GROUP STRUCTURE FOR CLASSICAL LATTICE SYSTEMS OF ARBITRARY SPIN

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Abstract

We equip lattice systems of arbitrary spin with group structures Harmonic analysis is used to derive low and high temperature expansions of the partition function as well as duality relations among different models.
The Asano contraction is formulated without using the Griffiths transformation into an equivalent spin $\frac{1}{2}$ system. A necessary and sufficient condition is given to obtain the partition function as the Asano contraction of smaller systems. For a given system with spin $p > \frac{1}{2}$, the group structure is not unique. The consequences of this fact are discussed in the case of spin 1 models for which we give analyticity domains.

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0. INTRODUCTION

In the following, we would like to deduce some consequences of the group structures which can be defined on classical lattice systems of arbitrary spin.[1,2,3,4]. For spin $\frac{1}{2}$ systems these structures have been extensively discussed by C. Gruber and D. Merlini.[1].

In the usual physical picture of higher spin systems or multicomponent systems, there is no obvious group structure. However, generalizing the natural group structures defined for spin $\frac{1}{2}$ systems, we got an abstract picture, called group picture, of arbitrary spin systems in terms of abelian groups. Using harmonic analysis on these groups, the idea is to gain as much properties as possible of arbitrary spin lattice systems in terms of this group picture. e.g. high - and low temperature expansions, duality relations between different systems, equations for the correlation functions, Asano contractions for the partition function. Having these informations in the group picture, simple transformations immediately yield the analogue in the physical picture. This will be done explicitly in the spin 1 case.

I. GROUP STRUCTURES ASSOCIATED WITH SYSTEMS OF ARBITRARY SPIN

Let Λ be a finite subset of the lattice $\mathcal{L}$. We associate at each site $i \in \Lambda$ a compact abelian group $\mathcal{G}_i$ and we label the values of the spin variable at the site i with the elements of this group. Thus if $\mathcal{G}_i = \mathbb{Z}_q$, the integers modulo q, we will describe either a q component system, a system of spin $\frac{q-1}{2}$ or any other system having q local configurations. Therefore the configuration space of the system is identified with

$$\begin{array}{ccc} \mathcal{G}_\Lambda & = & \underset{i\in\Lambda}{\times} \mathcal{G}_i \\ \Psi & & \Psi \\ N & = & (n_1, \dots, n_{|\Lambda|}) \end{array}$$

The algebra of observables $\mathcal{O}\!\!\mathcal{l}_\Lambda$ of the system is the set of continuous functions on $\mathcal{G}_\Lambda$. A natural basis of this algebra is the dual group $\hat{\mathcal{G}}_\Lambda = \underset{i\in\Lambda}{\times} \hat{\mathcal{G}}_i$. The Hamiltonian $-\beta H_\Lambda \in \mathcal{O}\!\!\mathcal{l}_\Lambda$ for a given system admits the Fourier decomposition

$$-\beta H_\Lambda = \sum_{\chi \in \hat{\mathcal{G}}_\Lambda} J(\chi)\,\chi \qquad \left(= \int_{\hat{\mathcal{G}}_\Lambda} d\chi\, J(\chi)\,\chi \right) \quad , \quad J(\chi) \in \mathbb{C}$$

The potentials $J(\chi)$ are the Fourier coefficients of $-\beta H_\Lambda$ defining the support of the interactions $\mathcal{J}$

$$\mathcal{J} = \{ \chi \; ; \; J(\chi) \neq 0 \}$$

The group $\overline{\mathcal{J}} \subset \mathcal{G}_\Lambda^\wedge$ generated by the elements of $\mathcal{J}$ is called group of interactions. Moreover, for technical reasons, we define a set of generating bonds $\mathcal{B}$ as a set $\mathcal{B}$ together with a mapping $\mathbb{T} : \mathcal{B} \longrightarrow \mathcal{G}_\Lambda^\wedge$ such that

i) $\forall \chi \in \mathcal{J}$ $\exists$ at least one $b \in \mathcal{B}$ and r integer such that $\chi = (\mathbb{T}(b))^r = \chi_b^r$

ii) $|\mathcal{B}|$ is minimal

Let us remark that

a) the choice of $\mathcal{B}$ is not unique;

b) dropping ii), we could choose $\mathcal{B} = \mathcal{J}$ which has been done by Greenberg [2], but this does not represent the best choice;

c) in the spin $\frac{1}{2}$ case there is no possible choice and $\mathcal{B} = \mathcal{J}$.

Using the bicharacter notation, the Hamiltonian writes

$$-\beta H_\Lambda(N) = \sum_{b \in \mathcal{B}} \sum_{r=1}^{d_b - 1} J_{b,r} \langle \chi_b^r ; N \rangle_\Lambda \qquad d_b = \text{order of } \chi_b$$

and $$J_{b,r} = \begin{cases} J(\chi) & \text{if } \chi_b^r \text{ is associated with } \chi \\ 0 & \text{otherwise} \end{cases}$$

and we have for the partition function

$$Z_\Lambda\{\mathcal{J}\} = \sum_{N \in \mathcal{G}_\Lambda} e^{-\beta H_\Lambda(N)} = \sum_{N \in \mathcal{G}_\Lambda} \prod_{b \in \mathcal{B}} e^{\sum_{r=1}^{d_b - 1} J_{b,r} \langle \chi_b^r ; N \rangle_\Lambda}$$

and for the correlation functions, we get with $\chi \in \mathcal{G}_\Lambda^\wedge$

$$\langle \chi \rangle_\Lambda = (Z_\Lambda\{\mathcal{J}\})^{-1} \sum_{N \in \mathcal{G}_\Lambda} \langle \chi ; N \rangle_\Lambda \, e^{-\beta H_\Lambda(N)}$$

Defining the internal symmetry group $\mathcal{S}$ by

$$\mathcal{S} = \{ S \; ; \; S \in \mathcal{G}_\Lambda \; , \; H_\Lambda(SN) = H_\Lambda(N) \quad \forall N \in \mathcal{G}_\Lambda \}$$

we get

Lemma : $\mathcal{J} = \overline{\mathcal{J}}^{\perp}$, $\mathcal{G}_{\Lambda}/\mathcal{J} \cong \overline{\mathcal{J}}^{\wedge}$

From the set of bonds $\mathcal{B}$, we introduce the group of graphs $\mathcal{G}_{\mathcal{B}}$:

$$\mathcal{G}_{\mathcal{B}} = \underset{b \in \mathcal{B}}{\times} \mathcal{G}_b \quad , \quad \mathcal{G}_b \cong \mathbb{Z}_{d_b}$$

The fundamental groups in the spin systems are $\mathcal{G}_{\Lambda}$, $\mathcal{G}_{\mathcal{B}}$ and their dual groups $\hat{\mathcal{G}_{\Lambda}}$, $\hat{\mathcal{G}_{\mathcal{B}}}$, respectively. Let us define two homomorphisms between these groups, which are related to the high temperature (low temperature) expansions of the partition function :

$$\pi : \mathcal{G}_{\mathcal{B}} \longrightarrow \hat{\mathcal{G}_{\Lambda}} \qquad\qquad \gamma : \mathcal{G}_{\Lambda} \longrightarrow \hat{\mathcal{G}_{\mathcal{B}}}$$
$$\cup \qquad\qquad\qquad \cup \qquad\qquad\qquad \cup \qquad\qquad\qquad \cup$$
$$L \rightsquigarrow \pi(L) = \prod_{b \in \mathcal{B}} \langle \chi_b^{\ell_b} ; \cdot \rangle_{\Lambda} \qquad\qquad N \rightsquigarrow \gamma(N)$$

The relation $\langle \gamma(N) ; L \rangle_{\mathcal{B}} = \langle \pi(L) ; N \rangle_{\Lambda}$ between bicharacters of $\mathcal{G}_{\mathcal{B}}$ and those of $\mathcal{G}_{\Lambda}$ defines the homomorphism γ . One easily verifies $\mathrm{Im}\,\pi = \overline{\mathcal{J}}$, $\ker \gamma = \overline{\mathcal{J}}^{\perp} = \mathcal{J}$. The kernel of π and the image of γ are by definition the high temperature group $\mathcal{K}$, the low temperature group Γ , respectively. Moreover, for these groups, we have the following isomorphisms

$$\mathcal{G}_{\mathcal{B}}/\mathcal{K} \cong \overline{\mathcal{J}} \ , \quad \mathcal{G}_{\Lambda}/\mathcal{J} \cong \Gamma \ , \quad \mathcal{G}_{\Lambda}/\mathcal{J} \cong \overline{\mathcal{J}}^{\perp} \ \text{ thus } \ (\mathcal{G}_{\mathcal{B}}/\mathcal{K})^{\wedge} \cong \Gamma$$

and finally

Lemma : $\mathcal{K} = \Gamma^{\perp}$, $\Gamma = \mathcal{K}^{\perp}$

2. IMPLICATIONS OF THESE GROUP STRUCTURES

2.1. The high temperature (H.T.) expansion of the partition function is obtained from the Fourier decomposition of the Boltzmann factors

$$e^{\sum_{\ell=1}^{d_b-1} J_{b,\ell} \langle \chi_b^{\ell} ; N \rangle_{\Lambda}} = \sum_{\ell'=0}^{d_b-1} \langle \chi_b^{\ell'} ; N \rangle_{\Lambda} \, f_{b,\ell'}$$

with $f_{b,\ell'}$ the Fourier coefficients of the Boltzmann factors. Thus putting $t_{b,\ell} = \frac{f_{b,\ell}}{f_{b,0}}$ and using the group $\mathcal{G}_{\mathcal{B}}$ we get

$$Z_\Lambda\{J\} = \prod_{b\in\mathcal{B}} f_{b,0} \sum_{N\in\mathcal{G}_\Lambda} \sum_{L\in\mathcal{G}_\mathcal{B}} \prod_{b\in\mathcal{B}} t_{b,\ell_b} \langle \pi(L); N\rangle_\Lambda$$

and with the orthogonality relations, the H.T. expansion becomes

$$Z_\Lambda\{J\} = |\mathcal{G}_\Lambda| \prod_{b\in\mathcal{B}} f_{b,0} \sum_{K\in\mathcal{K}} \prod_{b\in\mathcal{B}} t_{b,k_b}$$

2.2. The low temperature (L.T.) expansion of the partition function is also obtained using the properties of the homomorphism π and γ. With any $b\in\mathcal{B}$, let $L_b\in\mathcal{G}_\mathcal{B}$ s.t. $(L_b)_{b'} = \delta_{bb'}$. Hence

$$\langle \chi_b^r; N\rangle_\Lambda = \langle (\pi(L_b))^r; N\rangle_\Lambda = \langle \pi(L_b^r); N\rangle_\Lambda = \langle \gamma(N); L_b^r\rangle_\mathcal{B}$$

Thus

$$Z_\Lambda\{J\} = \sum_{N\in\mathcal{G}_\Lambda} \prod_{b\in\mathcal{B}} e^{\sum_{r=1}^{d_b-1} J_{b,r} \langle \gamma(N); L_b^r\rangle_\mathcal{B}}$$

which implies

$$Z_\Lambda\{J\} = |\mathcal{G}| \prod_{b\in\mathcal{B}} e^{\sum_{r=1}^{d_b-1} J_{b,r}} \sum_{\gamma\in\Gamma} \prod_{b\in\mathcal{B}} e^{\sum_{r=1}^{d_b-1} J_{b,r}(\langle \gamma; L_b^r\rangle_\mathcal{B} - 1)}$$

and is called the L.T. expansion of the partition function.

2.3. The isomorphism $(\mathcal{G}_\mathcal{B}/\mathcal{K})^\wedge \cong \Gamma$ tells us that these two expansions are related by means of the Poisson formulae [3].

2.4. Duality transformations among different models can be defined by identifying the high temperature group $\mathcal{K}$ with the configuration space $\mathcal{G}_{\Lambda^*}$ of the dual system. Choosing a minimal set of generators of $\mathcal{K}$ and associating to each generator a site i^*, the dual lattice is defined by $\Lambda^* = \{i^*\}$. Now we associate with the site i^* the group $\mathcal{G}_{i^*} = \mathbb{Z}_{q^*}$, q^* = order of the generator on the site i^*. The interactions in the dual system are computed taking into account that the H.T. expansion can also be viewed as the mean value of the function $\prod_{b\in\mathcal{B}} t_{b,k_b}$ over the group $\mathcal{G}_{\Lambda^*} = \times_{i^*\in\Lambda^*} \mathcal{G}_{i^*}$. The Fourier decomposition with respect to $\hat{\mathcal{G}}_{\Lambda^*}$ of the logarithm of this function directly gives the interactions for the dual model as well as the coupling constants. Since the choice of the generators of $\mathcal{K}$ is not unique, a given model has many duals. Taking generators with many b-components

different from zero, the dual interactions will have complicated N-body interactions. Thus one generally chooses generators of $\mathcal{K}$ with the less b-components possible.

2.5. Equations for the correlation functions :

Let $\chi \in \mathcal{G}_\Lambda^\wedge$ and take $j \in \Lambda$ such that $\langle \chi ; N_j \rangle_\Lambda \neq 1$ for some $N_j \in \mathcal{G}_j$. Moreover for any $N \in \mathcal{G}_\Lambda$ we define $N = \bar{N} \cdot N_j$ with $N_j \in \mathcal{G}_j$. Then we can write with $\mathcal{G}_{\Lambda/j} = \underset{\substack{i \in \Lambda \\ i \neq j}}{\times} \mathcal{G}_i$

$$\langle \chi \rangle_\Lambda = (Z_\Lambda\{J\})^{-1} \int_{\mathcal{G}_{\Lambda/j}} d\mu_{\Lambda/j}(\bar{N}) \langle \chi ; \bar{N} \rangle_{\Lambda/j} \prod_{\substack{b \in \mathcal{B} \\ \langle \chi_b ; N_j \rangle_\Lambda = 1}} e^{\sum_{r=1}^{\alpha_b - 1} J_{b,r} \langle \chi_b^r ; \bar{N} \rangle_{\Lambda/j}} \int_{\mathcal{G}_j} d\mu_j(N_j) \langle \chi ; N_j \rangle_j \prod_{\substack{b \in \mathcal{B} \\ \langle \chi_b ; N_j \rangle_\Lambda \neq 1}} e^{\sum_{r=1}^{\alpha_b - 1} J_{b,r} \langle \chi_b^r ; \bar{N} N_j \rangle_\Lambda}$$

Restoring the trace over $\mathcal{G}_\Lambda$, we get with $\chi = \bar{\chi} \cdot \chi_j$, $\bar{\chi} \in \mathcal{G}_{\Lambda/j}^\wedge$, $\chi_j \in \mathcal{G}_j^\wedge$ the following set of equations

$$\langle \chi \rangle_\Lambda = \left\langle \bar{\chi} \cdot \frac{\int_{\mathcal{G}_j} d\mu_j(N_j) \langle \chi_j ; N_j \rangle_j \prod_{\substack{b \in \mathcal{B} \\ \langle \chi_b ; N_j \rangle_\Lambda \neq 1}} e^{\sum_{r=1}^{\alpha_b - 1} J_{b,r} \langle \chi_b^r ; N_j \rangle_j \bar{\chi}_b^r}}{\int_{\mathcal{G}_j} d\mu_j(N_j) \prod_{\substack{b \in \mathcal{B} \\ \langle \chi_b ; N_j \rangle_\Lambda \neq 1}} e^{\sum_{r=1}^{\alpha_b - 1} J_{b,r} \langle \chi_b^r ; N_j \rangle_j \bar{\chi}_b^r}} \right\rangle_\Lambda$$

The Fourier expansion of the quotient yields with $\Upsilon = \bigcup_{\substack{b \in \mathcal{B} \\ b_j \neq 0}} \bigcup_{\substack{i \in \Lambda/j \\ b_i \neq 0}} i$

$$\langle \chi \rangle_\Lambda = \sum_{\tilde{\chi} \in (\times_{i \in \Upsilon} \mathcal{G}_i)^\wedge} Th(\hat{\chi}) \langle \bar{\chi} \, \tilde{\chi} \rangle_\Lambda$$

$Th(\tilde{\chi})$ being the Fourier coefficients of the quotient.

3. ASANO CONTRACTIONS

To define a polynomial, which is linear in each variable, associated with the L.T. expansion, let us introduce for any $b \in \mathcal{B}$ the α_b generalized activities

$$z_{b,m} = e^{\sum_{r=1}^{\alpha_b - 1} J_{b,r} (\langle m ; L_b^r \rangle_{\mathcal{B}} - 1)} \qquad m \in \mathbb{Z}_{\alpha_b}$$

(since $z_{b,0} = 1$ there are only $\alpha_b - 1$ independent ones). Thus this polynomial becomes

$$M(z_{\mathcal{B}}) = \sum_{M \in \mathcal{G}_{\mathcal{B}}} c_M z^M \quad , \quad z^M = \prod_{b \in \mathcal{B}} z_{b,m_b}$$

and $c_M = 1,(0)$ if $M \in \Gamma$, $(M \notin \Gamma)$, respectively. Let $\mathcal{B} = \bigcup_{i=1}^{n} \mathcal{B}_i$ be a finite covering of $\mathcal{B}$ and

$$P_i(z_{\mathcal{B}_i}) = \sum_{L_i \in \times_{b \in \mathcal{B}_i} \mathcal{G}_b} c_{i,L_i} z^{L_i} \quad , \quad z^{L_i} = \prod_{b \in \mathcal{B}_i} z_{b,\ell_{i_b}}$$

be a family of small polynomials

Definition : The polynomial $P(z_{\mathcal{B}}) = \sum_{L \in \mathcal{G}_{\mathcal{B}}} c_L z^L$ is the Asano contraction (A.C.) of the family of polynomials $\{P_i(z_{\mathcal{B}_i})\}$ if $c_L = \prod_{i=1}^{n} c_{i,L/\mathcal{B}_i}$ where $(L/\mathcal{B}_i)_b = \ell_b \, \delta_{b,\mathcal{B}_i}$, $\delta_{b,\mathcal{B}_i} = 1$ (0) if $b \in \mathcal{B}_i$ ($b \notin \mathcal{B}_i$).

This is a particular form of the usual A.C. which is defined in more general terms. Therefore we can apply Ruelle's theorem [5] without modification to relate the properties of zeros of the small polynomials to the ones of the contracted polynomial. The following theorem gives a necessary and sufficient condition that the polynomial associated to an expansion of the partition function is the A.C. of small polynomials :

Theorem Let $\mathcal{G} \subset \mathcal{G}_{\mathcal{B}}$ be any subgroup of $\mathcal{G}_{\mathcal{B}}$ and $\mathcal{B} = \bigcup_{i=1}^{n} \mathcal{B}_i$ be a finite covering of $\mathcal{B}$ then

$M(z_{\mathcal{B}}) = \sum_{R \in \mathcal{G}} z^R$ is the A.C. of $M(z_{\mathcal{B}_i}) = \sum_{R_i \in \mathcal{G}_i} z^{R_i}$, $\mathcal{G}_i \subset \times_{b \in \mathcal{B}_i} \mathcal{G}_b$ $i=1,\dots,n$

if and only if $\mathcal{G}^{\perp}$ = group generated by $\bigcup_{i=1} \mathcal{G}_i^{\perp}$

where $\mathcal{G}_i = \{ L_i \in \times_{b \in \mathcal{B}_i} \mathcal{G}_b \; ; \; (L_i)_b = \ell_b \; \forall b \in \mathcal{B}_i$ for some $L \in \mathcal{G} \}$

This is a straightforward generalization of a theorem by Slawny [6] proved for the spin $\frac{1}{2}$ case. In order to find the family of small polynomials, we proceed in the same way as in the spin $\frac{1}{2}$ case[7] : First find $\mathcal{G}^{\perp}$, take $\{\mathcal{G}_i^{\perp}\}_{i=1,\dots,n}$ a finite family of subgroups of $\mathcal{G}^{\perp}$ which generates $\mathcal{G}^{\perp}$, then the covering is given by

$$\mathcal{B}_i = \bigcup_{L_i \in \mathcal{G}_i^{\perp}} \{ b \; ; \; (L_i)_b \neq 0 \}$$

Remark that due to the fact that for any bond $b \in \mathcal{B}$ we have $\alpha_b - 1$ variables, we will necessarily have analyticity properties in $\alpha_b - 1$ complex variables. e.g. for a spin 1 model we get analyticity do-

mains in $e^{\beta h}$ and $e^{\beta\mu}$ simultaneously. Thus the above A.C. for higher spin systems is complementary to the ones introduced by Millard and Viswanathan using Griffiths transformation into an equivalent spin $\frac{1}{2}$ system [8] and their generalized A.C. given in [9] . Griffiths transformations and the resulting group structure have been discussed by Slawny [10] . They got analyticity domains in one complex variable, keeping the other interactions as real parameters.

4. APPLICATIONS TO SPIN 1 MODELS

Here, we limit ourselves to discuss the application to A.C., for other applications, we refer to [4] .

Let us consider the general spin 1 model $(\mathcal{G}_i = \mathbb{Z}_3 \ \forall i \in \Lambda)$ with field and two spin nearest neighbours (n.n.) interactions. Thus the set of bonds $\mathcal{B}$ satisfying i) and ii) is

$$\mathcal{B} = \{ b_r \ ; r \in \Lambda \} \cup \{ b^1_{rs} \ ; \ r,s \in \Lambda \ \ r,s \text{ n.n.} \} \cup \{ b^2_{r,s} ; \ r,s \in \Lambda \ , \ r,s \text{ n.n.} \}$$

and the coupling constants we have are the two fields $J_{b_r,1}$, $J_{b_r,2}$ and the four n.n. coupling constants $J_{b^1_{rs},1}$, $J_{b^1_{rs},2}$, $J_{b^2_{rs},1}$, $J_{b^2_{rs},2}$.

Taking $\mathcal{G} = \Gamma$ we have $\mathcal{G}^\perp = K$. The simplest generators of K are defined on the n.n. sets $\Lambda_i = \{r,s\} \subset \Lambda$ and the covering sets $\mathcal{B}_i$ are

$$\mathcal{B}_i = \{ b_r, b_s , b^1_{rs} , b^2_{r,s} \}$$

Each bond of $\mathcal{B}_i$ defining two generalized activities. Thus computing $\mathcal{G}_i$ and putting $z_1 = z_{b_r,1}$, $z_2 = z_{b_r,2}$, $z_3 = z_{b_s,1}$, $z_4 = z_{b_s,2}$, $z_5 = z_{b^1,1}$, $z_6 = z_{b^1,2}$, $z_7 = z_{b^2,1}$, $z_8 = z_{b^2,2}$ the small polynomials become for i = 1,...,n

$$M(z_{\mathcal{B}_i}) = M(z_1,\dots,z_8) = 1 + z_1z_5z_7 + z_1z_6z_8 + z_3z_5z_8 + z_4z_6z_7 + z_1z_3z_6 + z_1z_4z_8 + z_2z_3z_7 + z_2z_4z_5$$

Again we use A.C. to discuss the domains free of zeros of this poly

nomial and we find that $M(z_1, \dots, z_8)$ is the A.C. of 6 polynomials each depending on two variables only. The use of the theorem of Grace [11] combined with Ruelle's theorem [5] yields domains free of zeros of the small polynomials and finally of the polynomial associated with the L.T. expansion of the partition function.

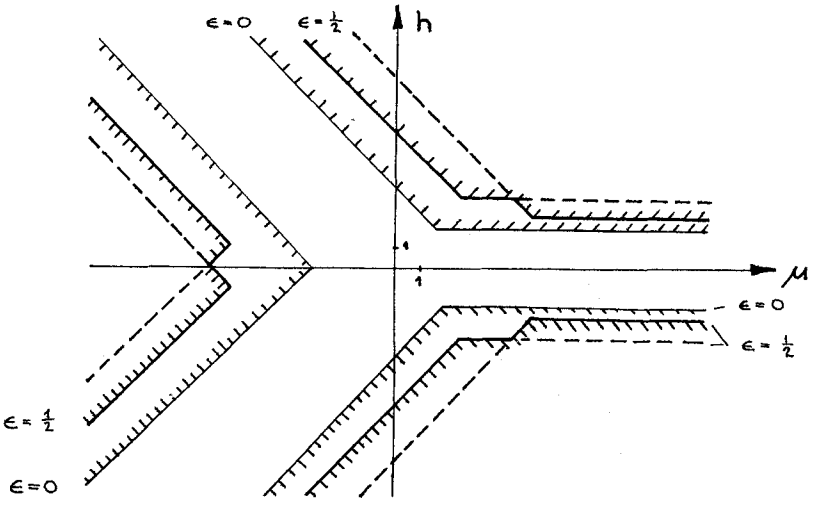
5. THE PHYSICAL AND GROUP PICTURE OF SPIN 1 MODELS

In the usual physical language, the spin variables of a spin 1 model are $\sigma_i \in \{0, \pm 1\} \quad \forall i \in \Lambda$, and the most general Hamiltonian with fields and n.n. interactions writes

$$-\beta H_\Lambda\{\sigma\} = \sum_{\substack{ij \\ n.n}} (\epsilon_{11}\sigma_i\sigma_j + \epsilon_{12}\sigma_i\sigma_j^2 + \epsilon_{21}\sigma_i^2\sigma_j + \epsilon_{22}\sigma_i^2\sigma_j^2) + h\sum_i \sigma_i + \mu\sum_i \sigma_i^2$$

The local transformation $\phi : \sigma_i \longrightarrow n_i \quad \forall i \in \Lambda$ where $n_i \in \{0,1,2\} = \mathbb{Z}_3$ maps the physical picture of any spin 1 model on the corresponding group picture. Thus $-\beta H_\Lambda\{\sigma\}$ can be viewed as a function over G_Λ and Fourier transforms immediately yield the coupling constants $J_{b,r}$ in terms of the ϵ_{jk}, h, μ and conversely. There are 6 different local transformations ϕ to equip the phase space of the physical picture with a group structure, which are exhausted by the elements of the permutation group of 3 elements. (e.g. $\phi(-1)=0$, $\phi(0)=1$, $\phi(1)=2$ etc.). Thus for the same spin 1 model, we get 6 different sets of generalized activities in terms of ϵ_{jk}, h, μ. Note that the small polynomials do not depend on ϕ but the domains free of zeros of the partition function in terms of the "physical" activities e^{μ}, e^{h}, $e^{\epsilon_{jk}}$ will depend on ϕ and in general we get for different maps different domains.

Taking as a typical spin 1 model the model of Lebowitz-Gallavotti [12] where $\epsilon_{11} = -\epsilon_{12} = \epsilon > 0$, $h, \mu \neq 0$ we find for fixed real values of ϵ the following analyticity regions (shaded) in h and μ of the partition function:



Similar analyticity domains are obtained for other spin 1 models. For the dilute Ising model, where $\epsilon_{11} > 0$, $\epsilon_{12} = \epsilon_{21} = \epsilon_{22} = 0$, $h, \mu \neq 0$ we improve bounds for the tricritical point given by Sarbach and Rys [13] .

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