

Effect of the nucleon-nucleon interaction on the fusion cross-section within the relativistic mean field formalism

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Abstract. We establish a relationship between the nucleon-nucleon interaction potential and the nuclear fusion reaction cross-sections at low energies. The axially deformed self-consistent relativistic mean field is used with the non-linear NL3* interaction parameter set. The Wong formula is used to estimate the fusion cross-section for $^{58}\text{Ni} + ^{58}\text{Ni}$ and $^{48}\text{Ca} + ^{238}\text{U}$ systems, which are known to display fusion hindrance phenomena. The results of the application of the nucleus-nucleus optical potential for the fusion cross-section from the recently developed effective relativistic NN -interaction (R3Y) potential is compared with the well-known, phenomenological M3Y potential. The results obtained from our present calculation for the R3Y interaction are reasonable good as compared to the M3Y potential concerning the available experimental at barrier energies. The present analysis pursues a full microscopic studies of fusion process at low energies by taking the R3Y potential along with the relativistic mean field density instead of taking the M3Y interaction within the double folding approach.

1. Introduction

In the last 70 years, a large body of theoretical and experimental work have been devoted to understanding the properties of atomic nuclei through the bare interaction between a pair of nucleons. Although a substantial development has been made to explain the nuclear force in terms of nucleon-nucleon (NN) interactions, remains an open problem at present. In principle, the central part of the NN -interaction is considered as a typical square-well, Gaussian or Yukawa potential of various ranges and strengths, which can be obtained from the observed phase shifts in an elastic-scattering processes [1]. A large number of interactions have been constructed by studying NN - scattering, but extensive modifications in the scattering behavior due to the presence of surrounding nucleons occur in a nucleus [2]. Further, the reconstruction of the NN -potential through particle exchanges is made possible by the development of quantum field theory [3]. The analytical derivation of a potential through particle exchange is important to understand the nuclear force as well as structural properties via the nucleus-nucleus optical potential for



the study of many nuclear phenomena such as nuclear radioactivity, nuclear scattering, nuclear fission and fusion process [4, 5, 6]. An alternative approach to NN -interactions at low energies has been formulated by Ekström *et. al.*, [7] in terms of an effective theory for non-relativistic nucleons, which involves a few basic coupling constants to reproduce the nucleon scattering data. Furthermore, the relativistic effective NN -interaction R3Y potential [4, 6] analogous to the M3Y one [8] can be derived from the relativistic mean field Lagrangian, which depends on the coupling constant among the interacting mesons and their masses [4, 6].

The nucleus-nucleus optical potential is quite important in studies of elastic scattering of light and heavy-ion (HI) systems, in particular for the simple one-dimensional barrier penetration model (BPM) of a fusion reaction, which is significantly influenced by the NN -potential, nuclear matter density distributions and the Coulomb potential [9, 6]. A microscopic description is required for calculating the nuclear potential that incorporates the physical process, especially fusion, in terms of the NN interaction [6]. At low energy, the system can fuse either by penetrating the interaction barrier or it must have sufficient energy to overcome the Coulomb barrier to be absorbed. In the present study, we consider reactions from different mass regions i.e $^{58}\text{Ni} + ^{58}\text{Ni}$, and $^{48}\text{Ca} + ^{238}\text{U}$, as their fusion excitation functions are available experimentally and also known for fusion hindrance [10, 11]. Below the Coulomb barrier, nuclear structure effects dominate the fusion dynamics, whereas the centrifugal potential suppresses the structure effects at above barrier energies. Here, one of the points of interest is to observe the ability of the relativistic R3Y potential along with the microscopic relativistic mean field density to estimate the nuclear interaction potential for the study of fusion reactions at low energies.

The paper is organized as follows: the relativistic mean-field formalism and the analytical expressions for the R3Y potential are given in Sec. 2. This section also including a brief description of the Wong formula. Sec. 3 presents the results of our calculations and discussions. A brief summary of the results obtained, together with concluding remarks, are given in Sec. 4.

2. The relativistic mean-field Theory

In the last few decades, the relativistic mean field theory has been applied successfully to study the structural properties of finite nuclei over the nuclear chart, including the unknown island of superheavy nuclei [6, 12, 13, 14]. We have used the microscopic self-consistent relativistic mean field (RMF) theory as a standard tool to study fusion using the Wong formula. The form of a typical relativistic Lagrangian density for a nucleon-meson many body system is, [6, 14, 15]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}\{i\gamma^\mu\partial_\mu - M\}\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_s\bar{\psi}\psi\sigma \\ & - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu - \frac{1}{4}\vec{B}^{\mu\nu}\cdot\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu \\ & - g_\rho\bar{\psi}\gamma^\mu\vec{\tau}\psi\cdot\vec{\rho}^\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^\mu\frac{(1-\tau_3)}{2}\psi A_\mu. \end{aligned} \quad (1)$$

The ψ are the Dirac spinors for the nucleons. The iso-spin and its third component are denoted by τ and τ_3 , respectively. Here g_σ , g_ω , g_ρ and $\frac{e^2}{4\pi}$ are the coupling constants for σ -, ω -, ρ - mesons and photon, respectively. The constant g_2 , and g_3 are coupling constants for the self-interacting non-linear σ -meson fields. The masses of the σ -, ω -, ρ - mesons and nucleons are m_σ , m_ω , m_ρ , and M , respectively. The quantity A_μ stands for the electromagnetic field. The vector field tensors for the ω^μ , $\vec{\rho}_\mu$ and photon are given by, $F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, and $\vec{B}^{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$, respectively. From the above Lagrangian density we obtain the field equations for the Dirac nucleons, and the meson fields, as

$$\left(-i\alpha\cdot\nabla + \beta(M + g_\sigma\sigma) + g_\omega\omega + g_\rho\tau_3\rho_3\right)\psi = \epsilon\psi,$$

$$\begin{aligned} (-\nabla^2 + m_\sigma^2) \sigma(r) &= -g_\sigma \rho_s(r) + g_2 \sigma^2(r) + g_3 \sigma^3(r), \\ (-\nabla^2 + m_\omega^2) \omega(r) &= g_\omega \rho(r); \quad (-\nabla^2 + m_\rho^2) \rho(r) = g_\rho \rho_3(r). \end{aligned} \quad (2)$$

In the limit of one-meson exchange, in a static baryonic medium, the single nucleon-nucleon potential for scalar (σ), and vector (ω , ρ) fields are given by,

$$V_\sigma = -\frac{g_\sigma^2}{4\pi} \frac{e^{-m_\sigma r}}{r} + \frac{g_2^2}{4\pi} r e^{-2m_\sigma r} + \frac{g_3^2}{4\pi} \frac{e^{-3m_\sigma r}}{r}; \quad V_\omega(r) = +\frac{g_\omega^2}{4\pi} \frac{e^{-m_\omega r}}{r}; \quad V_\rho(r) = +\frac{g_\rho^2}{4\pi} \frac{e^{-m_\rho r}}{r}. \quad (3)$$

The relativistic effective nucleon-nucleon interaction (V_{eff}^{R3Y}) taking into account the single-nucleon exchange effects can be written as, [4, 6],

$$V_{eff}^{R3Y}(r) = \frac{g_\omega^2}{4\pi} \frac{e^{-m_\omega r}}{r} + \frac{g_\rho^2}{4\pi} \frac{e^{-m_\rho r}}{r} - \frac{g_\sigma^2}{4\pi} \frac{e^{-m_\sigma r}}{r} + \frac{g_2^2}{4\pi} r e^{-2m_\sigma r} + \frac{g_3^2}{4\pi} \frac{e^{-3m_\sigma r}}{r} + J_{00}(E) \delta(r). \quad (4)$$

The effective R3Y NN -interaction is obtained from the scalar and vector parts of the meson fields, analogous to the M3Y potential [8]. Here the R3Y potential is derived for the NL3* force, which can predict nuclear matter properties as well as the properties of the finite nuclei at very high isospin asymmetries [6]. It is worth mentioning that the analytical expression for the R3Y interaction is only possible for interaction parameters that contain only linear and/or non-linear self-coupling terms. In the case of relativistic forces with cross-coupling terms (i.e., FSUGold, G1, G2, etc.), one has to obtain a numerical solution to generate NN -interactions. On the other hand, the M3Y effective interaction, obtained from a fit of the G-matrix elements based on the Reid-Elliott soft-core NN -interaction [8], in an oscillator basis, is the sum of three Yukawa's (M3Y) with ranges 0.25 fm for a medium-range attractive part, 0.4 fm for a short-range repulsive part and 1.414 fm to ensure the long-range tail of the one-pion exchange potential (OPEP). The widely used M3Y effective interaction ($V_{eff}^{M3Y}(r)$) is given by

$$V_{eff}^{M3Y}(r) = 7999 \frac{e^{-4r}}{4r} - 2134 \frac{e^{-2.5r}}{2.5r} + J_{00}(E) \delta(r), \quad (5)$$

where the strength is in MeV. One can find more details in Refs. [4, 5, 6]. The nuclear interaction potential, $V_n(R)$, between the projectile (p) and the target (t) nuclei is calculated from the RMF (NL3*) matter densities ρ_p and ρ_t using the well known double folding procedure [8] for the M3Y and the R3Y interaction potential, as

$$V_n(\vec{R}) = \int \rho_p(\vec{r}_p) \rho_t(\vec{r}_t) V_{eff}(|\vec{r}_p - \vec{r}_t + \vec{R}| \equiv r) d^3 r_p d^3 r_t. \quad (6)$$

Adding the Coulomb potential $V_C(R)$ ($=Z_p Z_t e^2/R$) results in a nucleus-nucleus interaction potential $V_T(R)$ [$=V_n(R) + V_C(R)$], used for calculating the fusion properties.

Wong Formula: In terms of the partial waves ℓ , the fusion cross-section for two nuclei colliding with a center-of-mass energy ($E_{c.m.}$), is given by [16]

$$\sigma(E_{c.m.}) = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{max}} (2\ell + 1) P_\ell(E_{c.m.}), \quad (7)$$

with $k = \sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}}$ and μ is the reduced mass. P_ℓ is the transmission coefficient for each ℓ which describes the penetration of the barrier $V_T^\ell(R)$. Using the Hill-Wheeler [17, 18] approximation, the penetrability P_ℓ , in terms of the barrier height $V_B^\ell(E_{c.m.})$ and curvature $\hbar\omega_\ell(E_{c.m.})$, is

$$P_\ell = \left[1 + \exp \left(\frac{2\pi(V_B^\ell(E_{c.m.}) - E_{c.m.})}{\hbar\omega_\ell(E_{c.m.})} \right) \right]^{-1}. \quad (8)$$

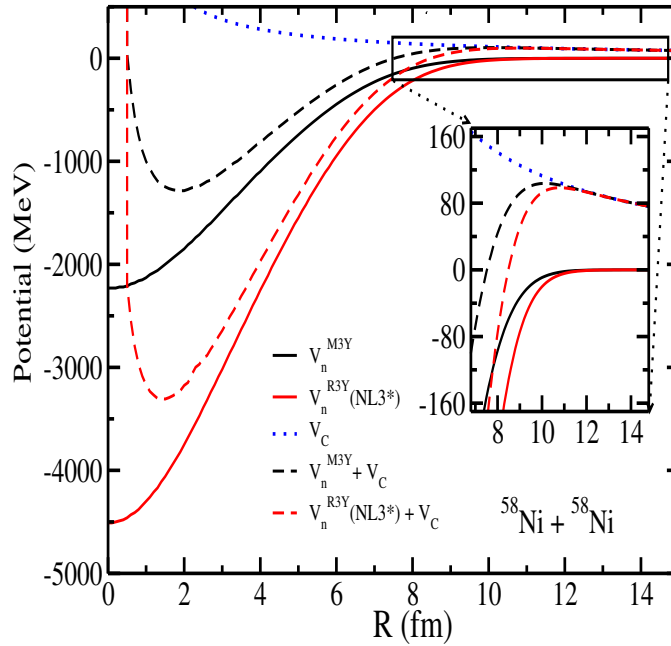


Figure 1. (Color online) The total nucleus-nucleus optical potential $V_T(R)$ and the individual contributions (the nuclear $V_n(R)$ (M3Y) and $V_n(R)$ (R3Y) for the NL3* parameter set, and the Coulomb $V_C(R)$ potential) as a function of radial distance for $^{58}\text{Ni} + ^{58}\text{Ni}$. See text for details.

Here, $\hbar\omega_\ell = \hbar \left[|d^2V_T^\ell(R)/dR^2|_{R=R_B^\ell} / \mu \right]^{1/2}$ is evaluated at the barrier position $R = R_B^\ell$ corresponding to the barrier height V_B^ℓ , and the R_B^ℓ obtained from the condition, $|dV_T^\ell(R)/dR|_{R=R_B^\ell} = 0$. Instead of solving Eq. (7) explicitly, which requires the complete ℓ -dependent potentials $V_T^\ell(R)$, Wong [16] carried out the ℓ -summation in Eq. (7) approximately under specific conditions: (i) $\hbar\omega_\ell \approx \hbar\omega_0$, and (ii) $V_B^\ell \approx V_B^0 + \frac{\hbar^2\ell(\ell+1)}{2\mu R_B^0{}^2}$, which assumes $R_B^\ell \approx R_B^0$ too. In other words, both V_B^ℓ and $\hbar\omega_\ell$ are obtained for $\ell = 0$. Using these approximations, and replacing the ℓ -summation in Eq. (7) by an integral, gives, on integration, the $\ell = 0$ barrier-based Wong formula [16],

$$\sigma(E_{c.m.}) = \frac{R_B^0{}^2 \hbar\omega_0}{2E_{c.m.}} \cdot \ln \left[1 + \exp \left(\frac{2\pi}{\hbar\omega_0} (E_{c.m.} - V_B^0) \right) \right]. \quad (9)$$

This is the simple formula used in the present work to calculate the fusion cross-section using the barrier characteristics, V_B^0 , R_B^0 and $\hbar\omega_0$ within the barrier penetration model. For details see Ref. [6, 19].

3. Details of the calculations and Results

The RMF calculations furnish the effect of fusion hindrance using the self-consistent relativistic mean field formalism via the Wong formula. In this regards, in the first step, we calculate the M3Y (using Eq. 5) and the microscopic R3Y (using Eq. 4) NN -potential for the NL3* interaction parameter set. The details of the relativistic effective NN -interactions for various forces and the phenomenological M3Y potential can be found in the Refs. [4, 5, 6]. In the second step, we calculate bulk properties such as the binding energy, quadrupole moment Q_{20} , the total matter density distribution, nuclear radii, and the single particle energy levels for nucleons. Instead of concentrating on the nuclear structure output profiles, we use the monopole component of the

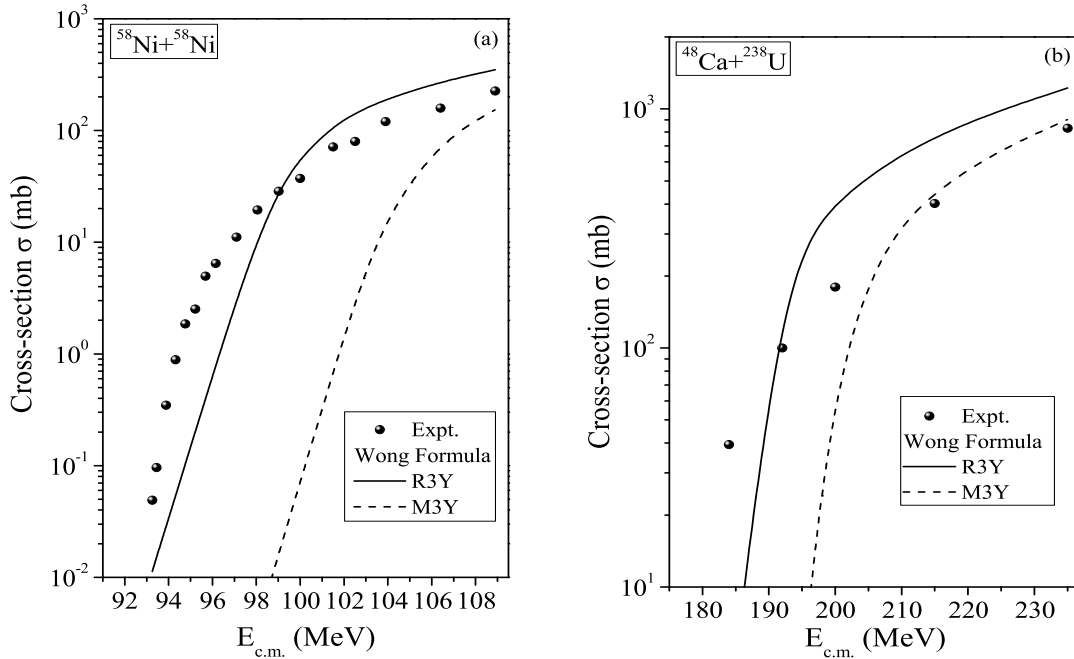


Figure 2. Fusion-evaporation cross-section as a function of the center-of-mass energy $E_{c.m.}$, for the R3Y using NL3* (solid line), and the M3Y (dashed line) potential along with the experimental data for (a) $^{58}\text{Ni} + ^{58}\text{Ni}$ [10] and (b) $^{48}\text{Ca} + ^{238}\text{U}$ [11]. See the text for details.

mean field densities for the target (t) and projectile (p) as the input to estimate the nuclear interaction potential using Eq. (6). The total interaction potentials $V_T(R) = V_n(R) + V_C(R)$ for the $^{58}\text{Ni} + ^{58}\text{Ni}$, and $^{48}\text{Ca} + ^{238}\text{U}$ systems are obtained for the M3Y and R3Y interactions for NL3* densities. As a representative case, the results for the nucleus-nucleus interaction potentials without Coulomb for the M3Y (solid black line), and the R3Y interactions for NL3* (solid red line) interaction parameters are displayed in Fig. 1. The total interaction potential (corresponding dashed line) along with the Coulomb potential V_C (blue dotted line) are also shown in Fig. 1. From the figure, we note that the nuclear potentials obtained from the M3Y differ significantly from the R3Y (NL3*), particularly in the central region while this difference decreases simultaneously with respect to the radial distance. Further, the heights of the barrier for M3Y interaction are a bit higher as compare to the R3Y (NL3*) case (seen more clearly in the inset of Fig. 1). For example, the R3Y (NL3*) is about 1 MeV more attractive compared to the M3Y as is illustrated in the inset of Fig. 1.

The barrier characteristics of the nuclear interaction potential i.e. the barrier height, position and frequency from the total interaction potential are used in the Wong formula for estimating the fusion reaction cross-section for the systems $^{58}\text{Ni} + ^{58}\text{Ni}$, and $^{48}\text{Ca} + ^{238}\text{U}$, known for fusion hindrance phenomena. Fig. 2 (a) shows the comparison of the fusion cross-section obtained for $^{58}\text{Ni} + ^{58}\text{Ni}$ around the Coulomb barrier with the experimental data [10]. The solid and dashed line are for the fusion cross-section from R3Y and M3Y potentials, respectively within the Wong formula for NL3* densities. From the figure, one can see that the fusion cross-section calculated using M3Y underestimates that of the R3Y interaction and the experimental data. In other words, the cross-section obtained for R3Y interaction with the NL3* interaction parameters is relatively superior to M3Y interaction when compared with the experimental data [10] at below barrier energies. Motivated by the observation, calculations are then pursued for $^{48}\text{Ca} + ^{238}\text{U}$ along with the experimental data [11], shown in Fig. 2 (b). From Fig. 2 (b), a similar conclusion can be drawn for the R3Y and M3Y potentials for this reaction system, as found for $^{58}\text{Ni} +$

^{58}Ni at below barrier energies. At above barrier energies, the M3Y interaction is slightly better than the R3Y interaction. However the estimate of the R3Y interactions, at energies above the barrier, can be improved by adopting the ℓ -summed Wong formula. More details of the ℓ -summed Wong formula for various fusion cross-sections can find in Refs. [6, 19, 20].

4. Summary and Conclusions

We have shown the effect of the nucleon-nucleon interaction potential on the fusion reaction cross-sections for $^{58}\text{Ni} + ^{58}\text{Ni}$, and $^{48}\text{Ca} + ^{238}\text{U}$, known for fusion hindrance phenomena. The axial deformed relativistic mean field with the NL3* force has been used along with the Wong formula to provide a transparent and analytic way to calculate the fusion cross-section by means of a convenient approach to the nucleus-nucleus optical potential. The RMF (NL3*) matter densities for target and projectile are used for calculating the corresponding nuclear potential within a double folding procedure, for the study of fusion at low energies. We find the R3Y interaction to be a better choice than the M3Y one for prediction of the cross-section of fusion reactions below the barrier. The present analysis pursues a full microscopic study by taking the R3Y potential along with the relativistic mean field densities within the Wong formula. More details of the work can be found in Refs. [6, 20].

Acknowledgments

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References

- [1] Brueckner K A and Watson K M 1953 *Phys. Rev.* **92** 1023.
- [2] Stoks V G J, Klomp R A M, Terheggen C P F and Swart J J de 1994 *Phys. Rev. C* **49** 2950.
- [3] Garcon M and Orden J W Van 2001 *Adv. Nucl. Phys.* **26** 293.
- [4] Singh B B, Bhuyan M, Patra S K and Gupta Raj K 2012 *J. Phys. G: Nucl. Part. Phys.* **39** 025101.
- [5] Sahu B B, Singh S K, Bhuyan M, Biswal S K and Patra S K 2014 *Phys. Rev. C* **89** 034614.
- [6] Bhuyan M and Kumar Raj 2018 *Phys. Rev. C* **98** 054610.
- [7] Ekström A, Baardsen G, Forssn C, Hagen G, Hjorth-Jensen M, Jansen G R, Machleidt R, Nazarewicz W, Papenbrock T, Sarich J and Wild S M 2013 *Phys. Rev. Lett.* **110** 192502.
- [8] Satchler G R and Love W G 1979 *Phys. Reports* **55** 183.
- [9] Stiliarid E, Bohlen H G, Frobrich P, Gebauer B, Kolbert D, Oertzen W von, Wilpert M and Th Wilpert 1989 *Phys. Lett. B* **223** 291.
- [10] Beckerman M *et al.* 1981 *Phys. Rev. C* **23** 1581.
- [11] Kozulin E M, Knyazheva G N, Itkis I M, Itkis M G, Bogachev A A, Chernysheva E V, Krupa L, Hanappe F, Dorvaux O, Stuttgé L, Trzaska W H, Schmitt C and Chubarian G 2014 *Phys. Rev. C* **90** 054608.
- [12] Serot B D and Walecka J D 1986 *Advances in Nuclear Physics* ed Negele J W and Vogt Erich vol 16 (Plenum Press, New York) p 1.
- [13] Carlson B V and Hirata D 2000 *Phys. Rev. C* **62** 054310.
- [14] Lalazissis G A, Karatzikos S, Fossion R, Arteaga D Pena, Afanasjev A V and Ring P 2009 *Phys. Lett. B* **671** 36.
- [15] Boguta J and Bodmer A R 1977 *Nucl. Phys. A* **292**, 413.
- [16] Wong C Y 1973 *Phys. Rev. Lett.* **31** 766.
- [17] Hill D L and Wheeler J A 1953 *Phys. Rev.* **89** 1102.
- [18] Thomas T D 1959 *Phys. Rev.* **116** 703.
- [19] Kumar Raj, Bansal M, Arun S K and Gupta Raj K 2009 *Phys. Rev. C* **80** 034618.
- [20] Bhuyan M, Kumar Raj, Jain D, Patra S. K and Carlson B V 2019 **under preparation**.