Study of wobbling bands in ^{139}Pm

Mehak Narula[1],* Pooja Siwach[2], and P. Arumugam[1]
[1] *Department of Physics, Indian Institute of Technology, Roorkee - 247667, INDIA and*
[2] *Department of Physics, University of Arizona, Tucson, Arizona, USA.*

Introduction

Wobbling motion is a special kind of nuclear rotation that indicates the presence of a triaxial shape. It appears when the nucleus rotates not exactly around its main axis but slightly away from it, leading to oscillations around the angular momentum direction [1]. Two main types of wobbling are known: longitudinal wobbling, where the unpaired particle's spin is aligned with the axis of the largest moment of inertia, and transverse wobbling, where it is oriented perpendicular to that axis. Such wobbling behavior has already been observed in several odd nuclei like ^{163}Lu, ^{135}Pr, and ^{187}Au. In the case of ^{139}Pm, it is a suitable candidate to study this motion due to the presence of the odd quasiparticle near high-j orbitals in triaxially deformed nucleus [2]. Recent work on ^{139}Pm indicates that the quadrupole bands in this nucleus may originate from transverse wobbling [3]. By examining how the wobbling frequency evolves with increasing spin in ^{139}Pm, one can gain deeper insight into the wobbling behavior of the nucleus.

In this work, we calculate the rotational energy of wobbling bands with $n = 0, 1, 2,$ and 3 for ^{139}Pm, where n is the wobbling quantum number, within the framework of modified particle-rotor model (MPRM) and from the excitation energies we can determine whether the nucleus behaves as a transverse or longitudinal wobbler.

Theoretical Formalism

The rotation–particle coupling in an odd-A nucleus can be described by combining the motion of the valence nucleon with the collective rotation of the even-even core using angular momentum algebra [4]. The Hamiltonian for a triaxially deformed particle–plus–rotor system using a mean field defined by the deformed Woods-Saxon potential can be written as

$$H = H_{\rm av} + H_{\rm pair} + H_{\rm rot}, \tag{1}$$

where $H_{\rm av} + H_{\rm pair}$ has eigenvalues corresponding to the quasiparticle energy ϵ_q. The Hamiltonian for a triaxial rotor is given by $H_{\rm rot} = \sum_{k=1}^{3} \frac{R_k^2}{2\mathcal{I}_k}$, where R_k are the angular momentum components and $\mathcal{I}_k$ are the corresponding moments of inertia along the three principal axes. The total wavefunction for a given spin (I, M) of the odd-even nucleus can be written as

$$\Psi^{IM} = \sum_{ljR} \frac{\phi^I_{ljR}(r)}{r} \, |ljR, IM\rangle, \tag{2}$$

where I is the total angular momentum with M its projection on the laboratory z-axis. $\phi^I_{ljR}(r)$ represents the radial wave function of the relative motion of the valence nucleon with respect to the core.

The matrix element for the total Hamiltonian given in Eq. 1 can be written as

$$\begin{aligned}\langle q'K', IM|H|qK, IM\rangle = \\ \epsilon_q \, \delta_{K'K}\delta_{q'q} + \sum_{lj\Omega'_p,\Omega} W^{K'K}_{j\Omega'_p\Omega_p} \\ \times \int dr \, f_{uv} \, \phi^{IK'*}_{lj\Omega'_p}(r) \, \phi^{IK}_{lj\Omega_p}(r),\end{aligned} \tag{3}$$

where q defines the particle state with quasiparticle energy ϵ_q. $W^{K'K}_{j\Omega'_p\Omega_p}$ is called the coupling matrix where K is projection of I on the third axis (body-fixed frame). f_{uv} is a fac-

*Electronic address: mehak@ph.iitr.ac.in

tor transforming particle states to quasiparticle states, with u and v being the occupation and vacancy probabilities, respectively.

Results and Discussion

The odd-Z nucleus ^{139}Pm is studied with a microscopic nonadiabatic quasiparticle approach. To investigate the energy of wobbling bands theoretically, we couple the valence quasiproton to ^{138}Nd, which is treated as the triaxial core. The single-particle energies of protons are calculated with Esbensen-Davids set of parameters [5] and the corresponding quasiparticle energies are obtained with BCS calculations. Single-particle and quasiparticle energies are evaluated as a function of the deformation parameters β_2, β_4, and γ. Here, β_2, β_4, and γ account for the quadrupole, the hexadecapole, and the triaxial deformations, respectively. These calculations are performed by varying β_2 $(0.0-0.4)$ at a fixed value of γ and by varying γ $(0° - 40°)$ at a fixed value of β_2. From these calculations, it is predicted that the negative parity states of ^{139}Pm predominantly originate from the $\pi h_{11/2}$ orbital near to $\beta_2 = 0.15$ and $\gamma = 15°$, and hence all those levels are included in our basis. These values of deformations are also predicted in Ref. [3], hence we have performed the calculations for $\beta_2 = 0.15$, $\beta_4 = 0.228\beta_2$, and $\gamma = 15°$.

The rotational energies are calculated for $n = 0, 1, 2,$ and 3 wobbling states. The yrast states $n = 0$ are denoted by $11/2_1, 15/2_1, 19/2_1, ...$, the single wobbling excitations $n = 1$ by $13/2_1, 17/2_1, 21/2_1, ...$, the double wobbling excitations $n = 2$ by $15/2_2, 19/2_2, 23/2_2, ...$, and triple wobbling excitations $n = 3$ by $17/2_2, 21/2_2, 25/2_2, ...$. The energy of these wobbling bands are represented in Table I along with the experimental energies for $n = 0$ and 1. The calculated level energies excellently reproduce the experimental values for both yrast and yrare bands.

The excitation energies $\Delta E_n = E_n - \bar{E}_{n=0}$ can be calculated using the energies listed in Table I. By studying how ΔE_n changes with spin I, we can explore whether the $n = 1, 2, 3$ bands show longitudinal or transverse

TABLE I: Rotational energies calculated for $n = 0, 1, 2,$ and 3 negative parity wobbling bands for nucleus ^{139}Pm.

I ($\hbar$)	$E_I - E_{11/2_1}$ (MeV)			I ($\hbar$)	$E_I - E_{11/2_1}$ (MeV)		
	n = 0		n = 2		n = 1		n = 3
	Calc.	Exp.	Calc.		Calc.	Exp.	Calc.
11/2	0.000	0.000	–	13/2	0.837	0.597	–
15/2	0.447	0.466	1.794	17/2	1.494	1.187	2.403
19/2	1.179	1.217	2.563	21/2	2.329	2.002	3.010
23/2	2.046	2.164	3.413	25/2	3.257	3.012	3.810
27/2	2.987	3.228	4.401				
31/2	3.982	4.195	5.481				
35/2	5.025	4.995	6.577				

wobbling behavior [3]. In addition, the reduced electromagnetic transition probabilities can also be calculated using the framework of MPRM, providing further insight into the nature of these wobbling bands.

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