + m_z^2 cos(2\beta)\left(\frac{1}{2} - \frac{2}{3}sin^2(\theta_w)\right)\mathcal{I} & \frac{m_u}{2}(A_t - \mu cot(\beta)\mathcal{I}) \\ \frac{m_t}{2}(A_t - \mu cot(\beta)\mathcal{I}) & m_{\tilde{U}}^2 + m_u^2 + \frac{2}{3}m_z^2 cos(2\beta) sin^2(\theta_w)\mathcal{I} \end{pmatrix} \tag{3-25}$$

$Down - Squarks$

$$M_b = \begin{pmatrix} m_{\tilde{Q}d}^2 + m_d^2 + m_z^2 cos(2\beta)\left(\frac{1}{2} - \frac{1}{3}sin^2(\theta_w)\right)\mathcal{I} & \frac{m_b}{2}(A_b - \mu tan(\beta)\mathcal{I}) \\ \frac{m_b}{2}(A_b - \mu tan(\beta)\mathcal{I}) & m_{\tilde{D}}^2 + m_d^2 + \frac{1}{3}m_z^2 cos(2\beta) sin^2(\theta_w)\mathcal{I} \end{pmatrix} \tag{3-26}$$

$ChargedSleptons$

$$M_{\tilde{l}} = \begin{pmatrix} m_{\tilde{l}e}^2 + m_\ell^2 - \frac{1}{2}m_z^2 cos(2\beta) cos(2\theta_w)\mathcal{I} & \frac{m_\tau}{2}(A_\tau - \mu tan(\beta)\mathcal{I}) \\ \frac{m_\tau}{2}(A_\tau - \mu tan(\beta)\mathcal{I}) & m_{\tilde{e}}^2 + m_\ell^2 - m_z^2 cos(2\beta) sin^2(\theta_w)\mathcal{I} \end{pmatrix} \tag{3-27}$$

$\mathcal{I}$ the 3×3 identity matrix. To date, no sparticle has been observed so, if exist, they have to have a mass in the TeV scale making the electroweak contribution negligible. It results in mass eigenstates fully dominated by the soft-breaking parameters and degenerate high-values masses. Usually the family mixing is neglected [21], so the mass matrix is a 2×2 family independent one which mixes the left and right counterparts of each sparticle.

However, the scalar doublets describe three mass matrices: one of the CP-even (h) and another one the CP-odd (η) counterparts together with a charged scalar mass matrix, which are given by:

$$\mathbf{M}_h = \frac{1}{2}\begin{pmatrix} m_{12}^2 tan(\beta) + m_z^2 cos^2(\beta) & -m_z^2 sin(\beta)cos(\beta) - m_{12}^2 \\ -m_z^2 sin(\beta)cos(\beta) - m_{12}^2 & m_{12}^2 cot(\beta) + m_z^2 sin^2(\beta) \end{pmatrix} \tag{3-28}$$

$$\mathbf{M}_\eta = \frac{1}{2}\begin{pmatrix} m_{12}^2 Tan(\beta) & -m_{12}^2 \\ -m_{12}^2 & m_{12}^2 Cot(\beta) \end{pmatrix} \quad \mathbf{M}_{CH} = \begin{pmatrix} m_{12}\frac{v_2}{v_1} + \frac{g^2 v_2^2}{4} & m_{12}^2 + \frac{g^2}{4}v_1 v_2 \\ m_{12}^2 + \frac{g^2}{4}v_1 v_2 & m_{12}\frac{v_1}{v_2} + \frac{g^2 v_1^2}{4} \end{pmatrix} \tag{3-29}$$

On the one hand,the masses of the CP-even scalars are obtained by direct diagonalization of the mass matrix resulting in $m_{h1/2} = \frac{1}{2}\left[m_A^2 + m_z^2 \pm \sqrt{m_A^4 + m_z^4 - 2m_A^2 m_z^2 cos(4\beta)}\right]$. On the other hand, there is one massless CP-odd scalar and one massless charged scalar, which are identified as the Goldstone bosons arising from the symmetry breaking $SU(2)_L \otimes U(1)_Y \Rightarrow U(1)_Q$ and provide a finite mass value to the Z and W gauge bosons respectively as indicates the Goldstone theorem revisited in section (2.2.3). In addition, the massive states are given by $m_\eta^2 = \frac{2m_{12}^2}{sin(2\beta)}$ and $m_{CH}^2 = m_{12}^2\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right) + m_w^2$

3.4 Family mixing

Now we can consider the third term on Eq. (3-5), which came from fermion kinetic terms, applied to the SM symmetry groups, it can be written explicitely as [32]:

$$\mathcal{L}_c = -\frac{1}{2}\psi_L^\dagger \left(\begin{pmatrix} g\not{W}^3 + g'Y\not{B} & g(\not{W}^1 - i\not{W}^2) \\ g(\not{W}^1 + i\not{W}^2) & -g\not{W}^3 + g'Y\not{B} \end{pmatrix}\right)\psi_L - g'Y\psi_L^C\not{B}\psi_L^C \tag{3-30}$$

where ψ can be any fermion with hypercharge Y, we define then the charged $W^\pm$ bosons as $W^\pm \equiv \frac{W^1 \mp W^2}{\sqrt{2}}$, with mass $\frac{gv}{2}$ found previously. Similarly the W^3 and B bosons can be rotated into the physical fields (photon and Z-boson) by a rotation of θ_w, which in fact is the angle that diagonalizes the gauge boson matrix, the result is:

$$\mathcal{L}_c = -\psi_L^\dagger \left(\begin{pmatrix} (g_L^Z)^1\not{Z} + (g_L^A)^1\not{A} & g\not{W}^+ \\ g\not{W}^- & (g_L^Z)^2\not{Z} + (g_L^A)^2\not{A} \end{pmatrix}\right)\psi_L - g_R^Z Y\psi_L^C\not{Z}\psi_L^C - g_R^A Y\psi_L^C\not{A}\psi_L^C \tag{3-31}$$

with

$$2(g_L^Z)^1 = g\cos\theta_w - g'Y\sin\theta_w \qquad 2(g_L^Z)^2 = -g\cos\theta_w - g'Y\sin\theta_w \tag{3-32}$$

$$2(g_L^A)^1 = g\sin\theta_w + g'Y\cos\theta_w \qquad 2(g_L^A)^2 = g\sin\theta_w - g'Y\cos\theta_w \tag{3-33}$$

$$g_R^Z = -g'Y\sin\theta_w \qquad g_R^A = -g'Y\cos\theta_w \tag{3-34}$$

ψ	g_L^Z	g_L^A	g_R^Z	g_R^A
ν_e , ν_μ, ν_τ	$\frac{g}{2\cos\theta_w}\left(\frac{1}{2}\right)$	0	0	0
e, μ, τ	$\frac{g}{2\cos\theta_w}\left(-\frac{1}{2}+\sin^2\theta_w\right)$	e	$\frac{g}{2\cos\theta_w}\left(\sin^2\theta_w\right)$	e
u, c, t	$\frac{g}{2\cos\theta_w}\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_w\right)$	$-\frac{2}{3}e$	$\frac{g}{2\cos\theta_w}\left(-\frac{2}{3}\sin^2\theta_w\right)$	$-\frac{2}{3}e$
d, s, b	$\frac{g}{2\cos\theta_w}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_w\right)$	$\frac{1}{3}e$	$\frac{g}{2\cos\theta_w}\left(\frac{1}{3}\sin^2\theta_w\right)$	$\frac{1}{3}e$

Table **3-2**: Current coupling constant for fermions

where the numerical superscript indicates the isospin component, the numerical specific coupling for each kind of fermion is shown in table (**3-2**). In this way, the interaction can be splited into a neutral current and a charged current depending of the electrical charge of the gauge boson the fermion is coupling with.

$$\begin{aligned}
\mathcal{L}_{NC} &= -(g_L^Z)^1(\psi_L^\dagger)^1 \not{Z}(\psi_L^\dagger)^1 - (g_R^Z)^1(\psi_L^{C\dagger})^1 \not{Z}(\psi_L^\dagger)^1 - (g_L^A)^1(\psi_L^\dagger)^1 \not{A}(\psi_L^\dagger)^1 - (g_R^A)^1(\psi_L^{C\dagger})^1 \not{A}(\psi_L^\dagger)^1 \\
&= -j_Z^\mu Z_\mu - j_A^\mu A_\mu \\
&= \mathcal{L}_{NC}^Z + \mathcal{L}_{NC}^A \qquad (3\text{-}35) \\
\mathcal{L}_{CC} &= -\frac{g}{2\sqrt{2}}((\psi_L^\dagger)^1)\gamma^\mu(1-\gamma^5)(\psi_L)^2 W_\mu^+ + H.C. \qquad (3\text{-}36)
\end{aligned}$$

The interaction with the photon actually makes the QED lagrangian, so regarding neutral currents it is of our interest just the interaction with the Z-boson. Since the hypercharge is family universal, the $-(g_L^Z)^k$ coupling is the same for all three generations, so we can promote the neutral interactions to three generations as:

$$\mathcal{L}_{NC}^Z = -\begin{pmatrix} f_L^{1\dagger} & f_L^{2\dagger} & f_L^{3\dagger} \end{pmatrix} \begin{pmatrix} g_L^Z(f_1) & 0 & 0 \\ 0 & g_L^Z(f_2) & 0 \\ 0 & 0 & g_L^Z(f_3) \end{pmatrix} \not{Z} \begin{pmatrix} f_L^1 \\ f_L^2 \\ f_L^3 \end{pmatrix} \qquad (3\text{-}37)$$

where the superscript indicates the family and no isospin index is written since the interaction is between equal isospin particles. Since the coefficient matrix is proportional to the identity ($g_L^Z(f_i)$ is the same for all three families), we get the same equation in the mass eigenbasis, forbidding interactions that change flavor, i.e. change between generations ($\tau \to Ze$) by the emission of a neutral boson (Z or A). Nevertheless, it can be possible if a new non-universal interaction is included, so the coefficient matrix would not be proportional to identity. This result is known as the Glashow-Iliopoulos-Maiani (GIM) mechanism [42] which guarantees the absence of Flavor Changing Neutral Currents (FCNC) in the SM, or at least its suppression. To present, there is no observed event consistent with a FCNC being a success of the SM, so any Beyond the SM theory must predict such a supression [49].

In the case of the charged currents, it is hypercharge independent but the interacting fermions have opposite isospin. Let's consider the case with quarks if we rotate to the mass eigenbasis, in that case we would get:

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} \begin{pmatrix} u & c & t \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \not{W} \begin{pmatrix} d \\ s \\ b \end{pmatrix} + H.C. \tag{3-38}$$

$$= -\frac{g}{2\sqrt{2}} \begin{pmatrix} u^1 & u^2 & u^3 \end{pmatrix} V_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \not{W} V_d^\dagger \begin{pmatrix} d^1 \\ d^2 \\ d^3 \end{pmatrix} + H.C. \tag{3-39}$$

$$= -\frac{g}{2\sqrt{2}} \begin{pmatrix} u^1 & u^2 & u^3 \end{pmatrix} V_{CKM} \not{W} \begin{pmatrix} d^1 \\ d^2 \\ d^3 \end{pmatrix} + H.C. \tag{3-40}$$

It turns out that up-like quarks and down-like quarks does not have a common basis, so the product of their rotations is not the identity ($V_u V_d^\dagger \neq \mathcal{I}$) the product of them is the Cabibbo-Kobayashi-Maskawa matrix (CKM) which represent the relative rotation between mass eigenbasis of quarks and in general allows flavor changing interactions i.e. a second generation quark can decay into a first generation one by changing its isospin. That matrix parametrize most of quark interactions and it is well measured, to present it takes the form [63]:

$$V_{CKM} = \begin{pmatrix} 0.97420 \pm 0.00021 & 0.2243 \pm 0.0005 & (3.94 \pm 0.36) \times 10^{-3} \\ 0.218 \pm 0.004 & 0.997 \pm 0.017 & (42.2 \pm 0.8) \times 10^{-3} \\ (8.1 \pm 0.5) \times 10^{-3} & (39.4 \pm 2.3) \times 10^{-3} & 1.019 \pm 0.025 \end{pmatrix} \tag{3-41}$$

In general, this matrix can be parametrized using three angles and one CP phase. However, since the quark mixing angles turns out to be small, a different parametrization can be used by introducing the Wolfenstein parameters, so it reads:

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & A\lambda^2 & 1 \end{pmatrix} \tag{3-42}$$

where according to the last results[63] are $A = 0.836 \pm 0.015$, $\lambda = 0.22453 \pm 0.00044$, $\rho = 0.122^{+0.018}_{-0.017}$ and $\eta = 0.355^{+0.012}_{-0.011}$.

In the case of lepton-neutrino mixing happens a similar fashion due to hypercharge universality. In that case, the relative rotation between lepton and neutrino mass eigenstates is known as the Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS) which is given analogously by

$V_{PMNS} = U_l U_\nu^\dagger$. There is however an important phenomenological difference.

Despite the CKM and PMNS matrices are mathematically analogous, the neutrino mass matrix entries are much smaller than the quark matrix entries, since experimentally it is known that neutrinos must have a very small mass. Consequently, the oscillations between quarks in the flavor basis happens in small times, corresponding to a length scale of the nuclear radius while neutrino oscillations require hundreds of kilometres as pointed out in the solar neutrino problem [6] together with the time dilation due to the relativistic speed of them. As a result, the flavor changes are highly suppressed ($l_\alpha \to l_\beta < 10^{-54}$) [17] so neutrino mass eigenstates as well as leptonic flavor changes are not often considered in discussions. To present, the PMNS matrix has the following values [25] at 3σ confidence level:

$$V_{PMNS} = \begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.243 \to 0.490 & 0.473 \to 0.674 & 0.651 \to 0.772 \\ 0.295 \to 0.525 & 0.493 \to 0.688 & 0.618 \to 0.744 \end{pmatrix} \tag{3-43}$$

	NO	IO
$\frac{\Delta m^2_{21}}{10^{-5} eV^2}$	$7.39^{+0.21}_{-0.20}$	$7.39^{+0.21}_{-0.20}$
$\frac{\Delta m^2_{3\ell}}{10^{-3} eV^2}$	$+2.523^{0.032}_{-0.030}$	$-2.509^{+0.032}_{-0.030}$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$33.56^{+0.77}_{-0.75}$
$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$48.6^{+1.1}_{-1.5}$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.65^{+0.13}_{-0.12}$
$\delta/^\circ$	222^{+38}_{-28}	285^{+24}_{-26}

Table **3-3**: Neutrino mixing parameters [25]

The elements of the PMNS matrix are determined mainly from neutrino oscillation experiments, by studying the three main sources of neutrinos (solar, atmospheric and reactor neutrinos) they get the mixing angles and the CP phase, given in table **3-3**. Nevertheless, they only provide mass differences instead of information about each mass eigenstate leaving as an unresolved questions the mass hierarchy of neutrinos. In general, two schemes are considered: the normal ordering (NO) ($m_1 < m_2 < m_3$) and the inverse ordering (IO) ($m_3 < m_1 < m_2$). To conclude this section, it is important to mention the mixing matrices standard parametrization in terms of 4 mixing angles and 1 CP phase, it reads:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -\sin\theta_{13} e^{i\delta_{CP}} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3-44}$$

4 The $U(1)_X$ Extension

4.1 General remarks

Although the main motivations lies in string theory, extra dimension grand unification among others, most of the theories predict a new gauge boson called Z'. In the most promising scenarios this particle exist from a $U(1)$ additional symmetry breaking [38], as part of a non-abelian higher symmetry ($SU(2)_R$, 331 models) [51], from Kaluza-Klein excitations in extra dimensions [8] or as a string resonance [26]. In the first two scenarios, the need of a higher symmetry group that breaks into an additional $U(1)$ gauge symmetry provides also a solution to the fermion mass hierarchy i.e. a natural explanation of the fermion masses. A new non-universal $U(1)_X$ interaction together with a $\mathcal{Z}_2$ parity are included into the MSSM based on the non-supersymmetric version of the model [44] which provides an scenario for solving the FMH problem based on the existence of two Higgs doublets and two scalar singlets, leaving the hiearchy understood partially from the VEV hierarchy. However, in the fermionic sector, a exotic up-like quark ($\mathcal{T}$), two down-like quarks ($\mathcal{J}^a$, $a = 1, 2$), two exotic leptons (E, $\mathcal{E}$), three right handed neutrinos (ν_L^C) and three heavy Majorana neutrinos (N_R) all of them interacting via the scalar singlets and generating a mass matrix texture compatible with a natural realization of FMH without inducing anomalies.

When a new symmetry is included in a theory, there is always the risk of inducing anomalies as shown in section 2.2.5. The inclusion of a new $U(1)$ symmetry leads to the following equations that are required to vanish.

Table **4-1**: Scalar content of the model, non-universal X quantum number, $\mathbb{Z}_2$ parity and hypercharge

Higgs Scalar Doublets	$X^{\pm}$	Y	Higgs Scalar Singlets	$X^{\pm}$	Y
$\hat{\Phi}_1 = \begin{pmatrix} \hat{\phi}_1^+ \\ \frac{\hat{h}_1+v_1+i\hat{\eta}_1}{\sqrt{2}} \end{pmatrix}$	$+{}^2/_3{}^+$	$+1$	$\hat{\chi} = \frac{\hat{\xi}_\chi+v_\chi+i\hat{\zeta}_\chi}{\sqrt{2}}$	$-{}^1/_3{}^+$	0
$\hat{\Phi}_2 = \begin{pmatrix} \hat{\phi}_2^+ \\ \frac{\hat{h}_2+v_2+i\hat{\eta}_2}{\sqrt{2}} \end{pmatrix}$	$+{}^1/_3{}^-$	$+1$	$\sigma = \frac{\hat{\xi}_\sigma+i\hat{\zeta}_\sigma}{\sqrt{2}}$	$-{}^1/_3{}^-$	0
$\hat{\Phi}'_1 = \begin{pmatrix} \frac{\hat{h}'_1+v'_1+i\hat{\eta}'_1}{\sqrt{2}} \\ \hat{\phi}_1^{-\prime} \end{pmatrix}$	$-{}^2/_3{}^+$	-1	$\hat{\chi}' = \frac{\hat{\xi}'_\chi+v'_\chi+i\hat{\zeta}'_\chi}{\sqrt{2}}$	$+{}^1/_3{}^+$	0
$\hat{\Phi}'_2 = \begin{pmatrix} \frac{\hat{h}'_2+v'_2+i\hat{\eta}'_2}{\sqrt{2}} \\ \hat{\phi}_2^{-\prime} \end{pmatrix}$	$-{}^1/_3{}^-$	-1	$\sigma' = \frac{\hat{\xi}'_\sigma+i\hat{\zeta}'_\sigma}{\sqrt{2}}$	$+{}^1/_3{}^-$	0

$$[\mathrm{SU}(3)_C]^2\,\mathrm{U}(1)_X \rightarrow \quad A_C = \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \tag{4-1}$$

$$[\mathrm{SU}(2)_L]^2\,\mathrm{U}(1)_X \rightarrow \quad A_L = \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L} \tag{4-2}$$

$$[\mathrm{U}(1)_Y]^2\,\mathrm{U}(1)_X \rightarrow \quad A_{Y^2} = \sum_{\ell,Q}\left[Y_{\ell_L}^2 X_{\ell_L} + 3Y_{Q_L}^2 X_{Q_L}\right] - \sum_{\ell,Q}\left[Y_{\ell_R}^2 X_{L_R} + 3Y_{Q_R}^2 X_{Q_R}\right] \tag{4-3}$$

$$\mathrm{U}(1)_Y\,[\mathrm{U}(1)_X]^2 \rightarrow \quad A_Y = \sum_{\ell,Q}\left[Y_{\ell_L} X_{\ell_L}^2 + 3Y_{Q_L} X_{Q_L}^2\right] - \sum_{\ell,Q}\left[Y_{\ell_R} X_{\ell_R}^2 + 3Y_{Q_R} X_{Q_R}^2\right] \tag{4-4}$$

$$[\mathrm{U}(1)_X]^3 \rightarrow \quad A_X = \sum_{\ell,Q}\left[X_{\ell_L}^3 + 3X_{Q_L}^3\right] - \sum_{\ell,Q}\left[X_{\ell_R}^3 + 3X_{Q_R}^3\right] \tag{4-5}$$

$$[\mathrm{Grav}]^2\,\mathrm{U}(1)_X \rightarrow \quad A_{\mathrm{G}} = \sum_{\ell,Q}\left[X_{\ell_L} + 3X_{Q_L}\right] - \sum_{\ell,Q}\left[X_{\ell_R} + 3X_{Q_R}\right]. \tag{4-6}$$

Although in the non-susersymmetric model these equations are satisfied when supersymmetry is imposed they are not because of the precense of Higgsinos in the fermion content. The simplest way of avoiding this problem is by doubling the scalar content, so the new scalar fields would behave as the conjugate ones. The final particle content of the model is shown in tables **4-2** and **4-1**

The scalar singlets σ and σ' do not acquire VEV. Therefore, they contribute to the generation of the lightest fermions masses at one loop level. However, the scalar singlets χ, χ' acquire a VEV at the TeV scale which breaks the $U(1)_X$ symmetry leading to the following spontaneous

Table **4-2**: Fermion content of the abelian extension, non-universal X quantum number and parity $\mathbb{Z}_2$.

Left-Handed Fermions	$X^{\pm}$	Right-Handed Fermions	$X^{\pm}$
SM Quarks			
$\hat{q}^1_L = \begin{pmatrix} \hat{u}^1 \\ \hat{d}^1 \end{pmatrix}_L$	$+1/3^{+}$	$\hat{u}^{1\,c}_L$	$-2/3^{+}$
		$\hat{u}^{2\,c}_L$	$-2/3^{-}$
$\hat{q}^2_L = \begin{pmatrix} \hat{u}^2 \\ \hat{d}^2 \end{pmatrix}_L$	0^{-}	$\hat{u}^{3\,c}_L$	$-2/3^{+}$
		$\hat{d}^{1\,c}_L$	$+1/3^{-}$
$\hat{q}^3_L = \begin{pmatrix} \hat{u}^3 \\ \hat{d}^3 \end{pmatrix}_L$	0^{+}	$\hat{d}^{2\,c}_L$	$+1/3^{-}$
		$\hat{d}^{3\,c}_L$	$+1/3^{-}$
SM Leptons			
$\hat{\ell}^e_L = \begin{pmatrix} \hat{\nu}^e \\ \hat{e}^e \end{pmatrix}_L$	0^{+}	$\hat{\nu}^{e\,c}_L$	$-1/3^{-}$
		$\hat{\nu}^{\mu\,c}_L$	$-1/3^{-}$
$\hat{\ell}^\mu_L = \begin{pmatrix} \hat{\nu}^\mu \\ \hat{\mu}^\mu \end{pmatrix}_L$	0^{+}	$\hat{\nu}^{\tau\,c}_L$	$-1/3^{-}$
		$\hat{e}^{e\,c}_L$	$+4/3^{-}$
$\hat{\ell}^\tau_L = \begin{pmatrix} \hat{\nu}^\tau \\ \hat{\tau}^\tau \end{pmatrix}_L$	-1^{+}	$\hat{e}^{\mu\,c}_L$	$+1/3^{-}$
		$\hat{e}^{\tau\,c}_L$	$+4/3^{-}$
Non-SM Quarks			
$\hat{\mathcal{T}}_L$	$+1/3^{-}$	$\hat{\mathcal{T}}^c_L$	$-2/3^{-}$
$\mathcal{J}^1_L$	0^{+}	$\hat{\mathcal{J}}^{c\ 1}_L$	$+1/3^{+}$
$\mathcal{J}^2_L$	0^{+}	$\hat{\mathcal{J}}^{c\ 2}_L$	$+1/3^{+}$
Non-SM Leptons			
$\hat{E}_L$	-1^{+}	$\hat{E}^c_L$	$+2/3^{+}$
$\hat{\mathcal{E}}_L$	$-2/3^{+}$	$\hat{\mathcal{E}}^c_L$	$+1^{+}$
Majorana Fermions		$\mathcal{N}^{1,2,3}_R$	0^{-}

symmetry breaking chain:

$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_X \xrightarrow{\chi} \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \xrightarrow{\Phi} \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q.$$

In the context of the supersymmetric theory, the gauge groups induce a $D-Term$ potential shown in Eq. (4-7) and the superpotential given in Eq. (4-8) divided into scalar (W_ϕ), quark (W_Q) and lepton (W_L) parts shown in Eqs. (4-9-4-11)

$$\begin{aligned} V_D = &\frac{g^2}{2}\Big[|\Phi_1^\dagger \Phi_2|^2 + |\Phi_1'^\dagger \Phi_2'|^2 + |\Phi_1'^\dagger \Phi_1|^2 + |\Phi_1'^\dagger \Phi_2|^2 + |\Phi_2'^\dagger \Phi_1|^2 + |\Phi_2'^\dagger \Phi_2|^2 \\ &- |\Phi_1|^2|\Phi_2|^2 - |\Phi_1'|^2|\Phi_2'|^2\Big] + \frac{g^2+g'^2}{8}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 - \Phi_1'^\dagger\Phi_1' - \Phi_2'^\dagger\Phi_2')^2 \\ &+ \frac{g_X^2}{2}\left[\frac{2}{3}(\Phi_1^\dagger\Phi_1 - \Phi_1'^\dagger\Phi_1') + \frac{1}{3}(\Phi_2^\dagger\Phi_2 - \Phi_2'^\dagger\Phi_2') - \frac{1}{3}(\chi^*\chi - \chi'^*\chi') - \frac{1}{3}(\sigma^*\sigma - \sigma'^*\sigma')\right]^2 \end{aligned} \tag{4-7}$$

$$W[\Phi] = W_\phi + W_Q + W_L \tag{4-8}$$

It is worth to mention that only the terms involving interactions among Higgs particles has been considered in the V_D potential since we are not interested in the sparticles mass generation. The full D-Term potential also include interactions among squarks, sleptons and Higgs particles which are not gauge singlets, leaving the Majorana sneutrinos as non interacting.

$$W_\phi = -\mu_1 \hat{\Phi}_1'\hat{\Phi}_1 - \mu_2\hat{\Phi}_2'\hat{\Phi}_2 - \mu_\chi \hat{\chi}'\hat{\chi} - \mu_\sigma \hat{\sigma}'\hat{\sigma} + \lambda_1 \hat{\Phi}_1'\hat{\Phi}_2\hat{\sigma}' + \lambda_2\hat{\Phi}_2'\hat{\Phi}_1\sigma \tag{4-9}$$

$$\begin{aligned} W_Q = &\hat{q}_L^1\hat{\Phi}_2 h_{2u}^{12}\hat{u}_L^{2\,c} + \hat{q}_L^2\hat{\Phi}_1 h_{1u}^{22}\hat{u}_L^{2\,c} + \hat{q}_L^3\hat{\Phi}_1 h_{1u}^{3k}\hat{u}_L^{k\,c} - \hat{q}_L^3\hat{\Phi}'_2 h_{2d}^{3j}\hat{d}_L^{j\,c} + \hat{q}_L^1\hat{\Phi}_2 h_{2T}^1\hat{\mathcal{T}}_L^c \\ &+ \hat{q}_L^2\hat{\Phi}_1 h_{1T}^2\hat{\mathcal{T}}_L^c - \hat{q}_L^1\hat{\Phi}'_1 h_{1J}^{1a}\hat{\mathcal{J}}_L^{a\,c} - \hat{q}_L^2\hat{\Phi}'_2 h_{2J}^{2a}\hat{\mathcal{J}}_L^{a\,c} + \hat{\mathcal{T}}_L\hat{\chi}' h_{\chi'}^T\hat{\mathcal{T}}_L^c - \hat{\mathcal{J}}_L^a\hat{\chi} h_\chi^{Jab}\hat{\mathcal{J}}_L^{b\,c} \\ &+ \hat{\mathcal{T}}_L\hat{\chi}' h_{\chi' u}^2\hat{u}_L^{2\,c} + \hat{\mathcal{J}}_L^a\hat{\sigma} h_\sigma^{Jaj}\hat{d}_L^{jc} + \hat{\mathcal{T}}_L\hat{\sigma}' h_{\sigma'}^{Tk}\hat{u}_L^{kc} \end{aligned} \tag{4-10}$$

$$\begin{aligned} W_L = &\hat{\ell}_L^p\hat{\Phi}_2 h_{2\nu}^{pq}\hat{\nu}_L^{q\,c} - \hat{\ell}_L^p\hat{\Phi}_2' h_{2e}^{p\mu}\hat{e}_L^{\mu\,c} - \hat{\ell}_L^\tau\hat{\Phi}_2' h_{2e}^{\tau r}\hat{e}_L^{r\,c} - \hat{\ell}_L^p\hat{\Phi}_1' h_{1E}^p\hat{E}_L^c + \hat{E}_L\hat{\chi}' g_{\chi' E}\hat{E}_L^c \\ &- \hat{E}_L\mu_E\hat{\mathcal{E}}_L^c + \hat{\mathcal{E}}_L\hat{\chi} g_{\chi\mathcal{E}}\hat{\mathcal{E}}_L^c - \hat{\mathcal{E}}_L\mu_{\mathcal{E}}\hat{E}_L^c + \hat{\nu}_L^{m\,c}\hat{\chi}' h_\chi'^{N\,mn}\hat{N}_L^{n\,c} + \frac{1}{2}\hat{N}_L^{m\,c} M_{mn}\hat{N}_L^{n\,c} \\ &+ \hat{E}_L\hat{\sigma} h_\sigma^{e^c p}\hat{e}_L^{cr} + \hat{\mathcal{E}}_L\hat{\sigma}' h_{\sigma'}^{e^c\mu}\hat{e}_L^{\mu c}, \end{aligned} \tag{4-11}$$

where $j = 1, 2, 3$ labels the down type singlet quarks, $k = 1, 3$ labels the first and third quark doublets, $a = 1, 2$ is the index of the exotic $\mathcal{J}_L^a$ and $\mathcal{J}_L^{ca}$ quarks, $p = e, \mu$, $q = e, \mu, \tau$, $r = e, \tau$ and m, n label the right handed and Majorana neutrinos. Finally, a soft breaking potential in included, since we are not interested in sparticle masses let's consider soft breaking terms for the scalar particles, gauginos and Higgsinos as shown in Eq. (4-12).

$$
\begin{aligned}
V_{soft} &= m_1^2\Phi_1^\dagger\Phi_1 + m_1'^2\Phi_1'^\dagger\Phi_1' + m_2^2\Phi_2^\dagger\Phi_2 + m_2'^2\Phi_2'^\dagger\Phi_2' + m_\chi^2\chi^\dagger\chi + m_\chi'^2\chi'^\dagger\chi' + m_\sigma^2\sigma^\dagger\sigma \\
&+ m_\sigma'^2\sigma'^\dagger\sigma' - \Big[\mu_{11}^2\epsilon_{ij}(\Phi_1'^i\Phi_1^j) - \mu_{22}^2\epsilon_{ij}(\Phi_2'^i\Phi_2^j) - \mu_{\chi\chi}^2(\chi\chi') + \mu_{\sigma\sigma}^2(\sigma\sigma') + \tilde{\lambda}_1\Phi_1'^\dagger\Phi_2\sigma' \\
&+ \tilde{\lambda}_2\Phi_2'^\dagger\Phi_1\sigma - \frac{2\sqrt{2}}{9}(k_1\Phi_1^\dagger\Phi_2\chi' - k_2\Phi_1^\dagger\Phi_2\chi^* + k_3\Phi_1'^\dagger\Phi_2'\chi - k_4\Phi_1'^\dagger\Phi_2'\chi'^*) + h.c.\Big] \\
&+ M_{\tilde{B}}\tilde{B}\tilde{B}^\dagger + M_{\tilde{B}'}\tilde{B}'\tilde{B}'^\dagger + M_{\tilde{W}^\pm}\tilde{W}^\pm\tilde{B}^{\pm\dagger} + M_{\tilde{W}}\tilde{W}_3\tilde{W}_3^\dagger
\end{aligned}
\tag{4-12}
$$

where the last terms, proportional to the coupling constants named k_1, k_2, k_3 and k_4, also breaks softly parity symmetry. This feature is required to avoid massless scalar particles as will be shown later. Althoug $F - term$ potential codifies mainly all sparticles interactions and off diagonal sparticle mass terms we can take only the associated with Higgs particles:

$$
\begin{aligned}
V_F &= \mu_1^2(\Phi_1^\dagger\Phi_1 + \Phi_1'^\dagger\Phi_1') + \mu_2^2(\Phi_2^\dagger\Phi_2 + \Phi_2'^\dagger\Phi_2') + \mu_\chi^2(\chi^*\chi + \chi'^*\chi') + +\mu_\sigma^2(\sigma^*\sigma + \sigma'^*\sigma') \\
&+ \Big(\lambda_1^2|\epsilon_{ij}\Phi_1'^i\Phi_2^j|^2 + \lambda_2^2|\epsilon_{ij}\Phi_2'^i\Phi_1^j|^2 + \lambda_1^2(\Phi_2^\dagger\Phi_2 + \Phi_1'^\dagger\Phi_1'\sigma'^*\sigma' + \lambda_2^2(\Phi_1^\dagger\Phi_1 + \Phi_2'^\dagger\Phi_2')\sigma^*\sigma \\
&- \lambda_1\mu_1\Phi_1^\dagger\Phi_2\sigma' - \lambda_1\mu_2\Phi_2'^\dagger\Phi_1'\sigma' - \lambda_2\mu_1\Phi_1'^\dagger\Phi_2'\sigma - \lambda_2\mu_2\Phi_2^\dagger\Phi_1\sigma - \lambda_1\mu_\sigma\epsilon_{ij}\Phi_1'^i\Phi_2^j \\
&- \lambda_2\mu_\sigma\epsilon_{ij}\Phi_2'^i\Phi_1^j + h.c.\Big)
\end{aligned}
\tag{4-13}
$$

4.2 Scalar and gauge boson masses

Prior to develop the fermion sector of the model, it is required to check the model consistency with the most relevant results such as the correct gauge boson masses and the compatibility of the lightest scalar with the observed $125.3GeV$ one [3]. First, we get the gauge boson mass matrix by considering the scalar particles kinetic terms as stated in section 3.3. Now, there is another contribution due to the $U(1)_X$ symmetry so the covariant derivative reads:

$$
D_\mu = \partial_\mu - igW_\mu^a T_a - ig'\frac{Y}{2}B_\mu - ig_X X B_\mu'. \tag{4-14}
$$

so the kinetic terms for the fields Φ_1, Φ_2, Φ_1', Φ_2', χ and χ' after SSB generates the following mass matrix for the neutral bosons in the basis (W_μ^3, B_μ, B_μ'):

$$M_0^2 = \frac{1}{4}\begin{pmatrix} g^2v^2 & -gg'v^2 & -\frac{2}{3}gg_Xv^2(1+\cos^2\beta) \\ * & g'^2v^2 & \frac{2}{3}g'g_Xv^2(1+\cos^2\beta) \\ * & * & \frac{4}{9}g_X^2\left[V_\chi^2+(1+3\cos^2\beta)v^2\right] \end{pmatrix}, \tag{4-15}$$

where we have done the following definitions

$$v^2 = v_1^2 + v_2^2 + v_1'^2 + v_2'^2 \tag{4-16}$$

$$\tan\beta = \frac{\sqrt{v_2^2+v_2'^2}}{\sqrt{v_1^2+v_1'^2}} \equiv \frac{V_2}{V_1} \tag{4-17}$$

$$V_\chi^2 \equiv v_\chi^2 + v_\chi'^2 \tag{4-18}$$

Moreover, the charged gauge bosons $W^\pm$ acquire a SM-like mass term given by $m_\omega = \frac{gv}{2}$ with v defined in Eq. (4-16). Since we already know that these bosons have a $80.4GeV/c^2$ mass we have a constraint for the doublet VEV's given by:

$$v^2 = v_1^2 + v_2^2 + v_1'^2 + v_2'^2 = (246.3GeV)^2 \tag{4-19}$$

However, the neutral gauge bosons mass matrix in Eq. (4-15) has null determinant, which implies that one particle is massless. It is identified as the photon. The other two mass eigenstates corresponds to the Z and Z' bosons whose masses are given in a first approximation by:

$$M_Z \approx \frac{gv}{2\cos\theta_W}, \tag{4-20}$$

$$M_{Z'} \approx \frac{g_X V_\chi}{3}, \tag{4-21}$$

beign θ_w the Weinberg angle defined as $\tan\theta_w = \frac{g'}{g}$. We can see that the model reproduces the correct masses for the gauge bosons and ensures a yet unobservable Z' boson since it comes from the breaking of the $U(1)_X$ symmetry at the TeV scale and provides a constraint for the electroweak VEV's.

4.2.1 Cp-even scalar particles

Now, let's consider the scalar potential that arise from Eqs. (4-7)-(4-13), it is given by:

$$
\begin{aligned}
V_s =& \frac{g^2}{2}\Big[|\Phi_1^\dagger\Phi_2|^2 + |\Phi_1'^\dagger\Phi_2'|^2 + |\Phi_1'^\dagger\Phi_1|^2 + |\Phi_1'^\dagger\Phi_2|^2 + |\Phi_2'^\dagger\Phi_1|^2 + |\Phi_2'^\dagger\Phi_2|^2 \\
&- |\Phi_1|^2|\Phi_2|^2 - |\Phi_1'|^2|\Phi_2'|^2\Big] + \frac{g^2+g'^2}{8}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 - \Phi_1'^\dagger\Phi_1' - \Phi_2'^\dagger\Phi_2')^2 \\
&+ \frac{g_X^2}{2}\left[\frac{2}{3}(\Phi_1^\dagger\Phi_1 - \Phi_1'^\dagger\Phi_1') + \frac{1}{3}(\Phi_2^\dagger\Phi_2 - \Phi_2'^\dagger\Phi_2') - \frac{1}{3}(\chi^*\chi - \chi'^*\chi') - \frac{1}{3}(\sigma^*\sigma - \sigma'^*\sigma')\right]^2 \\
&+ \mu_1^2(\Phi_1^\dagger\Phi_1 + \Phi_1'^\dagger\Phi_1') + \mu_2^2(\Phi_2^\dagger\Phi_2 + \Phi_2'^\dagger\Phi_2') + \mu_\chi^2(\chi^*\chi + \chi'^*\chi') + +\mu_\sigma^2(\sigma^*\sigma + \sigma'^*\sigma') \\
&+ \Big(\lambda_1^2|\epsilon_{ij}\Phi_1'^i\Phi_2^j|^2 + \lambda_2^2|\epsilon_{ij}\Phi_2'^i\Phi_1^j|^2 + \lambda_1^2(\Phi_2^\dagger\Phi_2 + \Phi_1'^\dagger\Phi_1')\sigma'^*\sigma' + \lambda_2^2(\Phi_1^\dagger\Phi_1 + \Phi_2'^\dagger\Phi_2')\sigma^*\sigma \\
&- \lambda_1\mu_1\Phi_1^\dagger\Phi_2\sigma' - \lambda_1\mu_2\Phi_2'^\dagger\Phi_1'\sigma' - \lambda_2\mu_1\Phi_1'^\dagger\Phi_2'\sigma - \lambda_2\mu_2\Phi_2^\dagger\Phi_1\sigma - \lambda_1\mu_\sigma\epsilon_{ij}\Phi_1'^i\Phi_2^j \\
&- \lambda_2\mu_\sigma\epsilon_{ij}\Phi_2'^i\Phi_1^j + h.c.\Big) + m_1^2\Phi_1^\dagger\Phi_1 + m_1'^2\Phi_1'^\dagger\Phi_1' + m_2^2\Phi_2^\dagger\Phi_2 + m_2'^2\Phi_2'^\dagger\Phi_2' + m_\chi^2\chi^\dagger\chi + m_\chi'^2\chi'^\dagger\chi' \\
&+ m_\sigma^2\sigma^\dagger\sigma + m_\sigma'^2\sigma'^\dagger\sigma' - \Big[\mu_{11}^2\epsilon_{ij}(\Phi_1'^i\Phi_1^j) - \mu_{22}^2\epsilon_{ij}(\Phi_2'^i\Phi_2^j) + \mu_{\chi\chi}^2(\chi\chi') + \mu_{\sigma\sigma}^2(\sigma\sigma') + \tilde{\lambda}_1\Phi_1'^\dagger\Phi_2\sigma' \\
&+ \tilde{\lambda}_2\Phi_2'^\dagger\Phi_1\sigma - \frac{2\sqrt{2}}{9}(k_1\Phi_1^\dagger\Phi_2\chi' - k_2\Phi_1^\dagger\Phi_2\chi^* + k_3\Phi_1'^\dagger\Phi_2'\chi - k_4\Phi_1'^\dagger\Phi_2'\chi'^*) + h.c.\Big]
\end{aligned} \tag{4-22}
$$

the coefficients have to fullfill the following conditions in order to have a non trivial minimum of the potential:

$$g^2 + 2\lambda_1^2 + 2\lambda_2^2 > 0 \tag{4-23}$$

$$9b^2 > 32ac \tag{4-24}$$

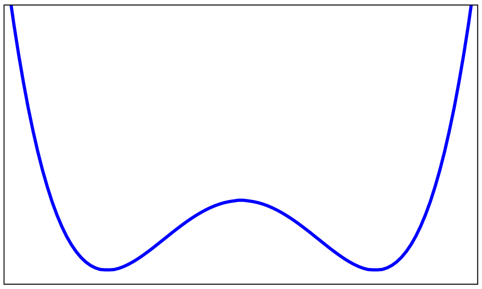
where

$$a = g^2 + 2\lambda_1^2 + 2\lambda_2^2 \tag{4-25}$$

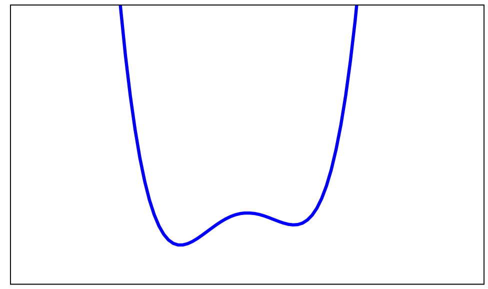
$$b = -2(\lambda_1(\mu_1+\mu_2) + \lambda_2(\mu_1+\mu_2) - \tilde{\lambda}_1 - \tilde{\lambda}_2 + \frac{2\sqrt{2}}{9}(k_1 - k_2 + k_3 - k_4)) \tag{4-26}$$

$$
\begin{aligned}
c =& 2\mu_1^2 + 2\mu_2^2 + 2\mu_\chi^2 + 2\mu_\sigma^2 + m_1^2 + m_2^2 + m_1'^2 + m_2'^2 + m_\chi^2 + m_\chi'^2 + m_\sigma^2 + m_\sigma'^2 \\
&- 2\mu_{11}^2 - 2\mu_{22}^2 - 2\mu_{\chi\chi}^2 - 2\mu_{\sigma\sigma}^2 - 2\lambda_1\mu_\sigma - 2\lambda_2\mu_\sigma
\end{aligned} \tag{4-27}
$$

the first condition ensures that the potential is bounded from bellow in the equal field direction, i.e. when all scalar fields are equal, while the second guaranties the existence of a non trivial minimum as shown in figure (**4-1**). It is common to find the scalar potential with trilinear interactions in models that involve scalar singlets as noted in [45]. However, in most cases those couplings are highly suppressed due to the restrictions on the mass matrix and because of the potential. The cubic terms, represented by the constant b , generates asymetries on the potential as shown in figure (**4-1**b). Nevertheless, since these couplings are small ($b \approx 0$) the asymmetry is supressed so the potential has a mexican hat shape



(a) Symmetric quartic potential representation .



(b) Effect of cubic terms in a quartic potential.

Figure **4-1**: Quartic potential graph, without cubic terms (a) and with asymmetries due to cubic terms (b)

approximately as shown in figure (**4-1**a). As will be shown later, the k_i couplings are needed in order to ensure unobserved heavy scalar states, being the only not suppressed trilinear couplings. We might still think that their values are close enough to be supressed in the differences since it is an interaction in a much higher energy scale. Due to the small character of b, the second condition is true if the constant c becomes negative, implying:

$$m_{H1}^2 + m_{H1}'^2 + m_{H2}^2 + m_{H2}'^2 + m_{H\chi}^2 + m_{H\chi}'^2 + m_{H\sigma}^2 + m_{H\sigma}'^2 < 2(\mu_{11}^2 + \mu_{22}^2 + \mu_{\chi\chi}^2 + \mu_{\sigma\sigma}^2)$$

where we have defined $m_{H\alpha}^{(\prime)2} = m_{\alpha}^{(\prime)2} + \mu_\alpha^2$ with $\alpha = 1, 2, \chi, \sigma$. Despite the latter restriction, the mass spectrum turns out to be independent of $m_{H\alpha}^{(\prime)2}$ thanks to the minima conditions that come off the potential:

$$\begin{aligned}
m_{H1}^2 + \frac{1}{8}(g^2 + g'^2)C_{EW} + \frac{g_X^2}{9}c_X - \mu_{11}\frac{v_1'}{v_1} + \frac{\lambda_2^2}{2}v_2'^2 &= 0 \\
m_{H1}'^2 - \frac{1}{8}(g^2 + g'^2)C_{EW} - \frac{g_X^2}{9}c_X - \mu_{11}\frac{v_1}{v_1'} + \frac{\lambda_1^2}{2}v_1'^2 &= 0 \\
m_{H2}^2 + \frac{1}{8}(g^2 + g'^2)C_{EW} + \frac{g_X^2}{18}c_X - \mu_{22}\frac{v_2'}{v_2} + \frac{\lambda_1^2}{2}v_2^2 &= 0 \\
m_{H2}'^2 - \frac{1}{8}(g^2 + g'^2)C_{EW} - \frac{g_X^2}{18}c_X - \mu_{22}\frac{v_2}{v_2'} + \frac{\lambda_2^2}{2}v_1^2 &= 0 \\
M_\chi^2 - \frac{g_X^2}{18}c_X - \mu_{\chi\chi}\frac{v_\chi'}{v_\chi} &= 0 \\
M_\chi'^2 + \frac{g_X^2}{18}c_X - \mu_{\chi\chi}\frac{v_\chi}{v_\chi'} &= 0,
\end{aligned} \tag{4-28}$$

with the definitions $C_{EW} = v_1^2 + v_2^2 - v_1'^2 - v_2'^2$ and $C_X = 2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 + v_\chi'^2 - v_\chi^2$. The latter conditions are obtained by taking the derivative of the potential with respect to each scalar field and then by replacing the fields by their respective VEV. In this way, the minima condition of the singlets σ and σ' do not provide any simplification but gives the following restrictions over the coupling constants:

$$v_1(v_2'\tilde{\lambda}_2 - v_2\lambda_2\mu_2) = v_1'(v_2'\lambda_2\mu_1 + v_2\lambda_1\mu_\sigma) \tag{4-29}$$

$$v_2(v_1'\tilde{\lambda}_1 - v_1\lambda_1\mu_1) = v_2'(v_1'\lambda_1\mu_2 + v_1\lambda_2\mu_\sigma) \tag{4-30}$$

By implementing a spontaneous symmetry breaking on the $U(1)_X$ symmetry and the electroweak symmetry by rewriting the scalar fields as perturbations from the VEV as shown in table **4-1**, mass terms arise. They can be written in the following matrix for the CP-even scalar particles in the basis $(h_1, h_1', h_2, h_2', \xi_\chi, \xi_\chi', \xi_\sigma, \xi_\sigma')$

$$\frac{1}{2}M_h^2 = \begin{pmatrix} M_{hh} & M_{h\xi} \\ M_{h\xi}^T & M_{\xi\xi} \end{pmatrix}. \tag{4-31}$$

M_{hh} is a 4×4 matrix containing the mixing of the h fields, related with the scalar doublets of the model. It can be written as:

$$M_{hh} = \begin{pmatrix} f_{4g}v_1^2 - \frac{v_2 f_{1k}}{9v_1} + \frac{v_1'\mu_{11}^2}{2v_1} & -f_{4g}v_1v_1' - \frac{\mu_{11}^2}{2} & f_{2g}v_1v_2 + \frac{f_{1k}}{9} & -f_{2g}v_1v_2' + \frac{1}{2}\lambda_2^2 v_1 v_2' \\ * & f_{4g}{v_1'}^2 - \frac{v_2' f_{2k}}{9v_1'} + \frac{v_1\mu_{11}^2}{2v_1'} & -f_{2g}v_1'v_2 + \frac{1}{2}\lambda_1^2 v_2 v_1' & f_{2g}v_1'v_2' + \frac{f_{2k}}{9} \\ * & * & f_{1g}v_2^2 - \frac{v_1 f_{1k}}{9v_2} + \frac{v_2'\mu_{22}^2}{2v_2} & -f_{1g}v_2v_2' - \frac{\mu_{22}^2}{2} \\ * & * & * & f_{1g}{v_2'}^2 - \frac{v_1' f_{2k}}{9v_2'} + \frac{v_2\mu_{22}^2}{2v_2'} \end{pmatrix} \tag{4-32}$$

It can be seen that the latter mixing matrix does not depend on the $m_{H\alpha}^{(\prime)}$ masses, they do not appear explicitly due to the minimum conditions stated above. As a consequence, the mixing is determined mainly by the μ_{ii} couplings, coming from the soft breaking potential rather than the superpotential parameters. However, the mixings between scalar doublets and singlets are written in the 4×4 $M_{h\xi}$ matrix and it is given by:

$$M_{h\xi} = \begin{pmatrix} \frac{1}{9}(k_2v_2 - g_X^2 v_1 v_\chi) & \frac{1}{9}(-k_1v_2 + g_X^2 v_1 v_\chi') & \frac{1}{2\sqrt{2}}(\tilde{\lambda}_2 v_2' - \lambda_2\mu_2 v_2) & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_1 v_2 + \lambda_2\mu_\sigma v_2') \\ \frac{1}{9}(-k_3v_2' + g_X^2 v_1' v_\chi) & \frac{1}{9}(k_4v_2' - g_X^2 v_1' v_\chi') & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_1 v_2' + \lambda_1\mu_\sigma v_2) & \frac{1}{2\sqrt{2}}(\tilde{\lambda}_1 v_2 - \lambda_1\mu_2 v_2') \\ \frac{1}{9}(k_2v_1 - \frac{1}{2}g_X^2 v_2 v_\chi) & \frac{1}{9}(-k_1v_1 + \frac{1}{2}g_X^2 v_2 v_\chi') & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_2 v_1 + \lambda_1\mu_\sigma v_1') & \frac{1}{2\sqrt{2}}(\tilde{\lambda}_1 v_1' - \lambda_1\mu_1 v_1) \\ \frac{1}{9}(-k_3v_1' + \frac{1}{2}g_X^2 v_2' v_\chi) & \frac{1}{9}(k_4v_1' - \frac{1}{2}g_X^2 v_2' v_\chi') & \frac{1}{2\sqrt{2}}(\tilde{\lambda}_2 v_1 - \lambda_2\mu_1 v_1') & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_2 v_1' + \lambda_2\mu_\sigma v_1) \end{pmatrix} \tag{4-33}$$

We can see that the mixing between these two sectors, both expected at a different energy scale, is governed by the trilinear couplings of the soft breaking potential. It implies that at the SUSY scale, scalar singlets and doublets are completely decoupled. Then, the SUSY breaking provides interactions between them and so the possibility of being observed at the right energy. Last but not least, the mixing matrix between Higgs singlets, $M_{\xi\xi}$, reads:

$$M_{\xi\xi} = \begin{pmatrix} \frac{g_X^2}{18}v_\chi^2 + \frac{v'_\chi \mu_{\chi\chi}^2}{2v_\chi} - \frac{k_{23}}{9v_\chi} & -\frac{g_X^2}{18}v_\chi v'_\chi - \frac{\mu_{\chi\chi}^2}{2} & 0 & 0 \\ * & \frac{g_X^2}{18}{v'_\chi}^2 + \frac{v_\chi \mu_{\chi\chi}^2}{2v'_\chi} - \frac{k_{14}}{9v'_\chi} & 0 & 0 \\ * & * & M_\sigma^2 + \frac{\lambda_2^2}{4}(v_1^2 + v_2'^2) & -\frac{\mu_{\sigma\sigma}}{2} \\ * & * & * & M_\sigma'^2 + \frac{\lambda_1^2}{4}(v_2^2 + v_1'^2) \end{pmatrix} \tag{4-34}$$

The following definitions have been done to give shorter expressions:

$$f_{ng} = \frac{g^2 + g'^2}{8} + \frac{n}{18}g_X^2 \qquad f_{1k} = k_2 v_\chi - k_1 v'_\chi \qquad f_{2k} = -k_3 v_\chi + k_4 v'_\chi \tag{4-35}$$

$$k_{23} = k_2 v_1 v_2 - k_3 v'_1 v'_2 \qquad k_{14} = -k_1 v_1 v_2 + k_4 v'_1 v'_2 \tag{4-36}$$

$$M_\sigma = \frac{1}{2}(\mu_\sigma^2 + m_\sigma^2) - \frac{g_X^2}{36}(2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 - v_\chi^2 + v_\chi'^2) \tag{4-37}$$

$$M'_\sigma = \frac{1}{2}(\mu_\sigma^2 + m_\sigma'^2) + \frac{g_X^2}{36}(2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 - v_\chi^2 + v_\chi'^2) \tag{4-38}$$

The high energy decoupling of the doublet and singlet sectors lead us to assume the hierarchy $\mu_{\chi\chi}, \mu_{\sigma\sigma}, M_\sigma, M'_\sigma \gg \mu_{11},\ \mu_{22} \gg k_i v_j \gg g_X^2 v_\chi v_j,\ g_X^2 v'_\chi v_j,\ g_X^2 v_\chi v'_j,\ g_X^2 v'_\chi v'_j,\ \lambda_i^2 v_i v_j^{(\prime)}$, where $i = 1,2,3,4$ and $j = 1,2$. Besides, no singlet has been observed so the $U(1)_X$ is expected at a much higher energy scale, implying that v_χ and v'_χ should be at least at the TeV scale. Thus, they satisfy $v_\chi, v'_\chi \gg v_j, v'_j$, where $j = 1,2$. It implies for the mixing matrices $\mathcal{O}(M_{\xi\xi}) \gg \mathcal{O}(M_{h\xi}) \gg \mathcal{O}(M_{hh})$ which is a favorable scenario to implement a type I seesaw mechanism [37], with a rotation matrix V, leading to a block-diagonal form of the matrix represented by $\tilde{M}_h^2$.

$$\frac{1}{2}\tilde{M}_h^2 = V\frac{1}{2}M_h^2 V^\dagger \approx \begin{pmatrix} \tilde{M}_{hh} & 0 \\ 0 & M_{\xi\xi} \end{pmatrix}, \qquad V = \begin{pmatrix} \mathbb{I} & M_{h\xi}M_{\xi\xi}^{-1} \\ -(M_{h\xi}M_{\xi\xi}^{-1})^T & \mathbb{I} \end{pmatrix}$$

The matrix rank for the M_{hh} submatrix is 4, which means that the four lightest eigenstates are massive and acquire their tree level mass from its mixings. Consequently, the seesaw contribution $M_{h\xi}M_{\xi\xi}^{-1}M_{h\xi}^T$ enters as small corrections to the tree level mass and can be neglected because of the order of magnitude of the involved parameters in each submmatrix. Thus, we can assume $\tilde{M}_{hh} \approx M_{hh}$ and the block diagonal mass matrix takes the form:

$$\frac{1}{2}\tilde{M}_h^2 \approx \begin{pmatrix} \tilde{M}_{hh} & 0 \\ 0 & M_{\xi\xi} \end{pmatrix} \tag{4-39}$$

In fact, all mass eigenstates are certainly massive since the mass matrix has rank 8 before and after the seesaw rotation as well as after the assumption of $\tilde{M}_{hh}$.

It is straightforward to get the scalar singlet masses since its 4×4 submatrix has a block diagonal form. In this model the χ, χ' scalars do not mix with σ and σ'. The resulting mass eigenvalues are:

$$m^2_{h8/7} = (M^2_\sigma + M'^2_\sigma) + \frac{1}{4}[\lambda^2_2(v^2_1 + v'^2_2) + \lambda^2_1(v^2_2 + v'^2_1)] \pm \sqrt{\mu^4_{\sigma\sigma} - \left((M^2_\sigma + M'^2_\sigma) + \frac{1}{4}[\lambda^2_2(v^2_1 + v'^2_2) - \lambda^2_1(v^2_2 + v'^2_1)]\right)^2}, \tag{4-40}$$

$$= (M^2_\sigma + M'^2_\sigma) \pm \sqrt{\mu^4_{\sigma\sigma} - (M^2_\sigma + M'^2_\sigma)^2}, \tag{4-41}$$

$$m^2_{h6} \approx \mu^2_{\chi\chi} \frac{v^2_\chi + v'^{\,2}_\chi}{v_\chi v'_\chi},$$

$$m^2_{h5} \approx \frac{g^2_X}{9}(v^2_\chi + v'^{\,2}_\chi) - \frac{2}{9}\frac{v_1 v_2(k_2 v'_\chi - k_1 v_\chi) + v'_1 v'_2(k_4 v_\chi - k_3 v'_\chi)}{v_\chi v'_\chi}, \tag{4-42}$$

m_{h5} and m_{h6} comes from the $\chi - \chi'$ submatrix and in general they would have an expression similar to the $m_{h8/7}$ masses. However, the latter expression are obtained due to our hierarchy choice among our parameters ($\mu_{\chi\chi}$ is the biggest parameter in the equation) so only the biggest eigenvalue must depend on it. The lightest one (m_{h5}) must depend mainly on the VEV's and the trilinear parameters which are expected to be much smaller than $\mu_{\chi\chi}$ together with the condition that they must reconstruct the trace. On the contrary, no assumption is made on the parameters involving the σ and σ' masses so the general formula is stated just by neglecting the electroweak contributions. In fact, due to the null VEV of these singlets there are no minimum condition relating M_σ, M'_σ and $\mu_{\sigma\sigma}$ so there is more freedom for a hierarchy choice among them. Anyway, they are expected to be at an unreachable energy scale for current experiments and do not represent the main focus of this work. Although they allow mass generation for some fermions as it will be shown later.

for obtaining the remaining four eigenvalues coming from the M_{hh} submatrix we consider that the heavy eigenstates must be function of the soft-SUSY breaking parameters μ_{11}, μ_{22} and k_i while the lightest eigenvalue must depend only on the VEV's since it must be identified with the SM Higgs particle. On the one hand, the heavy eigenstates are obtained by taking a small VEV approximation with the limit $v_1, v_2, v'_1, v'_2 \to 0$ on aditive terms. It causes the matrix rank to decrease to 3, verifying the hypothesis of a electroweak dependent lightest eigenvalue. From this approximation the two heavy states arise from the reduced matrix:

$$M_{hh}(v_i, v'_i \to 0) = \begin{pmatrix} \frac{\mu_{11}^2}{2}\frac{v'_1}{v_1} & -\frac{\mu_{11}^2}{2} & 0 & 0 \\ * & \frac{\mu_{11}^2}{2}\frac{v_1}{v'_1} & 0 & 0 \\ * & * & \frac{\mu_{22}^2}{2}\frac{v'_2}{v_2} & -\frac{\mu_{22}^2}{2} \\ * & * & * & \frac{\mu_{22}^2}{2}\frac{v_2}{v'_2} \end{pmatrix} \tag{4-43}$$

giving as a result the tree level eigenvalues:

$$m_{h3}^2 \approx \mu_{11}^2 \frac{v_1^2 + v_1'^2}{v_1 v'_1}, \qquad m_{h4}^2 \approx \mu_{22}^2 \frac{v_2^2 + v_2'^2}{v_2 v'_2}. \tag{4-44}$$

The next eigenvalue comes from approximating the exact solution of the matrix quartic order characteristic function, given by Ferrari's method [48] in order to get a leading term for its mass. If the characteristic polynomial has the form $Ax^4 + Bx^3 + Cx + D$ it can be proven that the leading contribution arising from the formula for the second eigenvalue is given by $x_2 \approx -\frac{C}{B}$ thanks to the chosen hierarchy among parameters. Considering only the terms proportional to $\mu_{11}^2\mu_{22}^2$ the eigenvalue becomes fully dependent on the parity violating terms, it reads:

$$m_{h2}^2 \approx \frac{2v^2(v_1 v_2(k_1 v'_\chi - k_2 v_\chi) + v'_1 v'_2(k_3 v_\chi - k_4 v'_\chi))}{9(v_1^2 + v_1'^2)(v_2^2 + v_2'^2)}. \tag{4-45}$$

It is worth to notice that the two heaviest eigenvalues can also be reproduced from the Ferrari's formula with the same stated assumptions. On the other hand, Ferrari's method provide an equation for the lightest eigenvalue which is identified as the SM Higgs particle. However, the resulting expression becomes too complicated even for thinking in approximations. Nevertheless, a different approach is used by considering the determinant. All previous eigenvalues were obtained in a tree level approximation by looking their dependence on $\mu_{11}^2\mu_{22}^2$. Likewise, if we consider the determinant dominant term it can be written as:

$$\begin{aligned} Det(\tilde{M}_{hh}) \approx \frac{\mu_{11}^2\mu_{22}^2}{2592}\Bigg[&(k_3 v_\chi - k_4 v'_\chi)\Bigg((9(g^2+g'^2)+16g_X^2)\frac{(v_1^2 - v_1'^2)^2}{v_1 v_2} + (9(g^2+g'^2)+4g_X^2)\frac{(v_2^2 - v_2'^2)^2}{v_1 v_2} \\ &+ 2(9(g^2+g'^2)+8g_X^2)\frac{(v_1^2 - v_1'^2)(v_2^2 - v_2'^2)}{v_1 v_2}\Bigg) + (k_1 v'_\chi - k_2 v_\chi)\Bigg((9(g^2+g'^2)+16g_X^2)\frac{(v_1^2 - v_1'^2)^2}{v'_1 v'_2} \\ &+ (9(g^2+g'^2)+4g_X^2)\frac{(v_2^2 - v_2'^2)^2}{v'_1 v'_2} + 2(9(g^2+g'^2)+8g_X^2)\frac{(v_1^2 - v_1'^2)(v_2^2 - v_2'^2)}{v'_1 v'_2}\Bigg)\Bigg] \end{aligned} \tag{4-46}$$

The lightest Higgs mass eigenvalue is found by dividing this expression by the tree heavy eigenstates (4-44)-(4-45). After doing some algebra, the result is:

$$m_{h1}^2 \approx \frac{g_X^2(2v_1^2+v_2^2-2v_1'^2-v_2'^2)^2}{9(v_1^2+v_2^2+v_1'^2+v_2'^2)} + \frac{(g^2+g'^2)(v_1^2+v_2^2-v_1'^2-v_2'^2)^2}{4(v_1^2+v_2^2+v_1'^2+v_2'^2)} \tag{4-47}$$

Let's define the angles $\tan^2\tilde{\beta} = \frac{v_1^2+v_2^2}{v_1'^2+v_2'^2}$, $\tan\beta_1 = \frac{v_1}{v_1'}$ and $\tan\beta_2 = \frac{v_2}{v_2'}$ so Eq. (4-47) is rewritten as:

$$\begin{aligned} m_{h1}^2 &= m_Z^2\left(cos^2 2\tilde{\beta} + \frac{4}{9}\frac{g_X^2}{g^2+g'^2}(cos2\beta_1+\cos 2\beta_2)^2\right) \\ &\approx m_Z^2\cos^2 2\tilde{\beta} + \Delta m_h^2 \end{aligned} \tag{4-48}$$

The first thing to notice is that it depends only on the electroweak VEV's and the coupling constants as expected as well as there is no dependence on the new physics' energy scale implied by v_χ and v'_χ nor soft susy breaking parameters like μ_{11} and μ_{22} which in general dominate the mass spectrum in SUSY theories. In fact, the theory with additional scalar singlets and D-terms due to supersymmetry, the correction term Δm_h^2 might be at the same tree level order but its experimental value is compatible with the NMSSM and USSM models. Supporting our assumptions and approximations, a small program in C++ with the use of the Armadillo package [57] was written to explore the parameter space and find the region in which a 125(GeV) value match the lightest eigenvalue and the others, including CP-odd and charged scalars, lie on the TeV scale. It was found that:

$$k_i \sim 10^3 \qquad 0 < \lambda_i, \tilde{\lambda}_i < 10^3 \tag{4-49}$$

$$\mu_{11}, \mu_{22} > 10^4 \qquad \mu_{\chi\chi}, \mu_{\sigma\sigma}, M_\sigma, M'_\sigma > 10^8 \tag{4-50}$$

$$v_\chi, v'_\chi > 10^3, \tag{4-51}$$

which in fact satisfy the considered parameter hierarchy. Besides, the difference between the numerical obtained eigenvalues and the tree level expressions stated above is no greater than 0.5%. Additionally, the singlet VEV's lower bound were fixed in such a way that guarantees the lightest eigenvalue to come mainly from a scalar doublets mixture. All in all, it can be seen that the tree level masses do not have a λ_i and $\tilde{\lambda}_i$ dependence, they would enter in higher order corrections which in fact is confirmed numerically. Consequently , the trilineal couplings λ_i, $\tilde{\lambda}_i$ have a negligible effect on the mass spectrum so we can consider values as small as required by phenomenology, being out parameter region compatible with the highly suppressed trilineal couplings assumption.

Furthermore, equation (4-47) provides restrictions for the g_X coupling if we consider that is must match a 125 GeV value. For instance, it is found that a 125 GeV Higgs boson can be reached for values of $g_X = 1.06\ g$, and a large list of possible values for v_i and v'_i.

For example considering $v_1 = 195$, $v_2 = 138$, $v_1' = 52$, $v_2' = 20$ and $g_X = 0.71$ a 125 GeV Higgs boson is found. More generally, in figure (**4-2**) we plot v_1' vs g_X by using the same equation at 95% of C.L. with 125.3 ± 0.4 GeV. v_1 is considered proportional to the top quark mass, v_2' at an intermediate value between the bottom quark and tau lepton masses, and $v_2 = \sqrt{v^2 - v_1^2 - v_1^{2\prime} - v_2^{\prime 2}}$. The result is written as a function of v_1' since it is not restricted directly by fermion mass hierarchy (FMH), as will be shown in the next section. To address FMH, the v_1 and v_2' domains were $[170\text{GeV}, 200\text{GeV}]$ and $[3\text{GeV}, 7\text{GeV}]$ respectively while v_2 has a wide range of allowed values since it is related with neutrino masses. Finally, the v_1' VEV is determined by the restriction (4-19), ($v_1' = \sqrt{v^2 - v_1^2 - v_2^2 - v_2^{\prime 2}}$), so in principle v_2 lies on the $0 < v_2 < 246\text{GeV}$ interval. Last by not least, the coupling parameter g_X was explored for the interval $[0, 1]$ where a perturbative regime for the interaction is allowed.

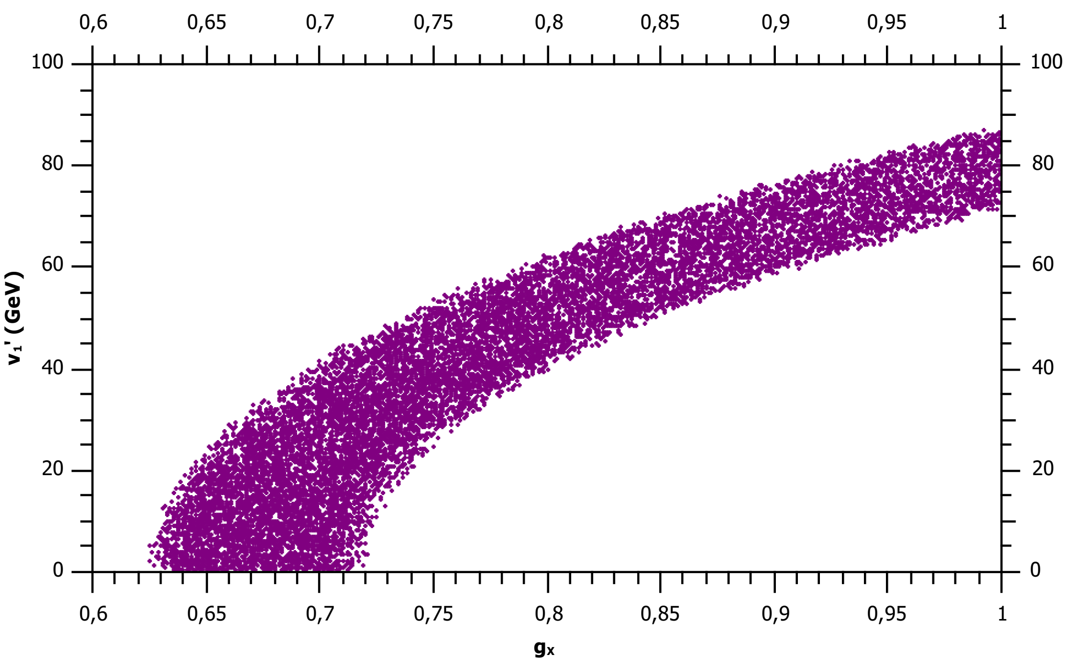


Figure **4-2**: Region in the parameter space v_1' vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.[5]

With this, an interesting fact arises. The possibility of having a null VEV for v_1' is allowed in the model, opening the possibility of having a inert doublet as a dark matter candidate for a future work. Anyway, a similar plot is shown in the figure (**4-3**), where the parameter space of v_2 vs g_X is now explored within the experimental constraints at 95% of confidence level. All in all, the conditions for a $125.3 GeV$ scalar particle exist and there are infinite ways for achieving it. The only condition is for g_X to be equal or greater than 0.63.

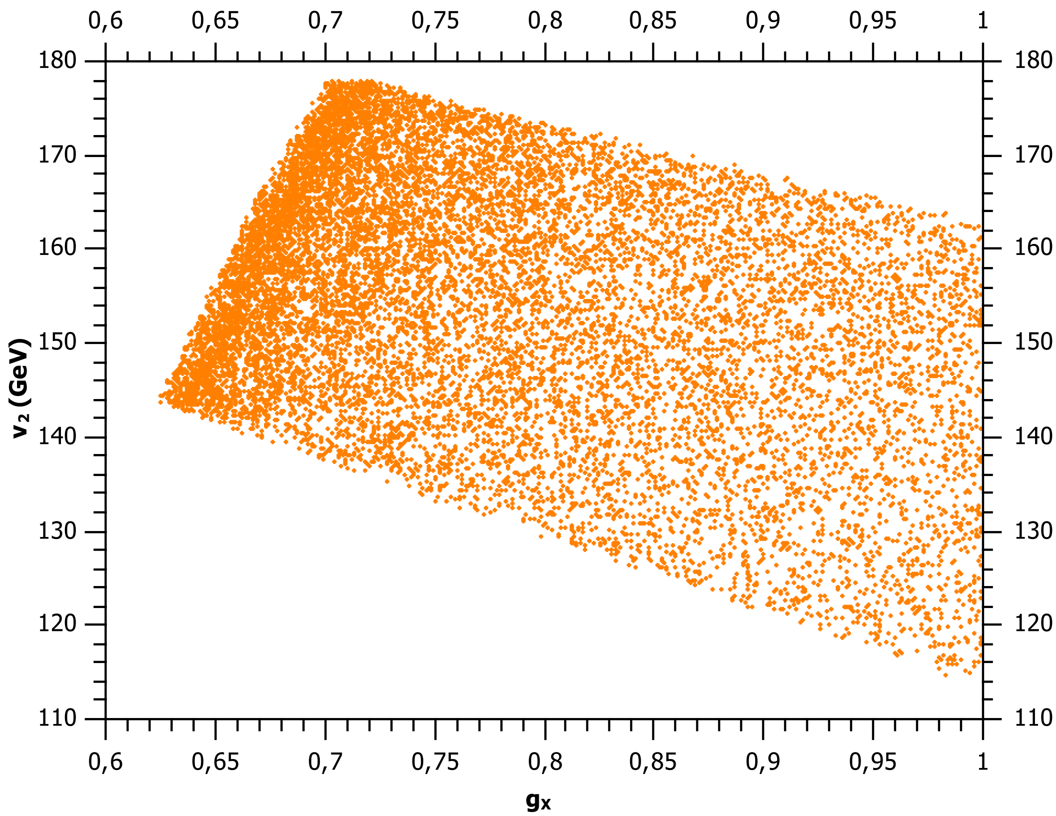


Figure **4-3**: Region in the parameter space v_2 vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.[5]

4.3 Fermion Masses

Now that we have checked the model consistency with a unique observed scalar particle, Higgs boson, an unobserved Z' gauge boson and the correct gauge boson masses. We are ready to introduce ourselves in the fermion sector, being of special interest the proposal of solving the FMH problem.

4.3.1 Charginos and Neutralinos

Let's consider first the sector of Majorana fermions which are characteristic of supersymmetry due to the promise of a Dark Matter candidate. It begins with the gaugino-higgsino mixing matrix that arise from their kinetic terms as was pointed out in section 3.3. The interaction terms in Eq.(3-5)

$$\sqrt{2} g f_{aji} (A_i^* \lambda^a P_L \psi_j + A_i \bar{\psi}_j P_R \lambda^a)$$

require A and ψ to be superpartners and λ a gaugino. Considering the eight scalar particles and superpartners with the associated five gauginos, one relative to each gauge boson, right after SSB we get the following matrix for the neutral fermionic fields:

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_{\tilde{B}} & 0 & 0 & \frac{g'v_1}{2} & -\frac{g'v_1'}{2} & \frac{g'v_2}{2} & -\frac{g'v_2'}{2} & 0 & 0 & 0 & 0 \\ 0 & M_{\tilde{W}} & 0 & -\frac{gv_1}{2} & \frac{gv_1'}{2} & -\frac{gv_2}{2} & \frac{gv_2'}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{\tilde{B}'} & \frac{2g_Xv_1}{3} & -\frac{2g_Xv_1'}{3} & \frac{g_Xv_2}{3} & -\frac{g_Xv_2'}{3} & -\frac{g_Xv_\chi}{3} & \frac{g_Xv_\chi'}{3} & 0 & 0 \\ \frac{g'v_1}{2} & -\frac{gv_1}{2} & \frac{2g_Xv_1}{3} & 0 & -\mu_1 & 0 & 0 & 0 & 0 & \frac{\lambda_2v_2'}{\sqrt{2}} & 0 \\ -\frac{g'v_1'}{2} & \frac{gv_1'}{2} & -\frac{2g_Xv_1'}{3} & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_1v_2}{\sqrt{2}} \\ \frac{g'v_2}{2} & -\frac{gv_2}{2} & \frac{g_Xv_2}{3} & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & \frac{\lambda_1v_1'}{\sqrt{2}} \\ -\frac{g'v_2'}{2} & \frac{gv_2'}{2} & -\frac{g_Xv_2'}{3} & 0 & 0 & -\mu_2 & 0 & 0 & 0 & \frac{\lambda_2v_1}{\sqrt{2}} & 0 \\ 0 & 0 & -\frac{g_Xv_\chi}{3} & 0 & 0 & 0 & 0 & 0 & -\mu_\chi & 0 & 0 \\ 0 & 0 & \frac{g_Xv_\chi'}{3} & 0 & 0 & 0 & 0 & -\mu_\chi & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\lambda_2v_2'}{\sqrt{2}} & 0 & 0 & \frac{\lambda_2v_1}{\sqrt{2}} & 0 & 0 & 0 & -\mu_\sigma \\ 0 & 0 & 0 & 0 & \frac{\lambda_1v_2}{\sqrt{2}} & \frac{\lambda_1v_1'}{\sqrt{2}} & 0 & 0 & 0 & -\mu_\sigma & 0 \end{pmatrix} \tag{4-52}$$

where $M_{\tilde{\chi}^0}$ is in the basis $(\tilde{B}, \tilde{W}_3, \tilde{B}', \tilde{h}_1, \tilde{h}_1', \tilde{h}_2, \tilde{h}_2', \xi_\chi, \xi_\chi', \xi_\sigma, \xi_\sigma')$. All terms that are not proportional to a coupling constant g comes from the soft breaking potential, like lambda terms, or from the superpotential, μ terms. The first thing to notice is the presence of the trilinear couplings λ_1 and λ_2 which are highly suppressed parameters that in principle can be neglected together with all electroweak contributions since mass eigenvalues are expected at a much higher energy scale. Secondly, in the case of scalar particles the terms μ_i coming from the superpotential were absorbed by the VEV conditions, playing an irrelevant role in mass generation. However, in the case of higgsinos these terms have a crucial role for their masses since are the only parameter who can be at a similar or greater scale than v_χ and v_χ'. In fact, these parameters provide the tree level eigenvalues, they are given by:

$$m_{\tilde{\chi}_1^0} = M_{\tilde{B}} \qquad m_{\tilde{\chi}_2^0} = M_{\tilde{W}} \tag{4-53}$$

$$m_{\tilde{\chi}_3^0} = \mu_1 \qquad m_{\tilde{\chi}_4^0} = -\mu_1 \tag{4-54}$$

$$m_{\tilde{\chi}_5^0} = \mu_2 \qquad m_{\tilde{\chi}_6^0} = -\mu_2 \tag{4-55}$$

$$m_{\tilde{\chi}_{10}^0} = \mu_\sigma \qquad m_{\tilde{\chi}_{11}^0} = -\mu_\sigma. \tag{4-56}$$

The resulting mass eigenstates are called "Neutralinos" and in general no assumption is made for the parameters which means that any eigenvalues can be the lightest one. In the case of $m_{\tilde{\chi}_7^0}$, $m_{\tilde{\chi}_8^0}$ and $m_{\tilde{\chi}_9^0}$, they are the roots of the polynomial $\mu_\chi \left(9M_{\tilde{B}'}\mu_\chi - 2v_\chi v_\chi' g_X^2\right) -$

$\lambda\left(g_X^2\left(v_\chi^2+v_\chi'^2\right)+9\mu_\chi^2\right)-9\lambda^2 M_{\tilde{B}'}+9\lambda^3=0$. Additionally, the MSSM gauginos seem to be decoupled due to its mass but they receive negligible electroweak corrections that couple them to Higgsinos. Nevertheless, in the case of charged particles, the resulting mass matrix in the basis $(\tilde{W}^+,\tilde{H}_1^+,\tilde{H}_2^+)$ from the right and $(\tilde{W}^-,\tilde{H}_1'^-,\tilde{H}_2'^-)$ from the left, the matrix reads:

$$M_{\tilde{\chi}^\pm}=\begin{pmatrix} M_{\tilde{W}^\pm} & \frac{gv_1}{\sqrt{2}} & \frac{gv_2}{\sqrt{2}} \\ \frac{gv_1'}{\sqrt{2}} & \mu_1 & 0 \\ \frac{gv_2'}{\sqrt{2}} & 0 & \mu_2 \end{pmatrix} \tag{4-57}$$

In this case the mass eigenvalues comes from the solution of a cubic polynomial, and since we do not have any estimate about their hierarchy no assumptions are made and the matrix is left stated. One interesting thing is the appearance of negative mass states in the Neutralino sector but it can be removed by a redefinition of the fields as pointed out in [11]. Consequently, the Neutralino sector presents degenerate masses at tree level.

4.3.2 Quark Masses at Tree Level

Let's consider the quark contribution of the superpotential shown in Eq. (4-10), it reads:

$$\begin{aligned} W_Q = {} & \hat{q}_L^1\hat{\Phi}_2 h_{2u}^{12}\hat{u}_L^{2\,c}+\hat{q}_L^2\hat{\Phi}_1 h_{1u}^{22}\hat{u}_L^{2\,c}+\hat{q}_L^3\hat{\Phi}_1 h_{1u}^{3k}\hat{u}_L^{k\,c}-\hat{q}_L^3\hat{\Phi}'_2 h_{2d}^{3j}\hat{d}_L^{j\,c}+\hat{q}_L^1\hat{\Phi}_2 h_{2T}^1\hat{\mathcal{T}}_L^c \\ & +\hat{q}_L^2\hat{\Phi}_1 h_{1T}^2\hat{\mathcal{T}}_L^c-\hat{q}_L^1\hat{\Phi}'_1 h_{1J}^{1a}\hat{\mathcal{J}}_L^{a\,c}-\hat{q}_L^2\hat{\Phi}'_2 h_{2J}^{2a}\hat{\mathcal{J}}_L^{a\,c}+\hat{\mathcal{T}}_L\hat{\chi}' h_{\chi'}^T\hat{\mathcal{T}}_L^c-\hat{\mathcal{J}}_L^a\hat{\chi} h_\chi^{Jab}\hat{\mathcal{J}}_L^{b\,c} \\ & +\hat{\mathcal{T}}_L\hat{\chi}' h_{\chi' u}^2\hat{u}_L^{2\,c}+\hat{\mathcal{J}}_L^a\hat{\sigma} h_\sigma^{Jaj}\hat{d}_L^{jc}+\hat{\mathcal{T}}_L\hat{\sigma}' h_{\sigma'}^{Tk}\hat{u}_L^{kc} \end{aligned} \tag{4-58}$$

where $j=1,2,3$ labels the down type singlet quarks, $k=1,3$ labels the first and third generation quark doublets, and $a=1,2$ is the exotic $\mathcal{J}_L^a$ and $\mathcal{J}_L^{ca}$ quarks index. It is worth to notice that this superpotential is identical in the non-supersymmetric case [44], the main difference lies on the existence of a pair of new scalars superfields Φ_1' and Φ_2' playing the role of conjugate fields $\tilde{\Phi}_i=i\sigma_2\Phi_i^*$. Consequently, it is expected for mass matrices to have an identical structure although mass spectrum might have slight differences due to the existencof three additional VEVs, in this way FMH can be explain due to a VEV hiearchy rather than a general Yukawa coupling values assignation. However, if we perform the superfield product and take only the fermion interactions non involving sparticles, the potential would read the same but using fields. Then, SSB produces bilinear terms that can be arranged in a mass matrix, in the case of up-like quarks it reads:

$$\mathcal{M}_{\mathcal{U}}=\begin{pmatrix} M_U & M_{UT} \\ M_{TU} & M_T \end{pmatrix} \tag{4-59}$$

where

$$M_U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & h_{2u}^{12} v_2 & 0 \\ 0 & h_{1u}^{22} v_1 & 0 \\ h_{1u}^{31} v_1 & 0 & h_{1u}^{33} v_1 \end{pmatrix} \qquad M_{UT} = \frac{1}{\sqrt{2}} \begin{pmatrix} h_{2T}^{1} v_2 \\ h_{1T}^{2} v_1 \\ 0 \end{pmatrix} \tag{4-60}$$

$$M_{TU} = \frac{v'_\chi}{\sqrt{2}} \begin{pmatrix} 0 & h_{\chi' u}^{2} & 0 \end{pmatrix} \qquad M_T = \frac{v'_\chi}{\sqrt{2}} g_{\chi' T}, \tag{4-61}$$

For obtaining the mass spectrum we need to diagonalize the squared mass matrix $M_U^2 = M_U M_U^\dagger$, which in this case provide the rotation matrices for the left-handed fermions, it reads:

$$M_U^2 = \frac{1}{2} \begin{pmatrix} v_2^2 \left((h_{2u}^{12})^2 + (h_{2T}^{1})^2 \right) & v_1 v_2 \left(h_{2u}^{12} h_{1u}^{22} + h_{2T}^{1} h_{1T}^{2} \right) & 0 & v_2 v'_\chi \left(h_{2u}^{12} h_{\chi' u}^{2} + h_{2T}^{1} g_{\chi' T} \right) \\ v_1 v_2 \left(h_{2u}^{12} h_{1u}^{22} + h_{2T}^{1} h_{1T}^{2} \right) & v_1^2 \left((h_{1u}^{22})^2 + (h_{1T}^{2})^2 \right) & 0 & v_1 v'_\chi \left(h_{1u}^{22} h_{\chi' u}^{2} + h_{1T}^{2} g_{\chi' T} \right) \\ 0 & 0 & v_1^2 \left((h_{1u}^{31})^2 + (h_{1u}^{33})^2 \right) & 0 \\ v_2 v'_\chi \left(h_{2u}^{12} h_{\chi' u}^{2} + h_{2T}^{2} g_{\chi' T} \right) & v_1 v'_\chi \left(h_{1u}^{22} h_{\chi' u}^{2} + h_{1T}^{2} g_{\chi' T} \right) & 0 & v'^2_\chi \left(\left(h_{\chi' u}^{2} \right)^2 + (g_{\chi' T})^2 \right) \end{pmatrix} \tag{4-62}$$

The diagonalization comes straightforward by using seesaw mechanism, all because the exotic quark $\mathcal{T}$ is expected to be much heavier than the already known quarks. Thus, the rotated mass matrix takes the form:

$$M_{U1}^2 = V_1 M_U^2 V_1^\dagger \tag{4-63}$$

$$\approx \frac{1}{2} \begin{pmatrix} v_2^2 r_1^2 & v_1 v_2 r_1 r_2 & 0 & 0 \\ v_1 v_2 r_1 r_2 & v_1^2 r_2^2 & 0 & 0 \\ 0 & 0 & v_1^2 \left((h_{1u}^{31})^2 + (h_{1u}^{33})^2 \right) & 0 \\ 0 & 0 & 0 & v'^2_\chi \left(\left(h_{\chi' u}^{2} \right)^2 + (g_{\chi' T})^2 \right) \end{pmatrix} \tag{4-64}$$

Where

$$V_1 = \begin{pmatrix} 1 & 0 & 0 & -\frac{v_2}{v'_\chi} r_{\chi 1}^2 \\ 0 & 1 & 0 & -\frac{v_1}{v'_\chi} r_{\chi 2}^2 \\ 0 & 0 & 1 & 0 \\ \frac{v_2}{v'_\chi} r_{\chi 1}^2 & \frac{v_1}{v'_\chi} r_{\chi 2}^2 & 0 & 1 \end{pmatrix} \tag{4-65}$$

$$r_1^2 = \frac{\left(h_{2T}^{1} h_{\chi' u}^{2} - h_{2u}^{12} g_{\chi' T} \right)^2}{\left(h_{\chi' u}^{2} \right)^2 + (g_{\chi' T})^2} \qquad r_{\chi 1}^2 = \frac{h_{2u}^{12} h_{\chi' u}^{2} + h_{2T}^{1} g_{\chi' T}}{\left(h_{\chi' u}^{2} \right)^2 + (g_{\chi' T})^2} \tag{4-66}$$

$$r_2^2 = \frac{\left(h_{1T}^{2} h_{\chi' u}^{2} - h_{1u}^{22} g_{\chi' T} \right)^2}{\left(h_{\chi' u}^{2} \right)^2 + (g_{\chi' T})^2} \qquad r_{\chi 2}^2 = \frac{h_{1u}^{22} h_{\chi' u}^{2} + h_{1T}^{2} g_{\chi' T}}{\left(h_{\chi' u}^{2} \right)^2 + (g_{\chi' T})^2} \tag{4-67}$$

Now we have two isolated mass eigenstates identified with a heavy quark singlet and the top quark. However, the 2×2 submatrix has null determinant implying that the up-quark turns out to be massless and the charm mass coincide with the matrix trace. Resulting in:

$$\begin{aligned} m_u^2 &\approx 0 & m_c^2 &= \frac{1}{2}(v_2^2 r_1^2 + v_1^2 r_2^2), \\ m_t^2 &= \frac{1}{2} v_1^2 \left[(h_{1u}^{31})^2 + (h_{1u}^{33})^2 \right], & m_T^2 &= \frac{1}{2} v_\chi'^{\,2} \left[(g_{\chi' T})^2 + (h_{\chi' u}^2)^2 \right]. \end{aligned} \tag{4-68}$$

In the case of down-like quarks the mass matrix takes the form:

$$\mathcal{M}_\mathcal{D} = \begin{pmatrix} M_D & M_{DJ} \\ M_{JD} & M_J \end{pmatrix} \tag{4-69}$$

where

$$\begin{aligned} M_D &= \frac{v_2'}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ h_{2d}^{31} & h_{2d}^{32} & h_{2d}^{33} \end{pmatrix} & M_{DJ} &= \frac{1}{\sqrt{2}} \begin{pmatrix} h_{1J}^{11} v_1' & h_{1J}^{12} v_1' \\ h_{2J}^{21} v_2' & h_{2J}^{22} v_2' \\ 0 & 0 \end{pmatrix} \\ M_{JD} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & M_J &= \frac{v_\chi}{\sqrt{2}} \begin{pmatrix} g_{\chi J}^{11} & g_{\chi J}^{12} \\ g_{\chi J}^{21} & g_{\chi J}^{22} \end{pmatrix}. \end{aligned} \tag{4-70}$$

Likewise, we diagonalize the squared mass matrix $\mathcal{M}_\mathcal{D} \mathcal{M}_\mathcal{D}^\dagger$ to get the unitary transformation for left handed fields. It reads:

$$\mathcal{M}_\mathcal{D}^2 = \frac{1}{2} \begin{pmatrix} M_D & M_{DJ} \\ M_{JD} & M_{JJ} \end{pmatrix} \tag{4-71}$$

where

$$M_D = \begin{pmatrix} v_1'^2 \left((h_{1J}^{11})^2 + (h_{1J}^{12})^2 \right) & v_1' v_2' \left(h_{1J}^{11} h_{2J}^{21} + h_{1J}^{12} h_{2J}^{22} \right) & 0 \\ v_1' v_2' \left(h_{1J}^{11} h_{2J}^{21} + h_{1J}^{12} h_{2J}^{22} \right) & v_2'^2 \left((h_{2J}^{21})^2 + (h_{2J}^{22})^2 \right) & 0 \\ 0 & 0 & v_2'^2 \left((h_{2d}^{31})^2 + (h_{2d}^{32})^2 + (h_{2d}^{33})^2 \right) \end{pmatrix} \tag{4-72}$$

$$M_{DJ} = \begin{pmatrix} -v_1' v_\chi \left(h_{1J}^{11} g_{\chi J}^{11} + h_{1J}^{12} g_{\chi J}^{12} \right) & -v_1' v_\chi \left(h_{1J}^{11} g_{\chi J}^{21} + h_{1J}^{12} g_{\chi J}^{22} \right) \\ -v_2' v_\chi \left(h_{2J}^{21} g_{\chi J}^{11} + h_{2J}^{22} g_{\chi J}^{12} \right) & -v_2' v_\chi \left(h_{2J}^{21} g_{\chi J}^{21} + h_{2J}^{22} g_{\chi J}^{22} \right) \\ 0 & 0 \end{pmatrix} = M_{JD}^T \tag{4-73}$$

$$M_{JJ} = \begin{pmatrix} v_\chi^2 \left((g_{\chi J}^{11})^2 + (g_{\chi J}^{12})^2 \right) & v_\chi^2 \left(g_{\chi J}^{11} g_{\chi J}^{21} + g_{\chi J}^{12} g_{\chi J}^{22} \right) \\ v_\chi^2 \left(g_{\chi J}^{11} g_{\chi J}^{21} + g_{\chi J}^{12} g_{\chi J}^{22} \right) & v_\chi^2 \left((g_{\chi J}^{21})^2 + (g_{\chi J}^{22})^2 \right) \end{pmatrix} \tag{4-74}$$

It can be seen that the exotic couple of quarks $\mathcal{J}^a$ depends on v_χ which is a high energy parameter. Thus, they make up a 2×2 heavy submatrix, allowing a straightforward block diagonalization via seesaw rotation U_1

$$\mathcal{M}^2_{D2} = U_1 {\mathcal{M}_{\mathcal{D}}}^2 U_1^\dagger \tag{4-75}$$

$$\approx \frac{1}{2}\begin{pmatrix} M_D - M_{DJ}M_{JJ}^{-1}M_{JD} & 0 \\ 0 & M_{JJ}\end{pmatrix} \tag{4-76}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_2'^2\left((h_{2d}^{31})^2+(h_{2d}^{32})^2+(h_{2d}^{33})^2\right) & 0 & 0 \\ 0 & 0 & 0 & v_\chi^2\left((g_{\chi J}^{11})^2+(g_{\chi J}^{12})^2\right) & v_\chi^2\left(g_{\chi J}^{11}g_{\chi J}^{21}+g_{\chi J}^{12}g_{\chi J}^{22}\right) \\ 0 & 0 & 0 & v_\chi^2\left(g_{\chi J}^{11}g_{\chi J}^{21}+g_{\chi J}^{12}g_{\chi J}^{22}\right) & v_\chi^2\left((g_{\chi J}^{21})^2+(g_{\chi J}^{22})^2\right)\end{pmatrix} \tag{4-77}$$

where

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & -\frac{v_1'}{v_\chi}n_1 & -\frac{v_1'}{v_\chi}n_2 \\ 0 & 1 & 0 & -\frac{v_2'}{v_\chi}n_3 & -\frac{v_2'}{v_\chi}n_4 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{v_1'}{v_\chi}n_1 & \frac{v_2'}{v_\chi}n_3 & 0 & 1 & 0 \\ \frac{v_1'}{v_\chi}n_2 & \frac{v_2'}{v_\chi}n_4 & 0 & 0 & 1 \end{pmatrix} \tag{4-78}$$

$$n_1 = \frac{g_{1J}^{11}g_{\chi J}^{22} - g_{1J}^{12}g_{\chi J}^{21}}{g_{\chi J}^{21}g_{\chi J}^{12} - g_{\chi J}^{11}g_{\chi J}^{22}} \qquad n_2 = \frac{g_{1J}^{12}g_{\chi J}^{11} - g_{1J}^{11}g_{\chi J}^{12}}{g_{\chi J}^{21}g_{\chi J}^{12} - g_{\chi J}^{11}g_{\chi J}^{22}} \tag{4-79}$$

$$n_3 = \frac{g_{1J}^{21}g_{\chi J}^{22} - g_{1J}^{22}g_{\chi J}^{21}}{g_{\chi J}^{21}g_{\chi J}^{12} - g_{\chi J}^{11}g_{\chi J}^{22}} \qquad n_4 = \frac{g_{1J}^{22}g_{\chi J}^{11} - g_{1J}^{21}g_{\chi J}^{12}}{g_{\chi J}^{21}g_{\chi J}^{12} - g_{\chi J}^{11}g_{\chi J}^{22}} \tag{4-80}$$

Due to the high energy scale in which the exotic quarks live, we can consider in a first approximations their mixing as negligible. All in all, we can see from the mass matrix in Eq. (4-75) that the down and strange quarks turns out to be massless whereas the other 3 quarks have a mass given by :

$$m_b^2 = \frac{1}{2}v_2'^2\left((h_{2d}^{31})^2+(h_{2d}^{32})^2+(h_{2d}^{33})^2\right) \qquad m_{J1}^2 = \frac{1}{2}v_\chi^2(g_{\chi J}^{11})^2 \qquad m_{J2}^2 = \frac{1}{2}v_\chi^2(g_{\chi J}^{22})^2 \tag{4-81}$$

$$m_d^2 \approx 0 \qquad m_s^2 \approx 0 \tag{4-82}$$

To sum up, the X quantum number restrictions over the potential makes up-like quarks to acquire mass from Φ_i Higgs fields while down-like quarks do it from Φ_i' fields and the diagonalization procedure comes from its non-SUSY counterpart [44] with the only difference on the VEVs. Despite we have three massless quarks, they can be obtained from radiative corrections as will be shown in the next section. Moreover, from mass expressions we have a hierarchy among VEVs by assuming $v_1 \approx m_t$ and $v_2' \approx m_b$ leaving the difference between charm and top quarks fully dependent on the Yukawa values, which in this case is more restricted than in the general SM scenario. In contrast, the exotic sector exist and have large masses thank to their coupling to heavy scalar singlets, giving them mass values expected at least in the TeV scale in consistence with recent experimental results that exclude exotic quarks with masses bellow $800\,GeV$ [60].

4.3.3 Quark masses at one loop level

It was found before that the lightest fermions, electron, up, down and strange quarks are tree level massless. However, it is in agreement with the model energy scale since their mass is considerably small in comparison. Nevertheless, they acquire a finite mass value through VEV insertions and σ and σ' mediated loop corrections thanks to the fact that these scalar do not acquire a VEV. The corrections have been calculated in [44] but now in this model shows up a second contribution coming from superpartners. In the case of the up-quark the one diagram is shown in figure **4-4**

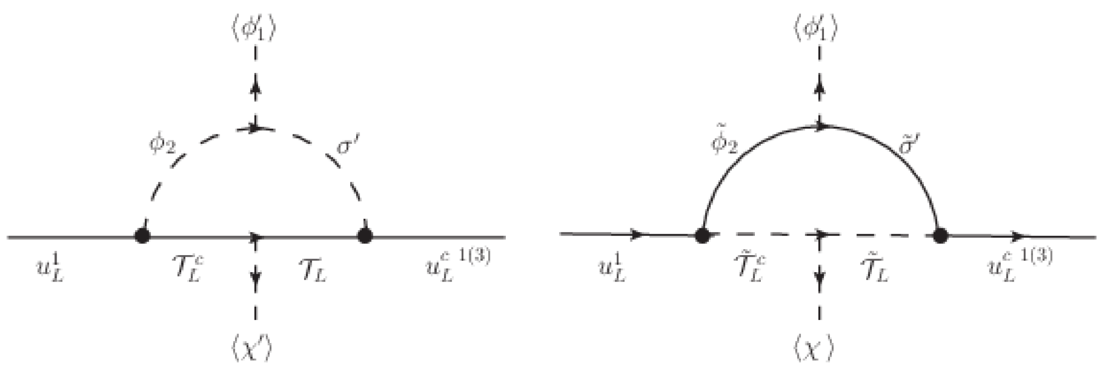


Figure **4-4**: One loop corrections to the quark up due to scalar singlets, exotic quarks, squarks and Higgsinos.

The interactions that makes possible the diagram belong to the superpotential and the soft breaking terms which in particular are:

$$W_Q \Rightarrow \hat{\mathcal{T}}_L \hat{\sigma}' h_{\sigma'}^{Tj} \hat{u}_L^{kc} + \hat{q}_L^1 \hat{\Phi}_2 h_{2T}^1 \hat{\mathcal{T}}_L^c + \hat{\mathcal{T}}_L \hat{\chi}' h_{\chi'}^T \hat{\mathcal{T}}_L^c \tag{4-83}$$

$$V_{soft} \Rightarrow \tilde{\lambda_1} \Phi_1'^{\dagger} \Phi_2 \sigma' + h.c. \qquad W_\phi \Rightarrow \lambda_1 \hat{\phi}_1' \hat{\phi}_2 \hat{\sigma}' + \mu_\chi \hat{\chi} \hat{\chi}'$$

The first diagram in figure **4-4** represents the non-SUSY contribution. The $\mathcal{T}$ quark is characterized by a small mixing with SM particles in agreement with the seesaw rotation that allowed the diagonalization. As a consequence, the exotic up-like quark is approximately the same in both flavor and mass basis. Then, the VEV insertion can be replaced by the mass eigenstate interacting with the two chiral up quarks instead of the infinite series of mass insertions as pointed out in [22]. However, it is more convenient to work with the scalars in the flavor basis resulting is a 3-point correction which reads:

$$v_1'\Sigma_{1k}^{NS}(p^2=0) = \frac{-1}{16\pi^2}\frac{v_1'}{\sqrt{2}}\frac{\tilde{\lambda}_1 h_{\sigma'}^{Tk}h_{2T}^1}{M_T}C_0\left(\frac{m_{h2}'}{M_T},\frac{m_\sigma'}{M_T}\right) \tag{4-84}$$

where M_T is the T exotic quark mass, m_{h2}' is the $(4,4)$ element of M_{hh} (Eq. 4-32), $m_{\sigma'}$ is the $(4,4)$ element of $M_{\xi\xi}$ and C_0 is the Passarino-Veltmann function evaluated for $p=0$ shown in equation (4-86) [15]. The second diagram in **4-4** represents the SUSY contributions due to superpartners, the scalar line is rotated to the mass eigenbasis. Thus, the diagram becomes to a 3-point correction where all 8 up-squarks mass eigenstates can run into the loop. Higgsinos are not rotated for simplicity. The final expression reads:

$$\begin{aligned} v_1'\Sigma_{1k}^{S}(p^2=0) = & \\ -\frac{1}{32\pi^2}\frac{v_1'}{\sqrt{2}}\sum_{m=1}^{8} & Z_U^{8m}Z_U^{4m}\lambda_1 h_{\sigma'}^{Tk}h_{2T}^1\times \\ \times & \left[\frac{(\tilde{m}_\sigma'+\tilde{m}_{h_2}')^2}{\tilde{M}_{T_m}^2}C_0\left(\frac{\tilde{m}_{h1}'}{\tilde{M}_{T_m}},\frac{\tilde{m}_\sigma'}{\tilde{M}_{T_m}}\right)+\tilde{m}_{h2}'^2B_0(0,\tilde{m}_\sigma',\tilde{M}_{T_m})+\tilde{m}_\sigma'^2B_0(0,\tilde{m}_{h2}',\tilde{M}_{T_m})\right] \end{aligned} \tag{4-85}$$

$$C_0(\hat{m}_1,\hat{m}_2) = \frac{1}{(1-\hat{m}_1^2)(1-\hat{m}_2^2)(m_1^2-\hat{m}_2^2)}\left[\hat{m}_1^2\hat{m}_2^2 Ln\left(\frac{\hat{m}_1^2}{\hat{m}_2^2}\right)+\hat{m}_2^2Ln(\hat{m}_2^2)-\hat{m}_1^2Ln(\hat{m}_1^2)\right], \tag{4-86}$$

where $\tilde{m}_\sigma'$ is the $(11,11)$ element of the neutralino mass matrix, $\tilde{m}_{h1}'$ the $(6,6)$ element of the neutralino mass matrix, $\tilde{M}_{T_m}^2$ $m=1,..,8.$ are the up-squark mass eigenvalues. $\tilde{T}_m$ are the squark mass eigenstates and Z_U the associated rotation matrix which relates the states $\tilde{T}_L$ and $\tilde{T}_L^c$ with the mass eigenstates. We assume the squak mass matrix basis in similar preserving the order of its fermion counterpart. The double fermion propagator makes that the diagram has a term proportional to the PV-Function C_{00}[54] that can be decomposed in terms of the scalar integrals C_0 and B_0[62].
taking into account these matrix elements, the up-quarks matrix at one-loop level, after the initial seesaw rotation (Eq. 4-63) reads:

$$M_{U2}^2 = V_1 M_U^2 V_1^\dagger \tag{4-87}$$

$$\approx \frac{1}{2}\begin{pmatrix} v_1'^2(\Sigma_{11}^2+\Sigma_{13}^2)+v_2^2r_1^2 & v_1v_2r_1r_2 & v_1v_1'(\Sigma_{11}h_{1u}^{31}+\Sigma_{13}h_{1u}^{33}) \\ v_1v_2r_1r_2 & v_1^2r_2^2 & 0 \\ v_1v_1'(\Sigma_{11}h_{1u}^{31}+\Sigma_{13}h_{1u}^{33}) & 0 & v_1^2\left((h_{1u}^{31})^2+(h_{1u}^{33})^2\right) \end{pmatrix} \tag{4-88}$$

where $\Sigma_{ik} = \Sigma_{ik}^{S} + \Sigma_{ik}^{NS}$ resumes the two kind of contributions. Now, a second seesaw rotation can be implemented since the top quark mass has much bigger mass in comparison with up and charm quarks resulting in:

$$M_{U3}^2 = V_2 M_{U2}^2 V_2^\dagger \tag{4-89}$$

$$\approx \frac{1}{2}\begin{pmatrix} v_1^2 v_1'^2 \frac{r_4^2}{2m_t^2} + v_2^2 r_1^2 & v_1 v_2 r_1 r_2 & 0 \\ v_1 v_2 r_1 r_2 & v_1^2 r_2^2 & 0 \\ 0 & 0 & v_1^2\left(\left(h_{1u}^{31}\right)^2 + \left(h_{1u}^{33}\right)^2\right) \end{pmatrix} \tag{4-90}$$

$$V_2 = \begin{pmatrix} 1 & 0 & -v_1 v_1' r_3 & 0 \\ 0 & 1 & 0 & 0 \\ v_1 v_1' r_3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad r_3 = \frac{(\Sigma_{11} h_{1u}^{31} + \Sigma_{13} h_{1u}^{33})}{2m_t^2} \tag{4-91}$$

$$r_4 = (\Sigma_{13} h_{1u}^{31} - \Sigma_{11} h_{1u}^{33}) \tag{4-92}$$

If we consider the tree level charm mass in Eq. (4-68), by reconstructing the trace we can write in a good approximation the up quark mass as the new term in the $(1,1)$ entry, so the quark spectrum reads:

$$m_u^2 \approx \frac{r_2^2 r_4^2 v_1'^2 v_1^4}{8m_c^2 m_t^2} \qquad m_c^2 \approx \frac{1}{2}\left(v_1^2 r_2^2 + v_2^2 r_1^2\right)$$
$$m_t^2 = \frac{1}{2} v_1^2 \left[(h_{1u}^{31})^2 + (h_{1u}^{33})^2\right], \qquad m_T^2 = \frac{1}{2} v_\chi'^{\,2} \left[(g_{\chi' T})^2 + (h_{\chi' u}^2)^2\right]. \tag{4-93}$$

Last but not least, we are interested in the rotation matrix. For a general symmetric 2×2 matrix in the form $\begin{pmatrix} a & c \\ c & b \end{pmatrix}$ with eigenvalues λ_1 and λ_2 the diagonalization can be performed by a rotation at an angle $\tan(2\theta) = \frac{2c}{a-b}$ [13]. However, the diagonalized matrix might not be ordered in ascending order depending on the relative sign of the denominator. Considering that $a < b$ and doing some algebra we can prove that:

$$V_3 = \begin{pmatrix} \cos\theta_{uc} & \sin\theta_{uc} & 0 & 0 \\ -\sin\theta_{uc} & \cos\theta_{uc} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \sin\theta_{uc} = -\frac{-b+\lambda_2}{c\sqrt{(\frac{\lambda_2 - b}{c})^2 + 1}} = -\frac{a-\lambda_1}{c\sqrt{(\frac{a-\lambda_1}{c})^2+1}} \tag{4-94}$$

$$= -\frac{2m_c^2 - r_2^2 v_1^2}{r_1 r_2 v_1 v_2 \sqrt{\left(\frac{r_2^2 v_1^2 - 2m_c^2}{r_1 r_2 v_1 v_2}\right)^2 + 1}} \tag{4-95}$$

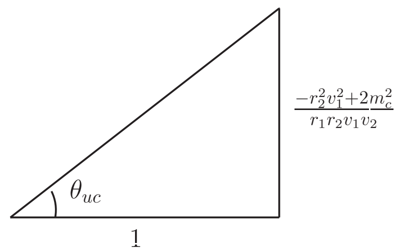


Figure **4-5**: triangle representation of the up-charm mixing

In the case of down quarks, we have four diagrams shown in **4-6** which contribute to the down and strange quarks. They are possible thanks to the following superpotential and soft breaking terms:

$$W_Q \Rightarrow \hat{\mathcal{J}}^a_L \hat{\sigma} h^{Jaj}_{\sigma} \hat{d}^{jc}_L + \hat{q}^1_L \hat{\Phi}'_1 h^{1a}_{1J} \hat{\mathcal{J}}^{a\,c}_L + \hat{q}^2_L \hat{\phi}'_2 h^{2a}_{2J} \hat{\mathcal{J}}^{a\,c}_L - \hat{\mathcal{J}}^a_L \hat{\chi} h^{Jab}_{\chi} \hat{\mathcal{J}}^{b\,c}_L \tag{4-96}$$

$$W_\phi \Rightarrow \lambda_1 \hat{\Phi}'_1 \hat{\Phi}_2 \hat{\sigma}' + \lambda_2 \hat{\Phi}'_2 \hat{\Phi}_1 \hat{\sigma} - \mu_\sigma \hat{\sigma}' \hat{\sigma} - \mu_\chi \hat{\chi}' \hat{\chi} \tag{4-97}$$

$$V_{soft} \Rightarrow \tilde{\lambda}_2 \Phi'_2 \Phi_1 \sigma \tag{4-98}$$

Similar to the case of up quarks, the contribution to down and strange quarks can be written as:

$$\begin{aligned} v'_2 \Sigma_{1j} &= v'_2 \Sigma^{NS}_{1j} + v'_2 \Sigma^{S}_{1j} \\ &= \frac{-1}{16\pi^2} \frac{v_2}{\sqrt{2}} \sum_{a=1}^{2} \frac{\lambda_1 h^{Jaj}_{\sigma} h^{1a}_{1J}}{M_{J^a}} C_0\left(\frac{m'_{h1}}{M_{J^a}}, \frac{m_\sigma}{M^a_J}\right) \\ &- \frac{1}{32\pi^2} \frac{v_2}{\sqrt{2}} \sum_{a=1}^{2} \sum_{q=1}^{10} \sum_{l=1}^{11} Z_D^{(8+a)q} Z_D^{(4+a)q} Z^{7l}_{\tilde{h}} Z^{8l}_{\tilde{h}} \mu_\sigma \lambda_1 h^{J^a j}_{\sigma} h^{la}_{lJ} \times \\ &\times \left[\frac{(\tilde{m}_{Hl} + \tilde{m}'_{h1})^2}{\tilde{M}^2_{D_q}} C_0\left(\frac{\tilde{m}'_{h1}}{\tilde{M}_{D_q}}, \frac{\tilde{m}_{Hl}}{\tilde{M}_{D_q}}\right) + \tilde{m}^2_{h2} B_0(0, \tilde{m}_{Hl}, \tilde{M}_{D_q}) + \tilde{m}^2_{Hl} B_0(0, \tilde{m}_{h_2}, \tilde{M}_{D_q}) \right] \end{aligned} \tag{4-99}$$

$$\begin{aligned} v'_1 \Sigma_{2j} &= v'_1 \Sigma^{NS}_{2j} + v'_1 \Sigma^{S}_{2j} \\ &= \frac{-1}{16\pi^2} \frac{v_1}{\sqrt{2}} \sum_{a=1}^{2} \frac{\tilde{\lambda}_2 h^{Jaj}_{\sigma} h^{1a}_{1J}}{M_{J^a}} C_0\left(\frac{m'_{h2}}{M_{J^a}}, \frac{m_\sigma}{M^a_J}\right) - \frac{1}{32\pi^2} \frac{v_1}{\sqrt{2}} \sum_{a=1}^{2} \sum_{q=1}^{10} Z_D^{(8+a)q} Z_D^{(4+a)q} \mu_\sigma \lambda_2 h^{J^a j}_{\sigma} h^{la}_{lJ} \\ &\times \left[\frac{(\tilde{m}_\sigma + \tilde{m}'_{h1})^2}{\tilde{M}^2_{D_q}} C_0\left(\frac{\tilde{m}'_{h2}}{\tilde{M}_{D_q}}, \frac{\tilde{m}_\sigma}{\tilde{M}_{D_q}}\right) + \tilde{m}^2_{h1} B_0(0, \tilde{m}_\sigma, \tilde{M}_{D_q}) + \tilde{m}^2_\sigma B_0(0, \tilde{m}_{h_1}, \tilde{M}_{D_q}) \right] \end{aligned} \tag{4-100}$$

where $\tilde{M}_{D_q}$ is the qth squark mass eigenvalue with $q = 1, .., 10.$, $\tilde{m}_{Hl}$ is the lth neutralino mass eigenvalues with $l = 1, .., 11.$ and all other mass terms correspond to the diagonal entry in their respective mass matrix, por instance m'_{h1} is the $(2,2)$ element in M_{hh} and $\tilde{m}_\sigma$ is the $(10,10)$ entry in the neutralino mass matrix. Finally, the index $a = 1,2.$ labels the exotic quarks. These new entries enters into the squared mass matrix after the initial seesaw rotation shown in Eq. (4-75) as:

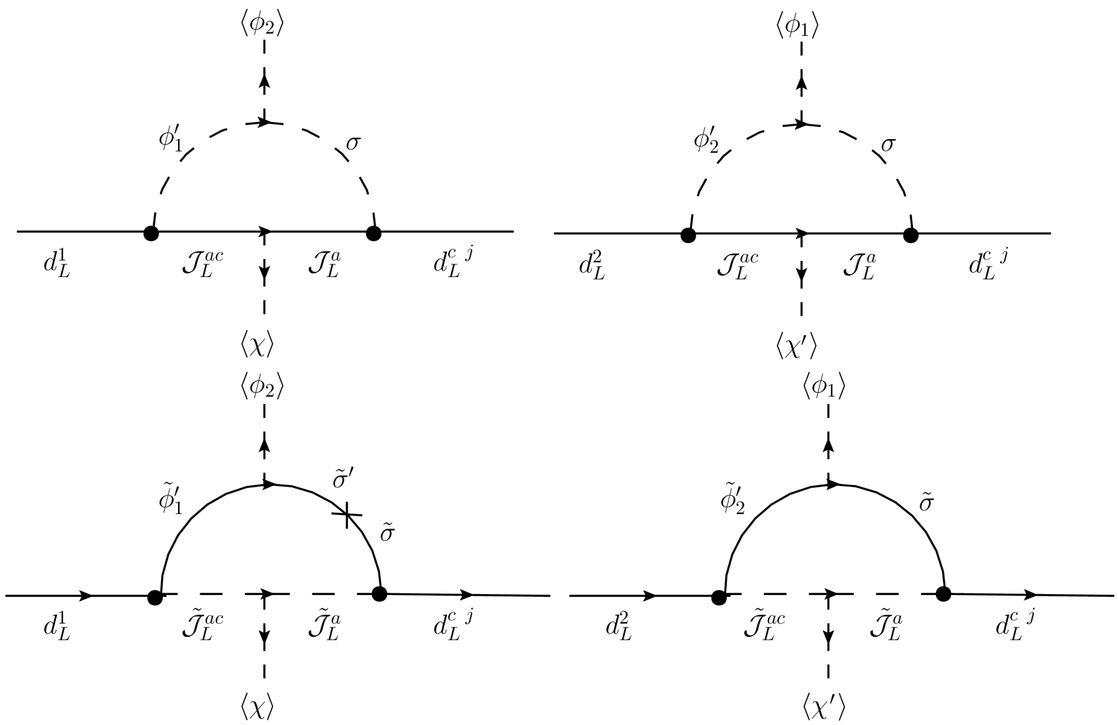


Figure **4-6**: One loop corrections to the quarks down and strange due to scalar singlets, exotic quarks, squarks and Higgsinos.

$$
\begin{aligned}
&M_D - M_{DJ}M_{JJ}^{-1}M_{JD} \equiv M_{D-SM} \\
&= \frac{v_2'^2}{2}\begin{pmatrix}
\left(\Sigma_{11}^2+\Sigma_{12}^2+\Sigma_{13}^2\right) & \frac{v_1'}{v_2'}\left(\Sigma_{11}\Sigma_{21}+\Sigma_{12}\Sigma_{22}+\Sigma_{13}\Sigma_{23}\right) & \left(\Sigma_{11}h_{2d}^{31}+\Sigma_{12}h_{2d}^{32}+\Sigma_{13}h_{2d}^{33}\right) \\
\frac{v_1'}{v_2'}\left(\Sigma_{11}\Sigma_{21}+\Sigma_{12}\Sigma_{22}+\Sigma_{13}\Sigma_{23}\right) & \frac{v_1'^2}{v_2'^2}\left(\Sigma_{21}^2+\Sigma_{22}^2+\Sigma_{23}^2\right) & \frac{v_1'}{v_2'}\left(\Sigma_{21}h_{2d}^{31}+\Sigma_{22}h_{2d}^{32}+\Sigma_{23}h_{2d}^{33}\right) \\
\left(\Sigma_{11}h_{2d}^{31}+\Sigma_{12}h_{2d}^{32}+\Sigma_{13}h_{2d}^{33}\right) & \frac{v_1'}{v_2'}\left(\Sigma_{21}h_{2d}^{31}+\Sigma_{22}h_{2d}^{32}+\Sigma_{23}h_{2d}^{33}\right) & \left((h_{2d}^{31})^2+(h_{2d}^{32})^2+(h_{2d}^{33})^2\right)
\end{pmatrix}
\end{aligned}
\tag{4-101}
$$

Due to the high bottom-quark mass, we perform a second seesaw rotations U_2 shown in Eq. (4-102) reducing the problem to the following decoupled 2×2 matrix:

$$U_2 = \begin{pmatrix} 1 & 0 & -\frac{v_2'^2 l_1}{2} & 0 & 0 \\ 0 & 1 & -\frac{v_1' v_2' l_2}{2} & 0 & 0 \\ \frac{v_2'^2 l_1}{2} & \frac{v_1' v_2' l_2}{2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad M_{D-ds} = \frac{1}{4m_b^2} \begin{pmatrix} t_{11} v_2'^4 & t_{12} v_1' v_2'^3 \\ t_{12} v_1' v_2'^3 & t_{22} v_1'^2 v_2'^2 \end{pmatrix} \tag{4-102}$$

$$l_1 = \frac{(\Sigma_{11} h_{2d}^{31} + \Sigma_{12} h_{2d}^{32} + \Sigma_{13} h_{2d}^{33})}{m_b^2} \qquad l_2 = \frac{(\Sigma_{21} h_{2d}^{31} + \Sigma_{22} h_{2d}^{32} + \Sigma_{23} h_{2d}^{33})}{m_b^2} \tag{4-103}$$

$$t_{11} = \frac{2m_b^2}{v_2'^2} \left(\Sigma_{11}^2 + \Sigma_{12}^2 + \Sigma_{13}^2 \right) - l_1^2 \tag{4-104}$$

$$t_{22} = \frac{2m_b^2}{v_2'^2} \left(\Sigma_{21}^2 + \Sigma_{22}^2 + \Sigma_{23}^2 \right) - l_2^2 \tag{4-105}$$

$$\begin{aligned} t_{12} &= (\Sigma_{11}\Sigma_{21} + \Sigma_{12}\Sigma_{22} + \Sigma_{13}\Sigma_{23}) \left((h_{2d}^{31})^2 + (h_{2d}^{32})^2 + (h_{2d}^{33})^2 \right) \\ &\qquad - \left(\Sigma_{11} h_{2d}^{31} + \Sigma_{12} h_{2d}^{32} + \Sigma_{13} h_{2d}^{33} \right) \left(\Sigma_{21} h_{2d}^{31} + \Sigma_{22} h_{2d}^{32} + \Sigma_{23} h_{2d}^{33} \right) \\ &= (\Sigma_{11}\Sigma_{21} + \Sigma_{12}\Sigma_{22} + \Sigma_{13}\Sigma_{23}) \frac{2m_b^2}{v_2'^2} - l_1 l_2 m_b^4 \end{aligned} \tag{4-106}$$

in the special case when the mixing t_{12} vanishes, the down and strange masses would match the diagonal entries of the matrix as pointed out in [44]. However, from the eigenvalues general form we can approximate the masses by considering that the matrix determinant is much smaller than the square of the trace:

$$m_d^2 \approx \frac{v_1'^2 v_2'^4 (t_{11} t_{22} - t_{12}^2)}{m_b^2 (t_{11} v_2'^2 + t_{22} v_1'^2)} \qquad m_s^2 \approx \frac{t_{11} v_2'^4 + t_{22} v_1'^2 v_2'^2}{2m_b^2} \tag{4-107}$$

where the mixing angle is given by eq. (4-94)with the rotation matrix given by:

$$U_3 = \begin{pmatrix} \cos\theta_{ds} & -\sin\theta_{ds} & 0 & 0 \\ \sin\theta_{ds} & \cos\theta_{ds} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \sin\theta_{ds} = -\frac{4m_b^2 m_s^2 - t_{22} v_1'^2 v_2'^2}{t_{12} v_1' v_2'^3 \sqrt{\left(\frac{4m_b^2 m_s^2 - t_{22} v_1'^2 v_2'^2}{t_{12} v_1' v_2'^3} \right))^2 + 1}} \tag{4-108}$$

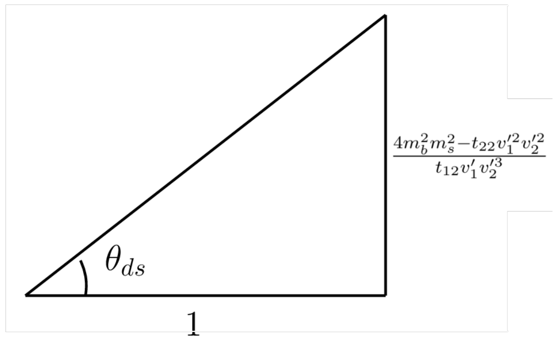


Figure **4-7**: triangle representation of the down-strange mixing.

Finally, putting all the results together the down-quark spectrum is:

$$m_d^2 \approx \frac{v_1'^2 v_2'^4 (t_{11} t_{22} - t_{12}^2)}{m_b^2 (t_{11} v_2'^2 + t_{22} v_1'^2)} \qquad m_s^2 \approx \frac{t_{11} v_2'^4 + t_{22} v_1'^2 v_2'^2}{2 m_b^2} \tag{4-109}$$

$$m_b^2 = \frac{1}{2} v_2'^2 \left((h_{2d}^{31})^2 + (h_{2d}^{32})^2 + (h_{2d}^{33})^2 \right) \qquad m_{J1}^2 = \frac{1}{2} v_\chi^2 (g_{\chi J}^{11})^2 \tag{4-110}$$

$$m_{J2}^2 = \frac{1}{2} v_\chi^2 (g_{\chi J}^{22})^2 \tag{4-111}$$

4.3.4 Lepton sector

Just as happened in the quark sector, the lepton superpotential comes from promoting fields into superfields in the non-SUSY counterpart potential [44]. Generating the same mass structure but with slight differences on the VEVs. It reads:

$$\begin{aligned} W_L = {} & \hat{\ell}_L^p \hat{\Phi}_2 h_{2\nu}^{pq} \hat{\nu}_L^{q\,c} - \hat{\ell}_L^p \hat{\Phi}_2' h_{2e}^{p\mu} \hat{e}_L^{\mu\,c} - \hat{\ell}_L^\tau \hat{\Phi}_2' h_{2e}^{\tau r} \hat{e}_L^{r\,c} - \hat{\ell}_L^p \hat{\Phi}_1' h_{1E}^p \hat{E}_L^c + \hat{E}_L \hat{\chi}' g_{\chi' E} \hat{E}_L^c \\ & - \hat{E}_L \mu_E \hat{\mathcal{E}}_L^c + \hat{\mathcal{E}}_L \hat{\chi} g_{\chi \mathcal{E}} \hat{\mathcal{E}}_L^c - \hat{\mathcal{E}}_L \mu_{\mathcal{E}} \hat{E}_L^c + \hat{\nu}_L^{j\,c} \hat{\chi}' h_\chi'^{N\,ij} \hat{N}_L^{i\,c} + \frac{1}{2} \hat{N}_L^{i\,c} M_{ij} \hat{N}_L^{j\,c} \\ & + \hat{E}_L \hat{\sigma} h_\sigma^{e^c p} \hat{e}_L^{cr} + \hat{\mathcal{E}}_L \hat{\sigma}' h_{\sigma'}^{e^c \mu} \hat{e}_L^{\mu c}, \end{aligned} \tag{4-112}$$

where $p = e, \mu$, $q = e, \mu, \tau$, $r = e, \tau$ and i, j label the right handed and Majorana neutrinos.

Charged lepton masses and 1-loop corrections

Once SSB takes place we can write the most general mass matrix as:

$$\mathcal{M}_E = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & h_{2e}^{e\mu} v_2' & 0 & h_{1e}^E v_1' & 0 \\ 0 & h_{2e}^{\mu\mu} v_2' & 0 & h_{1\mu}^E v_1' & 0 \\ h_{2e}^{\tau e} v_2' & 0 & h_{2e}^{\tau\tau} v_2' & 0 & 0 \\ \hline 0 & 0 & 0 & g_{\chi' E} v_\chi' & -\mu_E \\ 0 & 0 & 0 & -\mu_{\mathcal{E}} & g_{\chi \mathcal{E}} v_\chi \end{array} \right) \tag{4-113}$$

In the leptonic sector, we have two exotic singlets coupled by μ_E but the $\mathcal{E}$ fermion does not couple with any of the SM particles directly so it can be decoupled. The resulting 4×4 submatrix has almost exactly the same structure that up quarks masses. The difference lies in the absence of a coupling in the $(4,2)$ entry. As a consequence, rotations and mass eigenvalues can be obtained directly by comparison with the up-quark masses. Additionally, this tells us before hand that the electron turns out to be massless, so radiative corrections must be taken into account in this matrix. In figure **4-8** is shown the diagrams that contribute to the electron mass at one loop level

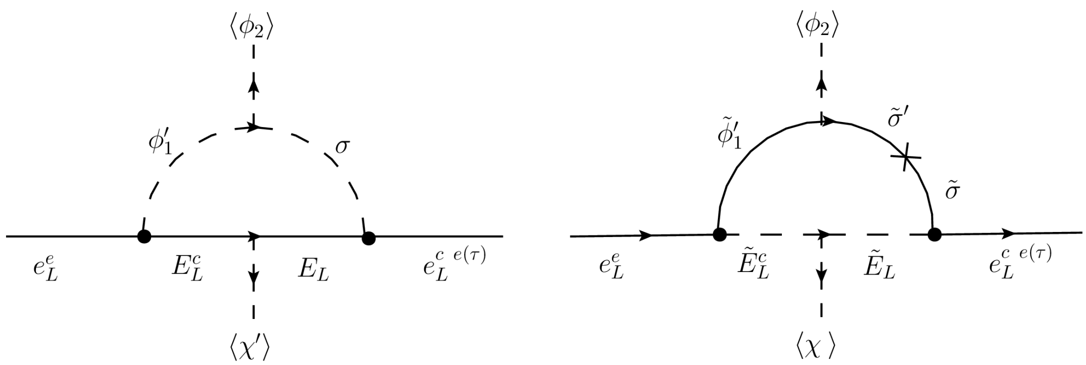


Figure **4-8**: One loop corrections to the leptons due to exotic fermions, sfermions and Higgsinos.

so the mass matrix preserves its similarity with up quarks mass matrix

$$\mathcal{M}_E^{1-Loop} = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} v_2\Sigma_{11} & h_{2e}^{e\mu} v_2' & v_2\Sigma_{13} & h_{1e}^{E} v_1' & 0 \\ 0 & h_{2e}^{\mu\mu} v_2' & 0 & h_{1\mu}^{E} v_1' & 0 \\ h_{2e}^{\tau e} v_2' & 0 & h_{2e}^{\tau\tau} v_2' & 0 & 0 \\ \hline 0 & 0 & 0 & g_{\chi' E} v_\chi' & -\mu_E \\ 0 & 0 & 0 & -\mu_{\mathcal{E}} & g_{\chi\mathcal{E}} v_\chi \end{array} \right) \tag{4-114}$$

The radiative corrections can be done thanks to the interactions terms given by:

$$W_\phi \Rightarrow \lambda_1 \Phi_1' \Phi_2 \sigma' - \mu_\sigma \hat{\sigma}' \hat{\sigma} - \mu_\chi \hat{\chi}' \hat{\chi} \qquad W_L \Rightarrow \hat{E}_L \hat{\sigma} h_\sigma^{e^c r} \hat{e}_L^{rc} + \hat{\ell}_L^e \hat{\Phi}_1' h_{1E}^e \hat{E}_L^c + \hat{E}_L \hat{\chi}' g_{\chi' E} \hat{E}_L^c, \tag{4-115}$$

where $r = e, \tau$, the couplings λ_1, $h_\sigma^{e^c k}$ and h_{1E}^e are dimensionless Yukawa coupling constants and μ_χ and μ_σ are mass unit parameters from the scalar potential. The first diagram in figure **4-8** illustrates the non-SUSY contribution which is given by:

$$v_2\Sigma^{NS}_{11(13)} = \frac{-1}{16\pi^2}\frac{v_2}{\sqrt{2}}\frac{\lambda_1\mu_\sigma h_\sigma^{e^c e(\tau)} h^e_{1E}}{M_E} C_0\left(\frac{m'_{h1}}{M_E}, \frac{m'_\sigma}{M_E}\right). \tag{4-116}$$

where M_E is the exotic charged fermion mass, m'_{h1} is the corresponding mass of the h'_1 field in flavor basis just like m'_σ is for the σ field and C_0 is the Veltmann-Passarino function evaluated for $p^2 = 0$ given by eq. (4-86). We recall that in this contribution a transformation to the mass eigenstate for the exotic fermion is not done because small mixing angles with SM particles are considered, making of this sector approximately decoupled. Furthermore, the SUSY contribution is given by:

$$v_2\Sigma^{S}_{11(13)}(p^2=0) = -\frac{1}{32\pi^2}\frac{v_2}{\sqrt{2}}\sum_{n=1}^{10}\sum_{k=1}^{2} Z_L^{9n} Z_L^{4n} Z_{\tilde{h}}^{10k} Z_{\tilde{h}}^{11k}\lambda_1\mu_\sigma h_\sigma^{e^c e(\tau)} h^e_{1E}\times \tag{4-117}$$

$$\times\left[\frac{(\tilde{m}_{\sigma k}+\tilde{m}'_{h_1})^2}{\tilde{M}^2_{L_n}} C_0\left(\frac{\tilde{m}'_{h1}}{\tilde{M}_{L_n}}, \frac{\tilde{m}_{\sigma k}}{\tilde{M}_{L_n}}\right) + \tilde{m}'^2_{h1} B_0(0, \tilde{m}'_\sigma, \tilde{M}_{L_n}) + \tilde{m}^2_{\sigma k} B_0(0, \tilde{m}'_{h1}, \tilde{M}_{L_n})\right]$$

where $\tilde{M}_{L_n}$ are the charged sleptons mass eigenvalues, $Z_{\tilde{h}}$ is the rotation matrix that connects $\tilde{\sigma}$ $(\tilde{\sigma'})$ with its mass eigenstates with eigenvalues $\tilde{m}_{hk}$ which are running inside the loop. Z_L is the rotation matrix that connects the exotic sleptons their respective eigenstates $\tilde{L}_n$ inside the loop. Mass terms without an index correspond to masses in the flavor basis i.e. the corresponding field diagonal element in the mass matrix. The resulting mass spectrum can be obtained by simply comparing it is given as follows:

$$m_e^2 = \frac{1}{2}v_2^2 v_2'^2 \frac{t_3^2}{2m_\tau^2} \qquad m_\mu^2 = \frac{1}{2}{v'_2}^2\left[(h^{e\mu}_{2e})^2 + (h^{\mu\mu}_{2e})^2\right], \tag{4-118}$$

$$m_\tau^2 = \frac{1}{2}{v'_2}^2\left[(h^{\tau e}_{2e})^2 + (h^{\tau\tau}_{2e})^2\right] \qquad m_E^2 = \frac{1}{2}g^2_{\chi' E}\ {v'_\chi}^2 \tag{4-119}$$

$$m_{\mathcal{E}}^2 = \frac{1}{2}g^2_{\chi\mathcal{E}}\ v_\chi^2 \tag{4-120}$$

with the associated rotation matrices given by:

$$V_1^\ell = \begin{pmatrix} 1 & 0 & 0 & -\frac{g_{\chi\mathcal{E}} h^E_{1e} v_\chi v'_1}{g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi - \mu_E\mu_{\mathcal{E}}} & -\frac{h^E_{1e}\mu_E v'_1}{\mu_E\mu_{\mathcal{E}} - g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi} \\ 0 & 1 & 0 & -\frac{g_{\chi\mathcal{E}} h^E_{1\mu} v_\chi v'_1}{g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi - \mu_E\mu_{\mathcal{E}}} & -\frac{h^E_{1\mu}\mu_E v'_1}{\mu_E\mu_{\mathcal{E}} - g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi} \\ 0 & 0 & 1 & 0 & 0 \\ \frac{g_{\chi\mathcal{E}} h^E_{1e} v_\chi v'_1}{g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi - \mu_E\mu_{\mathcal{E}}} & \frac{g_{\chi\mathcal{E}} h^E_{1\mu} v_\chi v'_1}{g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi - \mu_E\mu_{\mathcal{E}}} & 0 & 1 & 0 \\ \frac{h^E_{1e}\mu_E v'_1}{\mu_E\mu_{\mathcal{E}} - g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi} & \frac{h^E_{1\mu}\mu_E v'_1}{\mu_E\mu_{\mathcal{E}} - g_{\chi E} g_{\chi\mathcal{E}} v_\chi v'_\chi} & 0 & 0 & 1 \end{pmatrix} \tag{4-121}$$

$$V_2^\ell = \begin{pmatrix} 1 & 0 & -\frac{m_e^2}{t_3 v_2 v_2'} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \frac{m_e^2}{t_3 v_2 v_2'} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad t_3 = \Sigma_{11} h_{2e}^{\tau e} + \Sigma_{33} h_{2e}^{\tau\tau} \tag{4-122}$$

$$V_3^\ell = \begin{pmatrix} \cos\theta_{e\mu} & -\sin\theta_{e\mu} & 0 & 0 \\ \sin\theta_{e\mu} & \cos\theta_{e\mu} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \sin\theta_{e\mu} = -\frac{2m_\mu^2 - v_2'^2 (h_{2e}^{\mu\mu})^2}{h_{2e}^{e\mu} h_{2e}^{\mu\mu} v_2'^2 \sqrt{\left(\frac{v_2'^2 (h_{2e}^{\mu\mu})^2 - 2m_\mu^2}{h_{2e}^{e\mu} h_{2e}^{\mu\mu} v_2'^2}\right)^2 + 1}} \tag{4-123}$$

Despite v_2' it is suppressed by the adimensional factor Σ_{ij} so it can give mass the bottom quark, the electron and at tree level the μ and τ leptons. A estimate of some couplings can be done by considering the physical mass ratio of μ and τ masses which is approximately 0.14, then:

$$0.14 \approx \frac{\sqrt{(h_{2e}^{e\mu})^2 + (h_{2e}^{\mu\mu})^2}}{\sqrt{(h_{2e}^{\tau e})^2 + (h_{2e}^{\tau\tau})^2}} \tag{4-124}$$

Neutrino masses at tree level

Although charged leptons have a similar structure as for up-like quarks, neutrinos do not behave like down quarks. It is because, neutrino oscillations have proven for neutrinos to have mass while theory indicates a massless character. To date, many solutions have been raised to explain not only the existence but their very small value with the problem of not knowing if it is a Dirac or Majorana mass. Their small mass value and the absence of an observed right handed counterpart forces us to think in particles beyond actual observations that provide masses via (inverse)seesaw mechanism. In this scenario, both right handed and Majorana neutrinos are considered in the model arranged in a mass matrix in the basis $(\nu_L^q, \nu_L^{q\,C}, N_L^{i\,C})$, such matrix reads:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & M_M \end{pmatrix}, \tag{4-125}$$

where the block matrices are given by:

$$m_D = \frac{v_2}{\sqrt{2}} \begin{pmatrix} h_{2\nu}^{ee} & h_{2\nu}^{e\mu} & h_{2\nu}^{e\tau} \\ h_{2\nu}^{e\mu} & h_{2\nu}^{\mu\mu} & h_{2\nu}^{\mu\tau} \\ 0 & 0 & 0 \end{pmatrix}, \quad (M_D)^{ij} = \frac{v_\chi'}{\sqrt{2}} (h_\chi'^{\nu})^{ij}, \qquad (M_M)_{ij} = \frac{1}{2} M_{ij}. \tag{4-126}$$

The Inverse SeeSaw mechanism (ISS) works under the assumption of small Majorana coupling constants, $M_M \ll m_D \ll M_D$ [23]. Therefore, block diagonalization is done by a rotation matrix $\mathbb{V}_{SS}$ giving as a result a light and heavy majorana mass matrix in a block diagonal form:

$$\mathbb{V}_{SS}\mathcal{M}_\nu\mathbb{V}_{SS}^\dagger \approx \begin{pmatrix} m_{light} & 0 \\ 0 & m_{heavy} \end{pmatrix} \qquad \mathbb{V}_{SS} = \begin{pmatrix} I & -\Theta_\nu \\ \Theta_\nu^T & I \end{pmatrix} \tag{4-127}$$

$$\Theta_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_M \end{pmatrix}^{-1} \begin{pmatrix} m_D \\ 0 \end{pmatrix}, \tag{4-128}$$

where $m_{light} = m_D^T (M_D^T)^{-1} M_M (M_D)^{-1} m_D$ is the 3×3 mass matrix for the light left handed neutrinos and encodes the information of the PMNS matrix and m_{heavy} matrix involves the mixings of right handed and Majorana neutrinos, which is given by:

$$m_{heavy} \approx \begin{pmatrix} 0 & M_D^T \\ M_D & M_M \end{pmatrix}. \tag{4-129}$$

This structures are exactly the same as for the non-SUSY model and in a similar fashion we are going to consider the same scenario for neutrino masses. Even though, a different approach for the PMNS matrix reproduction will be shown in a further section. For simplicity and thinking in exotic neutrinos whose mass is big enough to be indistinguishable for us, we can take the particular case where M_D is diagonal and M_M is proportional to the identity to explore one of the possible scenarios of the model.

$$M_D = \frac{v_\chi}{\sqrt{2}} \begin{pmatrix} h_{N\chi 1} & 0 & 0 \\ 0 & h_{N\chi 2} & 0 \\ 0 & 0 & h_{\chi N 3} \end{pmatrix} \qquad M_M = \mu_N \mathbb{I}_{3\times 3}. \tag{4-130}$$

in this way, the light neutrino mass matrix takes the form

$$m_{\text{light}} = \frac{\mu_N v_2^2}{{h_{N\chi 1}}^2 v_\chi^2} \begin{pmatrix} (h_{2e}^{\nu e})^2 + \left(h_{2\mu}^{\nu e}\right)^2 \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu \mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \mu} \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu \mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \mu} \rho^2 & (h_{2e}^{\nu \mu})^2 + \left(h_{2\mu}^{\nu \mu}\right)^2 \rho^2 & h_{2e}^{\nu \mu} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu \mu} h_{2\mu}^{\nu \tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu \tau} \rho^2 & h_{2e}^{\nu \mu} h_{2e}^{\nu \tau} + h_{2\mu}^{\nu \mu} h_{2\mu}^{\nu \tau} \rho^2 & (h_{2e}^{\nu \tau})^2 + \left(h_{2\mu}^{\nu \tau}\right)^2 \rho^2 \end{pmatrix}, \tag{4-131}$$

where $\rho = h_{N\chi 1}/h_{N\chi 2}$. The same notation has been used for simplicity and facilitating the relationship between the non-SUSY and the current work. It turns out that the latter matrix m_{light} has zero determinant for every possible choice of M_D and M_M since m_D has null determinant, obtaining at least, one massless neutrino. Then, the diagonalization procedure comes from its singular value decomposition, providing a diagonal form whose entries are the positive square roots of the matrix $m_{light} m_{light}^\dagger$ which in general do not coincide with the m_{light} eigenvalues. From the characteristic polynomial for $m_{light} m_{light}^\dagger$ we can get an expression for the squared mass eigenvalues.

$$m_{\nu_1}^2 = 0 \qquad m_{\nu_2}^2 = \mu_\nu^2 \frac{A - \sqrt{A^2 - 4B}}{2} \qquad m_{\nu_3}^2 = \mu_\nu^2 \frac{A + \sqrt{A^2 - 4B}}{2} \tag{4-132}$$

$$m_{\nu_1}^2 = \mu_\nu^2 \frac{A + \sqrt{A^2 - 4B}}{2} \qquad m_{\nu_2}^2 = \mu_\nu^2 \frac{A - \sqrt{A^2 - 4B}}{2} \qquad m_{\nu_3}^2 = 0 \tag{4-133}$$

where

$$\begin{aligned}
A =& 2|{h_{2e}}^{\nu \mathrm{e}} {h_{2e}}^{\nu\mu} + \rho^2 {h_{2\mu}}^{\nu \mathrm{e}} {h_{2\mu}}^{\nu\mu}|^2 + 2|{h_{2e}}^{\nu \mathrm{e}} {h_{2e}}^{\nu\tau} + \rho^2 {h_{2\mu}}^{\nu \mathrm{e}} {h_{2\mu}}^{\nu\tau}|^2 \\
&+ |\left({h_{2e}}^{\nu \mathrm{e}}\right)^2 + \rho^2 \left({h_{2\mu}}^{\nu \mathrm{e}}\right)^2|^2 + 2|{h_{2e}}^{\nu\mu} {h_{2e}}^{\nu\tau} + \rho^2 {h_{2\mu}}^{\nu\mu} {h_{2\mu}}^{\nu\tau}|^2 \\
&+ |\left({h_{2e}}^{\nu\mu}\right)^2 + \rho^2 \left({h_{2\mu}}^{\nu\mu}\right)^2|^2 + |\left({h_{2e}}^{\nu\tau}\right)^2 + \rho^2 \left({h_{2\mu}}^{\nu\tau}\right)^2|^2 \\
B =& \rho^4 ({h_{2e}}^{\nu\tau} \left(h_{2e}^{\nu\tau *} \left(h_{2\mu}^{\nu \mathrm{e}} h_{2\mu}^{\nu \mathrm{e}*} + {h_{2\mu}}^{\nu\mu} h_{2\mu}^{\nu\mu *}\right) - {h_{2\mu}}^{\nu\tau *} \left(h_{2e}^{\nu \mathrm{e}*} h_{2\mu}^{\nu \mathrm{e}} + h_{2e}^{\nu\mu *} h_{2\mu}^{\nu\mu}\right)\right) \\
&+ h_{2e}^{\nu \mathrm{e}} \left({h_{2\mu}}^{\nu\mu} \left(h_{2e}^{\nu \mathrm{e}*} h_{2\mu}^{\nu\mu *} - h_{2e}^{\nu\mu *} h_{2\mu}^{\nu \mathrm{e}*}\right) + h_{2\mu}^{\nu\tau} \left(h_{2e}^{\nu \mathrm{e}*} h_{2\mu}^{\nu\tau *} - h_{2e}^{\nu\tau *} h_{2\mu}^{\nu \mathrm{e}*}\right)\right) \\
&+ h_{2e}^{\nu\mu} \left(h_{2\mu}^{\nu \mathrm{e}} \left(h_{2e}^{\nu\mu *} h_{2\mu}^{\nu \mathrm{e}*} - h_{2e}^{\nu \mathrm{e}*} h_{2\mu}^{\nu\mu *}\right) + h_{2\mu}^{\nu\tau} \left(h_{2e}^{\nu\mu *} h_{2\mu}^{\nu\tau *} - h_{2e}^{\nu\tau *} h_{2\mu}^{\nu\mu *}\right)\right))^2
\end{aligned} \tag{4-134}$$

where $\mu_\nu = \frac{\mu_N v_2^2}{{h_{N\chi 1}}^2 v_\chi^2}$. The mass spectrum has been written twice because the first line corresponds to normal ordering while the second line to inverse ordering which are pictorically shown in figure **4-3**. Despite we do not know the exact mass values, they must be in accordance with the squared mass difference shown in table **3-3**. Since the squared mass difference $m_{3\ell}$ is approximately the same for $\ell = 1, 2$, we can conclude that $m_{\nu 2}$ has a very small value in comparison with $m_{\nu 3}$ in the case of normal ordering implying $A^2 \gg 4B$. Thus, we can perform a Taylor series expansion resulting in the masses shown in Eq. (4-135). In the case of inverse ordering, the $m_{3\ell}$ means $A^2 - 4B \approx 0$ so $m_{\nu 1} \approx m_{\nu 2} \approx \frac{A}{2}$ both with a correction $\Delta = \sqrt{A^2 - 4B}$ resulting in the masses shown in Eq. (4-136). Finally, the squared mass differences are stated in table **4-3**

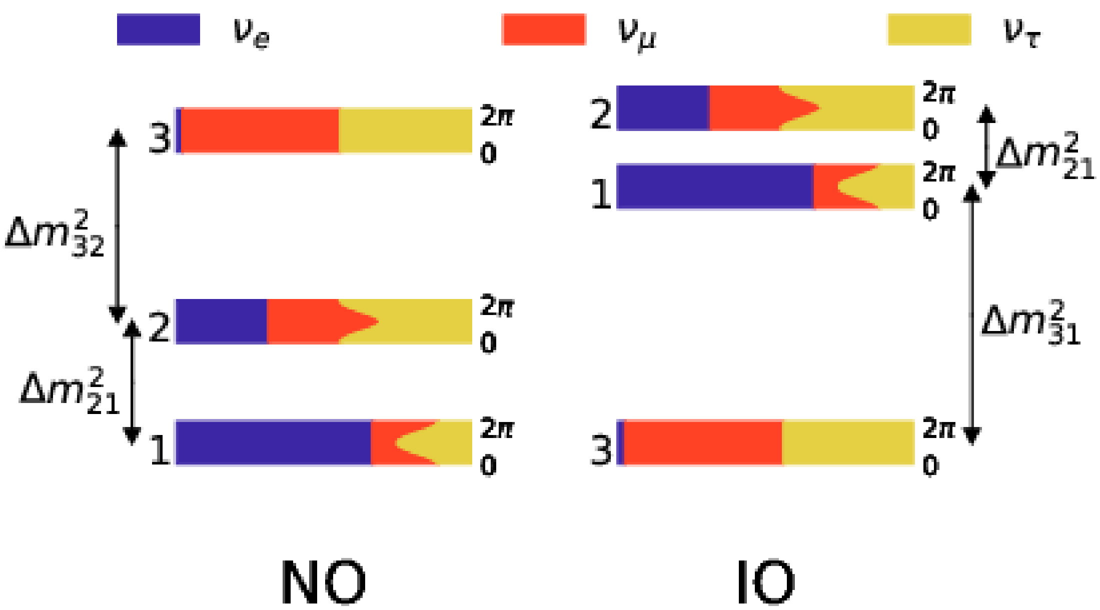


Figure **4-9**: Diagram for the neutrino mass eigenvalues hierarchy [2]

$$NO \qquad m_{\nu_1}^2 = 0 \qquad m_{\nu_2}^2 \approx \mu_\nu^2 \frac{B}{A} \qquad m_{\nu_3}^2 \approx \mu_\nu^2 A \tag{4-135}$$

$$IO \qquad m_{\nu_1}^2 = \mu_\nu^2 \frac{A-\Delta}{2} \qquad m_{\nu_2}^2 = \mu_\nu^2 \frac{A+\Delta}{2} \qquad m_{\nu_3}^2 = 0 \tag{4-136}$$

	Normal Ordering	Inverse Ordering
$\frac{\Delta m_{21}^2}{10^{-5} eV^2}$	$\mu_\nu^2 \frac{B}{A} = 7.39^{+0.21}_{-0.20}$	$\mu_\nu^2 \Delta \approx 7.39^{+0.21}_{-0.20}$
$\frac{\Delta m_{3\ell}^2}{10^{-3} eV^2}$	$\mu_\nu^2 \left(A - \frac{B}{A}\right) \approx +2.523^{0.032}_{-0.030}$	$-\frac{\mu_\nu^2 A}{2} \approx -2.509^{+0.032}_{-0.030}$

Table **4-3**: Conditions for reproducing the neutrino squared mass differences for normal and inverse ordering

4.4 Family mixing

In the previous section the fermion mass spectrum was determined analytically considering the observed physical masses hierarchy. Nevertheless, the yukawa couplings must also ensure the CKM and PMNS matrices reproduction which is not a trivial question despite the amount of parameters. There have been some works on fermion mass structures in order to determine the cases in which masses can be reproduced [28][29] or when the CKM and PMNS can [47] [46]. In the present work the problem of massless tree level particles is overcome thanks to the radiative corrections induced by the σ and σ' scalars and fermions these correction terms must be considered for an appropriate reproduction.

4.4.1 CKM matrix

In sections 4.3.2 and 4.3.3 mass diagonalization was done by specifying the rotation matrices, given in equations (4-65), (4-91) and (4-94) for up-like quarks and in equation (4-78), (4-102) and (4-108) for down-like quarks, then the final rotation matrices reads:

$$
\begin{aligned}
V &= V_3 V_2 V_1 \\
&= \begin{pmatrix}
\cos\theta_{uc} & \sin\theta_{uc} & \cos\theta_{uc} r_3 v_1 v_1' & -\frac{v_2 \cos\theta_{uc} r_{\chi 1}^2 + \sin\theta_{uc} r_{\chi 2}^2 v_1}{v_\chi'} \\
-\sin\theta_{uc} & \cos\theta_{uc} & -\sin\theta_{uc} r_3 v_1 v_1' & \frac{\sin\theta_{uc} r_{\chi 1}^2 v_2 - \cos\theta_{uc} r_{\chi 2}^2 v_1}{v_\chi'} \\
-r_3 v_1 v_1' & 0 & 1 & \frac{r_3 r_{\chi 1}^2 v_1 v_1' v_2}{v_\chi'} \\
\frac{r_{\chi 1}^2 v_2}{v_\chi'} & \frac{r_{\chi 2}^2 v_1}{v_\chi'} & 0 & 1
\end{pmatrix}
\end{aligned}
\tag{4-137}
$$

$$
\begin{aligned}
U &= U_3 U_2 U_1 \\
&= \begin{pmatrix}
\cos\theta_{ds} & \sin\theta_{ds} & -\frac{1}{2} v_2' \left(\sin\theta_{ds} l_2 v_1' + l_1 v_2' \cos\theta_{ds}\right) & -\frac{\sin\theta_{ds} n_3 v_2' + n_1 v_1' \cos\theta_{ds}}{v_\chi} & -\frac{\sin\theta_{ds} n_4 v_2' + n_2 v_1' \cos\theta_{ds}}{v_\chi} \\
-\sin\theta_{ds} & \cos\theta_{ds} & \frac{1}{2} v_2' \left(\sin\theta_{ds} l_1 v_2' - \cos\theta_{ds} l_2 v_1'\right) & \frac{\sin\theta_{ds} n_1 v_1' - \cos\theta_{ds} n_3 v_2'}{v_\chi} & \frac{\sin\theta_{ds} n_2 v_1' - \cos\theta_{ds} n_4 v_2'}{v_\chi} \\
\frac{l_1 v_2'^2}{2} & \frac{l_2 v_1' v_2'}{2} & 1 & -\frac{(l_1 n_1 + l_2 n_3) v_1' v_2'^2}{2 v_\chi} & -\frac{(l_1 n_2 + l_2 n_4) v_1' v_2'^2}{2 v_\chi} \\
\frac{n_1 v_1'}{v_\chi} & \frac{n_3 v_2'}{v_\chi} & 0 & 1 & 0 \\
\frac{n_2 v_1'}{v_\chi} & \frac{n_4 v_2'}{v_\chi} & 0 & 0 & 1
\end{pmatrix}
\end{aligned}
\tag{4-138}
$$

Despite these rotation matrices have different dimensions, the product can be done if V is extended by one row and one column full of zeros. By performing that product, taking only the first three rows and columns the theoretical CKM matrix reads:

$$
\begin{aligned}
V^{CKM} &= V U^\dagger \\
&= \begin{pmatrix}
\cos(\theta_{ds} - \theta_{uc}) & \sin(\theta_{ds} - \theta_{uc}) & V_{13} \\
-\sin(\theta_{ds} - \theta_{uc}) & \cos(\theta_{ds} - \theta_{uc}) & \frac{1}{2}\cos\theta_{uc} l_2 v_1' v_2' - \frac{1}{2}\sin\theta_{uc} l_1 v_2'^2 \\
V_{31} & \frac{1}{2} v_2' \left(\sin\theta_{ds} l_1 v_2' - \cos\theta_{ds} l_2 v_1'\right) & 1
\end{pmatrix}
\end{aligned}
\tag{4-139}
$$

$$
\begin{aligned}
V_{13} &= \frac{1}{2} l_1 \cos\theta_{uc} v_2'^2 + \frac{1}{2}\sin\theta_{uc} l_2 v_1' v_2' - r_3 v_1 v_1' \cos\theta_{uc} \\
V_{31} &= \cos\theta_{ds} r_3 v_1 v_1' - \frac{1}{2} v_2' (\sin\theta_{ds} l_2 v_1' + l_1 v_2' \cos\theta_{ds})
\end{aligned}
$$

where V_{13} and V_{31} have been defined for shortening the expression. We neglect the contributions proportional to n_i, $i = 1, 2, 3, 4$ and $r_{\chi j}$, $j = 1, 2$. because they are related with factor of v_χ^{-1} and $v_\chi'^{-1}$ which are highly suppressed. Moreover, since r_3 is a factor dependent on the radiative corrections it has a small value which can be neglected in entries 11, 12, 21, 22 and 33. Finally, the smallness of the CKM angles leads to the Wolfenstein parametrization shown in Eq. (3-42) so the r_3 contribution in the entries 23 and 32 is neglected as well to be consistent with it. Now, we are going to present the conditions over the parameters that reproduces the CKM matrix and quark masses.

From mass eigenstates, we choose the following parameters as random to parametrize the matrix:

$$0 < h^2_{\chi' u}, g_{\chi' T}, h^1_{2T}, h^2_{1T}, h^{33}_{1u} < 1 \tag{4-140}$$

$$h^1_{2T} h^2_{\chi' u} - \sqrt{\frac{2m_c^2 (g^2_{\chi' T} + (h^2_{\chi' u})^2)}{v_2^2}} < h^{12}_{2u} g_{\chi' T} < h^1_{2T} h^2_{\chi' u} + \sqrt{\frac{2m_c^2 (g^2_{\chi' T} + (h^2_{\chi' u})^2)}{v_2^2}} \tag{4-141}$$

where the second condition comes from the requirement of getting only real parameters. Given those parameters, then the other up-quarks parameters are fully determined by:

$$h^{22}_{1u} = \frac{1}{g_{\chi' T}} \left(\sqrt{\frac{2m_c^2 (g^2_{\chi' T} + (h^2_{\chi' u})^2) - v_2^2 (h^1_{2T} h^2_{\chi' u} - h^{12}_{2u} g_{\chi' T})^2}{v_1^2}} + h^2_{1T} h^2_{\chi' u} \right) \tag{4-142}$$

$$h^{31}_{1u} = \sqrt{\frac{2m_t^2}{v_1^2} - (h^{33}_{1u})^2} \tag{4-143}$$

$$\sin\theta_{uc} = \frac{1}{\sqrt{\frac{\left(r_2^2 v_1^2 - 2m_u^2\right)^2}{r_1^2 r_2^2 v_1^2 v_2^2} + 1}} \tag{4-144}$$

Then, by imposing r_3 to recreate the imaginary part in CKM matrix entries 13 and 31 it can be written:

$$r_3 = \frac{i2A\eta\lambda^3}{v_1 \left(\cos\theta_{ds} + \cos\theta_{uc}\right) v_1'} \tag{4-145}$$

r_3 also enters in the up quark mass so the radiative contributions Σ_1 and Σ_3 are complex numbers that must satisfy the following set of equations in order to avoid an imaginary mass:

$$Im[\Sigma_1] h^{31}_{1u} + Im[\Sigma_3] h^{33}_{1u} = 2m_t^2 r_3 \quad Re[\Sigma_1] h^{31}_{1u} + Re[\Sigma_3] h^{33}_{1u} = 0 \tag{4-146}$$

$$Im[\Sigma_3] h^{31}_{1u} - Im[\Sigma_1] h^{33}_{1u} = 0 \qquad Re[\Sigma_3] h^{31}_{1u} - Re[\Sigma_1] h^{33}_{1u} = \frac{2\sqrt{m_t^2 m_u^2}\sqrt{2m_u^2 - r_1^2 v_2^2 - r_2^2 v_1^2}}{v_1 v_1' \sqrt{2m_u^2 - r_2^2 v_1^2}}$$

The equation system always have a solution and as a consequence fully determines r_4 defined in Eq. (4-92) that ensures the correct up quark mass. It is also worth to mention that the up quark sector can be parametrized by using only the eigenvalues and r_4 but we recall that first the Yukawa couplings must ensure the mass eigenvalues. Next, the imposition of the 11 of the CKM matrix provides a relationship between θ_{uc} and θ_{ds} which read (and is shown in figure **4-10**)s:

$$sin\theta_{ds} = \sin\theta_{uc}V_{11}^{CKM} \pm \sqrt{\sin^2\theta_{uc}V_{11}^{CKM\,2} - V_{11}^{CKM\,2} - \sin^2\theta_{uc} + 1} \tag{4-147}$$

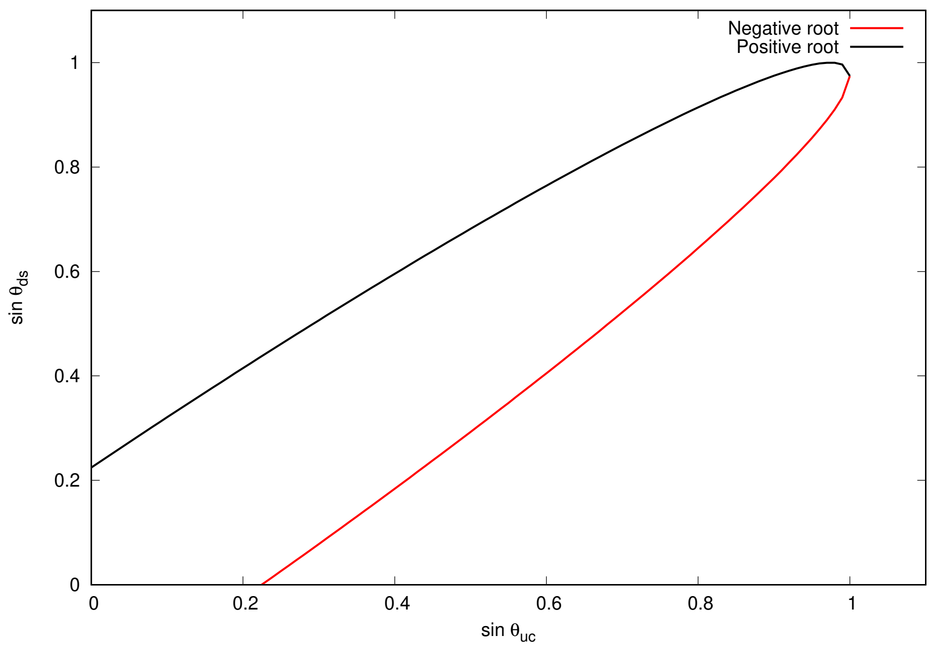


Figure **4-10**: Relationship between θ_{uc} and θ_{ds} with $V_{11}^{CKM} = 0.97420 \pm 0.00021$ [63]. 5σ region is very small to be observed.

In the case of down-like quarks, it can be parametrized by using only the mass eigenvalues and the parameter t_{22} defined in Eq. (4-105), the latter, t_{11} and t_{12} which can be rewritten as:

$$t_{22} = \frac{4\left(m_b^2 m_d^2 \sin^2\theta_{ds} - m_b^2 m_s^2 \sin^2\theta_{ds} + m_b^2 m_s^2\right)}{v_1'^2 v_2'^2} \tag{4-148}$$

$$t_{11} = \frac{4m_b^2 m_d^2 + 4m_b^2 m_s^2 - v_1'^2 v_2'^2 t_{22}}{v_2'^4} \tag{4-149}$$

$$t_{12} = \frac{\sqrt{t_{22}v_1'^2 v_2'^2 - 4m_b^2 m_d^2}\sqrt{4m_b^2 m_s^2 - t_{22}v_1'^2 v_2'^2}}{v_1' v_2'^3} \tag{4-150}$$

Since the latter values become uniquely determined, they represent a system of three equations for the radiative corrections:

$$\frac{2m_b^2}{v_2'^2}(\Sigma_{21}^2+\Sigma_{22}^2+\Sigma_{23}^2)-l_2^2m_b^2=t_{22} \tag{4-151}$$

$$\frac{2m_b^2}{v_2'^2}(\Sigma_{11}^2+\Sigma_{12}^2+\Sigma_{13}^2)-l_1^2m_b^2=t_{11} \tag{4-152}$$

$$(\Sigma_{11}\Sigma_{21}+\Sigma_{12}\Sigma_{22})\frac{2m_b^2}{v_2'^2}-l_1l_2m_b^2 \tag{4-153}$$

where l_1 and l_2 are determined from of entries 13 and 23 giving:

$$l_1=\frac{2(V_{23}^{CKM})\sin\theta_{uc}-Re[V_{13}^{CKM}]\cos\theta_{uc}}{v_2'^2} \qquad l_2=\frac{2(Re[V_{13}^{CKM}])\sin\theta_{uc}-V_{23}^{CKM}\cos\theta_{uc}}{v_1'v_2'} \tag{4-154}$$

In order to solve the 3×3 equation we choose three parameters whose value can be random, particulary we choose Σ_{13}, Σ_{23} and Σ_{22}. However, the equation system has solution only when these parameters are in the interval:

$$0<\Sigma_{13},\Sigma_{23},\Sigma_{22}<10^{-4} \tag{4-155}$$

The equation system is solved numerically since the analytical results are quite long to be presented, with these results, we can now solve the equations related with l_1, l_2 and the bottom mass, which reads:

$$m_b^2=\frac{v_2'^2}{2}((h_{2d}^{31})^2+(h_{2d}^{32})^2+(h_{2d}^{33})^2) \tag{4-156}$$

$$l_1=\frac{(\Sigma_{11}h_{2d}^{31}+\Sigma_{12}h_{2d}^{32}+\Sigma_{13}h_{2d}^{33})}{m_b^2} \tag{4-157}$$

$$l_2=\frac{(\Sigma_{21}h_{2d}^{31}+\Sigma_{22}h_{2d}^{32}+\Sigma_{23}h_{2d}^{33})}{m_b^2} \tag{4-158}$$

These equation system now provides the values of h_{2d}^{31}, h_{2d}^{32} and h_{2d}^{33}.
The latter conditions and procedure ensures the correct reproduction of the CKM matrix, leaving all other parameters to run freely. In this case they are:

$$0<h_{1J}^{11},h_{1J}^{12},h_{2J}^{21},h_{2J}^{22},g_{\chi J}^{11},g_{\chi J}^{12},g_{\chi J}^{21},g_{\chi J}^{22}<1 \tag{4-159}$$

However, small values of there parameters are recommended in order to ensure small corrections due to the first seesaw rotation U_1. All in all, from 27 different non zero entries in quark mass matrices, only 1 turns out to be complex, and 17 can be random numbers, which in general parametrizes the theoretical CKM matrix, mainly from the interactions with the exotic quarks. This result is nothing to be surprised since the number of variables and constrains tell us that before hand but in general provides an analytical approach that can be simplified to the case where less than 6 VEVs are considered. furthermore, the previous development was implemented in a small program on Mathematica 11, both numerically and theoretical. In both cases the CKM matrix was reproduced with exact values confirming our assumptions and developments.

4.4.2 PMNS Matrix

Now, in this section the procedure that describes the relationship among Yukawa couplings and PMNS parameters is described. First we focus our attention on the charged leptons, from the exact mass eigenvalues expressions we have the following restrictions:

$$h_{2e}{}^{\tau\tau} = \sqrt{\frac{2m_\tau^2}{v_2'^2} - \left(h_{2e}{}^{\tau\mathrm{e}}\right)^2} \tag{4-160}$$

$$h_{2e}{}^{\mathrm{e}\mu} = \frac{\sqrt{4m_\tau^2 m_\mu^2 - r_4^2 v_2^2 v_2'^2}\sqrt{2m_\mu^2 - \frac{8m_e^2 m_\tau^2 m_\mu^2}{r_4^2 v_2^2 v_2'^2}}}{2\sqrt{m_\tau^2 m_\mu^2}\, v_2'} \tag{4-161}$$

$$h_{2e}{}^{\mu\mu} = \frac{2\sqrt{2m_e^2 m_\tau^2 m_\mu^2}}{r_4 v_2 v_2'^2} \tag{4-162}$$

so in general they can be parametrized in terms of two parameters, r_4 and $h_{2e}^{\tau e}$, which means that couplings regarding interactions with exotic fermions can take a random value, which in general was considered as:

$$0 < g_{\chi E}, h_{1e}^{E}, h_{1\mu}^{E}, \mu_E, \mu_{\mathcal{E}}, g_{\chi\mathcal{E}} < 1 \tag{4-163}$$

Consequently, the electron-muon mixing angle is parametrized by r_4 as well by rewriting Eq. (4-123) in terms of the electron mass as:

$$\sin\theta_{e\mu} = -\frac{1}{\sqrt{\left(\frac{(h_{2e}{}^{\mu\mu})^2 - \frac{2m_e^2}{v_2'^2}}{h_{2e}{}^{\mathrm{e}\mu} h_{2e}{}^{\mu\mu}}\right)^2 + 1}} \tag{4-164}$$

This implies a restriction for the r_4 in order to being able of recreating all possible angles, a graph is shown in figure **4-11**. Consequently, r_4 is taken as a random value in the interval. In a similar fashion, Σ_1 is taken as a random number which together with r_4 determines uniquely Σ_3 and r_3 as follows:

$$\frac{2\sqrt{m_e^2}\sqrt{m_\tau^2}}{v_2 v_2'} < r_4 < \frac{2\sqrt{m_\mu^2}\sqrt{m_\tau^2}}{v_2 v_2'} \qquad 0 < \Sigma_1 < 10^{-4} \tag{4-165}$$

$$\Sigma_3 = \frac{\Sigma_1 {h_{2e}}^{\tau\tau} + r_4}{{h_{2e}}^{\tau \mathrm{e}}} \qquad r_3 = \frac{\Sigma_1 {h_{2e}}^{\tau \mathrm{e}} + \Sigma_3 {h_{2e}}^{\tau\tau}}{2m_\tau^2} \tag{4-166}$$

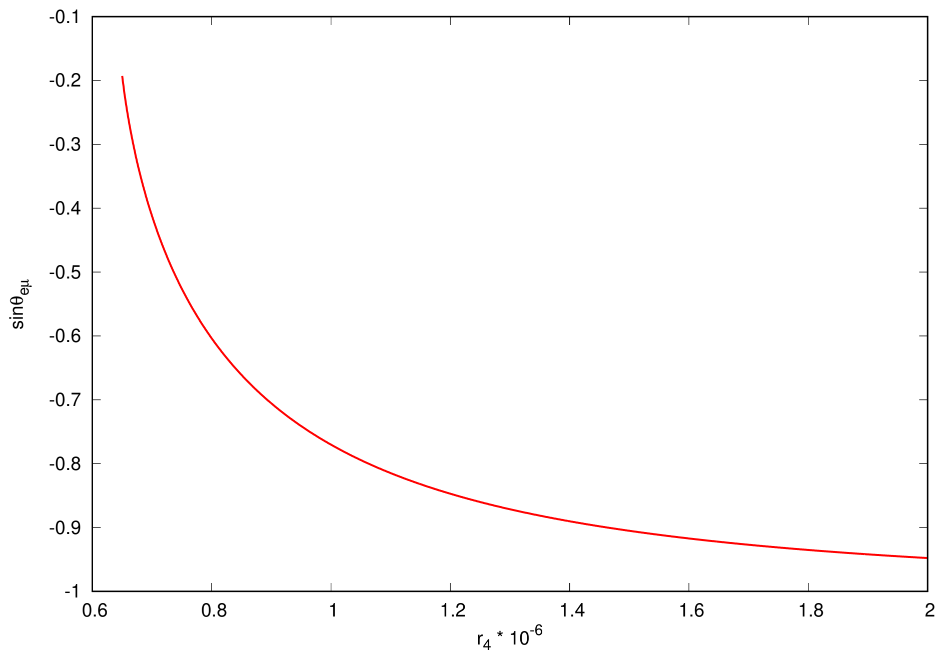


Figure **4-11**: $\sin\theta_{e\mu}$ as a function of r_4

In the case of $h_{2e}^{\tau e}$ it is taken as a random number in the interval $0 < h_{2e}^{\tau e} < 0.1$ for preventing $h_{2e}^{\tau\tau}$ to become imaginary. Then, by giving a random value to 9 parameters, where six are related to exotic fermions, we can reproduce the mass spectrum and generate all possible mixing angles for the eletron and muon particles. Leaving completelly determined the rotation matrix $V^\ell = V_3^\ell V_2^\ell V_1^\ell$. However, In section 4.3.4 the rotations matrices that diagonalizes the mass matrix via two seesaw rotations and a electron-muon mixing angle are given in Eqs. (4-121), (4-122) and (4-123). So the total rotation matrix reads:

$$V^{\ell} = V_3^{\ell} V_2^{\ell} V_1^{\ell}$$
$$= \begin{pmatrix} \cos\theta_{e\mu} & \sin\theta_{e\mu} & -\cos\theta_{e\mu} r_3 v_2 v_2' & -\sin\theta_{e\mu} r_{e\chi 3} + r_{e\chi 1}(-\cos\theta_{e\mu}) & -\sin\theta_{e\mu} r_{e\chi 4} - r_{e\chi 2}\cos\theta_{e\mu} \\ -\sin\theta_{e\mu} & \cos\theta_{e\mu} & \sin\theta_{e\mu} r_3 v_2 v_2' & \sin\theta_{e\mu} r_{e\chi 1} - \cos\theta_{e\mu} r_{e\chi 3} & \sin\theta_{e\mu} r_{e\chi 2} - \cos\theta_{e\mu} r_{e\chi 4} \\ r_3 v_2 v_2' & 0 & 1 & -r_3 r_{e\chi 1} v_2 v_2' & -r_3 r_{e\chi 2} v_2 v_2' \\ r_{e\chi 1} & r_{e\chi 3} & 0 & 1 & 0 \\ r_{e\chi 2} & r_{e\chi 4} & 0 & 0 & 1 \end{pmatrix} \tag{4-167}$$

where $r_{e\chi k},\ k = 1,2,3,4$ are the entries in the matrix V_1^{ℓ} which are proportional to $v_{\chi}^{(\prime)-1}$ which makes these contributions negligible. In contrast, the light neutrino mass matrix does not have a definite rotation matrix since we can not assume a hierarchy among parameters. Furthermore, since we are dealing with a Majorana mass matrix the diagonalization procedure slightly changes.

In section 3.4 we saw that PMNS matrix can be written as $V_{PMNS} = V^{\ell} V_{\nu}^{\dagger}$ where $V_{\nu}^{\dagger}$ is a unitary transformation that diagonalices the Majorana mass matrix m_{light} according to

$$m_{light}^{diag} = V_{\nu}^{*} m_{light} V_{\nu}^{\dagger} \tag{4-168}$$

the latter transformation is different because the mass matrix is in general complex and symmetric, but non hermitian. Thus, its diagonalization is done by considering its singular values m_k which are defined as the eigenvalues positive square root of the matrix $m_{light} m_{light}^{\dagger}$ [19] and coincide with the m_{light} eigenvalues iff the matrix is real.

Nevertheless, a general unitary PMNS parametrization can be written as

$$V_{PMNS}' = V^{\ell} V_{\nu}^{\dagger} \equiv P_{\ell} U P \tag{4-169}$$

where

$$P_{\ell} = \begin{pmatrix} e^{e\phi_e} & 0 & 0 \\ 0 & e^{i\phi_\mu} & 0 \\ 0 & 0 & e^{i\phi_\tau} \end{pmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i(\phi_3+\delta_{CP})} \end{pmatrix};\begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4-170}$$

being P a Majorana phase matrix [30], shown with two parametrizations, and U is the PMNS matrix in the standard parametrization shown in Eq. (3-44). In a very stringent scenario, the maximum number of independent parameters that might be present in an unitary matrix is 9. However, since we can rephase charged leptons (and neutral leptons in the case of Dirac masses) we can reduce the number of independent parameters to 6 (4) thanks to the fact that only phase differences are physical. Therefore, equation (4-169) indicates that the PMNS matrix is defined as a $V^{\ell} V_{\nu}^{\dagger}$ matrix in the original basis that we choose to parametrice as $P_{\ell} U P$. Consequently, the problem lies on determining the number of independent parameters present in $V^{\ell} V_{\nu}^{\dagger}$ to know the true number of parameters contained in V_{PMNS}'.

In fact, we do not truly know what is the real number of parameters present in the PMNS matrix, mainly because the phases in P_ℓ are unphysical but their exact value has to be known to perform the diagonalization. In general, we can consider a PMNS matrix who is dependent on 6 parameters (3 mixing angles, 1 CP phase and 2 Majorana phases) so a rephasing in the charged leptons eigenbasis is not needed which is the usual assumption. Nevertheless, in the most general case we can rephase them (i.e. a basis change) to get rid of P_ℓ. Although mass terms in the mass basis are invariant under a phase shift (which is not the case in the flavor basis since it introduces complex phases into the mass matrix and consequently makes no longer true the already mentioned diagonalization procedure) it doesn't leave invariant the V'_{PMNS} matrix but introduces a basis in which only the physical parameters are present despite the diagonalization must be done in the original basis as noted in [67]. In this new basis the PMNS matrix is reduced to:

$$V_{PMNS} = P_\ell V'_{PMNS} = UP = P_\ell V^\ell V_\nu^\dagger \tag{4-171}$$

In seems that we have introduced 3 new parameters in the right side of the equation. However, we need to remember that P_ℓ was a convenient parametrization, so before considering the exact form of the right hand side we need to know the values of the phases present in P_ℓ, which is not an easy task. In fact, we cannot remove completely the unphysical charged leptons phases because charged leptons are not diagonal in the flavor basis. Now, we can consider $V_\nu = V'^\dagger_{PMNS} V^\ell$ which imply that the PMNS matrix diagonalices the rotated neutrino mass matrix $V^{\ell *} m_{light} V^{\ell\dagger}$ and represents the relative rotation between neutrinos and charged leptons mass eigenstates. Taking it into account, the scheme goes from the diagonal form to the flavor basis matrix.

Consider the diagonalized mass matrix m_{light}^{diag}, where m_k can be the real or complex (if we include input Majorana phases) we unrotate this matrix by applying the PMNS matrix with the experimental values and then the inverse rotation of V^ℓ resulting in a numerical matrix M^ν which must be equal to m_{light} shown in Eq. (4-131)

$$\begin{aligned}
M^\nu &= V^{\ell T} P_\ell^T U^* m_{light}^{diag} U^\dagger P_\ell V^\ell \\
&= \begin{pmatrix} M_{11}^\nu & M_{12}^\nu & M_{13}^\nu \\ * & M_{22}^\nu & M_{23}^\nu \\ * & * & M_{33}^\nu \end{pmatrix} \\
&= \frac{\mu_N v_2^2}{{h_{N\chi 1}}^2 v_\chi^2} \begin{pmatrix} (h_{2e}^{\nu e})^2 + \left(h_{2\mu}^{\nu e}\right)^2 \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu\mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\mu} \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu\mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\mu} \rho^2 & (h_{2e}^{\nu\mu})^2 + \left(h_{2\mu}^{\nu\mu}\right)^2 \rho^2 & h_{2e}^{\nu\mu} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu\mu} h_{2\mu}^{\nu\tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\tau} \rho^2 & h_{2e}^{\nu\mu} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu\mu} h_{2\mu}^{\nu\tau} \rho^2 & (h_{2e}^{\nu\tau})^2 + \left(h_{2\mu}^{\nu\tau}\right)^2 \rho^2 \end{pmatrix}, \\
&\equiv \begin{pmatrix} (h_{2e}^{\nu e})^2 + \left(h_{2\mu}^{\nu e}\right)^2 \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu\mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\mu} \rho^2 & h_{2e}^{\nu e} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu\mu} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\mu} \rho^2 & (h_{2e}^{\nu\mu})^2 + \left(h_{2\mu}^{\nu\mu}\right)^2 \rho^2 & h_{2e}^{\nu\mu} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu\mu} h_{2\mu}^{\nu\tau} \rho^2 \\ h_{2e}^{\nu e} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\tau} \rho^2 & h_{2e}^{\nu\mu} h_{2e}^{\nu\tau} + h_{2\mu}^{\nu\mu} h_{2\mu}^{\nu\tau} \rho^2 & (h_{2e}^{\nu\tau})^2 + \left(h_{2\mu}^{\nu\tau}\right)^2 \rho^2 \end{pmatrix},
\end{aligned} \tag{4-172}$$

where $m_{light}^{diag} = diag(0, m_2 e^{2i\phi_2}, m_3 e^{2i(\phi_3+\delta_{CP})})$ for normal ordering and $m_{light}^{diag} = diag(m_1 e^{2i\alpha_1}, m_2 e^{2i\alpha_2}, 0)$ for inverse ordering, the factors $\frac{\mu_N v_2^2}{h_{N\chi^1}{}^2 v_\chi^2}$ and ρ^2 were removed by a redefinition of the coupling constants and we consider that the parameters can be complex in general in order to reproduce the PMNS matrix.

The above matrix M^ν is complex in all their entries whether Majorana phases are present or not. For that reason, at least 3 parameters must be complex, one in each column of m_D. The general purpose is to show that the model is able to reproduce the PMNS matrix, and for that reason we are going to consider this minimal case just like the CKM matrix where we were interested in the minimum number of complex parameters, for that reason we are going to consider this minimal scenario for a PMNS matrix that can be parametrized with 4 parameters i.e. there are no charged lepton phases. Writing the matrix in terms of magnitudes and phases it reads:

$$M \equiv \begin{pmatrix} (h_{2e}^{\nu e})^2 e^{2i\alpha} + (h_{2\mu}^{\nu e})^2 & h_{2e}^{\nu e} h_{2e}^{\nu\mu} e^{i(\alpha+\beta)} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\mu} & h_{2e}^{\nu e} h_{2e}^{\nu\tau} e^{i(\alpha+\gamma)} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\tau} \\ h_{2e}^{\nu e} h_{2e}^{\nu\mu} e^{i(\alpha+\beta)} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\mu} & (h_{2e}^{\nu\mu})^2 e^{2i\beta} + (h_{2\mu}^{\nu\mu})^2 & h_{2e}^{\nu\mu} h_{2e}^{\nu\tau} e^{i(\beta+\gamma)} + h_{2\mu}^{\nu\mu} h_{2\mu}^{\nu\tau} \\ h_{2e}^{\nu e} h_{2e}^{\nu\tau} e^{i(\alpha+\gamma)} + h_{2\mu}^{\nu e} h_{2\mu}^{\nu\tau} & h_{2e}^{\nu\mu} h_{2e}^{\nu\tau} e^{i(\beta+\gamma)} + h_{2\mu}^{\nu\mu} h_{2\mu}^{\nu\tau} & (h_{2e}^{\nu\tau})^2 e^{2i\gamma} + (h_{2\mu}^{\nu\tau})^2 \end{pmatrix}, \tag{4-173}$$

where the first row of m_D is made of complex numbers. Nevertheless, even if all parameters in m_D are complex it can always be rewritten as the above matrix by doing some new definitions, which only adds more algebra. However, the problem lies in solving the following system of equations

$$(h_{2e}^{\nu e} h_{2e}^{\nu\mu})^2 = (Re[M_{12}^\nu] - h_{2\mu}^{\nu e} h_{2\mu}^{\nu\mu})^2 + Im[M_{12}^\nu]^2 \tag{4-174}$$
$$(h_{2e}^{\nu e} h_{2e}^{\nu\tau})^2 = (Re[M_{13}^\nu] - h_{2\mu}^{\nu e} h_{2\mu}^{\nu\tau})^2 + Im[M_{13}^\nu]^2 \tag{4-175}$$
$$(h_{2e}^{\nu\mu} h_{2e}^{\nu\tau})^2 = (Re[M_{23}^\nu] - h_{2\mu}^{\nu\mu} h_{2\mu}^{\nu\tau})^2 + Im[M_{23}^\nu]^2 \tag{4-176}$$

$$(h_{2e}^{\nu e})^4 = (Re[M_{11}^\nu] - (h_{2\mu}^{\nu e})^2)^2 + Im[M_{11}^\nu]^2 \tag{4-177}$$
$$(h_{2e}^{\nu\mu})^4 = (Re[M_{22}^\nu] - (h_{2\mu}^{\nu\mu})^2)^2 + Im[M_{22}^\nu]^2 \tag{4-178}$$
$$(h_{2e}^{\nu\tau})^4 = (Re[M_{33}^\nu] - (h_{2\mu}^{\nu\tau})^2)^2 + Im[M_{33}^\nu]^2 \tag{4-179}$$

which provides several solutions for the real magnitude values. Unfortunately Mathematica was unable of solving the system of 6 equations so the last three were replaced in the first three, becoming a 3×3 system that the software was able to solve. additionally, the phases are obtained by:

$$\tan(2\alpha) = \frac{Im[M^\nu_{11}]}{Re[M^\nu_{11}] - (h^{\nu e}_{2\mu})^2} \quad (4\text{-}180)$$
$$\tan(2\beta) = \frac{Im[M^\nu_{22}]}{Re[M^\nu_{22}] - (h^{\nu\mu}_{2\mu})^2} \quad (4\text{-}181)$$
$$\tan(2\gamma) = \frac{Im[M^\nu_{33}]}{Re[M^\nu_{33}] - (h^{\nu\tau}_{2\mu})^2} \quad (4\text{-}182)$$

so the parameters can be found given a diagonal form for the mass matrix. Regarding the diagonal matrix, we only know mass differences and $m^2_{32} \approx m^2_{31} \equiv m^2_{3l}$ which means there are two possibilities for the mass eigenvalues since we do not know exactly which mass difference m^2_{3l} really is, then the possible mass eigenvalues are shown in table **4-4**.

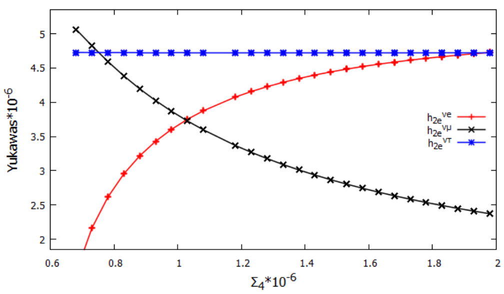
	m_1	m_2	m_3
NO	0	$\sqrt{m^2_{21}}$	$\sqrt{m^2_{3l}}$
NO	0	$\sqrt{m^2_{21}}$	$\sqrt{m^2_{3l} + m^2_{21}}$
IO	$\sqrt{-m^2_{3l}}$	$\sqrt{m^2_{21} - m^2_{3l}}$	0
IO	$\sqrt{-m^2_{3l} - m^2_{21}}$	$\sqrt{-m^2_{3l}}$	0

Table **4-4**: Neutrino mass eigenvalues for Normal and Inverse Ordering for a theory with one massless neutrino

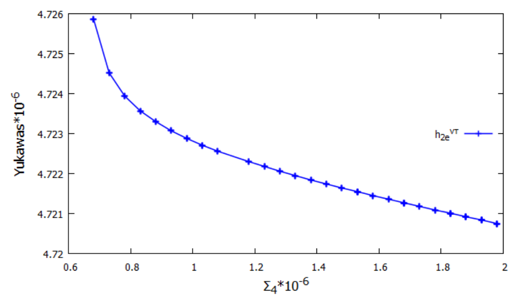
The system of equations were solved by implementing a Mathematica routine which showed many solutions to the system of equations. In general all six real parameters are of order $\sim 10^{-6}$. Then, the smallness of the couplings can be justified by the Majorana masses and the high energy breaking scale. Since a redefinition of the Yukawa coupling constants was made in Eq. (4-172), we can consider all dimensionless Yukawa couplings or order 1 while $\frac{\mu_N v_2^2}{{h_{N\chi 1}}^2 v_\chi^2} \sim 10^{-12} GeV$ which in general can be accomplished by the high energy breaking scale of the $U(1)_X$ symmetry together with a high right-handed neutrino mass. For a moderate value of $v_\chi \sim 10^3$ it means $\mu_N \sim 10^{-10} h^2_{N_{\chi 1}}$ showing that in general right handed neutrinos are much heavier than Majorana neutrinos as we assumed before. However, this allows to have a Majorana mass in the KeV scale and right-handed neutrinos with a mass with order $\sim 10^5 GeV$. Furthermore, since all parameters are of the same order implies that the $\rho = h_{N\chi 1}/h_{N\chi 2}$ parameter has to be of order 1 ($\rho \sim 1$) implying that there are two right handed neutrinos with similar masses. A graph representing the general behaviour of Yukawa couplings and phases is shown in figure **4-12** and **4-13**. We can see that in general de τ couplings tends to be around a fixed value, that is why an additional graph showing a more detailed view of $h^{\nu e}_{2e}$ and γ is shown. On the one hand, normal ordering scheme allows all parameters to be either positive or negative although figures **4-12**a and **4-13**a present its absolute value. On the other hand, in for an Inverse Ordering scheme either $h^{\nu\mu}_{2\mu}$ or $h^{\nu e}_{2\mu}$

and $h_{2\mu}^{\nu\tau}$ must be negative, but the same sign for all three couplings is not allowed. Again, in figure **4-13**a is shown its absolute value. The r_4 interval have been chosen in accordance with **4-11** so a general scan over the $\theta_{e\mu}$ is done.

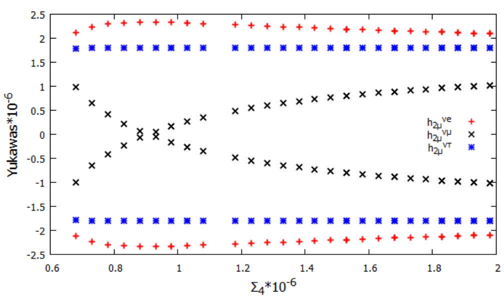
All in all, we have shown that given the neutrino mass eigenvalues and the Majorana phases, there is always a set of dimensionless Yukawa couplings which recreate the PMNS matrix. In this case, three complex parameters and three real parameters are the minimal set which reproduce the PMNS parameters with no additional Majorana phases. Nevertheless, general Majorana phases and charged lepton like phases can be included in the parametrization which consequently provides an appropriate set of parameters since the number of free parameters in the SM sector increases. It is also important to notice that the contributions due to exotic particles becomes negligible as a consequence of its dependence with the $U(1)_X$ scale. Finally, the relationship among Yukawa couplings and mixing angles has been already studied, and the relationship is neither trivial nor short, they can be seen however in [67]



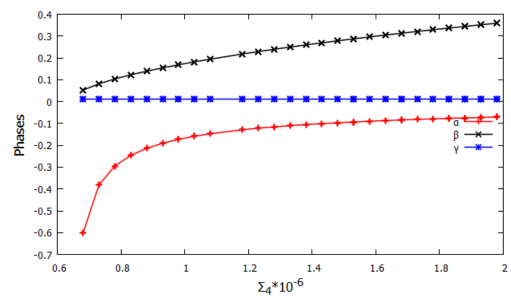
(a) $h_{2e}^{\nu e}$(red), $h_{2e}^{\nu\mu}$(black) and $h_{2e}^{\nu\tau}$(blue) as a function of r_4.



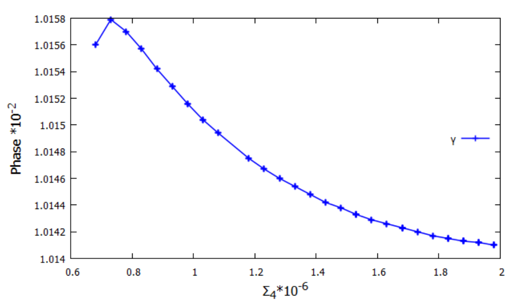
(b) $h_{2e}^{\nu\tau}$(blue) as a function of r_4.



(c) $h_{2\mu}^{\nu e}$(red), $h_{2\mu}^{\nu\mu}$(black) and $h_{2\mu}^{\nu\tau}$(blue) as a function of r_4.

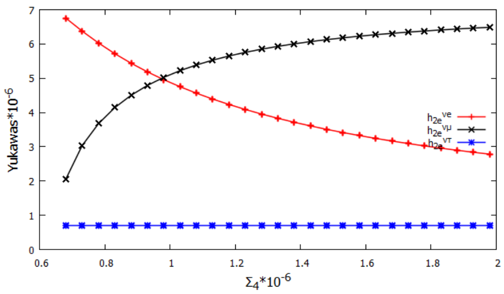


(d) α(red), β(black) and γ (blue) as a function of r_4.

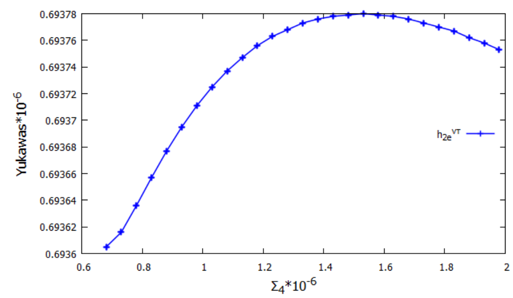


(e) γ phase as a function of r_4

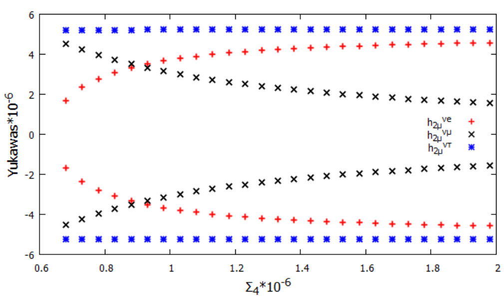
Figure **4-12**: Neutrino Yukawa couplings and phases values as a function of r_4 for a Normal Ordering Scheme for $\mu_E = 0.451771$, $\Sigma_1 = 4.482762 \times 10^{-6}$, $g_{\chi E} = 0.885898$, $g_{\chi\mathcal{E}} = 0.478386$, $\mu_{\mathcal{E}} = 0.975823$, $h_{1e}^{E} = 0.324576$, $h_{1\mu}^{E} = 0.171557$, $h_{2e}^{\tau e} = 0.0971024$, $h_{2e}^{\tau\tau} = 0.0740497$ (in general random numbers)



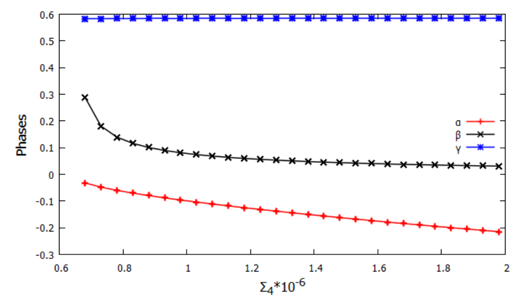
(a) $h_{2e}^{\nu e}$(red), $h_{2e}^{\nu\mu}$(black) and $h_{2e}^{\nu\tau}$(blue) as a function of r_4.



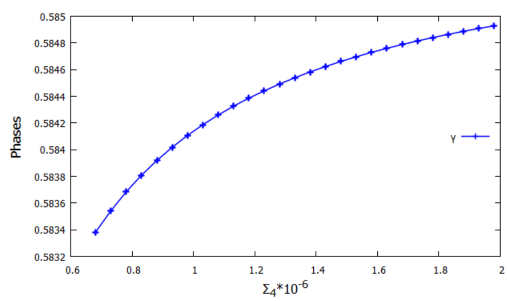
(b) $h_{2e}^{\nu\tau}$(blue) as a function of r_4.



(c) $h_{2\mu}^{\nu e}$(red), $h_{2\mu}^{\nu\mu}$(black) and $h_{2\mu}^{\nu\tau}$(blue) as a function of r_4.



(d) α(red), β(black) and γ (blue) as a function of r_4.



(e) γ phase as a function of r_4

Figure **4-13**: Neutrino Yukawa couplings and phases values as a function of r_4 for a Inverse Ordering Scheme for $\mu_E = 0.451771$, $\Sigma_1 = 4.482762 \times 10^{-6}$, $g_{\chi E} = 0.885898$, $g_{\chi\mathcal{E}} = 0.478386$, $\mu_{\mathcal{E}} = 0.975823$, $h_{1e}^{E} = 0.324576$, $h_{1\mu}^{E} = 0.171557$, $h_{2e}^{\tau e} = 0.0971024$, $h_{2e}^{\tau\tau} = 0.0740497$ (in general random numbers)

5 Concluding Remarks and Outlook

To date, it is a fact how successful the Standard Model of particle physics has been being with its original formulation being unchanged. On the one hand, it has a general scenario that can explain all observed phenomena by just adjusting the value of some model parameters which for some people is just how the universe is. On the other hand, some other people believe that the patters present in nature is not just a chaotic coincidence rather a manifestation of physics beyond the standard model. Of course, it is the case of Fermion Mass Hierachy. Aimed in giving a suitable explanation, a $U(1)_X$ extension has been proposed in such a way that avoids all kind of chiral anomalies due to the existence of opposite charged particles under the new symmetry group. Thus, it is consequent the presence of four Higgs doublets and four Higgs singlets as a response of the model non-SUSY counterpart which has half scalar particles.

The scalar sector is proved to be consistent with the observed Higgs boson and the absence of yet unobserved additional scalar particles consequence of the high energy breaking scale of the $U(1)_X$ symmetry which is mediated by the Vacuum Expectation Value acquired by the scalar singlets χ and χ' which are model free parameters while doublet VEVs are restricted by the charged W boson mass by $v_1^2 + v_1'^2 + v_2^2 + v_2'^2 = 243.3^2\ GeV^2$. The latter restriction allows to W and Z gauge bosons to have the correct mass while the Z' boson is proportional to v_χ and v'_χ which can explain its theorized elevated mass. However, the W mass restriction opens an infinite number of choice possibilities for each VEV although it is reduced to just one free parameter (v'_1) if we address Fermion Mass Hiearchy to be thank to multiple Higgs VEV rather than Yukawa couplings values. Additionally, the parameter region allowed by the observed Higgs boson includes the possibility of H'_1 to be an inert doublet but it cannot be ensured until the consequences on fermion and sparticles masses will be studied prior to think in the scalar particle as a Dark Matter candidate. It is not studied in this work but it is left as a future development.

Moreover, the model is provided with a $\mathcal{Z}_2$ symmetry which has to be broken, in this case by soft breaking terms, to prevent scalar particles lighter than the Higgs boson. In fact, without these soft breaking terms the seesaw mechanism is no longer valid on the scalar sector and a different approach has to be considered. Nevertheless, in the considered framework the lightest scalar particle, identified as the Higgs boson, receive additional contributions from the D-terms which explain more naturally its $125.3 GeV$ mass. In fact, its mass is proportional to the Z-boson mass plus a correction term of the same order in agreement with NMSSM and USSM models. The latter analogous to a 1-loop radiative correction to the Higgs mass due to stop particles, whose tree level order is unnatural but it is consistent with the elevated μ_{kk}, $k = 1, 2, \chi\sigma$ terms that prevents a large electroweak symmetry breaking.

Regarding the fermion sector, which is the main interest of the work, the SM fermion masses are reproduced successfully with approximately exact analytical formulas based in the application of concurrent seesaw mechanism due to the absence of observed exotic fermions, which naturally leads to the assumption being considerably heavy particles, even though they depend on v_χ and v'_{chi} in the flavor basis which supports the assumption. Likewise, we can explain heavy fermion masses by choosing VEV at a scale near its masses, which is the case of v_1 with the top quark and v'_2 with bottom quark and tau lepton, leading to restriction among Yukawa couplings and to an unconstrained v'_1 VEV. Nevertheless, the lightest fermions are massless at tree level, but at one loop level they are massive thanks to the coupling among exotic fermions and the inert scalar singlets σ and σ'. It is only at this level that v'_1 makes presence with the down and strange quarks. Furthermore, the different mass structure obeys a phenomenological texture in which the third generation fermion has a small coupling with first generation fermions and negligible coupling with the second one. However, charginos and Neutralinos are of special interest in SUSY models and as expected, there are governed by supoerpotential parameters and leads to degenerate masses and the contributions proportional to eletroweak VEVs turns out to be negligible to the TeV scale in which these particles are expected to exist.

Last but not least, the model is consistent with CKM and PMNS matrices without affecting the fermion physical masses. In the first case, all Yukawa couplings can be taken to be real, being the CP phase coming from the radiative corrections although a more general scenario can be considered. In the case of the PMNS matrix, no radiative corrections are needed but at least three parameters must be complex in order to reproduce the CP-violating phase and the Majorana phases which under this approach are considered input parameters. This model implies that the observed neutrinos are Majorana like thanks to the existence of a light Majorana neutrino and heavy right handed neutrinos which make possible neutrino masses via an inverse-seesaw mechanism. Moreover, the non-universal X-charge assignation leads to the presence of Z' mediated flavor changing neutral currents which is also left for a future work principally because of the high computational requirements of a supersymmetric non-universal model implementation of packages like Feynrules [4] or Lanhep [59]

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