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Scalar Field Configurations with Planar and Cylindrical Symmetry: Thick Domain Walls and Strings

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ABSTRACT

We study the gravitational field of static scalar field configurations representing thick, planar and cylindrical domain walls and thick, global cosmic strings by means of a new class of exact solutions of Einstein's equations for a scalar field with potential $V(\Phi) = V_0 \cos^{2(1-n)}(\Phi/f(n))$ $(0 < n < 1)$. The planar solution describes two static walls that are separated by a space time singularity. A particle horizon exists and the metric on the horizon becomes a static Kasner metric that also contains Minkowski space time. The cylindrical solutions may be interpreted either as a cylindrical wall or as a thick string. Both configurations have a particle horizon. A Minkowski space on the horizon is conical with a deficit angle of $\sim \pi$, which is negative for strings. Even for an energy scale $f << m_{pl}$ the energy density per length $G\mu$ is of the order one. In general, the string solution possesses no singularity, whereas the cylindrical wall has a singularity on the central axis.

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1. Introduction

As a relic of a symmetry breaking phase transition in the universe, scalar fields might play a significant role in the formation of cosmic structure. Among all possible scalar field configurations strings and domain walls have attracted most attention[1,2]. One can divide these objects in two classes: Infinitely thin strings and walls for which the thickness is small compared to some typical curvature radius and thick strings and walls with non-negligible thickness. Since the infinitely thin strings and walls are supposed to form as a result of a phase transition at a GUT scale very early in the universe, the strings and walls have a width small compared to their typical curvature radius. In this case one can examine the effects of strings and walls on the surrounding matter distribution under the assumption that the whole energy momentum is concentrated on a single line for strings and on a single two-dimensional plane for walls[3,4,5]. Since the dominant interaction of these infinitely thin strings and walls is gravitation, it is of utmost importance to study the gravitational effects created by scalar fields[6] which differ considerably from the gravitational effects of ordinary matter. In particular, it is desirable to impose constraints on any scenario of structure formation that involves scalar fields by the gravitationally induced distortion of the cosmic background radiation[7]

The gravitational field of infinitely thin strings is a conical flat space time[3] with a deficit angle determined by the mass per length of the string. The deflection of light in such a space time is independent from the impact parameter of the photons and the distortion of the background radiation is of step like nature[8]

The gravitational field of an infinitely thin wall has the remarkable property that particles are repelled[5] and that a particle horizon exists[4]. Another feature of thin walls whose significance in the cosmological context has not been clarified yet, is that no static solutions to Einstein equations with reflection symmetry exist[4,9]. It has been shown that the vacuum space times on both sides of these thin walls are different static Kasner metrics[9]. Only if the metric is allowed to depend on a time coordinate, reflection symmetric solutions exist. The cosmological significance of thin strings and walls has been extensively explored by means of numerical simulations[10,11]. In the present paper, however, we study thick walls and strings that might arise in a phase transition after recombination. Recently, such a model of structure formation was proposed[12], where scalar field configurations with intrinsic length scales of the order of Mpc arise after recombination to provide the seeds of galaxy formation. In order to put constraints on the parameters of this model one has to know the gravitational effect on the background radiation of various possible scalar field configurations for which the approximation of the energy momentum tensor by a $\delta-$function no longer holds. To study thick walls and strings in the context of this galaxy formation scenario is the foremost motivation of this paper.

The gravitational field of thick domain walls was studied in[13,14,15,16] and was shown to be repulsive far away from the wall. In[15] it was shown that, similar to the case of thin walls, no reflection symmetric thick walls exist and that the vacuum space time must be the Minkowski vacuum on one side of the wall and the Taub vacuum on the other side, provided the metric has two commuting space-like Killing vectors and, in addition, a rotational symmetry about any axis perpendicular to the wall. In[16] an exact solution for a thick wall with this three dimensional symmetry group was derived. The wall solution in the present paper has no rotational symmetry, i.e. it admits only two commuting Killing vectors, which is the difference between the assumptions on which the present paper and[16] is based. For a string with constant energy density matched to a vacuum solution it was shown[17,18] that the metric is conical far away from the center of the string with a deficit angle similar to that derived for an infinitely thin string.

In this paper we derive a class of static solutions with two commuting, space-like Killing vectors which can be interpreted either as a solution for a planar configuration without rotational symmetry about the axis perpendicular to the symmetry plane or as a solution for a cylindrical wall or an infinite string. The different interpretations of the solution depend on whether a coordinate along the integral curves of one of the Killing fields is periodic or not and on the choice of an integration constant. It turns out that the *plane symmetric* solution describes two static walls separated by a space time singularity. Such a singularity is to be expected because the attraction between the walls presumably excludes the existence of a smooth configuration of two static walls. The solution tends asymptotically to a Kasner metric which contains also Minkowski space time for a certain choice of the parameters. Each of the two walls is not reflection symmetric about the location of the maximum of the scalar field density, but the solution is in fact symmetric about the singular plane between the two walls. This double-wall solution has horizons whose distance from the center of each wall is $\propto 1/\sqrt{nG\rho_{max}}$ (ρ_{max} is the density in the center of the wall). The distance between the walls is exactly twice the horizon distance. If the parameters are chosen such that the metric becomes Minkowski space on the horizon, test particles can move over the entire range between the horizon and the singular plane and they are attracted towards the singular plane. If the metric is Kasner on the horizon, bound states are possible for massive particles. However, a particle cannot cross the singular plane because velocity and acceleration are infinite at the singularity.

Since *cylindrical symmetry* is also characterized by two commuting space-like Killing vectors it is possible, by means of a different interpretation of the coordinates, to obtain from the same solution a cylindrical wall configuration and an infinite string solution. The *cylindrical wall* has either a singularity on the central axis and tends to Minkowski space asymptotically or it is flat on the axis and tends to a singular Levi-Civita vacuum solution asymptotically. Similar to the planar case a particle horizon exists. The location of the maximum of the scalar field energy density is halfway between the central axis and the horizon.

Choosing a different value of a certain integration constant in the cylindrical solution yields a solution which describes a *thick string*, where the maximum of the energy density and the unbroken vacuum of the scalar field lies on the central axis. If the parameters in the string solution are chosen such that the metric becomes Minkowski on the horizon the space time is without singularities. This seems to contradict the predictions in[19], but all these investigations are based on a quartic scalar potential and the assumption that the derivatives of the metric tensor vanish on the string axis whereas in our case the potential is a power of $\cos(\Phi)$ and the derivatives of the metric are not zero on the axis. For the cylindrical wall and the string, the Minkowski space on the horizon is a conical space with a very weak dependence of the angular deficit on the energy per length.

In section 2 the Einstein equations for a static scalar field with two space-like Killing vectors are given. We derive a solution for a scalar field potential $V(\Phi) = V_0 \cos^{2(1-n)} (\Phi/f(n))$ by means of an ad-hoc ansatz, determine the asymptotic vacuum states and the singularity. In section 3, 4 and 5 we discuss the specific properties of the solution pertaining to the interpretation as a planar wall, a cylindrical wall and a string, respectively.

2. A class of planar and cylindrical solutions

2.1 Einstein equations

We are looking for solutions to Einstein equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} \tag{2.1}$$

where the source of the gravitational field is described by the energy momentum tensor for a scalar field Φ

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - g_{\mu\nu} \Big[\frac{1}{2}\, g^{\rho\sigma}\, \partial_\rho \Phi \partial_\sigma \Phi - V(\Phi) \Big] \ . \tag{2.2}$$

We denote the spatial coordinates by x^1, x^2, x^3 and the time coordinate by x^0. Solutions describing planar walls and cylindrical configurations shall be static and they shall admit two commuting space-like Killing vectors

$$\partial/\partial x^2\ , \quad \partial/\partial x^3 \tag{2.3}$$

The orbits of the symmetry group are either planes or cylinders, depending on whether both of the coordinates x^2, x^3 run from $-\infty$ to $+\infty$ or one of these coordinates is periodic. Using the freedom to transform the x^1 coordinate in an arbitrary way one can take the metric in the form

$$ds^2 = e^{2A(x^1)}\, (dx^0)^2 - e^{2B(x^1)}\, (dx^1)^2 - e^{B-A}\, (dx^2)^2 - e^{B-A+C(x^1)}\, (dx^3)^2 \tag{2.4}$$

where A, B, C are functions of x^1. For a scalar field $\Phi(x^1)$ the energy momentum tensor (2.2) is:

$$\begin{aligned} T_0{}^0 = \ T_2{}^2 = T_3{}^3 &= +\,\frac{1}{2}\, e^{-2B}\, \Phi'^2 + V(\Phi) \equiv \rho \\ T_1{}^1 &= -\,\frac{1}{2}\, e^{-2B}\, \Phi'^2 + V(\Phi) \equiv -p \end{aligned} \tag{2.5}$$

where 'prime' denotes d/dx^1 . The Einstein equations become

$$\begin{aligned} G_0{}^0 &= -\, e^{-2B} \ \Big[4B'' - B'^2 - 2A'B' - 4A'' + 3A'^2 + 2C'' + C'^2 + C'(B' - 3A')\Big]/4 = \ 8\pi G\ \rho \\ G_1{}^1 &= -\, e^{-2B} \ \Big[B'^2 + 2A'B' - 3A'^2 + C'(B' + A')\Big]/4 = -8\pi G\ p \\ G_2{}^2 &= -\, e^{-2B} \ \Big[2B'' - B'^2 - 2A'B' + 2A'' + 3A'^2 + 2C'' + C'^2\Big]/4 = \ 8\pi G\ \rho \\ G_3{}^3 &= -\, e^{-2B} \ \Big[2B'' - B'^2 - 2A'B' + 2A'' + 3A'^2\Big]/4 = 8\pi G\ \rho \end{aligned} \tag{2.6}$$

Now, $G_2{}^2 = G_3{}^3$ yields an equation for C

$$2C'' + C'^2 = 0 \tag{2.7}$$

which is solved by

$$e^C = (x^1 - \zeta)^2 \ , \qquad \zeta = \text{const} \tag{2.8}$$

Any constant multiple of this solution solves also (2.7), but the constant factor can always be absorbed in a redefinition of the x^3 coordinate. Note that the trivial solution of (2.7) $C = const$ leads to a metric with an additional rotational symmetry about the x^1-axis. A solution with such a three dimensional symmetry group has been derived in[16] . The scalar field equation $\Phi^{;\mu}{}_{;\mu} - dV/d\Phi = 0$ simplifies with (2.8) to:

$$e^{-2B} \ \left[\Phi'' + \frac{1}{(x^1 - \zeta)} \ \Phi' \right] - \frac{dV(\Phi)}{d\Phi} = 0 \ . \tag{2.9}$$

From (2.6) one immediately finds that

$$A'' + \frac{1}{(x^1 - \zeta)} A' = -8\pi G \ e^{2B} \ V(\Phi) \tag{2.10}$$

$$B'' = 3A'' - \frac{1}{(x^1 - \zeta)} \ (B' - 3A') \ . \tag{2.11}$$

Eqs. (2.9) - (2.11) and (2.8) are equivalent to the Einstein equations (2.6) and are sufficient to determine the functions A, B and Φ for a given $V(\Phi)$.

2.2 An exact solution

An ansatz for $B(x^1)$ that leads to a reasonable potential $V(\Phi)$ and an energy momentum tensor that vanishes for $|x^1| \to \infty$ is

$$e^B = \frac{\beta}{x^1 - \zeta} \left[\frac{(x^1 - \zeta)^c}{(x^1 - \zeta)^{2c} + 1} \right]^n \tag{2.12}$$

$(n, c, \beta = const)$. With $B(x^1)$ given by (2.12) , $A(x^1)$ and $V(x^1)$ can be calculated by (2.11) and (2.10) , respectively. The scalar field equation (2.9) (or equivalently one of eqs. (2.6)) yields $\Phi(x^1)$ and by eliminating x^1 from $\Phi(x^1)$ and $V(x^1)$ one gets V as a function of Φ . Carrying out all these steps one finally obtains the following solution to eqs. (2.9) - (2.11) :

$$e^{2A} = \frac{(x^1-\zeta)^{2nc/3+2k}}{\left[(x^1-\zeta)^{2c}+1\right]^{2n/3}} \tag{2.13}$$

$$e^{2B} = \beta^2\ \frac{(x^1-\zeta)^{2cn-2}}{\left[(x^1-\zeta)^{2c}+1\right]^{2n}} \tag{2.14}$$

$$e^{C} = (x^1-\zeta)^2 \tag{2.15}$$

$$\Phi - \Phi_0 = f\ \arcsin\left[\frac{(x^1-\zeta)^{2c}-1}{(x^1-\zeta)^{2c}+1}\right] \tag{2.16}$$

$$V(\Phi) = V_0\left[\frac{1}{2}\ \cos\left((\Phi-\Phi_0)/f\right)\right]^{2(1-n)} \tag{2.17}$$

$$f \equiv \left[\frac{n(1-n)}{12\pi G}\right]^{1/2} \quad , \quad V_0 \equiv \frac{nc^2}{6\pi G\beta^2} \quad , \quad k^2 \equiv \frac{1}{3}\left[\frac{4}{3}\ (nc)^2 - 1\right] \tag{2.18}$$

$$0 < n < 1 \quad , \quad (nc)^2 \geq \frac{3}{4} \tag{2.19}$$

$\beta, n, c, \zeta, \Phi_0$ are constants. We have already eliminated all integration constants that are associated with a mere rescaling of the coordinates. (This leaves us with dimensionless coordinates.) The physically meaningful constants are then n, c and β . n determines the energy scale f of the scalar field as well as the power of the cosine in the potential, c/β determines the amplitude of the potential and c the steepness of scalar field distribution. The energy density ρ , the pressure p along the x^1-axis and V as a function of x^1 are given by:

$$\rho = (2-n)\ V_0\ \frac{(x^1-\zeta)^{2c(1-n)}}{\left[(x^1-\zeta)^{2c}+1\right]^{2(1-n)}} \tag{2.20}$$

$$p = -n\ V_0\ \frac{(x^1-\zeta)^{2c(1-n)}}{\left[(x^1-\zeta)^{2c}+1\right]^{2(1-n)}} \tag{2.21}$$

$$V = V_0 \, \frac{(x^1-\zeta)^{2c(1-n)}}{\left[(x^1-\zeta)^{2c}+1\right]^{2(1-n)}} \tag{2.22}$$

It is interesting to note that the solution (2.13) - (2.22) is invariant under several transformations. A change of the sign $c \to -c$ leaves the metric and ρ, p, V invariant and changes only the sign of Φ : $(\Phi - \Phi_0) \to -(\Phi - \Phi_0)$. Since the sign of Φ has no physical meaning the sign of c can be chosen arbitrarily. To be specific, we assume henceforth $c > 0$. Another transformation

$$x^1 - \zeta \to \frac{1}{x^1-\zeta} \ , \ \ k \to -k \ , \ \ x^2 \to x^3 \ , \ \ x^3 \to x^2 \tag{2.23}$$

changes also only the sign of $\Phi - \Phi_0$. Therefore, a transformation (2.23) together with $c \to -c$ leaves the solution completely invariant. (2.23) transforms the region $x^1 \to \infty$ to the point $x^1 = \zeta$ and vice versa. Since (2.23) , which does not alter the physical interpretation of the solution, involves a change of the sign of k, one can conclude that the space time at $x^1 \to \infty$ and at $x^1 = \zeta$ must be different once the sign of k is fixed. Since the matter variables ρ, p, V vanish at $x^1 \to \infty$ and at $x^1 = \zeta$ this implies that the vacuum space time at $x^1 \to \infty$ and $x^1 = \zeta$ must be different.

Note that the scalar field energy density ρ , the pressure p along the x^1-axis and V are all proportional to each other. Thus, the location of possible extrema of density, pressure and potential energy all coincide. ρ, V have two maxima at $x^1 = \zeta + 1$ and $x^1 = \zeta - 1$. Consequently, since the pressure p along the x^1-axis is always negative, $p(x^1)$ has two minima at $x^1 = \zeta \pm 1$. All the matter variables ρ, p and V vanish at $x^1 = \zeta$ and at $|x^1| \to \infty$, i.e. the energy momentum tensor vanishes at these points and we have a vacuum space time at $x^1 = \zeta$ and at $|x^1| \to \infty$. At $x^1 = \zeta$ ρ, p, V are continuous but not all derivatives are finite. Which of the derivatives become infinite depends on the parameters n, c . For example, the first derivatives of ρ, p, V are zero at $x^1 = \zeta$ only if $2c(1-n) > 1$, otherwise they diverge. The scalar field Φ becomes $\Phi = \Phi_0 \pm \pi f/2$ for $x^1 \to \pm\infty$ and vanishes at the extrema of the density and the potential at $x^1 = \zeta \pm 1$. Note that the scalar field probes only a half-period of the cosine-potential, i.e. the cosine in (2.17) is positive for finite x^1 and becomes zero for $|x^1| \to \infty$ and at $x^1 = \zeta$. A more detailed discussion of the solution pertinent to the specific interpretations is left to sections 3, 4 and 5. Before we embark on this, we examine the asymptotic behavior of the solution and possible singularities.

2.3 ASYMPTOTICS AND SINGULARITIES

Realistic scalar field configurations should have a gravitational field that is flat far away from the peak of the energy density. In order to determine for which combination of the parameters the solution becomes flat space in some asymptotic region we examine the solution at the points $|x^1| \to \infty$ and $x^1 = \zeta$ where the energy momentum tensor vanishes. The vacuum space time at $|x^1| \to \infty$ and $x^1 = \zeta$ will be some general static vacuum metric with two commuting Killing vectors which becomes Minkowski space only for a particular choice of the parameters of our solution. The *general vacuum metric* in the coordinate system (2.4) obtained from (2.6) in the limit $\rho = p = V = 0$, is:

$$ds^2 = [x^1-\zeta]^{2a}\,(dx^0)^2 - [x^1-\zeta]^{2(b-1)}\,(dx^1)^2 - [x^1-\zeta]^{b-a-1}\,(dx^2)^2 - [x^1-\zeta]^{b-a+1}\,(dx^3)^2 \tag{2.24}$$

where the constants a, b satisfy

$$b + a = \pm\sqrt{4a^2+1} \tag{2.25}$$

The metric (2.24) is equivalent to the Kasner metric, since a coordinate transformation $\hat{x}^1 = (x^1 - \zeta)^b$ casts (2.24) into the well known form[20]

$$ds^2 = (\hat{x}^1)^{2p_1}(dx^0)^2 - (d\hat{x}^1)^2 - (\hat{x}^1)^{2p_2}(dx^2)^2 - (\hat{x}^1)^{2p_3}(dx^3)^2 \tag{2.26}$$

with $p_1 = a/b$, $p_2 = (b-a-1)/(2b)$, $p_3 = (b-a+1)/(2b)$ and $p_1 + p_2 + p_3 = (p_1)^2 + (p_2)^2 + (p_3)^2 = 1$. The metric (2.24) is Minkowski space time for $a = 0$ and $b = \pm 1$.

Now, we determine the asymptotic form of our metric (2.13) - (2.15) at the points $x^1 = \zeta$ and $|x^1| \to \infty$ and relate the constants a, b in the general vacuum metric (2.24) to the parameters in our solution. From that we can determine which choice of these parameters yields a flat space time. For $|x^1| \to \infty$ the metric (2.13) - (2.15) , (2.4), (2.8) becomes a vacuum space time of the form (2.24) with the parameters (note that $n, c > 0$):

$$a = -\frac{1}{3}nc + k \quad , \quad b = -nc \qquad \text{for} \quad |x^1| \to \infty \tag{2.27}$$

and for $x^1 \to \zeta$ the solution is asymptotically of the form (2.24) with

$$a = \frac{1}{3}nc + k \quad , \quad b = nc \qquad \text{for} \quad x^1 \to \zeta \tag{2.28}$$

The parameters in the asymptotic form of the solution (2.27) and (2.28) satisfy the relation (2.25) if the definition of k (2.18) is taken into account. (2.27) and

(2.28) show that we have Minkowski space ($a = 0, b = \pm 1$) either at x^1 or at $|x^1| \to \infty$ for the following combinations of parameters:

$$\text{Minkowski at } |x^1| \to \infty : nc = 1 \ , \ k = +\frac{1}{3} \tag{2.29}$$

$$\text{Minkowski at } x^1 = \zeta : nc = 1 \ , \ k = -\frac{1}{3} \tag{2.30}$$

Once the sign of k is fixed it is not possible that the solution becomes flat space at $|x^1| \to \infty$ and at $x^1 = \zeta$ simultaneously. For $nc \neq 1$ the vacuum space times at $|x^1| \to \infty$ and $x^1 = \zeta$ are different Kasner metrics.

Although the matter variables are finite in the whole coordinate range $-\infty < x^1 < +\infty$ some metric components become infinite or zero at $x^1 = \zeta$ or $|x^1| \to \infty$, depending on the parameters. However, there is no combination of the parameters for which the metric tensor is completely free of singularities and zero points. In the remaining part of this section we examine whether these infinities are merely singularities induced by the coordinate system or real space time singularities. A true space time singularity occurs at $|x^1| \to \infty$ or at $x^1 = \zeta$ if some curvature invariant diverges. The Ricci scalar

$$R = 2V_0(n-3)\,\frac{(x^1-\zeta)^{2c(1-n)}}{\left[(x^1-\zeta)^{2c}+1\right]^{2(1-n)}} \tag{2.31}$$

is finite everywhere since $n < 1$ and clearly cannot be used as an indicator for a singularity at $|x^1| \to \infty$ or $x^1 = \zeta$. In the following we use the square of the Riemann tensor $R_{\mu\nu\sigma\lambda}$ as a means to probe the singular behavior at the points in question. For our solution this invariant is:

$$R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda} = \pm\,\frac{16\ cn}{27\beta^4}\;\frac{1}{(x^1-\zeta)^{4cn}\left[(x^1-\zeta)^{2c}+1\right]^{4(1-n)}}$$

$$\times\left\{(x^1-\zeta)^{6c}\left[(x^1-\zeta)^{2c}+2\right]K-\left[2\,(x^1-\zeta)^{2c}+1\right]K\pm 12c^3n\,(2n^2-4n+3)\,(x^1-\zeta)^{4c}\right\} \tag{2.32}$$

$$K \equiv (c^2n^2-3)\,\sqrt{4c^2n^2-3}\ \pm\ 2\,(cn)^3 \tag{2.33}$$

The upper signs in (2.32) and (2.33) hold for $k > 0$ and the lower ones for $k < 0$. For $k = 0 \Leftrightarrow (cn)^2 = 3/4$ the two expressions coincide. In the case $nc \neq 1$, where the metric is neither flat at $|x^1| \to \infty$ nor at $x^1 = \zeta$, the invariant (2.32) diverges at $x^1 = \zeta$ for any k as $\propto (x^1 - \zeta)^{-4nc}$ and becomes zero for $|x^1| \to \infty$. In the more interesting case $nc = 1, k = +1/3$ where the space time is flat at $|x^1| \to \infty$ (see (2.29)) the invariant diverges at $x^1 = \zeta$. Vice versa, for $nc = 1, k = -1/3$ (see (2.30)) the invariant diverges at $|x^1| \to \infty$ and is zero at $x^1 = \zeta$. Thus, for $nc \neq 1$ the point $x^1 = \zeta$ is always singular and for $nc = 1$ the singularity is at $x^1 = \zeta$ if $k = +1/3$ or at $|x^1| \to \infty$ if $k = -1/3$.

To summarize this section, the solution we have found describes a scalar field configuration with two maxima of the energy density and the scalar potential at $x^1 = \zeta \pm 1$. The space time becomes a vacuum both at $|x^1| \to \infty$ and $x^1 = \zeta$ where the scalar field sits in the potential minimum. For $nc \neq 1$ these vacua are Kasner metrics and the point $x^1 = \zeta$ is a space time singularity. For $nc = 1$ and $k = +1/3$ the vacuum at $|x^1| \to \infty$ is Minkowski and $x^1 = \zeta$ is singular, for $nc = 1$ and $k = -1/3$ the vacuum at $x^1 = \zeta$ is Minkowski and the singularity is at $|x^1| \to \infty$. In the next three sections we discuss specific properties of the solution that are related to the different possible interpretations.

3. Planar domain walls

A metric with two space-like Killing vectors like (2.4) admits two different interpretations depending on the range of the coordinates. One has a planar space time if $-\infty < x^1, x^2, x^3 < +\infty$ and a cylindrical if the coordinates are bounded by $0 \leq x^1 < +\infty$, $-\infty < x^2 < +\infty$ and $0 \leq x^3 \leq 2\pi$. In this chapter we explore properties of the solution arising in the context of the interpretation as a planar domain wall configuration. For this purpose we rewrite the solution in coordinates $z \equiv x^1$, $x \equiv x^2$, $y \equiv x^3$ and $t \equiv x^0$, where the z coordinate runs perpendicular to the wall and x, y are coordinates parametrizing the planes parallel to the wall. We also set the constant $\zeta = 0$, since in the interpretation as a domain wall ζ is associated only with a translation of the entire wall along the z axis. Only in the cylindrical case ζ acquires a nontrivial meaning. Then the metric is

$$ds^2 = h^{2n/3} z^{2k}\, dt^2 - \beta^2 h^{2n} z^{-2} dz^2 - \beta^2 h^{2n/3} \left[z^{-k-1} dx^2 + z^{-k+1} dy^2\right] \qquad (3.1)$$

$$h(z) \equiv \frac{z^c}{z^{2c} + 1} \quad , \qquad (3.2)$$

the matter variables are

$$V = \rho/(2-n) = -p/n = V_0\, h^{2(1-n)} \qquad (3.3)$$

and the scalar field is

$$\Phi - \Phi_0 = f \ \arcsin \left[\frac{z^{2c} - 1}{z^{2c} + 1} \right] \quad . \tag{3.4}$$

k, f, c, n, β are related according to (2.18) . The density ρ and V have two maxima at $z = \pm 1$ where the scalar field sits in the maximum of $V(\Phi)$ and they vanish at $z = 0$ and $|z| \to \infty$, where Φ is in the minimum of the potential. Fig. 1 and Fig. 2 are graphs of the matter variables and the scalar field where the proper distance along the z-axis has been used instead of the coordinate distance z. Note that although the coordinate position $z = \pm 1$ of the extrema is independent of the parameters in the solution the corresponding proper distances do depend on the parameters, as one would expect. Because of these two different extrema of the matter variables the solution (3.1) - (3.4) represents two static, planar domain walls for which the energy momentum tensor vanishes at $|z| \to \infty$ and at $z = 0$. As far as we know this is the first time a double-wall solution has been found. The solution in[16] represents only a single wall. According to section 2.3 the plane $z = 0$ between the walls is singular (because the square of the Riemann tensor (2.32) blows up) if $nc \neq 1$ and the vacuum at $|z| \to \infty$ and $z = 0$ is a Kasner metric. In the case $nc = 1$, $k = +1/3$ the metric (3.1) is Minkowski space time at $|z| \to \infty$, but still singular at $z = 0$. For $nc = 1$, $k = -1/3$ the vacuum at $z = 0$ is flat space and a singular Kasner metric at $|z| \to \infty$. Clearly, the physically most reasonable case is the one where the space time becomes flat outside the walls ($nc = 1, k = +1/3$). The singularity between the walls at $z = 0$ in this case is not very surprising, because a smooth solution with two static walls seems very implausible on account of the gravitational attraction between the walls which renders any static configuration with more than one body impossible. One might speculate that this singularity arises from the collision of two moving walls. For some initial conditions the dynamical evolution of domain walls could lead asymptotically in time to a configuration described by the above static solution. However, in order to corroborate the statement that our static solution is some generic product of the evolution of domain walls one would have to prove the stability of this solution, which is presumably an intricate problem.

It is not yet clear whether the region $|z| \to \infty$ really is at an infinite proper distance from the centers of the walls. The proper distance s between two points z_1 and z_2 measured along a space like curve perpendicular to the wall is given by

$$s = \beta \int_{z_1}^{z_2} z^{-1} h^n \, dz \tag{3.5}$$

Therefore, the proper distance s_H between $z = 0$ and $z \to +\infty$ is

$$s_H = \frac{\beta\ (1+n)}{cn} 2^{-n} \sqrt{\pi}\ \frac{\Gamma(1+n/2)}{\Gamma(3/2+n/2)} \tag{3.6}$$

which is a finite quantity for $0 < n < 1$ (Γ is the usual Γ-function). This is an important result, because it means that any particle moving in the gravitational field of the walls can only travel a finite distance beyond the center of the walls ('particle horizon'). Such a particle horizon has been shown to exist also for infinitely thin domain walls[4] . It is also interesting to note that the proper distance between the vacuum at $z = 0$ and the center of the walls at $z = \pm 1$ is exactly $s_H/2$, i.e. the centers of the walls lie halfway between the horizons at $z = \pm\infty$ and the vacuum at $z = 0$ (see Fig.1). Nevertheless, the walls are not symmetric about the planes $z = \pm 1$, however, the solution is reflection symmetric about $z = 0$. By means of (3.3) and (2.18) one can relate the horizon size s_H to the maximum value of the scalar field density $\rho_{max} = \rho(z = \pm 1)$ $= V_0(2-n)\ 4^{(n-1)}$

$$s_H = \frac{1}{\sqrt{n\ G\rho_{max}}} (1+n)\ \sqrt{(2-n)/24}\ \frac{\Gamma(1+n/2)}{\Gamma(3/2+n/2)} \tag{3.7}$$

The fraction of Γ-functions $\gamma \equiv \Gamma(1+n/2)/\Gamma(3/2+n/2)$ in (3.6) and (3.7) lies in the range $\sqrt{\pi}/2 < \gamma < 2/\sqrt{\pi}$ for $0 < n < 1$. Thus, the horizon distance and the distance between the two walls varies as $\propto 1/\sqrt{n\ G\rho_{max}}$. Finally we want to relate the energy density per surface element σ to the parameters of the solution. The definition of energy is always ambigious in General Relativity, especially if the space time is not asymptotically flat. But a plausible, coordinate invariant definition of σ is provided by the integral of ρ over the element of proper length along the z-axis:

$$\sigma \equiv \int_0^\infty \rho\ \beta h^n z^{-1} dz = \frac{(2-n)\ nc}{\beta} \frac{2^n}{24\pi G} \frac{\Gamma(1/2)\ \Gamma(1-n/2)}{\Gamma(3/2-n/2)} \tag{3.8}$$

$$= \sqrt{n\ \rho_{max}/G}\ \sqrt{(2-n)/24}\ \frac{\Gamma(1-n/2)}{\Gamma(3/2-n/2)} \tag{3.9}$$

To get an impression how the gravitational field of the walls acts on surrounding matter, we briefly investigate the trajectories of test particles moving perpendicular to the wall. The first integrals of the geodesic equations for a particle on a curve $x^\mu = (t(\tau), z(\tau), 0, 0)$ are

$$\dot{t} = E\ z^{-2k} h(z)^{-2n/3} \tag{3.10}$$

$$\dot{z}^2 = \frac{1}{\beta^2}\, z^2 h(z)^{-2n} \left[E^2\ z^{-2k} h(z)^{-2n/3} - \mu^2\right] \tag{3.11}$$

'dot' denotes $d/d\tau$, E is a constant and $\mu^2 = 1, 0$ for massive and massless particles, respectively. Particles can be in a bound state if a turning point $\dot{z}^2 = 0$ exists. For massless particles ($\mu^2 = 0$) there is no turning point at finite $z \neq 0$. But the particles cannot cross the plane $z = 0$ if it is singular, as is the case for $nc \neq 1$ and $nc = 1, k = +1/3$. For massive particles, however, points $|z_T| < \infty$

$$E^2 - (z_T)^{2k}(h(z_T))^{2n/3} = 0 \tag{3.12}$$

where $\dot{z}^2 = 0$ can exist, depending on the parameter combination nc. In Fig. 3 $z^{2k} h(z)^{2n/3}$ is plotted for the three different cases where nc is larger smaller or equal to one. For $nc > 1$ every massive particle is in a bound state moving in the finite range $|z| \leq |z_T|$ which clearly corresponds to a finite proper distance from the wall. For $nc = 1$ bound states exist only for $E^2 < 1$ and particles with $E^2 > 1$ can move in the entire range of the horizon. The case $nc < 1$ is similar to the latter one, i.e. bound states exist only for $E^2 < 1$. However, even if there are no turning points, the horizon is the maximal distance a particle can traverse in any case. No particle can cross the singular plane $z = 0$ since the velocity and the acceleration diverge at this point.

4. Cylindrical domain walls

If we identify one of the coordinates in the orbit of the symmetry group, say x^3, at infinity, i.e. we assume that it is periodic, we get a cylindrical space time where x^1 is the distance from the cylinder axis, x^2 runs along the axis and x^3 is an azimuthal angle in the planes $x^2 = const.$ Setting the constant $\zeta = 0$ will allow us to interpret the solution as a cylindrical domain wall. In more familiar notation adapted to the cylindrical symmetry

$$r \equiv x^1 \quad , \quad z \equiv x^2 \quad , \quad \phi \equiv x^3$$

$$0 \leq r < \infty \quad , \quad -\infty < z < +\infty \quad , \quad 0 \leq \phi \leq 2\pi \tag{4.1}$$

the solution is:

$$ds^2 = g(r)^{2n/3} r^{2k} dt^2 - \beta^2 r^{-2} g^{2n} dr^2 - \beta^2 g^{2n/3} r^{-k-1} \left[dz^2 + r^2 d\phi^2\right] \tag{4.2}$$

$$g(r) \equiv \frac{r^c}{r^{2c}+1} \tag{4.3}$$

$$V = \rho/(2-n) = -p/n = V_0 \; g^{2(1-n)} \tag{4.4}$$

$$\Phi - \Phi_0 = f \;\; \arcsin\left[\frac{r^{2c}-1}{r^{2c}+1}\right] \tag{4.5}$$

The density, pressure and potential vanish both at $r = 0$ and $r \to \infty$ where the scalar field is in the minimum of the potential $V(\Phi)$. ρ and V attain its maximum at $r = 1$ whose proper distance s from the origin $r = 0$ is

$$s(r=1) = \beta \int\limits_0^1 r^{-1} g^n \; dr = \frac{\beta(1+n)}{cn} 2^{-n-1} \; \sqrt{\pi} \; \frac{\Gamma(1+n/2)}{\Gamma(3/2+n/2)} \quad . \tag{4.6}$$

In Fig. 4 and Fig. 5 the density and the scalar field distribution is depicted as a function of the proper distance in the radial direction. This behavior of the scalar field and the density characterizes a domain wall. Again, there is a horizon ($r \to \infty$) at a proper distance s_H (see (3.6)) from the central axis which is exactly twice the radius of the cylindrical wall. Thus, the location of the center of the wall is halfway between the origin $r = 0$ and the horizon $r \to \infty$. Note that s_H is related to $\rho_{max} = \rho(r = 1)$ according to (3.7) . For $nc \neq 1$ the space time on the axis $r = 0$ is singular since $R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda}$ diverges and becomes a Levi-Civita vacuum metric on the horizon. The Levi-Civita metric[20] is formally equivalent to the Kasner metric (2.24) , (2.25) if the coordinates are identified according to (4.1) . For $nc = 1, k = +1/3$ the space time on the horizon $r \to \infty$ is Minkowski and a singular Levi-Civita vacuum on the z-axis, whereas for $nc = 1, k = -1/3$ the metric is singular on the horizon and flat space on the z-axis.

Finally, we demonstrate an intriguing property of this geometry of a cylindrical domain wall in the case $nc = 1, k = 1/3$ (Minkowski space on the horizon $r \to \infty$). This property is closely related to the typical features of a conical space of an infinitely thin string. Consider the circumference U of a circle with center at $r = 0$ in a plane $z = const$ as it varies with radius

$$U = 2\pi\ \beta \frac{r^{2/3}}{\left[r^{2/n} + 1\right]^{n/3}} \tag{4.7}$$

This is a monotonically growing function of r whose limit at the horizon $r \to \infty$ is

$$U(r \to \infty) = 2\pi\ \beta = 2\pi\ s_H \frac{2^n}{(1+n)\sqrt{\pi}} \frac{\Gamma(3/2 + n/2)}{\Gamma(1 + n/2)} \tag{4.8}$$

s_H (see (3.6)) is equal to the proper radius of the circle on the horizon. Since the circumference U is smaller than the value $2\pi s_H$, which one expects in a flat space, the Minkowski space on the horizon is a conical space similar to the conical space created by an infinitely thin string. The relative angular deficit

$$\Delta \equiv \frac{2\pi s_H - U(r \to \infty)}{2\pi s_H} = 1 - \frac{2^n}{(1+n)\sqrt{\pi}} \frac{\Gamma(3/2 + n/2)}{\Gamma(1 + n/2)} \tag{4.9}$$

is determined by n which in turn is given by the energy scale f of the scalar field (see (2.18)). Note that the angular deficit used in the literature[3,17,18] is simply $2\pi\ \Delta$. Δ varies between the limiting values

$$\begin{aligned} \Delta =& 0.5 \qquad \text{for} \quad n \to 0 \\ \Delta =& 0.36 \qquad \text{for} \quad n \to 1 \end{aligned} \tag{4.10}$$

Note that for an energy scale f of the order of the Planck scale n is about 1/2 and for example for $f \approx 10^{15} GeV \quad n \approx 10^{-8}$. Thus, for an energy scale f well below the Planck scale, the circumference of a circle with proper radius equal to the horizon distance is about half the circumference of a circle in flat space. For thin strings the angular deficit is determined by the energy per unit length of the string. Therefore we try to relate the deficit Δ to the energy density per unit length μ along the cylinder axis. We define μ as the integral of the density ρ over the proper surface element in the $z = const$ plane:

$$\begin{aligned} \mu \equiv& \int_0^{2\pi} \int_0^{\infty} \rho\ \beta^2 g^{4n/3} r^{-(k+1)/2}\ dr d\phi \\ =& \frac{(2-n)\ nc}{6\ G} \frac{\Gamma(1 - n/3 + (1-k)/(4c))\ \Gamma(1 - n/3 - (1-k)/(4c))}{\Gamma(2 - 2n/3)} \end{aligned} \tag{4.11}$$

Γ is the standard Gamma-function. For the case where the space time on the horizon is Minkowski space, $nc = 1$, $k = +1/3$, μ becomes

$$\mu = \frac{(2-n)}{6G} \frac{\Gamma(1-n/6)\,\Gamma(1-n/2)}{\Gamma(2-2n/3)} \tag{4.12}$$

Thus, n determines μ as well as the angular deficit Δ uniquely. By eliminating n from (4.12) and (4.9) one gets Δ as a function of the energy density per length of the cylinder. Since $0.36 < \Delta < 0.5$ has no strong dependence on the parameter n the dependence on the energy per length is very weak. The energy density per length μ (4.12) varies between the limits

$$\begin{aligned} \mu G &= \frac{1}{3} \qquad \text{for} \quad n \to 0 \\ \mu G &= 0.37 \qquad \text{for} \quad n \to 1 \end{aligned} \tag{4.13}$$

The parameter n determines the energy scale f of the scalar field. Thus, the energy per length of the cylinder μG is always of the order one and fairly independent of the energy scale f.

5. Infinite string

If the coordinate ranges are chosen as in the cylindrical wall solution (see (4.1)) and if the integration constant $\zeta = -1$, the solution (2.13) - (2.22) represents an infinite string along the z-axis where the energy density has a maximum at $r = 0$ and the scalar field sits in the maximum of $V(\Phi)$. With the definitions (4.1) the string solution is

$$ds^2 = q^{2n/3}(r+1)^{2k}dt^2 - \beta^2 q^{2n}(r+1)^{-2}dr^2 - \beta^2 q^{2n/3}(r+1)^{-k-1}\left[dz^2 + (r+1)^2 d\phi^2\right] \tag{5.1}$$

$$q(r) \equiv \frac{(r+1)^c}{(r+1)^{2c}+1} \tag{5.2}$$

$$V = \rho/(2-n) = -p/n = V_0\; q^{2(1-n)} \tag{5.3}$$

$$\Phi - \Phi_0 = f\;\arcsin\left[\frac{(r+1)^{2c}-1}{(r+1)^{2c}+1}\right] \tag{5.4}$$

Fig. 6 and Fig. 7 are plots of the density and the scalar field as a function of proper distance in the radial direction. Note that the first derivatives of Φ and the metric are not zero at the origin $r = 0$.

Since the string metric differs from the cylindrical wall metric merely by the choice of an integration constant, the string and the wall have some properties in common. There is also a horizon, i.e. the proper distance s_H between the axis $r = 0$ and $r \to \infty$ is finite

$$s_H = \sqrt{\pi}\,\frac{\beta(1+n)}{nc}\,2^{-n-1}\,\frac{\Gamma(1+n/2)}{\Gamma(3/2+n/2)} =$$

$$= \frac{1}{2\sqrt{nG\rho_{max}}}(1+n)\sqrt{(2-n)/24}\,\frac{\Gamma(1+n/2)}{\Gamma(3/2+n/2)} \tag{5.5}$$

and half the horizon distance for a cylindrical wall. For the choice $nc = 1, k = 1/3$ the space time becomes flat space on the horizon $r \to \infty$ and all components of the metric tensor are finite everywhere, i.e. the solution is singularity free. For $nc = 1, k = -1/3$ the space time becomes a singular Levi-Civita vacuum on the horizon. For all the other possibilities ($nc \neq 1$) the invariant (2.32) is finite everywhere, which does not imply that the solution has no singularities at $r \to \infty$ since there might be other invariants which blow up. The absence of a singularity in the case where the metric is flat on the horizon seems to contradict the proof in[19] that any string solution must have a singularity. However, these statements are based on a quartic scalar field potential and the assumptions that the metric is Minkowski on the z-axis and that the first derivatives of the metric tensor vanish on the string axis. All these presuppositions required for the proof are not fulfilled by the present string solution which, we surmise, renders the proof in[19] not relevant to our solution.

The circumference U of a circle in the $z = const$ plane with center at $r = 0$ increases with r as

$$U = 2\pi\,\beta\,\frac{(r+1)^{2/3}}{\left[(r+1)^{2/n}+1\right]^{n/3}} \tag{5.6}$$

if the space time is flat on the horizon ($nc = 1, k = +1/3$). Note that U does not tend to zero if the radius of the circle goes to zero $r \to 0$. This strange property may be due to the fact that the geometry on the string axis is not Minkowski space and that the derivatives of the metric at $r = 0$ are not zero. On the horizon $r \to \infty$ the circumference $U \to 2\pi\beta$. But, since for the string the proper radius of the circle on the horizon is only half the value of the corresponding value for the cylindrical wall (compare (3.6) and (5.5)) $U(r \to \infty)$ now becomes

$$U(r \to \infty) = 2\pi \; s_H \; \frac{2^{n+1}}{(1+n)\sqrt{\pi}} \; \frac{\Gamma(3/2+n/2)}{\Gamma(1+n/2)} \tag{5.7}$$

which is twice the value (4.8) . Consequently, the relative angular deficit, defined in (4.9) , becomes negative and varies between

$$\begin{aligned} \Delta =& 0 \qquad \text{for} \quad n \to 0 \\ \Delta =& -0.28 \qquad \text{for} \quad n \to 1 \end{aligned} \tag{5.8}$$

A negative Δ means that the circumference is larger than the circumference of a circle with the same radius in ordinary flat space. This result seems quite strange especially since the gravitational field of the string (5.1) for $r \to \infty$ is the same as the gravitational field of the cylindrical wall (4.2) . But, as mentioned above, the circumference U does not vanish at $r = 0$ in the string metric, which presumably leads to such oddities on the horizon. If one defines the 'true' radius of the circle such that the circumference vanishes on the axis, the deficit would clearly be the same as in the case of a cylindrical wall. Finally, we give the energy density per length of the string ϵ defined as the integral of ρ over the proper surface element in the $z = const$ plane between $r = 0$ and $r \to \infty$

$$\epsilon = \frac{(2-n)\, nc^2}{4G(3c - nc + 3k - 3)} \; 2^{2n/3} \; F(1,\; n/3 + (1-k)/(2c),\; 2 - n/3 - (1-k)/c;\; -1) \tag{5.9}$$

where $F(a, b, c; z)$ is the hypergeometric function. For the case $nc = 1, k = +1/3$ this becomes

$$\epsilon = \frac{2-n}{12G\;(1-n)} \; 2^{2n/3} \; F(1, 2n/3, 2-n, -1) \tag{5.10}$$

Therefore, for an energy scale f well below the Planck mass , i.e. for $n \to 0$, the energy per length becomes $\epsilon G = 1/6$.

6. Summary and concluding remarks

We have discussed a new solution of the Einstein equations for scalar fields describing thick, planar and cylindrical domain walls and infinite thick strings. All configurations possess a particle horizon. The planar solution represents two static walls divided by a plane where the space time becomes a singular Kasner metric in general. However, for one particular choice of the parameters ($nc = 1, k = -1/3$) the vacuum on this plane can be Minkowski space, whereby a singularity appears on the horizon. If the parameters are chosen such that the space time is flat on the horizon, a singularity between the walls is unavoidable. Similar to the planar walls, the cylindrical wall is singular on the axis if the metric is flat on the horizon. Again, for special parameters the space time is flat on the axis and becomes singular on the horizon. It was shown that a flat space on the horizon is conical with an angular deficit determined by n or $f(n)$, the energy scale of the scalar field, which in turn is related to the energy density per length. The relative angular deficit depends very weakly on n and varies between 0.36....0.5. This is very large compared to an infinitely thin string with typical deficit of the order of 10^{-6}. The energy per length μG of the cylindrical wall is also determined by n and is for the entire range of n always of the order one. The metric for the thick string is quite similar to the cylindrical wall but seems to be more pathological, since for any choice of the parameters the first derivatives of the metric and the scalar field do not vanish on the string axis. Consequently, the circumference of a circle with zero radius does not vanish and the angular deficit of a circle on the horizon becomes negative, i.e. the circumference of a circle is larger than the circumference in ordinary flat space.

In order to get an impression of the possible length scales of the objects we are discussing, we take the numerical values for the scalar field potential of the model in[12] and plug them into our solutions. For $f \approx 10^{15} GeV$ and a neutrino mass $m_\nu \approx 10^{-2} eV$ the parameter n becomes $n \approx 10^{-8}$. Note that the parameter c of our solution is determined by n, solely by demanding that the solution becomes flat space on the horizon ($nc = 1, k = 1/3$). In the model V_0 is determined by the neutrino mass $V_0 \approx {m_\nu}^4$ and we get for the horizon size $s_H \approx 10^8 Mpc$ which is much larger than the horizon of the present universe. Thus, since s_H is also the typical distance between the two walls in the planar solution and the typical thickness of the walls and strings, already a single wall or string created by such a late-time phase transition proposed in[12], would dominate the entire universe. Only a larger f and/or a larger m_ν could reduce the typical length scales such that a dominance of the whole universe is avoided. For example for $f \approx 10^{16} GeV$

and $m_\nu \approx 1eV$ the horizon size becomes $s_H \approx 10^2 Mpc$.

Two other issues not addressed in this paper but relevant in a cosmological context, are the question of stability and the effect on the cosmic background radiation. According to Derrick's theorem[21] one-dimensional static scalar fields in Minkowski space ('kink' solution) are stable, whereas static 3D configurations cannot be stable in flat space. Since one cannot expect that gravitational effects stabilize scalar fields, one would surmise that 3D self-gravitating scalar fields are in general unstable too. However, our solutions depend only on one spatial coordinate. Since the corresponding flat space configurations are stable, there is at least a chance that our solutions are stable. In general, this question requires a detailed perturbation analysis. However, for the application of these scalar fields in the context of a scenario for structure formation, the stability is not that important, because the domain walls and strings are supposed to provide only strong temporary gravitational seeds for baryon clustering. Such a scenario would work if the domain walls survive a certain period during which enough baryonic matter accretes onto the walls to account for the observed masses of galaxies and clusters. The basic requirement for a viable scenario involving scalar fields after recombination is that the decay time of the domain walls is larger than the typical time scale of accretion. The question how these scalar fields effect the cosmic background is an important topic of future research which will enable us to put constraints on the model of a late time phase transition by comparing the gravitationally induced distortion with the observed isotropy.

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FIGURE CAPTIONS

1. Density vs. proper distance along the z-axis for two domain walls ($n = 0.5, c = 2.0, k = +1/3$). The dashed lines are the horizons at distance s_H.
2. The scalar field Φ vs. proper distance along the z-axis for two domain walls ($n = 0.5, c = 2.0, k = +1/3$). The dashed lines are the horizons.
3. The g_{tt} component of the metric (3.1) which determines the turning points for test particles for the planar walls according to (3.12).
4. Density vs. proper distance perpendicular to the axis for a cylindrical domain wall ($n = 0.5, c = 2.0, k = +1/3$). The dashed line is the horizon.
5. The scalar field vs. proper distance perpendicular to the axis for a cylindrical domain wall ($n = 0.5, c = 2.0, k = +1/3$). The dashed line is the horizon.
6. Density vs. proper distance perpendicular to the axis for a thick string ($n = 0.5, c = 2.0, k = +1/3$). The dashed line is the horizon.
7. The scalar field vs. proper distance perpendicular to the axis for a thick string ($n = 0.5, c = 2.0, k = +1/3$). The dashed line is the horizon.

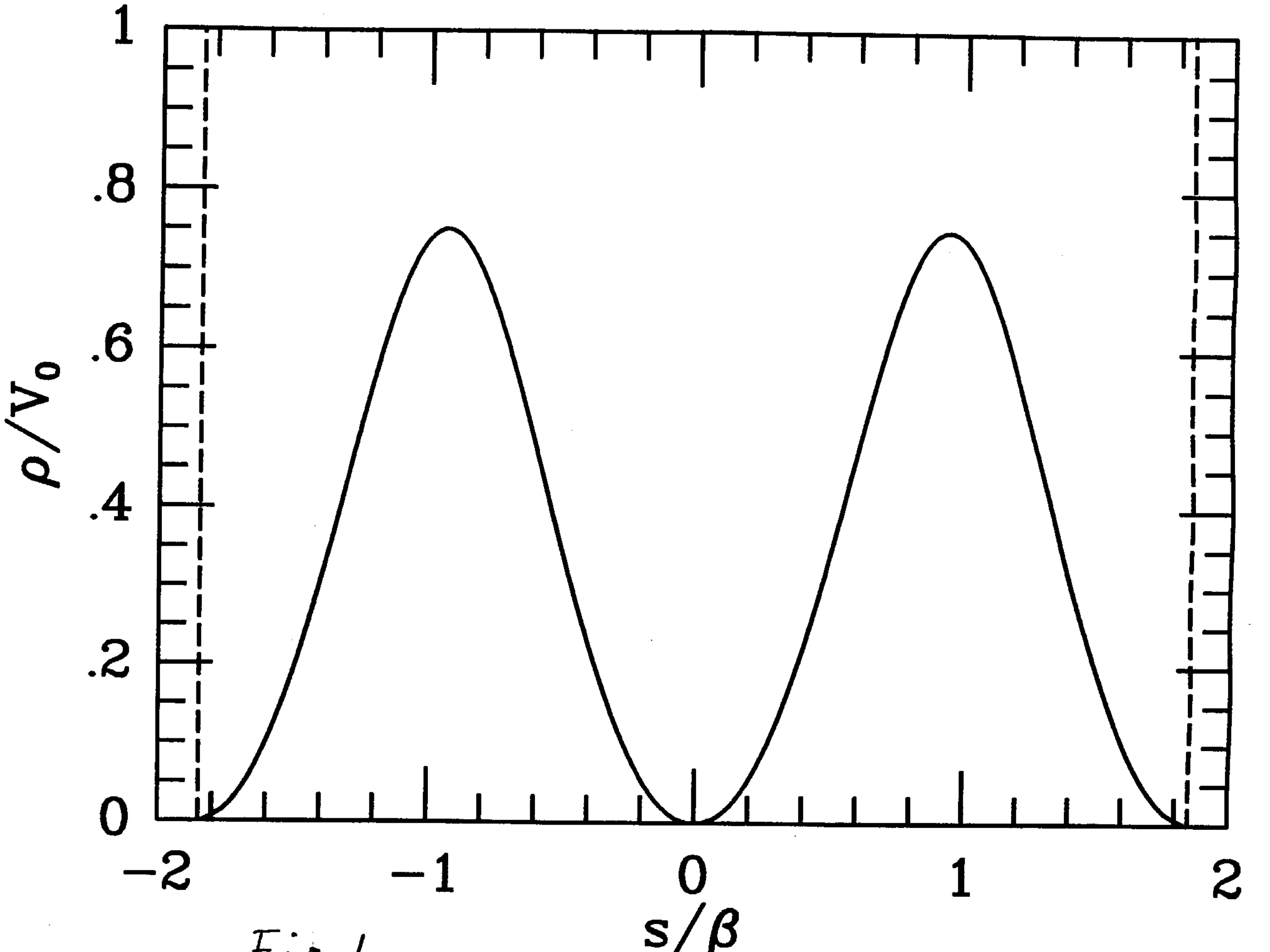

Fig. 1

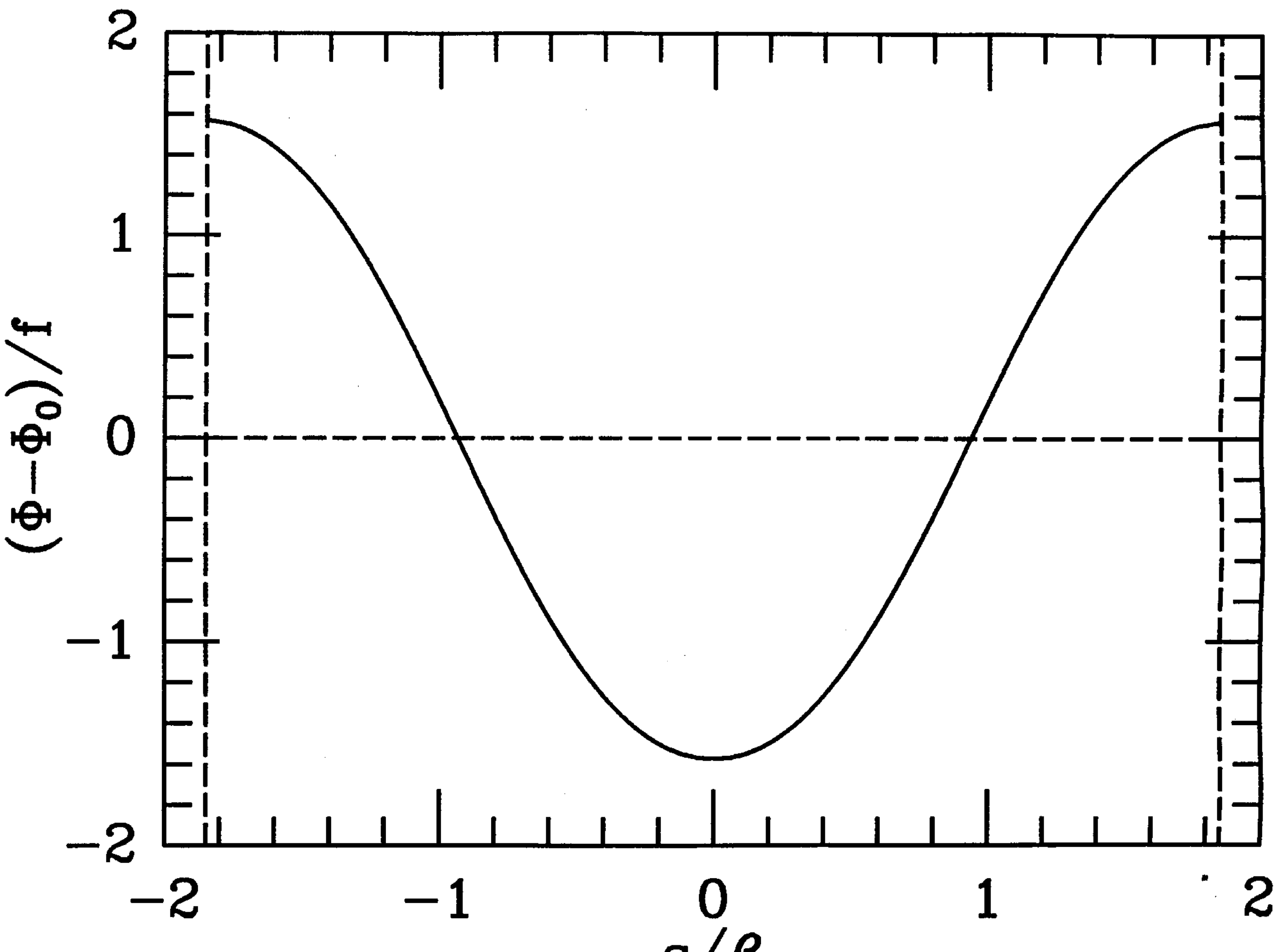

Fig. 2

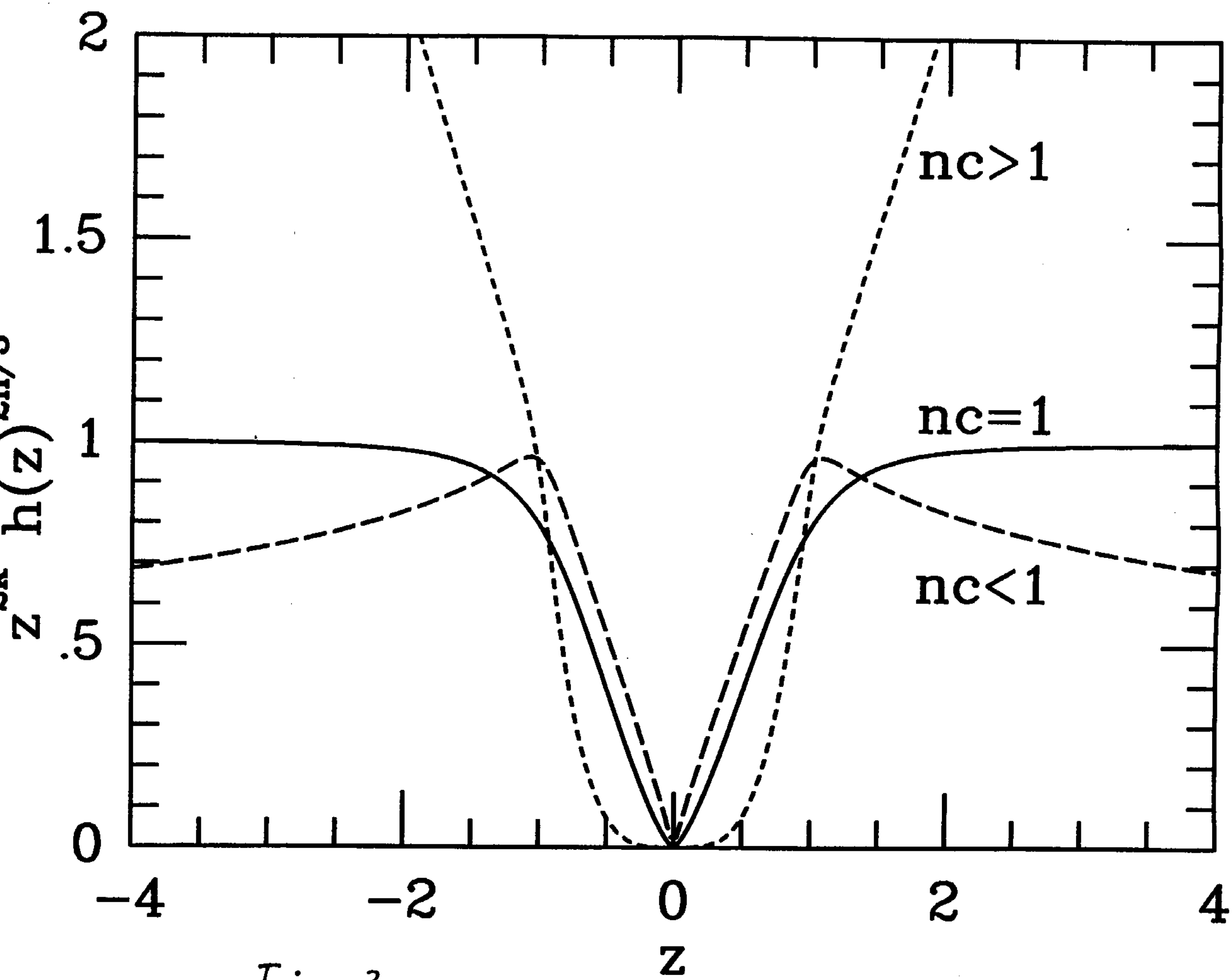

Fig. 3

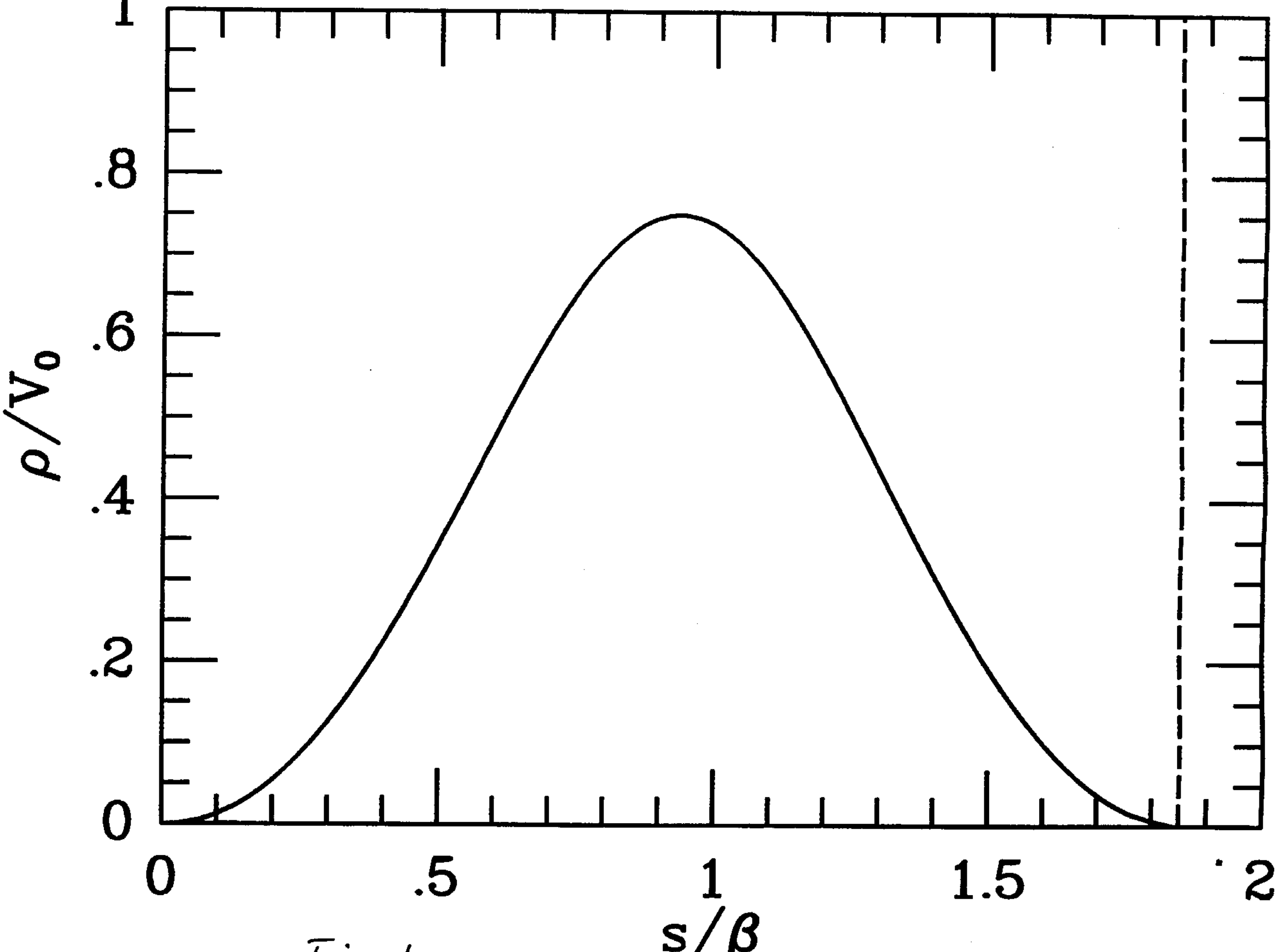

Fig. 4

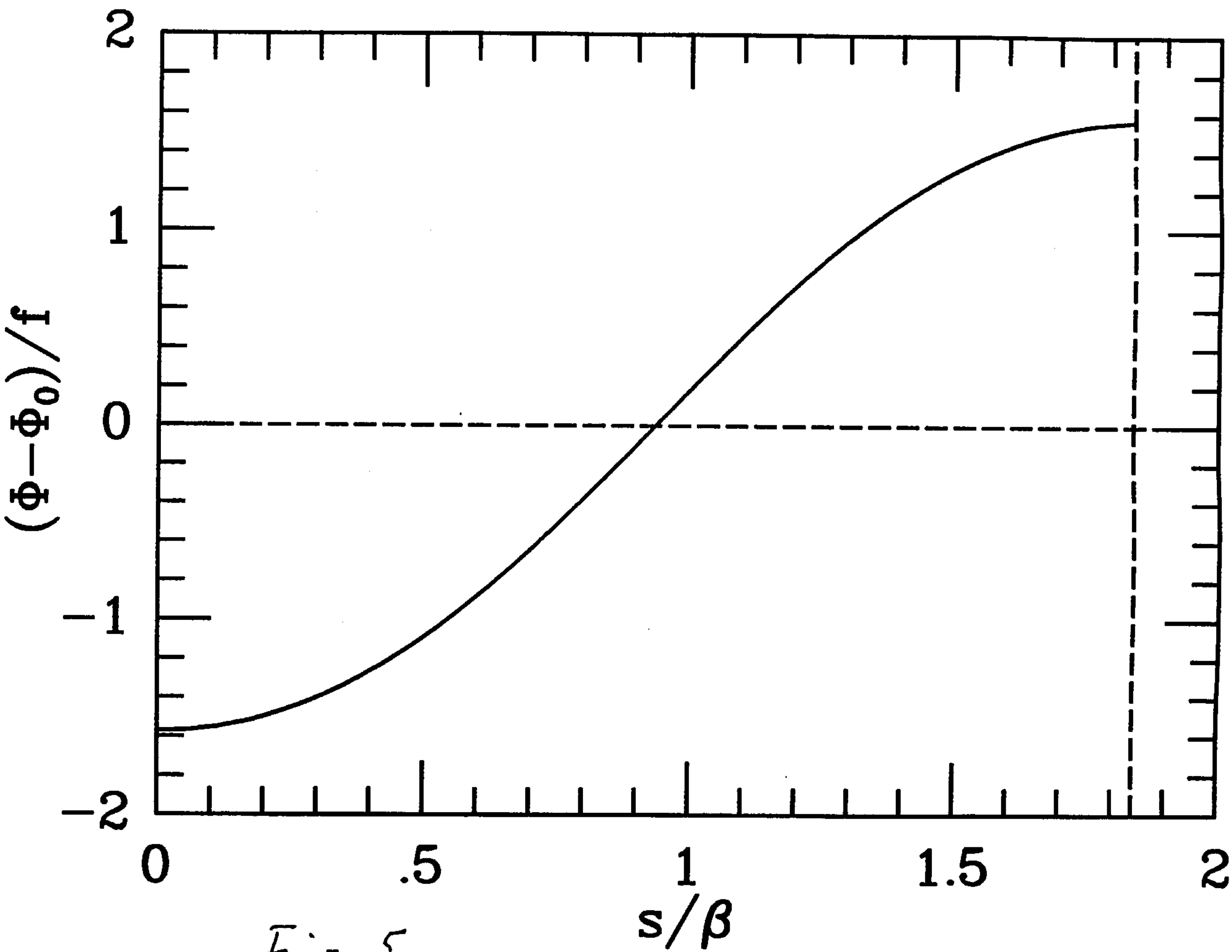

Fig. 5

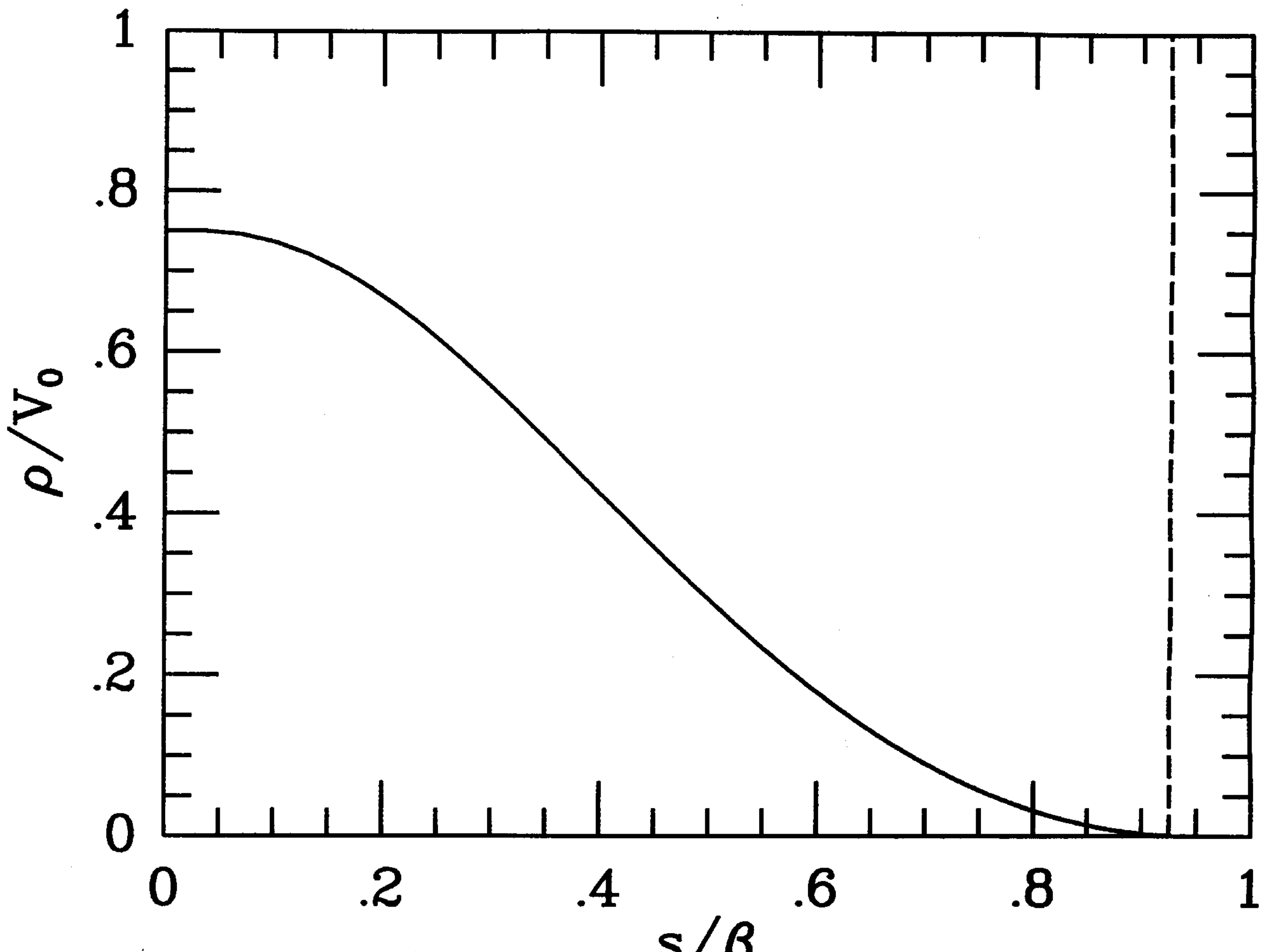

Fig. 6

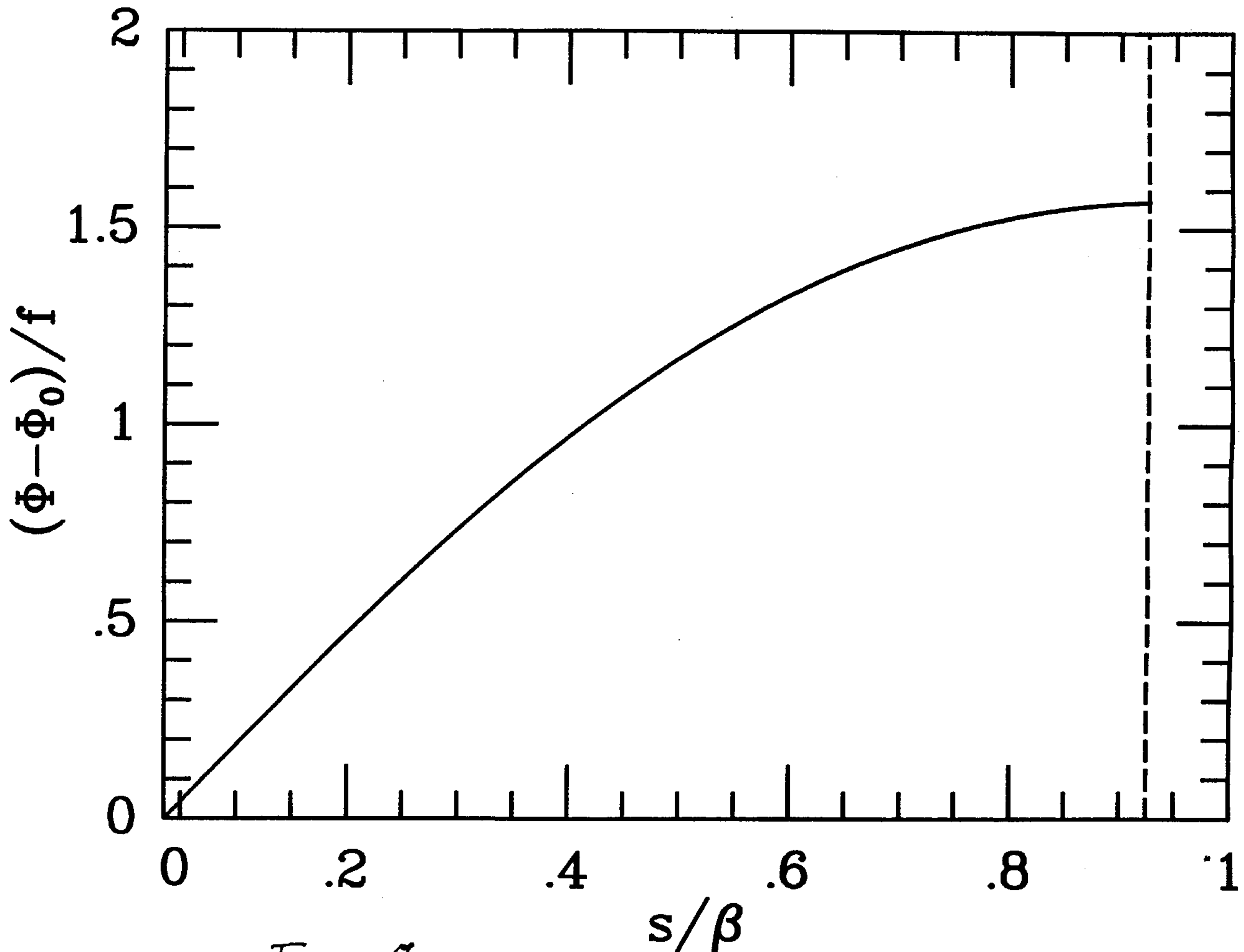

Fig 7